

as much time is taken in the earlier years gaining a mastery of certain tools from analysis before the student embarks on a study of partial differential equations that does not sacrifice the rigour his earlier training leads him to demand. Nevertheless, most of the material is well within the capabilities of honours students in their junior years, and with careful planning, standards of preparation need not be compromised. An advantage of the text under review is that its presentation does facilitate its introduction at such a time, for it requires a knowledge of multivariate analysis and ordinary differential equations that can be gained by a student after two years at university. When he uses the text concurrently with a course in complex variables the student will be ready to understand the development of special functions that comprise the last third of the book.

The material covered in Part 1 is as follows: formulation of p.d.e. and their classification with solutions by the method of characteristics, Green's functions, and separation of variables with the theory of eigenfunction expansions; potential theory; integral equations. Part 2 covers Bessel functions, spherical harmonics, Hermite and Laguerre Polynomials. The style of presentation is very similar to that used by such earlier Russian authors as Petrovskii, Tychanov and Samarski in their texts; although these texts are used by physics and engineering science students in Russia, there is no lack of mathematical detail. One main difference here is that the concept of generalized functions, adequately introduced in an appendix, is used throughout. Also, only elementary functions plus the error function are used to illustrate the theory in the first part; this avoids departing from the main argument of the theory to develop particular properties, though it does thereby limit the range of examples considered.

Particular criticisms can be made: there is no mention of integral transforms, which are certainly powerful tools in the solution of the heat conduction equation. The short chapter on the derivation of partial differential equations is somewhat brief and, in places, misleading when the physical assumptions required are not even mentioned. This section would need to be supplemented. Further, although all the theorems required in the logical development of the subject are stated, on occasion this is done without proof (e.g. the existence of an infinite set of eigenvalues for a regular Sturm-Liouville system). This is understandable; however, (in the translation at least) no mention is made of where a suitable proof can be found, so that the reader is left unaware of the intricacies involved. In fact, there is only a list of a dozen (standard) texts in English to serve as references; the job of translation, it seems, should include filling in such gaps in more detail if the books are to be helpful for students.

There are a number of problems at the end of each chapter. The translation is good and there are few misprints, though in places there is confusion over notation (e.g. both $\nabla^2\phi$ and $\Delta\phi$ are used to represent the Laplacian of the function ϕ , while ∇^2x is also used to represent an increment $x_2 - x_1$). To conclude, this book can definitely be recommended to the student, not necessarily as the basic text, but certainly as collateral reading. It will put him in contact with the excellent style of many of the present Russian authors.

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Lectures on the numerical solution of linear, singular and nonlinear differential equations, by D. Greenspan. Prentice Hall Inc., 1968. 185 pages. U.S. \$6.95.

This book is based on lectures given by the author at a series of summer

conferences at the University of Michigan and is concerned with the use of finite difference methods to solve linear, singular and nonlinear partial differential equations. The approach is essentially that of a survey, with emphasis being placed on problems which arise in mathematical physics. The value of the book is enhanced considerably by the inclusion of an extensive bibliography.

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Function theoretic methods in partial differential equations, by Robert P. Gilbert, Academic Press, New York, 1969. 311 pages. U.S. \$17.50.

Probably one of the most frustrating and irritating fates to befall a mathematician is to make a significant discovery and then have it attributed to someone else. This is the situation in which the author of the monograph under review finds himself and hopefully this book will serve to set the record straight. The discovery in question is the "envelope method" first obtained by the author in 1958 and since then often attributed mistakenly to either Landau, Bjorken, or Polkinghorne and Screaton. This method is a generalization of Hadamard's Multiplication of Singularities Theorem to functions of several complex variables and with the author playing a leading role in its development has had wide applications to both the analytic theory of partial differential equations and to basic problems arising in the theory of potential scattering.

The book is divided into five chapters. Chapter One is devoted to an introduction to the theory of several complex variables and in particular the derivation of the "envelope method". In Chapter Two integral operator techniques in conjunction with the "envelope method" are used to study the analytic properties of harmonic functions of $(p + 2)$ variables. This material is designed to bridge the gap between traditional treatises on partial differential equations and the function theoretic approach. Chapter Three is a clear presentation of the elegant integral operator method developed by Bergman and Vekua to investigate linear elliptic partial differential equations with analytic coefficients. Extensions of this approach are made to the study of nonlinear elliptic equations satisfying generalized Goursat data. Chapter Four is devoted to the study of certain classes of singular elliptic partial differential equations which are related to Weinstein's generalized axially symmetric potential equation. The use of the author's "envelope method" plays a central role here in obtaining theorems concerning the location of singularities and growth conditions for entire solutions. The final chapter is concerned with illustrating how function theoretic methods (and in particular the "envelope method") may be used to study the scattering problems which arise in quantum mechanics and quantum field theory.

The book is clearly written and well organized. It should become a standard in the field along with Bergman's Integral Operators in the Theory of Linear Partial Differential Equations [Springer, 1961] and Vekua's New Methods for Solving Elliptic Equations [Wiley, 1967].

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