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LINEAR GROUPS: ON NON-CONGRUENCE SUBGROUPS AND PRESENTATIONS

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The central theme of this thesis is the calculation of explicit presentations for certain linear groups both for their intrinsic interest and for their use in finding non-congruence subgroups in SL_2 of quadratic imaginary number fields.

Let $GL(n, R)$ denote the group of $n \times n$ invertible matrices with entries from a ring R with unity. When R is commutative $SL(n, R)$ is the subgroup of $GL(n, R)$ consisting of matrices with determinant 1.

Also let $\mathbb{Z}(\omega_d)$ be the ring of imaginary quadratic integers of the form $a + b\omega_d$; $a, b \in \mathbb{Z}$ and $\omega_d = \sqrt{d}$ if $d \neq 1 \bmod 4$, otherwise $\omega_d = \frac{1}{2}(1+ d)$, with $0 > d \in \mathbb{Z}$. When d has no square factors $\mathbb{Z}(\omega_d)$ is the full ring of integers in the imaginary quadratic number field of discriminant d. Let $\mathbb{Z}_n(\omega_d)$ be $\mathbb{Z}(\omega_d)$ factored by the principal ideal generated by $n \in \mathbb{Z}$.

The first chapter of the thesis is a computation of the simple factor groups in the composition series of $SL(m, \mathbb{Z}_n(\omega_d))$. Using results from the first chapter the second gives a solution of the 'congruence subgroup problem' for $SL(2, \mathbb{Z}(\omega_d))$ (which is an alternative one to Serre's) by giving examples of non-congruence subgroups.

The third and fourth chapters deal with presentations of certain $GL(n, R)$ and $SL(n, R)$. Early in Chapter Three the theorem used to give

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results in Chapter Four is developed, and using this a simple presentation of $SL(2, \mathbb{Z}_{p^r})$ is calculated.

Chapter Four, based on Chapter Three, gives a presentation of $GL(n, L)$ for various n, when L is a certain type of ring of linear operators. Let A be an ideal of the ring of bounded linear operators mapping a Banach space (over a field F) into itself and let $L = \{A+\lambda I \mid A \in A, \lambda \in F\}$. If the resolvent of each operator of A is dense in F then L is universal for $GE(2)$. If F is either $\mathbb{R}$ or $\mathbb{C}$ and A is the ideal of operators with finite dimensional range then L is a universal $GE(n)$-ring.

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