The Rate of Turn Required for Geographically Fixed Turns: A Formula and Fast-Time Simulations

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This paper concerns the derivation of a formula to follow geographically fixed turns in a homogeneous current or tidal stream. Until now, various well-founded but limited approximations have been used. In principle, all these approximations are based on the formula $v = \omega \times R$. One result of this research is the development and use of a fast-time simulation program. The initial aim was to illustrate to trainees the consequences and, in particular, the possible dangers of these approximations. The fast-time simulation program can be used in support of real-time simulation. Comparisons with real-time simulations carried out by the Dutch Pilots' Corporation (STODEL) indicate that the fast-time simulations generate turning-circle diameters that differ by a maximum 4 percent. The relationship with path-prediction is also dealt with. The possibility of applying the developed formulae in practice and for passage planning is currently under investigation. The fast-time simulation program has not been developed for one specific ship: apart from the use of an assumed position of the pivoting point at 1/3 of the ship's length from the bow, it does not take hydrodynamic effects into account.

KEY WORDS

1. Simulation. 2. Marine. 3. Training.

1. INTRODUCTION. To follow a track plotted on a chart, a course to steer must be determined by navigators and pilots to make the actual track over the ground match the desired track. The effect of current is usually taken into account using a vector sum. The magnitude and direction of the current-vector can be taken from tidal stream atlases or from actual measurements. For the effect of wind, as well as an estimation based on practical experience, eventual use can also be made of approximations such as those laid down by Isherwood or Blendermann. In cases where both wind and current are present, extremely complex hydrodynamic forces can arise. Until now, it has only been possible for navigators and pilots to guess the effects of these from experience. Path-prediction using a mathematical model of the ship, reliable under all circumstances, could allow for this (van Breda and Passenier, 1998). Currently this possibility is being researched world-wide. The results are quite definitely optimistic but are not yet completely reliable in practice. New problems

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arise, such as the effect of current differences in the vertical in both magnitude and/or direction. Influence of current on ships with a marginal under-keel clearance must also be considered. Just above the sea bed, the so called boundary layer effect manifests itself (it is in fact a decrease in the velocity of the current due to friction with the sea bed). In this way the phenomenon, already mentioned, of a current that does not always have the same speed when measured along the vertical can just as easily arise. As a result of this uncertainty, navigators and pilots will almost always use incorrect estimations in their calculations.

Nonetheless the vector sum, used in practice since time immemorial, has until now produced satisfactory results. What is odd, however, is that this method is only applied to straight tracks. In this paper, the use of this same method will be examined, in combination with a rate of turn indicator, when turning. The effect of wind will not be considered, only the influence of current will be examined. The angle between the track over the ground and the heading is referred to in this paper as the current drift angle (α) and should not be confused with a local drift angle caused by rate of turn!

In the early 1970s, both Swedish and Dutch pilots took the initiative to follow a curved track in a turn, based on a sound theoretical method (Gylden, van Hilten et al., 1994, van Roon, 1988). The basis of this comprises the mechanics formula for a circular movement ($v = \omega \times R$), and some trigonometric ratios, such that the precise moment of starting the turn (wheel-over-point or wheel-over-line) can be determined. Refinements can be achieved by applying corrections for the position of the radio navigation aid antenna in relation to the ship's pivoting point, the slow increase of the rate of turn (caused by the moment of inertia about the vertical axis) and the presence of current (van Hilten et al., 1994).

The approach used for training has previously focused on the following aspects:

(a) The manoeuvre-technical aspects; what the ship can and cannot do. Given the sometimes marginal under-keel clearance in narrow waterways, experience (as well as the manoeuvring characteristics of the ship) plays an important part.
(b) The navigation-technical aspects of the manoeuvre such as correlating the calculations with the output of the navigation systems.

Vital to this training is that there should be, above all, an understanding of the manoeuvre-technical aspects: after all, ships often sail in restricted or narrow waterways. From the very start, this topic has been part of various simulator training exercises for navigation officers and pilots.

In one of his publications (Spaans, 1980), Prof. Spaans examined the relationship between the manoeuvring characteristics available on board and the establishment of the wheel-over-line, as well as the influence of wind and current. In his paper, and from other sources, there are strong indications that it is vitally important to control the process continuously once a turn has been started (turn control). The constant radius method is gradually becoming common practice. In nautical colleges, more attention is being given to this concept. Since the grounding of the *Nedlloyd Honshu* in the Suez Canal, attention has been drawn, by the *Raad voor de Scheepvaart* (the Dutch equivalent of the US National Transportation Safety Board), to the question of initiating a turn and the notion 'rate of turn'.

From the considerable experience related to this subject amongst pilots, the following uncertainties have come to the surface over the years,:

- What speed is relevant to determining the rate of turn? (Speed through the water, readings from the Doppler log or perhaps the (D)GPS receiver?)
- Should any sort of correction be made for the presence of current and if so how?
- Turns followed over the earth's surface do not appear to be circular arcs: as a result, every so often a buoy has been hit (on the simulator!).
- Sometimes problems are experienced turning into harbours.
- There are reports that keeping to an exact rate of turn does not always achieve the aim.

A conclusion should certainly not be drawn from this summary that the present approach does not make any sense. If regular control is made during the turn, subsequent errors will be smaller, and the track can be attained perfectly. However, a troublesome point is that, with some methods, a circular arc cannot be followed. The arc is deformed by the direction of the current, its velocity and the ship's speed. At the Royal Netherlands Naval College (RNLNC) and amongst Dutch pilots, the need arose for further study in this field. This was undertaken so that instructors, together with their trainees, could look at all the possibilities and reach a decision on what the most acceptable and, more importantly, practical method is for following any given bend. Above all, the simulations give excellent insight into the phenomenon known as current drift angle. The research completed at the RNLNC and STODEL has shown the following:

(a) a precise calculation of the rate of turn is required in cases where it is necessary to follow a circular arc in relation to the earth's surface in the following circumstances:
   (i) constant speed through the water,
   (ii) increasing speed through the water,
   (iii) decreasing speed through the water.

(b) a fast-time simulation program in Matlab can be developed, to emulate all present methods, as well as the exact (and so in fact only correct) method and a method making use of (D)GPS speed. In addition, tables can be made of the various relevant parameters (including the rate of turn).

2. DERIVATION OF THE FORMULA FOR THE REQUIRED RATE OF TURN.

To give an idea of where this is leading, let us begin by looking at the components that make up the rate of turn.

(a) Without current, wind or speed alteration the known method is:

$$\omega = V/R.$$

where ω is the change in track (dTr) per unit of time (dt) or dTr/dt. Based on the pivoting point, we can establish that the momentary track direction (Tr) coincides with the heading (C). The heading change per unit of time (dC/dt) is therefore equal to dTr/dt.

(b) Where there is current, then at every point along the curved track the current drift angle (α) will be different (because the track direction changes constantly). As well as the rate of turn needed to make the turn without current, an extra measure of rate of turn comes into play due to the ever changing current drift angle: namely $d\alpha/dt$.

(c) If at the same time the speed also changes, affecting the current drift angle, an additional rate of turn should be considered. Although this additional part does not have a separate name during the derivation, for the time being we will call it $d\alpha_1/dt$.

The above can be summed up as follows:

$$dC/dt = dTr/dt + d\alpha/dt + d\alpha_1/dt \tag{1}$$

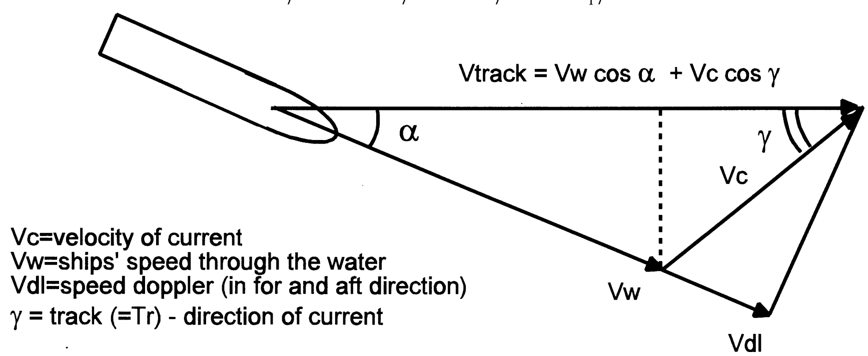

Figure 1. Applicable velocity vectors.

From Figure 1, it can be seen that :

$$Vw \sin\alpha = Vc \sin\gamma.$$

The variables dependent on time in the case of changes are principally: α and γ. In case of changes in speed, the situation is the same with Vw being included for convenience. The constants are: velocity of current (Vc) and direction of current. Differentiating with respect to time gives:

$$Vw \times \cos\alpha \times \alpha' + Vw' \times \sin\alpha = Vc \times \cos\gamma \times \gamma'.$$

$$\alpha' = (Vc \times \cos\gamma \times \gamma' - Vw' \times \sin\alpha)/(Vw \times \cos\alpha). \tag{2}$$

α' now represents the values read at (1):

$$d\alpha/dt + d\alpha_1/dt$$

Total rate of turn (dC/dt) now comes to:

$$dTr/dt + \alpha'$$

Thus for $Tr = \gamma +$ direction of current:

$$dTr/dt = d\gamma/dt = \gamma'$$

and the total rate of turn comes to:

$$\alpha' + \gamma' \tag{3}$$

$$\gamma' = dTr/dt = Vtrack/R \tag{4}$$

(where R = radius of turning circle and $Vtrack = Vw\cos\alpha + Vc\cos\gamma$)

It follows from (2), (3) and (4) that the total rate of turn =

$$Vtrack/R + (Vc \times \cos\gamma \times (Vtrack/R) - Vw' \times \sin\alpha)/(Vw \times \cos\alpha) \tag{5}$$

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This can be interpreted as:

$$\mathrm{ROT} = (\mathrm{Vtrack}/\mathrm{R}) \times (1 + (\mathrm{Vc} \times \cos\gamma)/(\mathrm{Vw} \times \cos\alpha)) - (\mathrm{Vw}'/\mathrm{Vw}) \times \tan\alpha \quad (6)$$

(with $\mathrm{Vtrack} = \mathrm{Vw}\cos\alpha + \mathrm{Vc}\cos\gamma$)

In terms of a *connection between speed doppler* (Vdl), the following can be derived:

$$\mathrm{Vdl} = \mathrm{Vtrack} \times \cos\alpha \quad (7)$$

It follows from (6) and (7) that:

$$\mathrm{ROT} = \frac{\mathrm{Vdl}^2}{\mathrm{Vw}} \times \frac{1}{(\cos\alpha)^3} \times \frac{1}{\mathrm{R}} - \frac{\mathrm{Vw}'}{\mathrm{Vw}} \times \tan\alpha \quad (8)$$

When Vw is constant, Vw′ (acceleration) is zero and in this case the last term can be left out. The term $\mathrm{Vdl}^2/\mathrm{Vw}$ can be regarded as a sort of imaginary speed that should be multiplied by a factor of $1/(\cos\alpha)^3$. Where the current drift angle is small (not much current, relatively high speed and small angle between track and direction of current), the term $1/(\cos\alpha)^3$ has little influence on the outcome of the formula.

The relationship at (6) is derived between: ROT on the one side and the values Vw, acceleration, track direction, radius of turning circle (constant), velocity of current (constant) and direction of current (constant) on the other. For every point throughout the turn, the ROT can thus be determined. Secondarily a connection is made at (8) with the known Vdl.

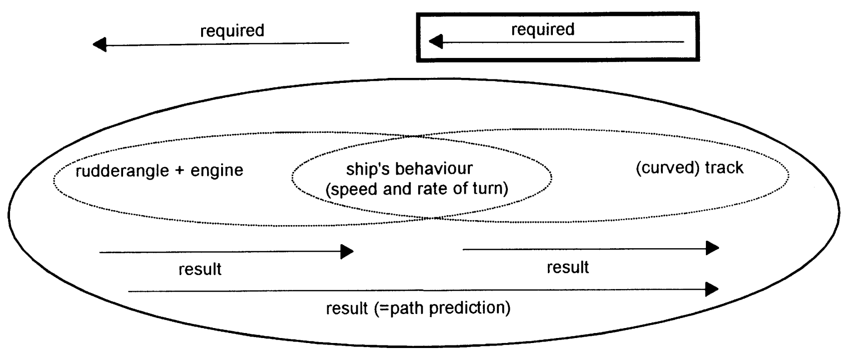

Figure 2. Path prediction modelling.

3. RELATIONSHIP WITH PATH-PREDICTION. The section contained in the solid ellipse in Figure 2 belongs to the subject of 'mathematical models' and, in spite of the fact that this has not been 100% researched, does not belong to this research. The issue of how much rudder and revolutions are needed to achieve a given result is something pilots and navigation officers will have to resolve. This is certainly the case when considering a ship's behaviour in shallow water, which is a problem yet to be fully solved. What this paper is about, however, is that section shown within the rectangle in Figure 2: namely, what the ship has to do (speed and rate of turn) to follow a given (curved) track.

4. THE SITUATION AT SEA. From a number of real-time simulator runs, whereby the rate of turn is determined as exactly as possible with the assistance of the

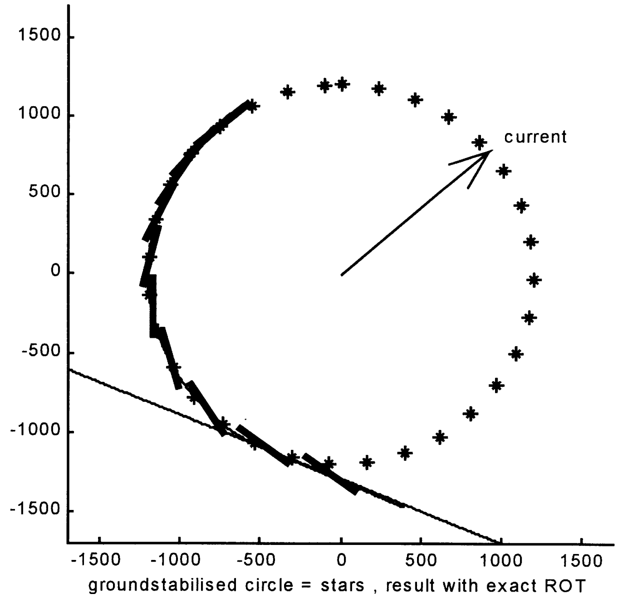

Figure 3. Rate of turn using exact formula.

formula derived for this, the results have been compared with the plot of the fast-time simulation. In spite of the fact that in fast-time simulation, regardless of the assumed position of the pivoting point – no hydrodynamic effects are taken into account, the similarities are remarkably easy to identify. For application at sea, the first (simple) version of the program has been used for some time by pilots in IJmuiden in support of briefing and debriefing real-time simulation. From the point of view of professional training, the results are most satisfying. Figures 3 and 4 show two plots of a fast-time simulation at sea. They are from a VLCC leaving the emergency anchorage off Hook of Holland after weighing anchor. The inputs used for the simulation are as follows:

Length overall (m)	360
Distance DGPS antenna-pivoting point (m)	180
Distance bow-pivoting point (m)	120
Direction of current	050
Velocity of current (knots)	2
Initial speed through the water (knots)	2
Final speed (knots)	8
Acceleration-time (linear, in minutes)	15
Initial track	230
Final track (Maas channel)	112
Radius geographical turn (m)	1200
Plot interval (seconds)	100

In Figure 3, the rate of turn is determined from the previously derived exact formula. In Figure 4, one of the possible rules of thumb, namely ROT = $(Vdl^2/Vw)/R$, has been used. The result is plain to see: the ship comes a long way off the desired track.

Table 1 shows the various values relating to the first plot; again every 100 seconds. If required, the program also shows the developing rate of turn graphically.

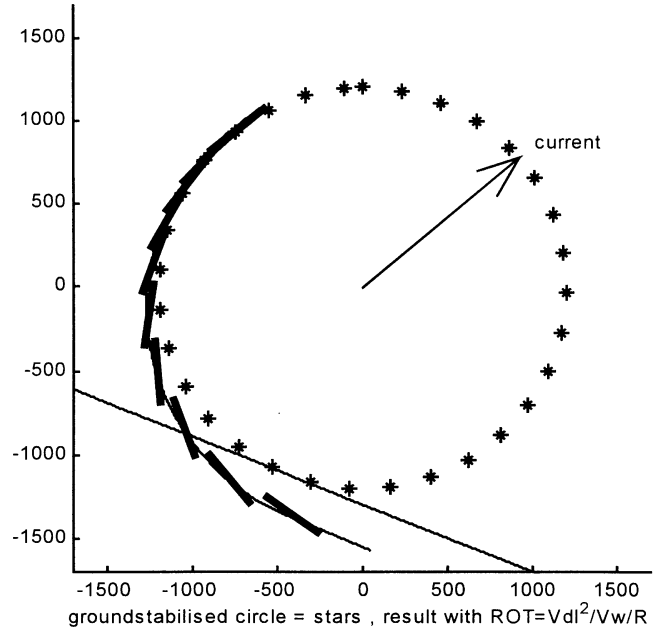

Figure 4. Rate of turn using $(Vdl^2/Vw)/R$.

Table 1. Rate of turn (ROT) by developed formula.

Time (sec)	Track	Alfa	Heading	Speed through water	Speed at DGPSAe	Course at DGPSAe	(1-cos (Δtrack)) × R	(1-cos (Δhdg)) × R	Actual ROT
0	230·00	0·00	230·00	2·01	0·01	230·06	1763	1494	0·00
100	228·43	0·96	229·39	3·27	1·27	232·72	1734	1481	−0·94
200	223·93	2·77	226·70	4·38	2·39	229·58	1648	1428	−2·31
300	216·85	4·89	221·74	5·34	3·36	223·15	1508	1323	−3·64
400	207·47	7·15	214·62	6·16	4·23	214·19	1314	1177	−4·90
500	196·01	9·43	205·45	6·82	5·01	203·07	1075	984	−6·13
600	182·63	11·57	194·20	7·34	5·73	190·03	802	757	−7·40
700	167·39	13·32	180·72	7·71	6·43	175·20	518	510	−8·82
800	150·34	14·37	164·71	7·93	7·13	158·64	259	266	−10·45
900	131·47	14·31	145·79	8·00	7·83	140·39	69	72	−12·32
1000	112·97	12·87	125·84	8·00	8·50	112·97	0	0	0·00

5. ENTERING A HARBOUR FROM A RIVER (FLOW). A logical consequence of this application in a homogeneous current field is the search for a method whereby the current is *partially* navigated. Such is the case when entering a harbour from a river flow: as soon as you leave the river the current reduces. In order to simulate this sequence of events, bespoke software has been written. In this simulation, a discontinuity in the current picture is, as yet, still assumed: ie. the abrupt (therefore not gently along one gradient) lessening of the current. These situations occur in practice: the so called, current rip or tide rip.

After starting the program, as in previously described simulations at sea, it is necessary to input various data. An important extra detail is that an acceptable

FIGURE 5

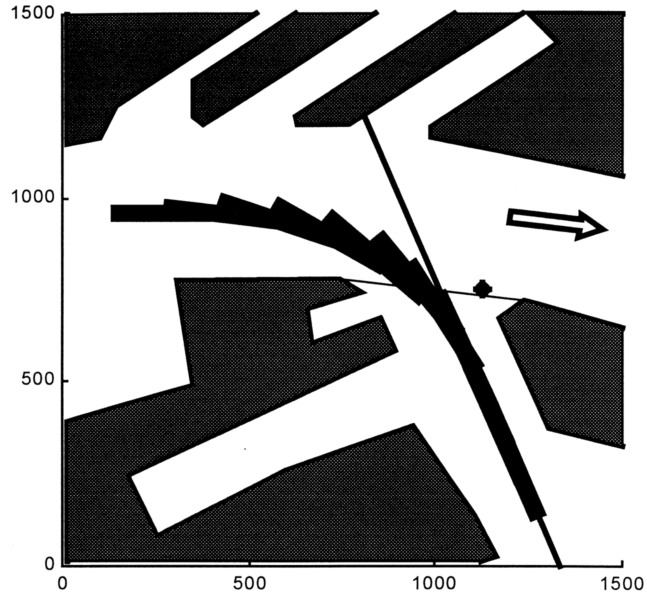

Figure 5. Plot from the simulation entering harbour.

remainder from the change of heading must be determined which, once in the harbour entrance, can still be completed. This is of course dependent on the turning ability of the ship and the width of the harbour basin. Figures 5 and 6 show, consecutively, the plot from a run and the graphical representation of the rate of turn. The tidal stream on the river runs from West to East. The ship has to turn from an easterly course to starboard to enter the harbour. The thin line represents the limit of the tidal stream. In the harbour basin to the South of this line there is therefore no current. Given that an attempt is made to follow a circular arc for the part of the turn in the river, the ship has a particular angle of approach for passing the tide rip. From that position and the relevant course, after a kink in the track, the ship follows an earth stabilised circular arc once more until the final heading is reached. The program recommends a position for initialising the manoeuvre (including correcting for the time and distance necessary for building up the required rate of turn). Certain approaches to this already exist and have been used for a long time. The main aim of the program is to show the effect of the current, the consequences of the approach used, to recommend the position to initialise the manoeuvre and to show the rate of turn needed. The latter is interesting in terms of passage planning. The question is, is the maximum needed value of rate of turn feasible for a specific ship?

The inputs used for this simulation are:

Length over all (m)	300
Distance bow-pivoting point (m)	100
Direction of current (degrees)	098
Velocity of current (knots)	1
Constant speed through water (knots)	3·5
Initial heading (degrees)	090

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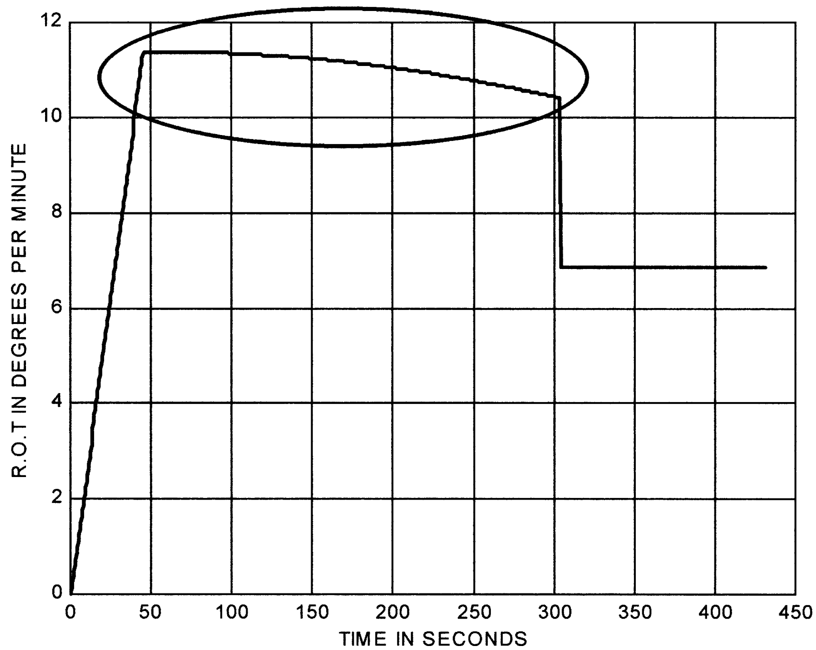

Figure 6. Rate of turn against time.

Final track (in harbour, in degrees)	157
Radius geographical turn (m)	900
Remainder course change in harbour	15
Plot interval (seconds)	60

Note on Figure 5, that the line between the western and eastern side of the harbour entrance is the tide rip, the thick line (track 157°) shows the desired track inside the harbour. Figure 6 shows a plot of the rate of turn versus time. The sudden leap, after approximately 300 seconds, is the result of passing the tide rip.

This jump in the rate of turn is impossible to realise in practice. However, the ship has almost entered the harbour at that moment, and no longer needs to make a large course alteration so the phenomenon is no longer really relevant. It is the first section (shown in the ellipse) of the turn, where the information is to do with rate of turn, that is especially important: can the rate of turn (albeit with the help of tugs) be achieved?

6. USE OF DGPS SPEED IN DETERMINING THE RATE OF TURN. When using the speed reading from the DGPS equipment to determine the rate of turn, the utmost caution must be taken. If a large distance exists between the GPS antenna and the ship's pivoting point, and the radius of the bend is small, the antenna is liable to a large angle of drift. Thus the speed (over the ground) from the antenna can be affected which also influences the turn established. The consequences of which, as simulations have shown, can be considerable.

7. CONCLUSIONS. The Royal Netherlands Naval College and STODEL have, based on a special algorithm derived for this purpose, developed a simple and

user friendly fast-time simulation program for comparing and contrasting the various practical methods for establishing the rate of turn in a ground-stabilised bend. The same algorithm might be useful in the improvement of automatic pilots, which also use the desired rate of turn as a function of the direction and velocity of the current. Practical experience plays an important part in the method used as does the use of ship's manoeuvring characteristics.

The comparisons with real-time simulations carried out so far by the Dutch Pilots' Corporation show that the fast-time simulations show little or no differences in this. Comparisons with real-time simulations with a Netherlands Navy supplier (HNLMS Zuiderkruis) were also completed. Although a scientific study to compare the two methods of simulation would require a more rigorous approach, a comparison of the plots show a maximum difference in turning-circle diameters of only 4 percent.

Allowing trainees to carry out real-time simulations demands a lot of time and is thus expensive. In this way the fast-time simulations could save money. From the didactic point of view it can be assumed that showing the results of all sorts of estimations consecutively, can provide a quick and clear insight into the topic.

The next (hoped for) step is the simulation of these kind of situations on an electronic chart. Further investigations into situations with non-homogeneous current-patterns have just started.

Whilst the authors are convinced of the benefits of using simulators for training purposes, they would leave you with this final thought taken from the conclusions reached during the 1998 PIANC congress [6]:

> …In the field of risk analysis of vessel manoeuvring, more problems are encountered. The usefulness and in fact indispensability of simulations in the design of ports and port approaches was clearly demonstrated. *However, it is felt that there is a need for calibrated and validated simulation manoeuvring models.*

Certainly in the simulation of manoeuvres such as docking, undocking, passing bridges and locks etc., a critical look should be taken at this model's potential!

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