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RESEARCH PAPER

Synthesis of nonuniformly spaced linear array of parallel and collinear dipole with minimum standing wave ratio using evolutionary optimization techniques

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In this paper, the author proposes a method based on two recent evolutionary algorithms (EAS): particle swarm optimization (PSO) and differential evolution (DE) to design nonuniformly placed linear arrays of half-wavelength long dipoles. The objective of the work is to generate pencil beam in horizontal (for parallel array) and vertical (for collinear array) plane with minimum standing wave ratio (SWR) and fixed side lobe level (SLL). Dynamic range ratio (DRR) of current amplitude distribution is kept at a fixed value. Two different examples have been presented having different array alignments. For both the configurations parallel and collinear, the excitation distribution and geometry of individual array elements are perturbed to accomplish the designing goal. Coupling effect between the elements is analyzed using induced electromotive force (EMF) method and minimized in terms of SWR. Numerical results obtained from both the algorithms are statistically compared to present a comprehensive overview. Beside this, the article also efficiently computes the trade-off curves between SLL, beam width, and number of array elements for nonuniformly spaced parallel array. It featured the average element spacing versus SWR curve for nonuniformly separated arrays. Furthermore, minimum achievable SLL performances of uniformly and nonuniformly spaced parallel arrays are compared for same average spacing in the proposed work.

Keywords: Particle swarm optimization (PSO), Differential evolution (DE), Standing wave ratio (SWR), Induced electromotive force (EMF)

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I. INTRODUCTION

Motivating the exploration of the better design technique is an essential need for increased antenna performances. Antenna engineers find an existing design that may have the desired electromagnetic characteristics. If this structure has an analytical expression that precisely predicts its performance, we try to find the optimal parameters. This paper describes the synthesis of nonuniformly excited and nonuniformly spaced linear arrays. The analysis of nonuniformly spaced linear arrays was first proposed by Unz [1], who developed a matrix formulation to obtain the current distribution necessary to generate a desired radiation pattern. Array geometry was calculated either by thinning array elements selectively or positioning the array elements randomly along the desired direction.

Skolnik employed dynamic programming for zeroing elements [2]. Mailloux and Cohen [3] utilized the statistical thinning of arrays with quantized element weights to improve side lobe level (*SLL*) performance. The genetic algorithm and simulated annealing were used to thin an array [4–7]. Razavi and Forooragi [8] used pattern search algorithm for array thinning.

Harrington [9] developed an iterative method to reduce the sidelobe levels of uniformly excited and nonuniformly spaced linear arrays (NULSAs). Literature described in [10–12] proposed different analytical methods for nonuniformly spaced array synthesis. In [10], the particle swarm optimization was applied for optimization of nonuniformly spaced antenna arrays and *SLL* was reduced. In [11], with neural network and in [12] with least mean square, nonuniformly spaced array was synthesized. Most works consider the minimization of the *SLL* at a fixed beam width without considering mutual coupling effect. In a few recent works driving point impedance matching has been derived with unequal spacing of elements [13].

In this work, two different antenna optimization problems of designing unequally spaced parallel and collinear arrays are presented. Radiation patterns are synthesized to obtain a specified *SLL* and dynamic range ratio *(DRR)* at a fixed beamwidth. Coupling effect between different array elements is also taken into account. Induced electromotive force method is used to estimate the coupling effect of individual elements in terms of *VSWR*. Reported algorithms are applied to compute the

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excitation and geometry of the individual elements in order to optimize the array performances.

An improved particle swarm optimization (PSO)-based technique is proposed to accommodate these complex design problems of coupling compensation [14–18]. Proposed method is adapted introducing velocity modulation technique where maximum particle velocity decreases as the number of iterations increases in order to favor the exploitation. Another improved differential evolution (DE)-based technique is also applied to the same problem and results obtained from both the algorithms are compared [19–22]. In the proposed DE scheme the scale factor and cross over rate are tuned depending on the fitness of individual population member. As the DE vector moves near to the optima it should suffer from lesser perturbation. So scale factor reduces decreasing the mutation step sizes. At the same time crossover rate also decreases allowing more genetic information to be passed to the offspring. On the contrary if the vector goes away from the optimal region DE parameters are tuned automatically for providing adequate population diversity.

Beside this the article presented a trade-off solution between different array objectives. It plotted the trade-off curves between minimum achievable *SLL* and number of elements for specified beam width. It also computed the curve featuring standing wave ratio *(SWR)* versus average array spacing for unequally spaced array. Moreover, the article presented a comparison of *SLL* performances of equally spaced linear array (ELSA) and NULSA for same average spacing.

II. FORMULATION

In this paper, two different array alignments are presented.

A) Parallel dipole array

Consider a linear array of $2N$ half-wavelength long center-fed very thin parallel dipole antennas along the x-axis with inter-element spacing $d_{n,n-1}$ between any two consecutive dipoles as shown in Fig. 1. Excitation and geometry both are assumed symmetric with respect to the center of the array in order to generate symmetric broadside pencil beam patterns in azimuth (x–y) plane.

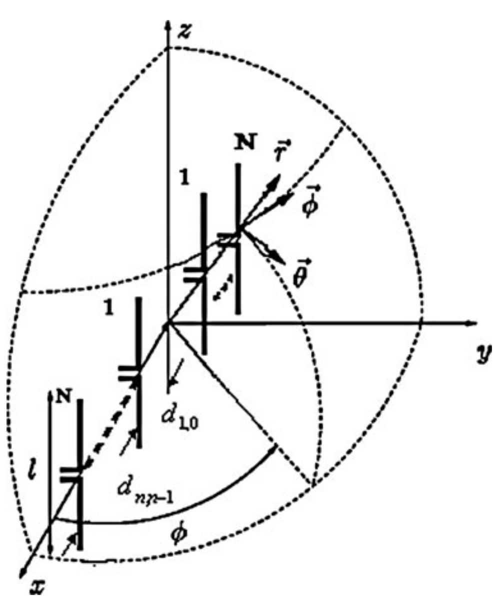

Fig. 1. Linear array of parallel dipoles along-axis.

The far-field pattern $F(\phi)$ in the horizontal xy plane in absence of any ground plane is given by equation (1) as in [23]. Element pattern has been assumed omnidirectional in horizontal plane in the absence of ground plane:

$$F(\phi) = \sum_{n=1}^{N} 2I_n \cos\left[kd_n \cos\phi\right]. \tag{1}$$

B) Collinear dipole array

Next, a collinear array of $2N$ number of identical half-wavelength dipoles spaced a distance $d_{n,n-1}$ apart (center to center) along the Z-axis as shown in Fig. 2 is considered. Excitation and geometry both are assumed symmetric with respect to the center of the array. Assuming sinusoidal current distribution of a very thin half-wavelength dipole directed along the Z-axis, the element pattern can be calculated using

$$Elepat(\theta) = \frac{\cos(0.5\pi\cos\theta)}{\sin\theta}. \tag{2}$$

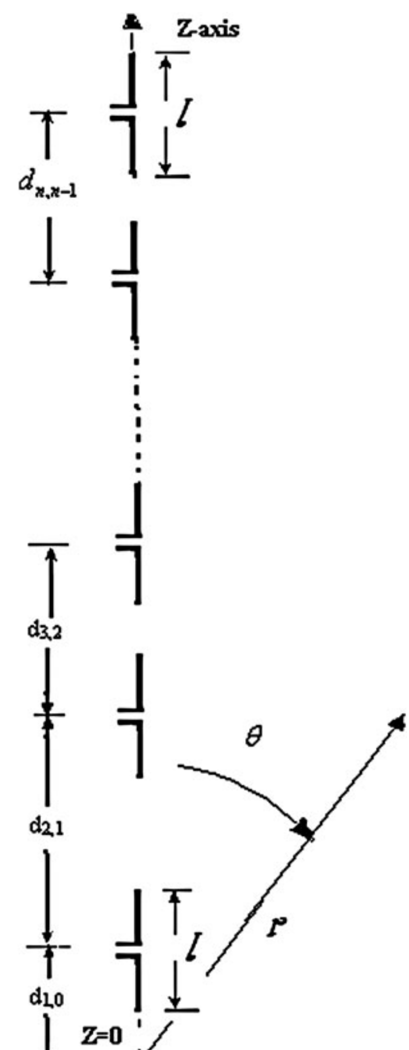

Fig. 2. Collinear dipole array.

The far-field pattern $F(\theta)$ in the principal plane (yz plane) considering the element pattern is given by

$$F(\theta) = \sum_{n=1}^{N} 2I_n \cos\left[kd_n \cos\theta\right] \times Elepat(\theta). \quad (3)$$

Normalized power pattern in dB for both the cases can be expressed as follows:

$$P(\gamma) = 10 \log_{10}\left[\frac{|F(\gamma)|}{|F(\gamma)|_{max}}\right]^2 = 20 \log_{10}\left[\frac{|F(\gamma)|}{|F(\gamma)|_{max}}\right], \quad (4)$$

where $\gamma = \phi$ for parallel array and $\gamma = \theta$ for collinear array. For both the cases n is the element number, $k = 2\pi/\lambda$, the free-space wave number, λ is the wavelength at the design frequency, d_n is the distance of center of the nth element from origin, ϕ and θ are the azimuth and polar angles of the far field, I_n is the complex excitation current of nth element, $[V]$ is the voltage matrix of size $N \times 1$ obtained from the given expression

$$V = ZI, \quad (5)$$

where Z is the mutual impedance matrix of size N. Self-impedances Z_{nn} and mutual impedances Z_{nm} are calculated using Hansen's expressions [24] (which assume the current distribution on the dipoles to be sinusoidal).

Using the rigorous electric field formulation of Schelkunoff and Friis [25], and the geometry of Fig. 3, the mutual impedance can be written as

$$Z_{mn} = -30\left[\left\{\int_{h}^{1/2+h} \sin\beta(z-h) + \int_{1/2+h}^{l+h} \sin\beta(l+h-z)\right\} \times\left(\frac{-je^{-j\beta r_1}}{r_1} + \frac{-je^{-j\beta r_2}}{r_2} + \frac{2j\cos\beta l e^{-j\beta r_0}}{r_0}\right)dz\right], \quad (6)$$

where $r_0 = \sqrt{d^2+z^2}$, $r_1 = \sqrt{d^2+(l/2-z)^2}$, $r_2 = \sqrt{d^2+(l/2+z)^2}$.

Mutual impedance is approximated by putting $h = 0$ and $l = \lambda/2$ for parallel alignment and $d = 0$ and $l = \lambda/2$ for collinear arrangement and used for our design.

It is obvious that the value of $Z_{n,m}$ depends on the geometry of the dipoles and their mutual geometric relations. Improved PSO- and DE-based techniques are used to optimize the proposed antenna arrays shown in Figs 1 and 2. The radiation patterns produced by these arrays are required to satisfy the condition of minimum bearable SWR and specified SLL and DRR value. In order to optimize the arrays according to the above three conditions, a cost function J is formed as a weighted sum of three respective terms, as given by the following equation:

$$J = w_1 * (SLL - SLL_d)^2 + w_2 * SWR_{max} + w_3 * (DRR - DRR_d)^2, \quad (7)$$

where SWR_{max} is the maximum SWR offered by the array elements (SWR is different for every array element). SLL, SLL_d, DRR, DRR_d are obtained and desired values of corresponding terms. DRR is computed from the given expression

$$DRR = \max(I_n)/\min(I_n). \quad (8)$$

The characteristic impedance Z_0 of the transmission line that feeds the element for efficient radiation is considered 50 Ω. Reflection coefficient at the input of the nth element is derived by the expression

$$R_n = (Z_{n,n} - Z_0)/(Z_{n,n} - Z_0). \quad (9)$$

Using R_n SWR is calculated at the input of the nth element,

$$SWR = (1 + |R_n|)/(1 - |R_n|). \quad (10)$$

For obtaining impedance matching condition, the maximum tolerable value of SWR is set at 2. The coefficients w_1, w_2, and w_3 are weight factors and they describe the importance of the corresponding terms that compose the cost function. Proposed optimization techniques attempt to minimize the cost function to meet the desired pattern specification.

This paper carried out a simultaneous optimization of excitation and geometry to reduce SLL and SWR value. To generate desired pencil beam, excitation current amplitude is varied in the range 0–1. Excitation current phase is kept fixed at 0 degree. Spacing is computed randomly within the range 0.4–0.8 for the parallel array and from 0.7 to 1.1 for collinear array. All the array parameters are assumed symmetric about the center of the array. Algorithms run independently for several iterations to optimize both the configurations, parallel and collinear.

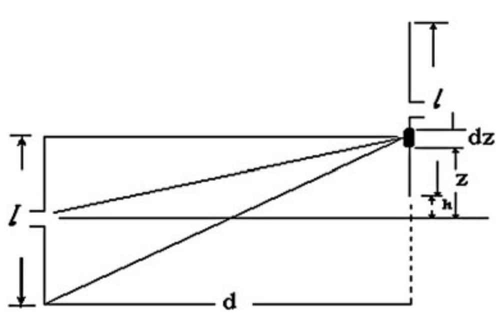

Fig. 3. Two antennas separated by d and staggered by h.

III. OVERVIEW OF PSO

PSO is a robust stochastic evolutionary computation technique based on the movement and intelligence of swarms looking for the most fertile feeding location [14–18]. PSO's foundation is based on the principle that each solution can be represented as a particle in a swarm. Each agent has a position and velocity vector and each position coordinate represents a parameter value. The algorithm requires a fitness evaluation function that assigns a fitness value to each particle position. The position with the best fitness value in the entire run is called the global best. Each agent also keeps track of its best fitness value. The location of this value is called its personal best. Each agent is initialized with a random position and random velocity. The velocity in each of n dimensions is accelerated toward the global best and its own personal best to converge to the desired solution.

Table 1. Desired and obtained result for parallel array.

Algorithms	PSO	Modified PSO	Modified DE
SLL (dB)	30.6409	30.4887	30.39
SWR	1.3639	1.2301	1.1218
DRR	6.7870	7.0112	6.8613

In PSO velocity and position of each particle is updated using the following equations:

$$V_{id}^{t} = w{*}V_{id}^{t-1} + c_1{*}rand1_{id}^{t}{*}(pbest_{id}^{t-1} - X_{id}^{t-1}) + c_2{*}rand2_{id}^{t}{*}(gbest_{id}^{t-1} - X_{id}^{t-1}), \quad (11)$$

$$V_{id}^{t} = \min(V_d^{max},\ \max(V_d^{min}, V_{id}^{t})), \quad (12)$$

$$X_{id}^{t} = X_{id}^{t-1} + V_{id}^{t}, \quad (13)$$

where c_1, c_2 are acceleration constants, equal to 1.4945, w is the inertia weight decreases linearly from 0.9 to 0.4 up to 80% of the maximum number of iterations and thereafter it remains at 0.4 for rest of the iterations, and *rand*1 and *rand*2 are two uniformly distributed random numbers between 0 and 1.

Equation (12) is used to clamp the velocity along each dimension within a specified region if they try to cross the desired domain of interest. The maximum velocity is set to the upper limit of the dynamic range of the search. Later it is linearly modified from V_d^{max} to $0.1{*}V_d^{max}$ over the full range of search [15]. Thus the modification introduced in the particle velocity improves the balance between exploration and exploitation and leads to a better PSO. Position clipping technique is avoided in modified PSO algorithm. Moreover, the cost evaluations of errant particles (positions outside the domain of interest) are discarded to improve the speed of the algorithm.

IV. DIFFERENTIAL EVOLUTION

DE proposed by Storn and Price [19, 20], Storn *et al.* [21], and Das *et al.* [22] is well known as a simple and efficient method of global optimization over continuous spaces.

DE utilizes NP *D*-dimensional parameter vectors as a population for each generation *G*:

$$\vec{X}_{i,G} = [x_{1,i,G}, x_{2,i,G}, x_{3,i,G}, \ldots, x_{D,i,G}]. \quad (14)$$

For each parameter there may be a definite region where better search results are likely to be found. The initial population should cover the entire search space constrained by the specified minimum and maximum bounds:

$$\vec{X}_{min} = \{x_{1,min}, x_{2,min}, \ldots, x_{D.min}\} \text{ and}$$
$$\vec{X}_{max} = \{x_{1,max}, x_{2,max}, \ldots, x_{D.max}\}.$$

Hence the *j*th component of the *i*th vector can be initialized as

$$x_{j,i,0} = x_{j,min} + rand_{i,j}(0, 1)(x_{j,max} - x_{j,min}), \quad (15)$$

where $rand_{i,j}$ (0,1) is a uniformly distributed random number lying between 0 and 1. The subsequent steps of DE are mutation, crossover, and selection, which are explained in the following subsections.

A) Mutation

DE creates a donor vector $\vec{V}_{i,G}$ corresponding to the best individual $\vec{X}_{best,G}$ that generates minimum cost value in the population at generation *G* through mutation:

$$\vec{V}_{i,G} = \vec{X}_{best,G} + F_i(\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G}). \quad (16)$$

The indices r_1^i and r_2^i are mutually exclusive and randomly chosen integers. F_i is called scaling factor that is tuned automatically depending on the value of the cost function generated by each vector.

If the objective function value of any vector nears the objective function value attained by $\vec{X}_{best,G}$, F_i is estimated as follows:

$$F_i = 0.8{*}\left(\frac{\Delta J_i}{\delta + \Delta J_i}\right), \quad (17)$$

where $\delta = 10^{-14} + \Delta J_i/10$, $\Delta J_i = J(X_i) - J(X_{best})$ and $\Delta J_i < 2.4$.

The expression results a lesser value of F_i causing lesser perturbation in the solution. So it will undergo a

Table 2. Current amplitude, spacing, and *SWR* value for parallel array.

n	PSO			Modified PSO			Modified DE		
	Current amplitude	Spacing (from origin)	SWR_n	Current amplitude	Spacing (from origin)	SWR_n	Current amplitude	Spacing (from origin)	SWR_n
1	0.9115	0.2	1.3251	0.72996	0.2	1.2300	0.6877	0.2	1.0780
2	1	0.6491	1.3504	0.9782	0.7361	1.0355	0.8460	0.7508	1.1180
3	0.99471	1.0929	1.0498	0.92068	1.3615	1.0098	0.6175	1.2182	1.0960
4	1	1.6087	1.0852	0.76413	1.9643	1.0813	0.8148	1.764	1.1094
5	0.68544	2.1202	1.0762	0.56663	2.5637	1.0084	0.5618	2.3076	1.0582
6	0.60002	2.5693	1.1890	0.52112	3.1383	1.0558	0.6225	2.8566	1.0796
7	0.63223	3.0877	1.0484	0.39552	3.7767	1.0083	0.6337	3.5963	1.1119
8	0.32488	3.6531	1.0916	0.26575	4.4246	1.1370	0.4458	4.3427	1.0642
9	0.17528	4.0931	1.3639	0.21856	5.0237	1.0030	0.3183	5.1261	1.1218
10	0.14734	4.5331	1.3589	0.13952	5.7035	1.1640	0.1233	5.8694	1.0690

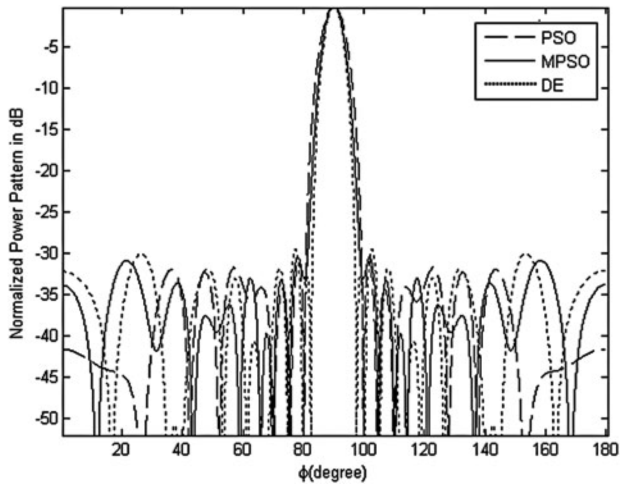

Fig. 4. Normalized absolute power patterns in dB for parallel array.

fine search within a small neighborhood of the suspected optima.

If $\Delta J_i > 2.4$ F_i is selected obeying the following relation:

$$F_i = 0.8*(1 - e^{-\Delta J_i}). \tag{18}$$

Equation (18) results a greater value of F_i that ultimately boosts the exploration ability of the algorithm within the specified search volume.

B) Crossover

To increase the potential diversity of the population, crossover operation is introduced. In crossover the donor vector exchanges its components with the target vector $\vec{X}_{i,G}$ to obtain the trial vector $U_{i,G}$:

$$u_{j,i,G} = \begin{cases} v_{j,i,G} & \text{if } (rand_{i,j}(0,1) \le Cr_i \text{ or } j = j_{rand}), \\ x_{j,i,G} & \text{otherwise.} \end{cases} \tag{19}$$

$rand_{i,j}$ (0,1) $\in [0, 1]$ is a uniformly distributed random number and Cr_i is a constant called crossover rate. $j_{rand} \in [1,2,\ldots, D]$ is a randomly chosen index, which ensures that $\vec{U}_{i,G}$ gets at least one parameter from $V_{i,G}$ and does not become exact replica of the parent vector. The number of parameters inherited from the donor has a (nearly) binomial distribution.

The parameter Cr_i is updated automatically depending on the value of the cost function produced by the donor vector. If the donor vector yields a cost value lesser than the minimum value attained by that population, Cr_i value is chosen high to pass more genetic information into the trial vector otherwise it remains small. Cr_i is determined accordingly:

$$Cr_i = \begin{cases} Cr_{const} & \text{if } J(\vec{V}_i) \le J(\vec{X}_{best}), \\ Cr_{min} + \dfrac{Cr_{max}}{1 + \Delta J_i} & \text{otherwise,} \end{cases} \tag{20}$$

where $\Delta J_i = |J(\vec{V}_i) - J(\vec{X}_{best})|$, $Cr_{min} = 0.1$, $Cr_{max} = 0.7$, and $Cr_{const} = 0.95$.

C) Selection

Selection is introduced to decide whether the target vector survives to the next generation. The trial vector is compared with the target vector using the following criterion:

$$\vec{X}_{i,G+1} = \begin{cases} \vec{U}_{i,G} & \text{if } J(\vec{U}_{i,G}) \le J(\vec{X}_{i,G}), \\ \vec{X}_{i,G} & \text{if } J(\vec{U}_{i,G}) > J(\vec{X}_{i,G}). \end{cases} \tag{21}$$

If the new trial vector yields an equal or lower cost value, it substitutes the corresponding target vector in the next generation. Otherwise the target is retained in the population.

Table 3. Desired and obtained result for collinear array.

Algorithms	PSO	Modified PSO	Modified DE
SLL (in dB)	29.5	29.9	30.01
SWR	1.6844	1.7103	1.5896
DRR	6.89	6.53	6.9979

V. RESULTS AND DISCUSSIONS

Two linear arrays consist of 20 dipoles of length 0.5λ and radius 0.005λ are considered. In the first case, synthesis of parallel array

Table 4. Current amplitude, spacing, and *SWR* value for collinear array.

n	PSO			Modified PSO			Modified DE		
	Current amplitude	Spacing (from $z = 0$)	SWR_n	Current amplitude	Spacing (from $z = 0$)	SWR_n	Current amplitude	Spacing (from $z = 0$)	SWR_n
1	0.9061	0.45	1.6407	1	0.45	1.6556	0.9944	0.45	1.5896
2	0.869	1.2547	1.6038	0.9309	1.2273	1.6538	0.9205	1.3347	1.4734
3	0.8408	2.0099	1.629	0.7653	1.9532	1.7103	0.9257	2.2054	1.4780
4	0.6921	2.8241	1.5759	0.7957	2.7314	1.6743	0.8391	3.1074	1.4831
5	0.6175	3.6585	1.5813	0.674	3.4937	1.6272	0.6722	4.0589	1.4690
6	0.4967	4.451	1.5779	0.6188	4.2902	1.5752	0.6286	4.8711	1.5222
7	0.4073	5.2828	1.563	0.4796	5.1243	1.5197	0.4393	5.798	1.4476
8	0.3273	6.0977	1.6109	0.3608	6.0141	1.4876	0.3985	6.6836	1.4871
9	0.1325	6.8622	1.6423	0.278	6.9029	1.5135	0.3513	7.6222	1.5328
10	0.1313	7.6753	1.6844	0.153	7.7221	1.581	0.1421	8.5669	1.4156

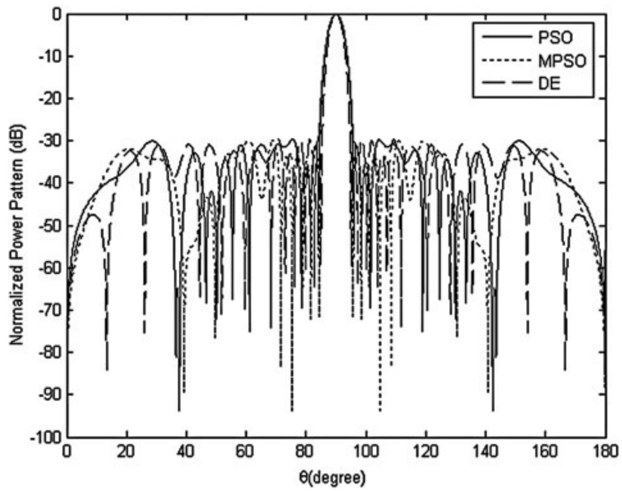

Fig. 5. Normalized absolute power patterns in dB for collinear array.

has been illustrated. To generate pencil beam, excitation current amplitude and inter-element spacing is varied in the range 0–1 and 0.4–0.8 wavelengths, respectively. Phase is kept at 0°. Desired *DRR* value of amplitude distribution is prefixed at 7.

Because of symmetry, only 10 amplitudes and 9 inter-element distances are to be optimized. Algorithms are designed to generate a vector of 19 real values between 0 and 1. The first 10 values of the vector are mapped and scaled to the desired amplitude weight (0–1) range and the last 9 values are mapped and scaled to the desired intermediate spacing weight (0.4–0.8) range. It is assumed that the first element is placed at a prefixed distance (0.2) from the origin.

All three algorithms (PSO, modified PSO, and modified DE) are run for 100 iterations to compute amplitude weight and element spacing of each element. Table 1 shows the desired and obtained values of *SLL*, *SWR*, and *DRR* of the array in absence of ground plane. There is a good agreement between desired and synthesized results using all the algorithms. Table 2 presents the excitation, spacing from origin, and *SWR* value of each array element. Because of symmetry, remaining 10 elements are also be excited with the same parameters. Using the proposed technique *SLL* value can be further lowered along with a very good *SWR*.

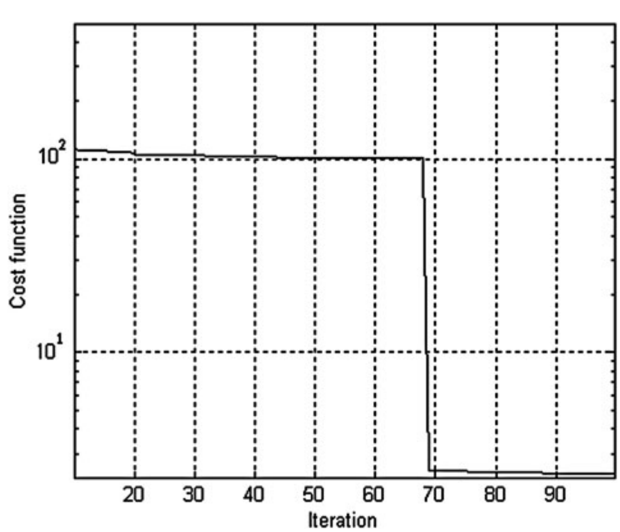

Fig. 6. Convergences curve of the cost function using modified PSO.

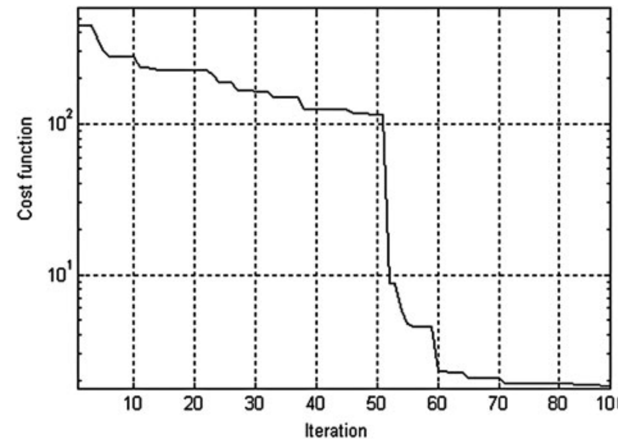

Fig. 7. Convergences curve of the cost function using DE.

Radiation patterns using the optimized data are plotted above. Figure 4 shows the normalized absolute power patterns (pencil-beam) in dB for nonuniformly spaced parallel array. Patterns are shown in phi space ranging from 0 to 180°.

Next, the collinear array consists of 20 wire dipoles is analyzed. To optimize the array, excitation amplitude is set within the interval [0,1] and spacing is varied in the range 0.7–1.1 wavelength. Inter-element distance is measured from center to center where first element is placed at a prefixed distance (0.45) from the origin. Algorithms are run for 100 iterations and results are presented in Tables 3 and 4.

Ten elements are considered for optimization because of symmetry like before. Remaining 10 elements are excited with the same excitation and geometry.

Radiation patterns using the optimized data are plotted above. Figure 5 shows the normalized absolute power patterns (pencil-beam) in dB for nonuniformly spaced collinear array. Patterns are shown in θ space ranging from 0 to 180°.

Lowering the desired value of *SLL* it can be lowered further for the same beamwidth and same number of array elements. Parallel configuration is more effective to compensate mutual coupling effect than collinear one.

All three algorithms are run independently to justify their effectiveness. Convergence curves of the two best performing algorithms are presented in Figs 6 and 7. It is seen that DE algorithm converges faster and proved most effective to yield the global minimum compared to its other two competitors. It produces similar results over repeated runs, which is an indication of its robustness. Among the remaining two algorithms modified particle swarm optimization performed better. Use of velocity clipping technique in MPSO significantly improves its performances compared to standard PSO. Although the modifications introduced in PSO and DE slow down the execution speed of the algorithms compared to standard PSO.

Standard PSO, modified PSO, and DE are compared in a statistical manner. Since the distribution of the best objective function values do not follow a normal distribution, the Wilcoxon rank sum test was used to compare the objective function mean values, standard deviations, and *P* values of each algorithm [26] and those values are listed in table 5. Each algorithm was executed for 50 times and the best result for each run is considered.

DE yields statistically better results compared to other two optimization approaches. *P* values obtained through the rank

Table 5. Mean cost values, standard deviations, and *P* values.

Parallel array				**Collinear array**			
Algorithms	**PSO**	**Modified PSO**	**Modified DE**	**Algorithms**	**PSO**	**Modified PSO**	**Modified DE**
Mean cost function	1.41416	1.27981	1.16559	Mean cost function	1.8913	1.7631	1.604
Standard deviation	0.017374	0.001127	0.000504	Standard deviation	0.001538	0.001284	0.001046
P value	7.7734e − 10	4.8934e − 7	NA	*P* value	2.7482e − 11	6.88824e − 8	NA

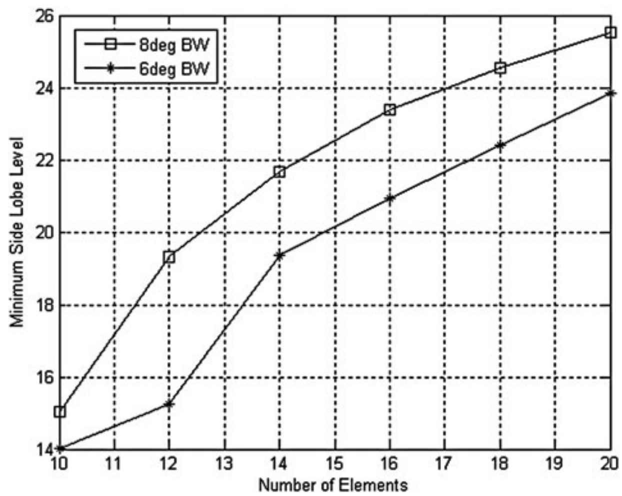

Fig. 8. Minimum achievable *SLL* versus *N* for different BW using NULSA.

sum test between the best algorithm and each of the contestants are presented. Here NA stands for "not applicable" and it occurred for the best performing algorithm. As the *P* values calculated for both the cases are less than 0.05 (5% significance level), null hypotheses is rejected and the results produced by DE are considered statistically significant.

The article also studied the behavior of the trade-off curve between different array objectives. Figure 8 shows the tradeoff between minimum achievable *SLL* and number of elements for nonuniformly spaced parallel array. Patterns are simulated for different pre-specified beamwidths (6 and 8°). As the 3 dB beamwidth increases the tradeoff between *SLL* and number of antenna elements *N* becomes improved.

Figure 9 illustrates the tradeoff between *VSWR* and average element spacing for the nonuniformly spaced array. It is seen that minimum bearable *VSWR* is obtained when the average element spacing is set within an optimum range relative to the element excitation.

Finally Fig. 10 shows the improvement in tradeoff between minimum achievable *SLL* and average element spacing for NULSA compared to ESLA. In ESLA grating lobe appears for $d > 0.95\lambda$. However NULSA produces a reasonable *SLL* beyond this region. All the experiments are carried out assuming that the excitations of individual element are equal and all unity.

The main objective of this paper is to illustrate the importance of the evolutionary multi-objective optimization techniques in the design of antenna arrays. The algorithms are applied to improve the radiation characteristics and compensate the coupling effect for nonuniformly spaced parallel and collinear array. Trade-off solutions presented in this section will make the array more suitable for using in wireless sensor networks. Proposed concept can be further extended in designing sparse array where maximum spatial resolution is needed incorporating the concept of Golomb ruler [27–29]. Coupling effect of these minimum redundancy arrays can be reduced by suitably computing the element excitation weight.

V. CONCLUSION

The use of optimization techniques based on PSO and DE in the synthesis of nonuniformly spaced linear array of half wave parallel and collinear dipoles is discussed here. Numerical results show that DE converges faster and with more certainty compared to other optimization approaches. It yields minimum fitness value and offers better statistical accuracy. Both the array alignments are analyzed in order to obtain lower value of *SLL* and *SWR*. In both the cases unknown

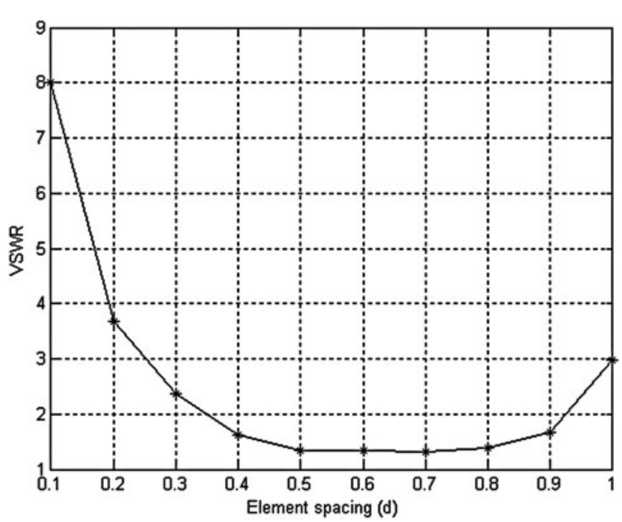

Fig. 9. *VSWR* for different average element spacing for NULSA.

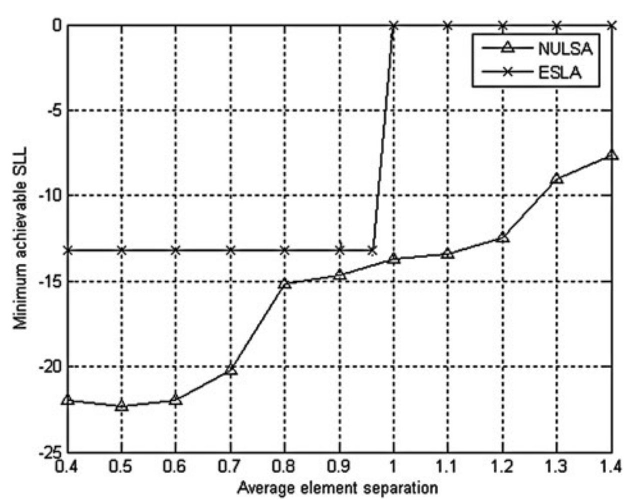

Fig. 10. Minimum achievable *SLL* versus average element spacing for ESLA and NULSA.

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excitation and unknown spacing are used and phase is prefixed at 0°. Result shows that the parallel array arrangement is more effective to minimize *SWR*. The excitation and geometry both are symmetric in nature that greatly simplifies the feed network. Mutual impedance matrix is calculated using induced EMF method. Fixing the *DRR* of excitation current amplitude to a lower value with little compromise on the design specifications further reduces the effect of coupling. It is seen that array performance is significantly enhanced by perturbing the inter-element spacing. The work also provides a trade-off solution between *SLL-BW* and number of array elements. *SWR* value is estimated for different average element separation for unequally spaced array. Finally, the work presents a comparison of *SLL* performances of ELSA and NULSA for same number of elements. Application of more powerful global search algorithm in antenna synthesis can be a topic of future research. Proposed technique is capable of optimizing more complex geometries and therefore is suitable for many applications in communication area.

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