

# Scaling and mechanism of the propagation speed of the upstream turbulent front in pipe flow

Haoyang Wu<sup>1</sup> and Baofang Song<sup>2,†</sup>

<sup>1</sup>Center for Applied Mathematics, Tianjin University, Tianjin 300072, PR China

<sup>2</sup>State Key Laboratory of Turbulence and Complex Systems, College of Engineering, Peking University, Beijing 100871, PR China

(Received 28 September 2023; revised 11 November 2023; accepted 25 November 2023)

The scaling and mechanism of the propagation speed of turbulent fronts in pipe flow with the Reynolds number has been a long-standing problem in the past decades. Here, we derive an explicit scaling law for the upstream front speed, which approaches a power-law scaling at high Reynolds numbers, and we explain the underlying mechanism. Our data show that the average wall distance of low-speed streaks at the tip of the upstream front, where transition occurs, appears to be constant in local wall units in the wide bulk-Reynolds-number range investigated, between 5000 and 60 000. By further assuming that the axial propagation of velocity fluctuations at the front tip, resulting from streak instabilities, is dominated by the advection of the local mean flow, the front speed can be derived as an explicit function of the Reynolds number. The derived formula agrees well with the speed measured by front tracking. Our finding reveals a relationship between the structure and speed of a front, which enables a close approximation to be obtained of the front speed based on a single velocity field without having to track the front over time.

**Key words:** shear-flow instability, transition to turbulence, pipe flow

## 1. Introduction

Front formation and propagation are important processes in nonlinear systems involving reaction, diffusion and advection, such as combustion, neural systems, epidemics and turbulent flows. In pipe flow, a localized turbulent region expands via the propagation of the upstream front (UF) and downstream front (DF) into the laminar region, where the flow is stable to infinitesimal perturbations (Meseguer & Trefethen 2003; Chen, Wei & Zhang 2022*b*) – see [figure 1](#) for an illustration. The front speeds determine the expansion rate of the turbulent region and consequently the wall friction. Therefore, front speed is an important characteristic of pipe-flow turbulence and, together with the front structure, has

† Email address for correspondence: [baofang.song@pku.edu.cn](mailto:baofang.song@pku.edu.cn)

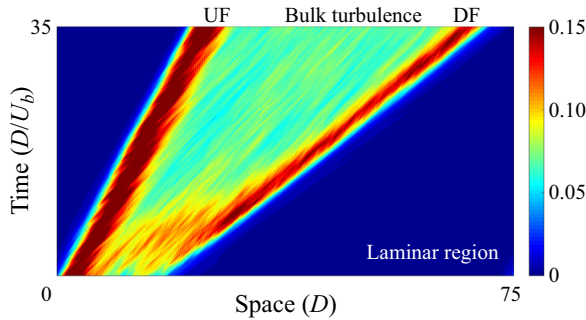


Figure 1. The expansion of turbulence at  $Re = 5000$ . Contours of the magnitude of transverse velocity (averaged over the pipe cross-section) are plotted in the space (pipe axis) and time plane. The length unit in space is the pipe diameter  $D$  and the time unit is  $D/U_b$ , with  $U_b$  being the bulk speed of the flow. The main stream is from left to right, and time is vertically up. Red shows highly turbulent and blue shows laminar regions. The different slopes of the two red strips (fronts) indicate different front speeds, i.e. the turbulent region expands.

been the subject of many studies in the past six decades (Lindgren 1957, 1969; Wygnanski & Champagne 1973; Darbyshire & Mullin 1995; Shan *et al.* 1999; Durst & Ünsal 2006; van Doorne & Westerweel 2008; Nishi *et al.* 2008; Duguet, Willis & Kerswell 2010; Holzner *et al.* 2013; Barkley *et al.* 2015; Barkley 2016; Song *et al.* 2017; Rinaldi, Canton & Schlatter 2019; Chen, Xu & Song 2022a; Wang & Goldenfeld 2022). Yet the mechanism that determines the front speed and the scaling of the speed with the Reynolds number ( $Re$ ) remains largely unclear to date.

Most relevant studies have focused on the narrow regime of transition from localized puffs to expanding turbulent states, i.e. slugs, at relatively low Reynolds numbers of  $O(10^3)$ . Owing to difficulties of measuring front speed at high  $Re$  (Chen *et al.* 2022a), especially for the DF, only Wygnanski & Champagne (1973) and Chen *et al.* (2022a) considered higher  $Re$  at  $O(10^4)$ . Reasonable agreement has been obtained among existing measurements for the UF speed, showing a monotonically decreasing trend as  $Re$  increases – see Chen *et al.* (2022a) and Avila, Barkley & Hof (2023) for the latest literature reviews. However, Wygnanski & Champagne (1973) and Chen *et al.* (2022a) reported opposite speed trends above  $Re \simeq 10\,000$  for the DF. By direct numerical simulations (DNS) up to  $Re = 10^5$ , Chen *et al.* (2022a) reported fits  $\tilde{c}_{UF} = 0.024 + (Re/1936)^{-0.528}$  for the UF and  $\tilde{c}_{DF} = 1.971 - (Re/1925)^{-0.825}$  for the DF, and argued for monotonic trends of the two front speeds with  $Re$  at high Reynolds numbers. Although fitting the measured speeds well, these are pure data fits with a prescribed form, but the underlying mechanism was left unexplained. In these fits and throughout this paper, the reference length and velocity are the pipe diameter  $D$  and bulk speed  $U_b$ , respectively. The Reynolds number is defined as  $Re = U_b D / \nu$ , where  $\nu$  is the kinematic viscosity. We will consider only the UF speed in this work.

Other than direct measurements of the front speed by tracking the front along the pipe axis, as in most studies, a few theoretical attempts have been made. For example, based on an energy flux analysis of the front region, without considering the dynamic transition process, Lindgren (1969) predicted an asymptotic speed of 0.69 as  $Re \rightarrow \infty$ . However, this prediction was questioned by both experimental (Wygnanski & Champagne 1973) and numerical (Chen *et al.* 2022a) measurements, which showed much lower speeds than the asymptotic prediction of Lindgren (1969). Barkley *et al.* (2015) and Barkley (2016) used a theoretical model to investigate the front in the transitional regime, which captures

the large-scale dynamics of the fronts successfully. The asymptotic analysis explains the front speed as a combination of the advection of the bulk turbulence and propagation with respect to the bulk turbulence (see [figure 1](#)). However, as a generic model for one-dimensional reaction–diffusion–advection systems, the model does not account for the three-dimensionality of the transition for pipe flow and does not give an explicit relationship between the front speed and  $Re$ . Besides, the  $Re$  range considered was too narrow for a scaling with  $Re$  to be established. Similar problems apply to the stochastic prey–predator model recently proposed by Wang & Goldenfeld ([2022](#)), which nevertheless can reproduce the basic phenomenology of the front of pipe flow in the transitional regime. In a word, there is still a big gap between experiments and theory.

In this paper, our goal is to derive a scaling law of the front speed by accounting for the transition at the front.

## 2. Results

Our starting point is the observation of our earlier work (Chen *et al.* [2022a](#)) that transition to turbulence occurs continuously at the tip of a front (see the supplementary movie available at <https://doi.org/10.1017/jfm.2023.1015>), maintaining a characteristic propagation speed and a characteristic shape of the front against the distortion of the mean shear. For a UF, the front tip refers to its most upstream point. We propose that an evaluation of the radial position of the transition point at the front tip is crucial for determining the front speed. The questions are how to quantitatively determine this position and how to relate it to the front speed.

Before presenting our results, the set-up of the flow system and some notation should be explained. The flow is incompressible and constant-mass-flux-driven, and is solved in cylindrical coordinates, where  $(r, \theta, z)$  denote radial, azimuthal and axial coordinates, respectively, and  $u_r$ ,  $u_\theta$  and  $u_z$  denote the respective fluctuating velocity components in the three directions. The axial length of the pipe domain is  $17.5D$  for  $Re \leq 10\,000$  and  $5D$  for higher  $Re$ . Readers are referred to Chen *et al.* ([2022a](#)) for more details about the simulation of a front in a short periodic pipe.

We first show the flow structure at the tip of the UF. In [figure 2\(a\)](#), contours of the magnitude of transverse velocity fluctuation in the  $z$ – $r$  plane show that turbulence is concentrated near the wall and gradually spreads out towards the pipe centre while going downstream (see the supplementary movie for more details). [Figure 2\(b\)](#) shows the distribution of maximum  $|u_z|$  and  $|u_r|$  in the  $r$ – $\theta$  cross-section along the pipe axis. These curves also reflect that the flow is non-turbulent on the upstream side of the front tip and turbulent on the downstream side. [Figure 2\(c\)](#) shows that the flow features nearly straight and streamwise-elongated low-speed (blue) and high-speed (red) streaks on the upstream side, whereas the flow structure is less regular on the downstream side of the front tip. These plots (especially [figure 2b](#)) suggest that the tip of the UF should sit in the rough interval  $z \in (3, 3.5)$ . [Figure 2\(d\)](#) shows the contours of  $u_z$  in the  $r$ – $\theta$  plane at  $z = 3.18$ . Alternating high-speed (red spots) and low-speed streaks (blue spots) can be seen close to the wall, while the flow is laminar in the core region of the pipe.

Although a quantitative description of the transition mechanism at the UF tip is still lacking, the consensus seems to be that the transition is caused by instabilities of the low-speed streaks (Shimizu & Kida [2009](#); Duguet *et al.* [2010](#); Hof *et al.* [2010](#)), which may consist of more fundamental substructures according to Jiang *et al.* ([2020a,b](#)). The instabilities here possibly coincide with those (either modal or non-modal) proposed for explaining either subcritical transition or the self-sustaining mechanism of shear-flow

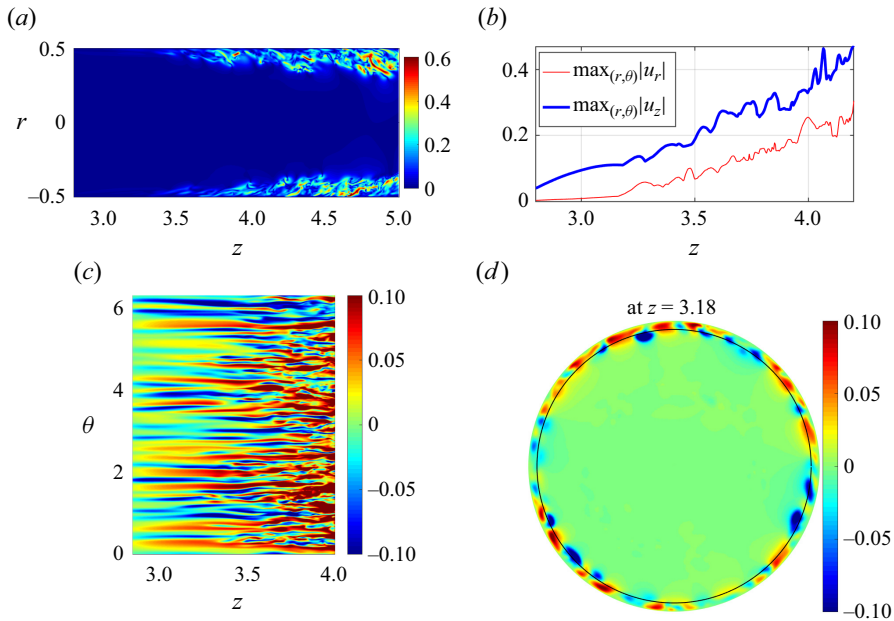


Figure 2. Structure of a UF. (a) The tip part (the most upstream part) of a UF in a  $z$ - $r$  plane at  $Re = 40\,000$ . Contours of the transverse velocity  $(u_r^2 + u_\theta^2)^{1/2}$  are plotted in the  $z$ - $r$  cross-section. The main flow is from left to right, the blue shows regions of low velocity fluctuation and the red shows regions of high velocity fluctuation. (b) The maximum of  $|u_r|$  and  $|u_z|$  in the  $r$ - $\theta$  cross-section plotted along the pipe axis. (c) Contours of  $u_z$  in the  $z$ - $\theta$  plane at  $r = 0.47$ , which show low-speed (blue) and high-speed (red) streaks. (d) Contours of  $u_z$  in the  $r$ - $\theta$  cross-section at  $z = 3.18$ . The circle, at  $r = 0.47$ , shows the radial position of the  $z$ - $\theta$  plane in panel (c). This circle approximately shows the average position of low-speed streaks (blue spots) at this axial position.

turbulence (Swearingen & Blackwelder 1987; Hamilton, Kim & Waleffe 1995; Zikanov 1996; Waleffe 1997; Schoppa & Hussain 1998, 2002; Meseguer 2003). In the following, we will establish a connection between the low-speed streaks and the front speed based on a few hypotheses, the first of which reads as follows:

- H1 *The wall distance of the transition point at the front tip, in local wall units, is independent of the Reynolds number statistically.*

This should be reasonable because the transition takes place near the wall, so that the wall distance of the transition point can be expected to scale with the wall length, which is the only length scale that can be derived from viscosity and wall shear. We will verify this hypothesis by measuring the wall distance of low-speed streaks as a proxy of that of the transition point.

At a turbulent front, the flow is axially developing, so that low-speed streaks are not parallel to the pipe wall but oblique, i.e. the wall distance varies along a streak. Figure 3(a,b) suggest that low-speed streaks are gradually lifted up away from the wall while going downstream. To show this variation more quantitatively, we take the following approach to determine the wall distance of low-speed streaks at a given axial location. In an  $r$ - $\theta$  cross-section, low-speed streaks can be detected by setting a proper threshold in  $u_z$ , and regions enclosed by contour lines of the specified threshold can be considered as (the cross-sections of) low-speed streaks – see the magenta contour lines in figure 3(a) with a threshold  $-0.04$ . (See Appendix A for a discussion on the threshold selection.) Then, the

## Speed of the upstream turbulent front in pipe flow

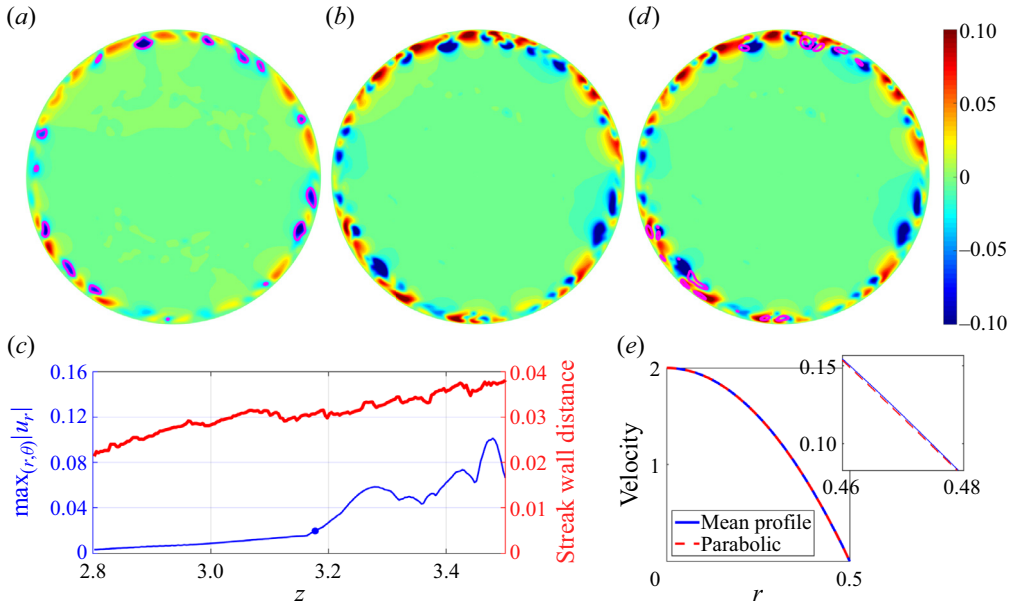


Figure 3. (a–d) Axial variation of the low-speed streaks and (e) mean velocity profiles near the front tip. The flow field is the same as that shown in figure 2. (a) Contours of  $u_z$  at  $z = 3.0$ . The contour level of  $-0.04$  is plotted in magenta to highlight the low-speed streaks. (b) Contours at  $z = 3.3$ . (c) The variation of the average wall distance of streaks (thick red) and  $\max_{(r,\theta)} |u_r|$  (thin blue) along the pipe axis. (d) Also at  $z = 3.3$ , and the contour level  $0.025$  of the transverse velocity  $(u_r^2 + u_\theta^2)^{1/2}$  is plotted in magenta to highlight the transverse velocity fluctuations. (e) The mean velocity profile at  $z = 3.3$ , i.e.  $U(r) = \langle u_x(r, \theta, 3.3) \rangle_\theta$ , where  $\langle \cdot \rangle_\theta$  means the average in the azimuthal direction. The parabolic profile is plotted as a broken line for comparison. The small window inset highlights the deviation between the two in the near-wall region.

nominal wall distance of a streak can be defined as the wall distance of the minimum of  $u_z$  within the streak. The average wall distance is calculated as the arithmetic mean of the wall distances of all the streaks detected in this pipe cross-section.

Figure 3(c) shows more quantitatively that the wall distance of streaks increases on going downstream. Therefore, it is necessary to determine the axial position of the front tip for finally determining the wall distance of the streaks at the front tip. We use  $\max_{(r,\theta)} |u_r|$ , which is a function of  $z$ , as an indicator of the local flow state. This curve is smooth and slowly varying in the laminar region and wiggles around in the turbulent region – see figures 2(b) and 3(c). The axial location of the front tip can be estimated by the position separating the smooth and wiggling parts of the curve of  $\max_{r,\theta} |u_r|$ . We use an algorithm that detects abrupt changes of a curve for this purpose, which is built-in as the function *findchangepts* in MATLAB R2018a (see Appendix B for a brief description of the algorithm). The blue dot in figure 3(c) shows the separating point determined using this algorithm.

Figure 4(a) shows the average wall distance of streaks  $y = 0.5 - r$  at the front tip in outer units. The larger  $Re$ , the smaller  $y$ , which can be expected. Figure 4(b) shows  $y^+$ , the wall distance in local wall length units  $\sqrt{\nu/\tau_w}$ , where  $\tau_w$  is the local wall shear stress. Considering that the azimuthally averaged velocity profiles at these axial locations are nearly parabolic (see figure 3e),  $\tau_w$  is simply approximated by the value of the parabolic profile. It appears that  $y^+$  stays nearly constant in the wide  $Re$  range considered, which

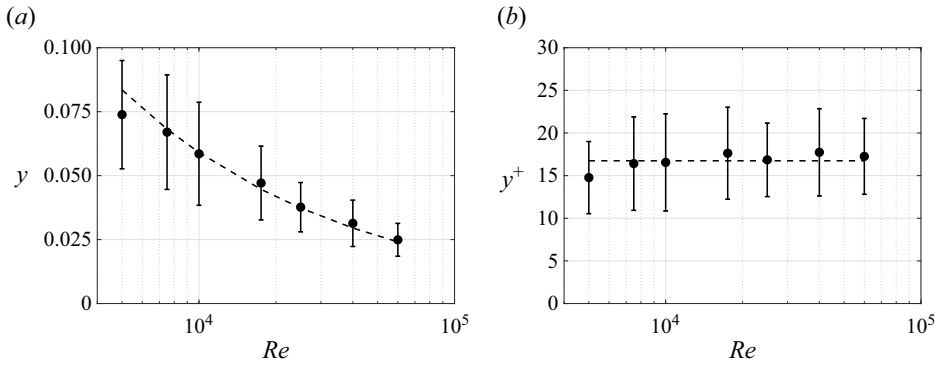


Figure 4. The average wall distance of the low-speed streaks at the tip of the UF. (a) The distance in the outer length units. (b) The distance in the local wall units. At each  $Re$ , about 10–20 velocity snapshots are collected, giving approximately 100–200 low-speed streaks for the statistics. The standard deviation is plotted as the error bars. The dashed lines are (2.1) taking  $A = 16.7$ , which is the average of  $y^+$  over all  $Re$  shown in panel (b).

supports our hypothesis H1 given the crucial role that low-speed streaks play in the transition.

Assuming this  $Re$  independence, we derive the scaling law for the speed of the UF as follows. First, take the wall distance of the transition point at the front tip to be  $y_F^+ = A$ , where  $A$  is independent of  $Re$ . Then, in outer units, we have

$$y_F = y_F^+ / Re_\tau = A / Re_\tau. \tag{2.1}$$

The local mean flow speed, i.e. the azimuthally averaged streamwise velocity at the radial position of the transition point, in outer units, can be approximated by the local laminar value

$$U(y_F) \approx 2 - 8(0.5 - y_F)^2 = 8(A / Re_\tau - A^2 / Re_\tau^2), \tag{2.2}$$

given that the mean velocity profile is nearly parabolic at the front tip. As the relationship between  $Re$  and  $Re_\tau$  is

$$Re_\tau = \sqrt{-\left. \frac{dU(r)}{dr} \right|_{r=0.5}} Re = 2\sqrt{2Re} \tag{2.3}$$

for a parabolic velocity profile, we have

$$U(y_F) \approx 2\sqrt{2} A Re^{-0.5} - A^2 Re^{-1}. \tag{2.4}$$

Now we come to our further hypotheses:

H2 *The wall distance of velocity perturbations at the front tip, resulting from streak instabilities, can be closely approximated by the wall distance of the streaks.*

H3 *The front speed is determined by the axial propagation speed of these velocity perturbations, which approximately equals the local mean flow speed.*

H2 should be reasonable, especially when perturbations appear at the flanks of the streaks. In fact, the data seem to support this hypothesis – see figure 3(d) where most of the strong perturbation region, enclosed by magenta contour lines, seems to be at the flanks of the low-speed streaks. H3 is based on our presumption that streak instabilities generate streamwise vortices, which further generate streaks while being advected downstream,

## Speed of the upstream turbulent front in pipe flow

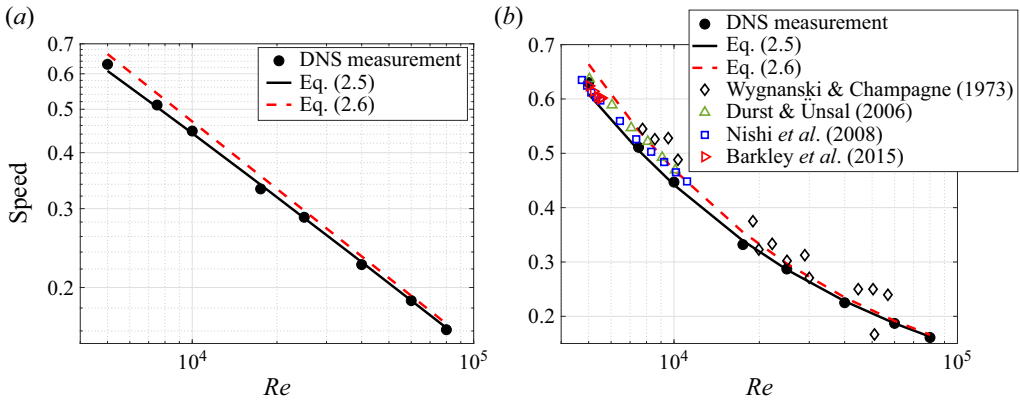


Figure 5. (a) Comparison of derived and measured front speeds. The circles show the speeds measured by front tracking using DNS (Chen *et al.* 2022a). The solid black line shows the approximation (2.5) with  $A = 16.7$ , which is the average of the  $y^+$  (black circles in figure 4b) over all  $Re$  values. The dashed red line shows the approximation (2.6) with  $A = 16.7$  also. (b) The same plot in linear scale for the speed where some data sets from the literature falling in this  $Re$  range are also included.

seeding new transition and closing the self-sustaining cycle of the dynamics at the front tip. Therefore, the propagation speed of these vortices likely determines that of the front tip and consequently the front speed. The propagation of vortical structures, at least in fully developed wall turbulence above the viscous sublayer, was shown to be dominated by the advection of the local mean flow (Wu & Moin 2008; Del Álamo & Jiménez 2009; Pei *et al.* 2012).

Following these hypotheses, we finally have an approximation of the front speed as

$$c_{UF} \approx U(y_F) \approx 2\sqrt{2}A Re^{-0.5} - A^2 Re^{-1}, \quad (2.5)$$

and an asymptotic approximation at large  $Re$ ,

$$c_{UF} \approx U(y_F) \approx 2\sqrt{2}A Re^{-0.5}, \quad (2.6)$$

where  $A$  can be approximated by the wall distance of low-speed streaks at the front tip.

Figure 5 concludes the speed measurements and our derivation. The filled circles are the DNS data from Chen *et al.* (2022a) (up to  $Re = 60\,000$ ) and the open symbols show the literature data in the  $Re$  range investigated here. In order to show that the formula is predictive, DNS at  $Re = 80\,000$  is performed here, and the front speed is measured by front tracking and plotted as filled circles also. The black solid line shows our derivation (2.5) by setting  $A = 16.7$ , which is the average of  $y^+$  of streaks at all  $Re$  values as shown in figure 4(b). The relative error of the prediction is on the level of a few per cent compared to the DNS measurement. The red dashed line shows the asymptotic speed (2.6) with the same  $A$ . Some former experimental measurements are also included in the figure. It should be noted that this formula can also be considered as a model for the front speed with only one parameter  $A$ , which has a physical meaning and, more precisely, should be interpreted as the wall distance of the transition point at the front tip. This formula can be used for other  $Re$  after calibrating the parameter  $A$  at one  $Re$  with the measured front speed.

Now we revisit the fit  $\tilde{c}_{UF} = 0.024 + (Re/1936)^{-0.528}$  given by Chen *et al.* (2022a). This was obtained by assuming a form of  $a + bRe^\beta$  without any explanation of the underlying physics. In other words, this form is not unique. Besides, the small constant 0.024 implies that the front speed would not approach zero as  $Re$  approaches infinity, which

was unexplained and seems counter-intuitive. It is probably just a result of measurement errors and the specific prescribed form of the fit. In contrast, our derivation (2.5) makes no assumption on the specific form of the formula. It follows naturally from the dynamics we observed at the front tip, with a few hypothetical but reasonable assumptions of the physics.

Our derivation (2.5) may suffer larger errors at lower  $Re$ . The low-speed streaks would be larger in transverse size at lower  $Re$ ; therefore, the position of a streak estimated simply by the position of the minimum of  $u_z$  in each streak becomes less representative. Besides, the streak position may not exactly coincide with the position of velocity fluctuations resulting from the streak instability. But these positions are close to each other at sufficiently high  $Re$ , so that our derivation will be more accurate.

As for the DF, the front speed is probably determined by the advection of the local mean flow at the front tip also. However, transition to turbulence occurs close to the pipe centre (Chen *et al.* 2022a) and the transition may not be triggered by streak instabilities as known for near-wall turbulence. Therefore, the location of the transition point may not scale with the wall length units, and cannot be explicitly related to  $Re$  as shown here for the UF at the present. This problem has to be left for future studies.

### 3. Conclusions

In summary, the speed of the UF of pipe-flow turbulence was derived as an explicit function of  $Re$  based on the dynamics at the front tip. To our knowledge, this is the first such work since the seminal measurements and theoretical analysis of Lindgren (1957, 1969) about six decades ago. The agreement with speed measurements (see figure 5) suggests that the mechanism proposed here captures the core of the physics, i.e. the front speed is largely determined by the advection of velocity fluctuations by the local mean flow at the front tip where transition takes place. This mechanism may also apply to turbulent fronts in other shear flows where turbulence propagates into the subcritical laminar flow region. Although the local mean flow is different in higher dimensions such as planar shear flows (see e.g. Duguet & Schlatter 2013; Tao, Eckhardt & Xiong 2018; Tuckerman, Chantry & Barkley 2020; Klotz, Pavlenko & Wesfreid 2021), our work will be helpful for elucidating the physics of front propagation in those flows.

**Supplementary movie.** A supplementary movie is available at <https://doi.org/10.1017/jfm.2023.1015>.

**Funding.** The authors acknowledge financial support from the National Natural Science Foundation of China under grant numbers 12272264 and 91852105. The work is also supported by The Fundamental Research Funds for the Central Universities, Peking University. Simulations were performed on Tianhe-2 at the National Supercomputer Centre in Guangzhou and on Tianhe-1(A) at the National Supercomputer Centre in Tianjin.

**Declaration of interests.** The authors report no conflict of interest.

**Author ORCID.**

Baofang Song <https://orcid.org/0000-0003-4469-8781>.

### Appendix A. Threshold for detecting streaks

The results presented in the main text take the threshold of  $-0.04$ . Here we explain the selection of this value. Figure 6 shows the contour levels of  $-0.02$ ,  $-0.04$  and  $-0.06$  in the  $r$ - $\theta$  cross-section at  $z = 3.18$ , which is the same position as shown in figure 2(d) of the main text, plotted as magenta lines. It can be seen that  $-0.02$  cannot very well separate adjacent streaks, whereas  $-0.06$  may miss out many streaks. We checked multiple velocity snapshots and Reynolds numbers and found this to be often the case. The threshold  $-0.04$



## Speed of the upstream turbulent front in pipe flow

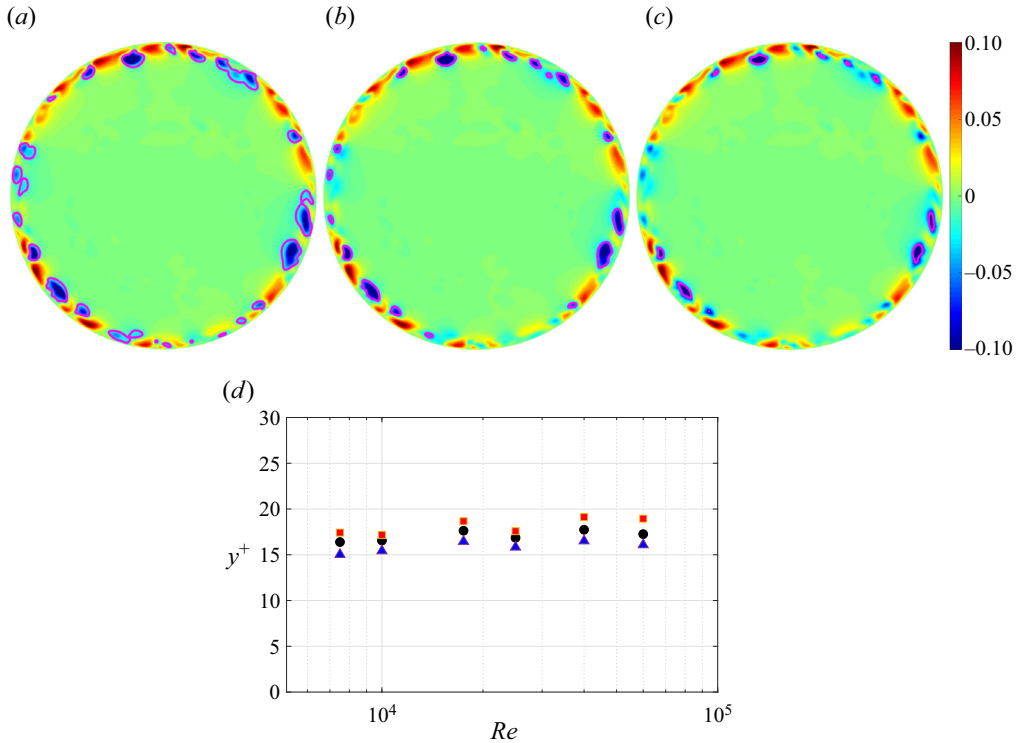


Figure 6. (a–c) Thresholds for detecting low-speed streaks:  $-0.02$ ,  $-0.04$  and  $-0.06$ , respectively. Contours of streamwise velocity fluctuation in the  $r-\theta$  cross-section for  $Re = 40\,000$ , as also shown in figure 2 of the main text. Contour levels of  $-0.02$ ,  $-0.04$  and  $-0.06$  are plotted as magenta lines. (d) The average wall distance of low-speed streaks in wall units determined with thresholds of  $-0.02$  (blue triangles),  $-0.04$  (black circles) and  $-0.06$  (red squares) in  $u_z$  for detecting the streaks.

is a reasonable choice because, in most cases, it separates streaks well and is able to detect most of the low-speed streaks.

It can be expected that the value of this threshold will affect the average position of the streaks. A higher threshold may drop out weaker streaks and only retain stronger streaks. Stronger streaks are often more lifted up away from the wall (as can be seen in figure 6), and, therefore, a higher threshold will give a larger average wall distance of the streaks. Here we measured the average wall distance  $y^+$  of the streaks determined using thresholds  $-0.02$  and  $-0.06$ ; see the blue triangles and red squares, respectively, in figure 6(d). It can be seen that  $-0.02$  gives slightly lower and  $-0.06$  gives slightly higher  $y^+$  compared to the black circles (with a threshold of  $-0.04$  for detecting streaks). But the important point is that  $y^+$  also appears to be a constant in the  $Re$  range considered using either threshold for detecting the streaks.

### Appendix B. The algorithm for determining the axial location of the front tip

In the main text, we use the algorithm that is built-in as the function *findchangepts* in MATLAB to detect abrupt changes in a signal sequence  $[x_1, x_2, \dots, x_n]$ . The key is to

minimize the target function

$$J(k) = \sum_{i=1}^{k-1} \{x_i - \text{mean}([x_1, x_2, \dots, x_{k-1}])\}^2 + \sum_{i=k}^n \{x_i - \text{mean}([x_k, x_{k+1}, \dots, x_n])\}^2 \quad (\text{B1})$$

by modifying the index  $k$ . The resulting  $k$  is regarded as the separation point of the slowly varying and abruptly varying parts of the sequence. The data sequence of  $\max_{r,\theta} |u_r|$ , containing both the laminar and turbulent parts on the upstream and downstream sides of the front tip, respectively, is fed in as the input. The output will be taken as the point separating the laminar part and the turbulent part of the curve, which we define as the axial location of the front tip. Readers are referred to the documentation of MATLAB (version R2018a) for more details about the algorithm.

#### REFERENCES

- AVILA, M., BARKLEY, D. & HOF, B. 2023 Transition to turbulence in pipe flow. *Annu. Rev. Fluid Mech.* **55**, 575–602.
- BARKLEY, D. 2016 Theoretical perspective on the route to turbulence in a pipe. *J. Fluid Mech.* **803**, P1.
- BARKLEY, D., SONG, B., MUKUND, V., LEMOULT, G., AVILA, M. & HOF, B. 2015 The rise of fully turbulent flow. *Nature* **526**, 550–553.
- CHEN, K., XU, D. & SONG, B. 2022a Propagation speed of turbulent fronts in pipe flow at high Reynolds numbers. *J. Fluid Mech.* **935**, A11.
- CHEN, Q., WEI, D. & ZHANG, Z. 2022b Linear stability of pipe Poiseuille flow at high Reynolds number regime. *Commun. Pure Appl. Maths* **76**, 1868–1964.
- DARBYSHIRE, A.G. & MULLIN, T. 1995 Transition to turbulence in constant-mass-flux pipe flow. *J. Fluid Mech.* **289**, 83–114.
- DEL ÁLAMO, J.C. & JIMÉNEZ, J. 2009 Estimation of turbulent convection velocities and corrections to Taylor's approximation. *J. Fluid Mech.* **640**, 5–26.
- VAN DOORNE, C.W.H. & WESTERWEEEL, J. 2008 The flow structure of a puff. *Phil. Trans. R. Soc. Lond. A* **367**, 489–507.
- DUGUET, Y. & SCHLATTER, P. 2013 Oblique laminar-turbulent interfaces in plane shear flows. *Phys. Rev. Lett.* **110**, 034502.
- DUGUET, Y., WILLIS, A.P. & KERSWELL, R.R. 2010 Slug genesis in cylindrical pipe flow. *J. Fluid Mech.* **663**, 180–208.
- DURST, F. & ÜNSAL, B. 2006 Forced laminar to turbulent transition in pipe flows. *J. Fluid Mech.* **560**, 449–464.
- HAMILTON, J.M., KIM, J. & WALEFFE, F. 1995 Regeneration mechanisms of near-wall turbulent structures. *J. Fluid Mech.* **287**, 317–348.
- HOF, B., DE LOZAR, A., AVILA, M., TU, X. & SCHNEIDER, T.M. 2010 Eliminating turbulence in spatially intermittent flows. *Science* **327**, 1491–1494.
- HOLZNER, M., SONG, B., AVILA, M. & HOF, B. 2013 Lagrangian approach to laminar-turbulent interfaces. *J. Fluid Mech.* **723**, 140–162.
- JIANG, X.Y., LEE, C.B., CHEN, X., SMITH, C.R. & LINDEN, P.F. 2020a Structure evolution at early stage of boundary-layer transition: simulation and experiment. *J. Fluid Mech.* **890**, A11.
- JIANG, X.Y., LEE, C.B., SMITH, C.R., CHEN, J.W. & LINDEN, P.F. 2020b Experimental study on low-speed streaks in a turbulent boundary layer at low Reynolds number. *J. Fluid Mech.* **903**, A6.
- KLOTZ, L., PAVLENKO, A.M. & WESFREID, J.E. 2021 Experimental measurements in plane Couette–Poiseuille flow: dynamics of the large- and small-scale flow. *J. Fluid Mech.* **912**, A24.
- LINDGREN, E.R. 1957 The transition process and other phenomena in viscous flow. *Ark. Fys.* **12**, 1–169.
- LINDGREN, E.R. 1969 Propagation velocity of turbulent slugs and streaks in transition pipe flow. *Phys. Fluids* **12**, 418–425.
- MESEGUER, A. 2003 Streak breakdown instability in pipe Poiseuille flow. *Phys. Fluids* **15**, 1203–1213.
- MESEGUER, A. & TREFETHEN, L.N. 2003 Linearized pipe flow to Reynolds number  $10^7$ . *J. Comput. Phys.* **186**, 178–197.

## *Speed of the upstream turbulent front in pipe flow*

- NISHI, M., ÜNSAL, B., DURST, F. & BISWAS, G. 2008 Laminar-to-turbulent transition of pipe flows through puffs and slugs. *J. Fluid Mech.* **614**, 425–446.
- PEI, J., CHEN, J., SHE, Z.-S. & HUSSAIN, F. 2012 Model for propagation speed in turbulent channel flows. *Phys. Rev. E* **86**, 046307.
- RINALDI, E., CANTON, J. & SCHLATTER, P. 2019 The vanishing of strong turbulent fronts in bent pipes. *J. Fluid Mech.* **866**, 487–502.
- SCHOPPA, W. & HUSSAIN, F. 1998 Formation of near-wall streamwise vortices by streak instability. *AIAA Paper* 98, 3000.
- SCHOPPA, W. & HUSSAIN, F. 2002 Coherent structure generation in near-wall turbulence. *J. Fluid Mech.* **453**, 57–108.
- SHAN, X., MA, B., ZHANG, Z. & NIEUWSTADT, F.T.M. 1999 Direct numerical simulation of a puff and a slug in transitional cylindrical pipe flow. *J. Fluid Mech.* **387**, 39–60.
- SHIMIZU, M. & KIDA, S. 2009 A driving mechanism of a turbulent puff in pipe flow. *Fluid Dyn. Res.* **41**, 045501.
- SONG, B., BARKLEY, D., HOF, B. & AVILA, M. 2017 Speed and structure of turbulent fronts in pipe flow. *J. Fluid Mech.* **813**, 1045–1059.
- SWEARINGEN, J.D. & BLACKWELDER, R.F. 1987 The growth and breakdown of streamwise vortices in the presence of a wall. *J. Fluid Mech.* **182**, 255–290.
- TAO, J.J., ECKHARDT, B. & XIONG, X.M. 2018 Extended localized structures and the onset of turbulence in channel flow. *Phys. Rev. Fluids* **3**, 011902(R).
- TUCKERMAN, L.S., CHANTRY, M. & BARKLEY, D. 2020 Patterns in wall bounded shear flows. *Annu. Rev. Fluid Mech.* **52**, 343–367.
- WALEFFE, F. 1997 On a self-sustaining process in shear flows. *Phys. Fluids* **9**, 883–900.
- WANG, M. & GOLDENFELD, N. 2022 Stochastic model for quasi-one-dimensional transitional turbulence with streamwise shear interactions. *Phys. Rev. Lett.* **129**, 034501.
- WU, X. & MOIN, P. 2008 A direct numerical simulation study on the mean velocity characteristics in turbulent pipe flow. *J. Fluid Mech.* **608**, 81–112.
- WYGNANSKI, I.J. & CHAMPAGNE, F.H. 1973 On transition in a pipe. Part 1. The origin of puffs and slugs and the flow in a turbulent slug. *J. Fluid Mech.* **59**, 281–335.
- ZIKANOV, O.Y. 1996 On the instability of pipe Poiseuille flow. *Phys. Fluids* **8**, 2923–2932.