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Lazy Cops and Robbers on Hypercubes

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We consider a variant of the game of Cops and Robbers, called Lazy Cops and Robbers, where at most one cop can move in any round. We investigate the analogue of the cop number for this game, which we call the lazy cop number. Lazy Cops and Robbers was recently introduced by Offner and Ojakian, who provided asymptotic upper and lower bounds on the lazy cop number of the hypercube. By coupling the probabilistic method with a potential function argument, we improve on the existing lower bounds for the lazy cop number of hypercubes.

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1. Introduction

The game of Cops and Robbers (defined, along with all the standard notation, at the end of this section) is usually studied in the context of the *cop number*, the minimum number of cops needed to ensure a winning strategy. The cop number is often challenging to analyse; establishing upper bounds for this parameter is the focus of Meyniel's conjecture that the cop number of a connected n-vertex graph is $O(\sqrt{n})$. For additional background on Cops and Robbers and Meyniel's conjecture, see the book [9] and the surveys [3, 5, 4].

A number of variants of Cops and Robbers have been studied. For example, we may allow a cop to capture the robber from a distance k, where k is a non-negative integer [6, 7], play on edges [10], allow one or both players to move with different speeds [2, 11] or to teleport, allow the robber to capture the cops [8], or make the robber invisible or drunk [12, 13]. See Chapter 8 of [9] for a non-comprehensive survey of variants of Cops and Robbers.

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We are interested in slowing the cops down to create a situation akin to chess, where at most one chess piece can move in a round. Hence, our focus in the present article is a recent variant introduced by Offner and Ojakian [15], where at most one cop can move in any given round. We refer to this variant, whose formal definition appears in Section 1.1, as *Lazy Cops and Robbers*; the analogue of the cop number is called the *lazy cop number*, and is written $c_L(G)$ for a graph G. In [15] it was proved for the hypercube Q_n that

$$2^{\lfloor \sqrt{n}/20 \rfloor} \leqslant c_L(Q_n) = O(2^n \log n / n^{3/2}).$$

We mention in passing that [15] introduced a number of variants of Cops and Robbers, in which some fixed number of cops (perhaps more than one) can move in a given round. We focus here on the extreme case in which only one cop moves in each round, but it seems likely that our techniques generalize to other variants.

In this short paper, we consider Lazy Cops and Robbers on hypercubes. In Theorem 2.1, by using the probabilistic method coupled with a potential function argument, we improve on the lower bound for the lazy cop number of hypercubes given in [15].

1.1. Definitions and notation

We consider only finite, undirected graphs in this paper. For background on graph theory, the reader is directed to [17].

The game of *Cops and Robbers* was independently introduced in [14, 16] and the cop number was introduced in [1]. The game is played on a reflexive graph; that is, each vertex has at least one loop. Multiple edges are allowed, but make no difference to the play of the game, so we always assume there is exactly one edge joining adjacent vertices. There are two players, consisting of a set of *cops* and a single *robber*. The game is played over a sequence of discrete time-steps or *turns*, with the cops going first on turn 0 and then playing on alternate time-steps. A *round* of the game is a cop move together with the subsequent robber move. The cops and robber occupy vertices; for simplicity, we often identify the player with the vertex they occupy. We refer to the set of cops as C and the robber as R. When a player is ready to move in a round they must move to a neighbouring vertex. Because of the loops, players can *pass*, or remain on their own vertices. Observe that any subset of C may move in a given round. The cops win if after some finite number of rounds, one of them can occupy the same vertex as the robber (in a reflexive graph, this is equivalent to the cop landing on the robber). This is called a *capture*. The robber wins if he can evade capture indefinitely. A *winning strategy for the cops* is a set of rules that if followed, result in a win for the cops. A *winning strategy for the robber* is defined analogously. As stated earlier, the game of *Lazy Cops and Robbers* is defined almost exactly as Cops and Robbers, with the exception that at most one cop moves in any round.

If we place a cop at each vertex, then the cops are guaranteed to win. Therefore, the minimum number of cops required to win in a graph G is a well-defined positive integer, named the *lazy cop number* of the graph G. We write $c_L(G)$ for the lazy cop number of a graph G.

2. Hypercubes

In [15], Offner and Ojakian provided asymptotic lower and upper bounds on $c_L(Q_n)$. More precisely, they showed that $c_L(Q_n) = \Omega(2^{\sqrt{n}/20})$ and $c_L(Q_n) = O(2^n \log n/n^{3/2})$. In this section, we asymptotically improve the lower bound. Our main result is the following.

Theorem 2.1. *For all $\varepsilon > 0$, we have that*

$$c_L(Q_n) = \Omega\left(\frac{2^n}{n^{5/2+\varepsilon}}\right).$$

Thus, the upper and lower bounds on $c_L(Q_n)$ differ by only a polynomial factor.

Proof. We present a winning strategy for the robber provided that the number of cops is not too large. Let $\varepsilon \in (0,1)$ be fixed, and suppose there are $k = k(\varepsilon, n)$ cops (where k will be chosen later). We introduce a potential function that depends on each cop's distance to the robber. Let N_i represent the number of cops at distance i from the robber. With $\rho = \rho(n) = o(n)$, $\rho \to \infty$ as $n \to \infty$, a function to be determined later (but such that $n/2 - \rho$ is a positive integer), we let

$$P = \sum_{i=1}^{n} N_i w_i$$

where, for $1 \leqslant i \leqslant n/2 - \rho$,

$$w_i = A \cdot \binom{n-2}{i}^{-1} \prod_{j=1}^{i} (1+\varepsilon_j), \qquad A = \frac{n-2}{1+\varepsilon_1},$$

and

$$\varepsilon_i = \frac{2+\varepsilon}{n-2i-2} = o(1).$$

It will be desired that the sequence w_i is decreasing. Since for $1 \leqslant i \leqslant i_0 = n/2 - \rho$ we have

$$\begin{aligned}
\frac{w_{i-1} - w_i}{w_i} &= \frac{n-1-i}{i}(1+\varepsilon_i)^{-1} - 1 \\
&\geqslant \frac{n-1-i_0}{i_0}(1+\varepsilon_{i_0})^{-1} - 1 \\
&= \left(1 + \frac{2\rho}{n/2} + o(\rho/n)\right)\left(1 + \frac{2+\varepsilon}{2\rho} + o(1/\rho)\right)^{-1} - 1 \\
&= \left(1 + \frac{4\rho}{n} + o(\rho/n)\right)\left(1 - \frac{2+\varepsilon}{2\rho} + o(1/\rho)\right) - 1 \\
&= \frac{4\rho}{n} - \frac{2+\varepsilon}{2\rho} + o(\rho/n) + o(1/\rho),
\end{aligned}$$

the desired property holds for $1 \leqslant i \leqslant n/2 - \rho$ provided that, say, $\rho \geqslant \sqrt{n}$. (In fact, ρ will have to be slightly larger than that.) For $n/2 - \rho \leqslant i \leqslant n$, we let w_i decrease linearly from

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$w_{n/2-\rho}$ to $w_n = 0$. Formally, for such i, we have

$$w_i = (n-i) \cdot \frac{w_{n/2-\rho}}{n/2+\rho}.$$

We say that a cop at distance i from the robber has *weight* w_i; this represents that cop's individual contribution toward the potential. In particular, we have $w_1 = 1$ and $w_2 = (1+o(1))2/n$. First, let us note that if the cops can capture the robber on their turn, then immediately before the cops' turn we must have $P \geqslant 1$, since some cop must be at distance 1 from the robber. Our goal is to show that the robber can always enforce that right before the cops' move

$$P \leqslant 1 - \frac{3}{n}, \tag{2.1}$$

from which it would follow that the robber can evade the cops indefinitely. Initially, we may assume that all cops start at the same vertex; the robber places himself at the vertex at distance n from the cops. Therefore, $P = 0$, so (2.1) holds. Suppose that before the cops make their move, the potential function satisfies (2.1); we consider a few cases.

Case 1. Suppose that on the cops' turn, a cop moves to some vertex adjacent to the robber, creating a 'deadly' neighbour for the robber. The robber's strategy is to move away from this 'deadly' vertex, but to do so in a way that maintains the invariant (2.1). To show that this is possible, we compute the expected change in the potential function if the robber were to choose his next position at random from among all neighbours other than the deadly one.

Suppose that before the robber's move,

$$P_1 = \sum_{i=2}^{n/2-\rho-1} N_i w_i$$

and

$$P_2 = \sum_{n/2-\rho}^{n} N_i w_i.$$

Then by (2.1), we have that $P_1 + P_2 + w_2 \leqslant 1 - 3/n$, where the extra w_2 accounts for the weight of the cop who moved to the robber's neighbourhood.

Consider a cop, C, at distance i from the robber, where $2 \leqslant i \leqslant n/2 - \rho - 1$. Before the robber's move, C has weight w_i. Let w_C represent the expected weight of C after the robber's move. If C's vertex and the deadly vertex differ on the deadly coordinate (that is, the coordinate in which the robber and his deadly neighbour differ), then

$$w_C = \frac{i-1}{n-1} w_{i-1} + \frac{n-i}{n-1} w_{i+1},$$

whereas if they agree on this coordinate, then

$$w_C = \frac{i}{n-1} w_{i-1} + \frac{n-1-i}{n-1} w_{i+1}.$$

Since $w_{i-1} > w_{i+1}$, we may bound w_C from above as follows:

$$\begin{aligned}
w_C &\leqslant \frac{i}{n-1} w_{i-1} + \frac{n-1-i}{n-1} w_{i+1} \;\leqslant\; \frac{i}{n-2} w_{i-1} + \frac{n-2-i}{n-2} w_{i+1} \\
&= \frac{i}{n-2} \cdot A \cdot \binom{n-2}{i-1}^{-1} \prod_{j=1}^{i-1} (1+\varepsilon_j) + \frac{n-2-i}{n-2} \cdot A \cdot \binom{n-2}{i+1}^{-1} \prod_{j=1}^{i+1} (1+\varepsilon_j) \\
&= \left(\frac{i}{n-2} (1+\varepsilon_i)^{-1} \frac{(i-1)!(n-2-i+1)!}{(n-2)!} \right. \\
&\quad \left. + \frac{n-2-i}{n-2} (1+\varepsilon_{i+1}) \frac{(i+1)!(n-2-i-1)!}{(n-2)!} \right) \cdot A \cdot \prod_{j=1}^{i} (1+\varepsilon_j).
\end{aligned}$$

Since

$$(1+\varepsilon_i)^{-1} = 1 - \varepsilon_i + \varepsilon_i^2 - \varepsilon_i^3 + \cdots \leqslant 1 - \varepsilon_i + \varepsilon_i^2$$

and

$$1 + \varepsilon_{i+1} = 1 + \varepsilon_i \left(1 + \frac{2}{n-2i-4} \right) \leqslant 1 + \varepsilon_i + \varepsilon_i^2,$$

we get

$$\begin{aligned}
w_C &\leqslant w_i \left(\frac{n-i-1}{n-2} (1 - \varepsilon_i + \varepsilon_i^2) + \frac{i+1}{n-2} (1 + \varepsilon_i + \varepsilon_i^2) \right) \\
&= w_i \left(1 + \frac{2}{n-2} - \varepsilon_i \left(\frac{n-i-1}{n-2} - \frac{i+1}{n-2} \right) + \varepsilon_i^2 \left(\frac{n-i-1}{n-2} + \frac{i+1}{n-2} \right) \right) \\
&= w_i \left(1 + \frac{2}{n-2} - \frac{2+\varepsilon}{n-2i-2} \cdot \frac{n-2i-2}{n-2} + \varepsilon_i^2 (1 + o(1)) \right) \\
&\leqslant w_i \left(1 - \frac{\varepsilon/2}{n} \right).
\end{aligned}$$

This last inequality holds as long as, say,

$$\varepsilon_i^2 \leqslant \frac{\varepsilon/4}{n}.$$

Since $i \leqslant n/2 - \rho - 1$, we have

$$\varepsilon_i^2 \leqslant \left(\frac{2+\varepsilon}{2\rho} \right)^2,$$

and so we will take $\rho = \rho(\varepsilon, n)$ such that

$$\rho^2 \geqslant \frac{4}{\varepsilon} \cdot \left(\frac{2+\varepsilon}{2} \right)^2 \cdot n = \frac{(2+\varepsilon)^2}{\varepsilon} \cdot n. \tag{2.2}$$

Hence, after the robber's move, the expected sum of the weights of such cops has decreased by a multiplicative factor of at least

$$\left(1 - \frac{\varepsilon/2}{n} \right),$$

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making it at most

$$P_1 \cdot \left(1 - \frac{\varepsilon/2}{n}\right). \tag{2.3}$$

In addition, the cop that moved to the neighbourhood of the robber would again be at distance 2, making her weight

$$w_2 = (1 + o(1))\frac{2}{n}. \tag{2.4}$$

Before dealing with cops at distance at least $n/2 - \rho$, let us estimate the weight of a single cop at distance $n/2 - \rho$:

$$w_{n/2-\rho} = (1 + o(1))n \cdot \binom{n-2}{n/2-\rho}^{-1} \prod_{i=1}^{n/2-\rho} \left(1 + \frac{2+\varepsilon}{n-2i-2}\right). \tag{2.5}$$

We bound the product term in (2.5) by

$$\begin{aligned}
\prod_{i=1}^{n/2-\rho} \left(1 + \frac{2+\varepsilon}{n-2i-2}\right) &\leqslant \exp\left(\sum_{i=1}^{n/2-\rho} \frac{2+\varepsilon}{n-2i-2}\right) \\
&= \exp\left(\frac{2+\varepsilon}{2} \sum_{i=\rho}^{n/2} \frac{1}{i} + O(1)\right) \\
&= \exp\left(\frac{2+\varepsilon}{2}(\ln(n/2) - \ln \rho + O(1))\right) \\
&= O\left(\left(\frac{n}{\rho}\right)^{1+\varepsilon/2}\right).
\end{aligned}$$

To bound the binomial term, we note that

$$\binom{n-2}{n/2-\rho} = \Theta\left(\binom{n}{n/2-\rho}\right),$$

and approximate:

$$\begin{aligned}
\binom{n}{n/2-\rho} &= \frac{n!}{(n/2-\rho)!(n/2+\rho)!} \\
&= \frac{\sqrt{2\pi n}(n/e)^n}{\sqrt{2\pi(n/2-\rho)}\left(\frac{n/2-\rho}{e}\right)^{n/2-\rho}\sqrt{2\pi(n/2+\rho)}\left(\frac{n/2+\rho}{e}\right)^{n/2+\rho}}(1+o(1)) \\
&= \Theta\left(\frac{2^n}{\sqrt{n}}\right) \cdot \left(1 - \frac{2\rho}{n}\right)^{-n/2+\rho} \left(1 + \frac{2\rho}{n}\right)^{-n/2-\rho} \\
&= \Theta\left(\frac{2^n}{\sqrt{n}}\right) \cdot \exp\left(\left(\frac{-2\rho}{n} - \frac{(2\rho/n)^2}{2} + o((\rho/n)^2)\right)\left(-\frac{n}{2} + \rho\right)\right) \\
&\quad \times \exp\left(\left(\frac{2\rho}{n} - \frac{(2\rho/n)^2}{2} + o\left((\rho/n)^2\right)\right)\left(-\frac{n}{2} - \rho\right)\right) \\
&= \Theta\left(\frac{2^n}{\sqrt{n}}\right) \cdot \exp\left(-(1+o(1))\frac{2\rho^2}{n}\right).
\end{aligned}$$

Now take $\rho(n)$ to be minimal such that $\rho \geqslant c_\varepsilon\sqrt{n}$ and $n/2-\rho$ is an integer, where, referring to (2.2), we set $c_\varepsilon = (2+\varepsilon)/\sqrt{\varepsilon}$. Then we have that

$$w_{n/2-\rho} = O\left(n \cdot \frac{\sqrt{n}}{2^n} \cdot n^{1/2+\varepsilon/4}\right) = O\left(\frac{n^{2+\varepsilon/4}}{2^n}\right).$$

Now let C be a cop at distance i from the robber, where $n/2-\rho \leqslant i \leqslant n$. Before the robber's move, C has weight w_i. Since the w_i are decreasing, we have that the change in weight of C is bounded above by $w_{i-1}-w_i$. For $i \geqslant n/2-\rho+1$, this quantity is equal to

$$\frac{w_{n/2-\rho}}{n/2+\rho}.$$

The largest increase comes when $i = n/2-\rho$. To bound this increase, we see that

$$\begin{aligned}
\frac{w_{n/2-\rho-1}}{w_{n/2-\rho}} &= \frac{\binom{n-2}{n/2-\rho}}{\binom{n-2}{n/2-\rho-1}} \cdot \frac{1}{1+\varepsilon_{n/2-\rho}} = \frac{n/2+\rho-1}{n/2-\rho} \cdot \frac{1}{1+(2+\varepsilon)/(2\rho-2)} \\
&\leqslant \left(1+\frac{2\rho-2}{n}\right) \cdot \left(1+\frac{2\rho}{n}+O\left(\frac{\rho}{n}\right)^2\right) \cdot \left(1-\frac{2+\varepsilon}{2\rho-2}+\left(\frac{2+\varepsilon}{2\rho-2}\right)^2\right) \\
&\leqslant 1+O\left(\frac{\rho}{n}+\frac{1}{\rho}\right).
\end{aligned}$$

By our definition of ρ, this is $1+O(1/\sqrt{n})$. Thus we have that

$$w_{n/2-\rho-1} - w_{n/2-\rho} = O\left(\frac{w_{n/2-\rho}}{\sqrt{n}}\right) = O\left(\frac{n^{3/2+\varepsilon/4}}{2^n}\right).$$

So if we let the total number of cops be $k = O(2^n/n^{5/2+\varepsilon})$, then we have that the total increase in weight of cops at distance at least $n/2-\rho$ is at most

$$O\left(\frac{2^n}{n^{5/2+\varepsilon}} \cdot \frac{n^{3/2+\varepsilon/4}}{2^n}\right) < \frac{\varepsilon/4}{n}.$$

So the total weight of such cops after the robber's move is at most

$$P_2 + \frac{\varepsilon/4}{n}. \tag{2.6}$$

Thus, after the robber's random move, combining estimates (2.3), (2.4) and (2.6), we can bound the total expected weight from above by

$$\begin{aligned}
&P_1 \cdot \left(1-\frac{\varepsilon/2}{n}\right) + w_2 + P_2 + \frac{\varepsilon/4}{n} \\
&\quad \leqslant \left(1-\frac{3}{n}-w_2-P_2\right) \cdot \left(1-\frac{\varepsilon/2}{n}\right) + w_2 + P_2 + \frac{\varepsilon/4}{n} \\
&\quad \leqslant 1-\frac{3}{n}-\frac{\varepsilon/4}{n}+O\left(\frac{1}{n^2}+\frac{P_2}{n}\right) \\
&\quad \leqslant 1-\frac{3}{n}.
\end{aligned}$$

To get the last line, we used the fact that

$$P_2 = O(2^n/n^{5/2+\varepsilon} \cdot w_{n/2-\rho}) = O(n^{-1/2-3\varepsilon/4}) = o(1).$$

Some deterministic move produces a potential at least as low as the expectation, so the robber may maintain the invariant, as desired.

Case 2. Suppose now that on the cops' turn, some cop C^* moves to a vertex at distance 2 from the robber. The reader should note that at this point there might be other cops at distance 2 from the robber; we only suppose that on the cops' turn, one particular cop has moved from distance 3 to distance 2. As in Case 1, we will see what happens if the robber moves away from C^*. In this case, there are two 'deadly' coordinates for the robber. The robber will flip a coordinate randomly amongst the other $n-2$ choices.

As before, suppose that before the robber moves, P_1 represents the total weight of all cops at distance i with $2 \leqslant i \leqslant n/2-\rho-1$ other than the cop C^* who moved to distance 2. Let P_2 represent the total weight of all cops at distance at least $n/2-\rho$. Since C^* was at distance 3 before its move, we have that $P_1+P_2+w_3 \leqslant 1-3/n$. As in Case 1, for a cop $C \neq C^*$ at distance $2 \leqslant i \leqslant n/2-\rho-1$, we have that the expected weight after the robber's move satisfies

$$w_C \leqslant \frac{i}{n-2}w_{i-1} + \frac{n-2-i}{n-2}w_{i+1}.$$

So again we can bound the total expected weight of such cops from above by

$$P_1 \cdot \left(1 - \frac{\varepsilon/2}{n}\right).$$

The estimate for the change in P_2 remains the same, so we can bound the expected total weight after the robber's move from above by

$$\begin{aligned} &P_1 \cdot \left(1-\frac{\varepsilon/2}{n}\right) + w_3 + P_2 + \frac{\varepsilon/4}{n} \\ &\quad\leqslant \left(1 - \frac{3}{n} - w_3 - P_2\right)\cdot\left(1-\frac{\varepsilon/2}{n}\right) + w_3 + P_2 + \frac{\varepsilon/4}{n} \\ &\quad\leqslant 1 - \frac{3}{n} - \frac{\varepsilon/4}{n} + O\left(\frac{1}{n^2} + \frac{w_3}{n} + \frac{P_2}{n}\right) \\ &\quad\leqslant 1 - \frac{3}{n}. \end{aligned}$$

This time, in addition to our bound on P_2, we have used that $w_3 = o(1/n)$.

Case 3. Suppose now that some cop moves to a vertex at distance $i \geqslant 3$ from the robber. Keep in mind that, again, we allow for the possibility that other cops are at distance 2 from the robber. The resulting increase in the potential function is at most $w_3 = O(1/n^2)$, so the new potential function has value at most $1-3/n+o(1/n)$. Now, by the calculations from Case 1, the robber can move so that the total weight of all cops at distances 2 through $n/2-\rho-1$ decreases by a multiplicative factor of

$$\left(1 - \frac{\varepsilon/2}{n}\right).$$

Once again, define P_2 to be the weight of all cops at distance at least $n/2 - \rho$ before the robber moves. Then, after the robber's move, the potential is at most

$$\left(1 - \frac{3}{n} - P_2 + o\left(\frac{1}{n}\right)\right) \cdot \left(1 - \frac{\varepsilon/2}{n}\right) + P_2 + \frac{\varepsilon/4}{n} \leqslant 1 - \frac{3}{n}. \qquad \square$$

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