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Current-driven drift wave instability in a collisional dusty negative ion plasma

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Abstract. The excitation of drift waves by an electron current parallel to the magnetic field is investigated in a nonuniform plasma composed of electrons, positive ions, negative ions, and massive, negatively charged dust. Electrostatic drift waves with frequencies smaller than the ion gyrofrequencies and wavelengths larger than the ion gyroradii are considered. Linear kinetic theory is used, and collisions of charged particles with neutrals are taken into account. The present results may be relevant to laboratory collisional magnetoplasmas containing negative ions and dust.

1. Introduction

The electrostatic drift wave is one of the fundamental modes of a nonuniform magnetized plasma (e.g. Kadomtsev 1965; Krall 1968). Drift wave instabilities can arise due to the energy in the gradient drifts as well as other mechanisms including the free energy associated with particles streaming along magnetic field lines. Drift waves excited by electron currents parallel to a magnetic field have been discussed theoretically (e.g. Bogdankevich and Rukhadze 1966; Dupree 1967; Ellis and Motley 1971, 1974) and identified in collisional and collisionless regimes (see, e.g. Ellis and Motley 1974; Hatakeyama et al. 2011) .

There has been recent interest in plasmas containing both negative ions and dust from the perspective of basic physics (see eg., Mamun et al. 2009, and refs. therein) as well as technological applications since such plasmas can occur in industrial plasma processing devices (see e.g. Ostrikov et al. 2001). Motivated by forthcoming experiments on dusty plasmas in large magnetic fields (Thomas et al. 2012), it is of interest to consider drift wave instabilities in dusty magnetoplasmas. There have been a number of prior theoretical studies on drift wave instabilities driven by density gradients or velocity shear in plasmas containing negative ions or massive negatively charged dust (e.g. Shukla et al. 1991; Rosenberg and Krall 1994; Ichiki et al. 2009; Shukla and Rosenberg 2009; Saleem 2010; Knist et al. 2011).

In this brief note we consider the effect of negative ions and massive, negatively charged dust on the current-driven drift wave instability in a nonuniform collisional magnetoplasma, using linear kinetic theory. Frequencies smaller than the ion gyrofrequencies are considered, and collisions of charged particles with neutrals are taken into account. This extends our prior work done in collaboration with Padma Shukla on the collisional drift wave instability in a multi-ion species dusty magnetoplasma (Shukla and Rosenberg 2009) to include an electron current and ion-collisional damping.

This note is organized as follows. Section 2 gives the analysis, dispersion relation, and analytic frequency and growth rates. Section 3 gives a brief summary and discussion.

2. Analysis

We consider a weakly ionized, nonuniform dusty magnetoplasma composed of electrons, singly charged positive and negative ions, and negatively charged dust grains. The condition of charge neutrality at equilibrium is given by

$$n_{e0}(x) + Z_d n_{d0}(x) = n_{+0}(x) - n_{-0}(x), \tag{1}$$

where n_{j0} is the number density of particle species j, with $j = e, +, -, d$ denoting electrons, positive ions, negative ions, and dust, respectively, and Z_d is the dust charge state. A slab model plasma is considered, with an external magnetic field in the z-direction, $B_0\hat{\mathbf{z}}$, and an electric field in the -z-direction, $-E_0\hat{\mathbf{z}}$, which leads to drifts U_{0j} of the charged particles along the magnetic field.

The equilibrium densities of the charged particle species have gradients in the x-direction, $\partial n_{j0}/\partial x$, assumed to be maintained by some external sources. The electron and ion gyroradii are much smaller than their respective gradient scale lengths, while the dust gyroradius is large. We consider frequencies smaller than the gyrofrequency $\omega_{cj} = eB_0/m_j c$ of the ions. The massive dust grains that are nonuniformly distributed are taken to be stationary on the time scale of the instability. Therefore, we neglect the dust dynamics, so that dust contributes to the dispersion relation only via its effect on equilibrium charge neutrality via (1) or related density gradient effects, viz.,

$$\frac{1}{n_{e0}}\frac{\partial N_0}{\partial x} = \frac{\delta_+}{n_{+0}}\frac{\partial n_{+0}}{\partial x} - \frac{\delta-}{n_{-0}}\frac{\partial n_{-0}}{\partial x}, \tag{2}$$

where $N_0 = n_{e0} + Z_d n_{d0}$, $\delta_+ = n_{+0}/n_{e0}$, and $\delta_- = n_{-0}/n_{e0}$. It is assumed that $Z_d n_{d0} < n_{e0}$ to avoid dust-related electron depletion effects on grain charging (e.g. reviews in Shukla and Mamun 2002; Shukla and Eliasson 2009).

Electrostatic waves are considered with perturbed electric field $\mathbf{E}_1 = -\nabla\phi(x)\exp[i(k_y y + k_z z - \omega t)]$, with $k_z \ll k_y$, $\omega \ll \omega_{c+}$ and $\omega \ll \omega_{c-}$. In a local analysis, using a BGK collision term, the linear kinetic dispersion relation can be obtained as (see e.g. Yamada and Hendel 1978; Alexandrov et al. 1984)

$$1 + \chi_e + \chi_+ + \chi_- \approx 0, \tag{3}$$

where

$$\chi_j = \frac{k_{Dj}^2}{k^2}\left[1 + \left(1 - \frac{\omega_{Dj}^*}{\bar{\omega} + i\nu_j}\right)\zeta_j Z(\zeta_j)\Gamma_0(b_j)\right] \times \left[1 + \frac{i\nu_j}{\sqrt{2}k_z v_j}\Gamma_0(b_j)Z(\zeta_j)\right]^{-1}. \tag{4}$$

Here, the diamagnetic drift frequency is $\omega_{Dj}^* = \mathbf{k}\cdot\mathbf{u}_{Dj}$, where the diamagnetic drift velocity is

$$\mathbf{u}_{Dj} = \frac{T_j c}{q_j B_0 n_{j0}}\frac{\partial n_{j0}}{\partial x}\hat{\mathbf{y}}, \tag{5}$$

where q_j is the particle charge. Here also $k_{Dj} = (4\pi n_{j0} q_j^2/T_j)^{1/2}$ is the Debye wavenumber of particle species j, $v_j = (T_j/m_j)^{1/2}$ is the thermal speed, T_j is the temperature, $\rho_j = v_j/\omega_{cj}$ is the particle gyroradius, $b_j = k_y^2\rho_j^2$, and $\Gamma_0(b_j) = I_0(b_j)\exp(-b_j)$, with I_0 being the modified Bessel function of zero order. In addition, Z is the plasma dispersion function (Fried and Conte 1961), $\zeta_j = (\bar{\omega} + i\nu_j)/\sqrt{2}k_z v_j$ with $\bar{\omega} = \omega - k_z U_{0j}$, and ν_j is the particle collision frequency due primarily to collisions with neutrals. In the following, we neglect the ion drifts along the magnetic field that are assumed to be much smaller than U_{0e} (consideration of possible ion–ion streaming instabilities being left for future work). Also, we assume the electron and positive ion densities decrease in the x-direction. Thus the diamagnetic drift velocity of the electrons is in the y-direction, so that $\omega_{De}^* > 0$ for a wave propagating in the y-direction.

We obtain analytic solutions of (3) in the nonresonant limit for the ions, and in both the collisional and kinetic limits for the electrons. For the ions, we consider the limits $\zeta_\pm \gg 1$ and $b_\pm \ll 1$. Thus the ion susceptibilities become

$$\chi_\pm \approx \frac{k_{D\pm}^2}{k^2}\left[\frac{\omega_{D\pm}^*(1 - A_\pm) + \omega A_\pm + i\nu_\pm A_\pm}{\omega + i\nu_\pm A_\pm}\right], \tag{6}$$

where

$$A_\pm = b_\pm - \frac{1}{2\zeta_\pm^2}.$$

In $A_\pm$, the term $b_\pm$ corresponds to the Larmor radius effect, while the other term corresponds to the parallel compression term in the collisionless limit.

For the electrons, we consider the small Larmor radius limit $b_e \ll 1$ with $\Gamma_0(b_e) \approx 1$. In the collisional regime, with $\nu_e \gg k_z v_e$, $|\bar{\omega}|$, ω_{De}^*, but with $\nu_e|\bar{\omega}| \ll k_z^2 v_e^2$, the electron susceptibility becomes

$$\chi_e \approx \frac{k_{De}^2}{k^2}\left[1 + i\frac{(\bar{\omega} - \omega_{De}^*)\nu_e}{k_z^2 v_e^2}\right]. \tag{7a}$$

On the other hand, in the kinetic regime, where $\zeta_e \ll 1$, the electron susceptibility becomes

$$\chi_e \approx \frac{k_{De}^2}{k^2}\left[1 + i\sqrt{\frac{\pi}{2}}\frac{(\bar{\omega} - \omega_{De}^*)}{k_z v_e}\right]. \tag{7b}$$

We solve (3) in the limit of weak growth or damping, with $\omega = \omega_r + i\gamma$, with $|\gamma| \ll \omega_r$, and in the regime where $A_\pm \approx b_\pm$. Then, using (6) and (7a), we find for the current-driven collisional drift wave instability the real frequency

$$\omega_r \approx -\frac{k_y T_e}{eB_0}\frac{1}{n_{e0}}\frac{\partial N_0}{\partial x}\left(1 + \delta_+\frac{T_e}{T_+}b_+ + \delta_-\frac{T_e}{T_-}b_-\right)^{-1}, \tag{8}$$

where we have neglected k^2/k_{De}^2 in comparison with the other terms in the denominator of (8). Note that the denominator in (8) is just $1 + k_y^2\rho_{s+}^2 + k_y^2\rho_{s-}^2$, where $\rho_{sj} = (\delta_j T_e/T_j)^{1/2}\rho_j$ is the ion sound gyroradius. The frequency given in (8) is the same as that given by (13a) in Shukla and Rosenberg (2009). The imaginary part of the frequency is given by

$$\gamma \approx \frac{\omega_r}{\omega_{D0}^*} \times \left[\frac{(\omega_{De}^* - \omega_r + k_z U_{0e})\nu_e\omega_r}{k_z^2 v_e^2} - \nu_+ b_+ C_+ - \nu_- b_- C_-\right], \tag{9}$$

with

$$C_\pm = \left[1 + \delta_\pm\frac{T_e}{T_\pm} + \delta_\mp\frac{T_e}{T_\mp}\frac{\omega_{D\mp}^*}{\omega_r}\right].$$

Here $\omega_{D0}^* = -(k_y T_e c/eB_0)\partial N_0/\partial x$. The imaginary part of the frequency given in (9) extends expression (13b) in Shukla and Rosenberg (2009) to include the parallel electron current and damping due to ion collisions. We note that in the limit of no dust, no negative ions, and equal temperatures, (8) and (9) resemble those given in Yamada and Hendel (1978) or Ellis and Marden-Marshall (1979) for the current-driven collisional drift instability.

In the kinetic regime for the electrons, the real frequency is the same as that given in (8). The only term that changes in the imaginary part of the frequency given by (9) is the first term in the brackets, which becomes multiplied by $\sqrt{\pi/2}(k_z v_e/\nu_e)$ and thus does not have a dependence on the electron collision frequency.

As discussed in Shukla and Rosenberg (2009), the presence of dust can affect the frequency and growth rate of the collisional drift wave instability via the term $\partial N_0/\partial x$. If the dust density gradient is in the opposite direction to the electron density gradient, that may lead to a reduction in the mode frequency as well as an increase in the growth rate in the current free case. However, it has been shown that the magnitude of the dust charge state is reduced in the presence of negative ions (Mamun and Shukla 2003; D'Angelo 2004; Annaratone and Allen 2005; Kim and Merlino 2006; Merlino and Kim 2006), which would tend to reduce the effect of dust.

To isolate the effects of negative ions on the collisional current-driven drift instability, we consider the case with no dust. The frequency is the same as (8) but with N_0 replaced by n_{e0}. The imaginary part of the frequency becomes, in the limit $(\delta_\pm T_e/T_\pm)b_\pm \ll 1$,

$$\gamma \approx \frac{\nu_e \omega_{De}^*}{k_z^2 v_e^2}\left[\omega_{De}^*\left(\delta_+\frac{T_e}{T_+}b_+ + \delta_-\frac{T_e}{T_-}b_-\right) + k_z U_{0e}\right] - \nu_+ b_+ C_+ - \nu_- b_- C_-. \tag{10}$$

From (8), it can be seen that the presence of negative ions tends to reduce the wave frequency, assuming the electron density gradient scale length does not change, due to the enhanced ion sound gyroradius effect (Shukla and Rosenberg 2009). This reduction in frequency is more pronounced in a plasma where $T_e \gg T_-$. Another effect of negative ions is to lead to additional ion-collisional damping. As can be seen from (10), in a plasma where the ion collision frequencies are comparable to the electron diamagnetic drift frequency, the current-free collisional drift wave instability could be quenched in this wavelength regime if $(\nu_e \omega_{De}^*/k_z^2 v_e^2) < 1$. However, if there is an additional source of free energy, such as the electron parallel drift U_{0e} considered here, that could overcome the enhanced ion-collisional damping and drive the instability.

3. Summary and discussion

An analysis of the current-driven drift wave instability in a collisional dusty negative ion plasma was presented, taking into account collisions of charged particles with neutrals. The instability is driven by a combination of the free energies in the density gradients and the electron drift along the magnetic field. Linear kinetic theory was used, and analytic expressions were obtained for the frequency and growth rates in certain limits. The frequency regime considered is smaller than the ion gyrofrequencies. It is assumed the massive dust is stationary on the time scale of the instability, so the effect of the charged dust appears via equilibrium charge density gradients that can affect the frequency and growth rate of the instability. As discussed in Shukla and Rosenberg (2009) and Knist et al. (2011), the theory shows that the presence of negative ions can lead to a reduction in the wave frequency. In addition, the presence of negative ions increases the ion-collisional damping. When the ion collision frequencies are comparable to the drift wave frequency, the additional free energy source provided by the electron current may drive instability. However, the presence of negative ions could increase the critical electron drift.

Future work should investigate nonlocal effects and finite geometries (e.g. Ellis and Marden-Marshall 1979), as well as effects of flows associated with an electric field perpendicular to the magnetic field, due, for example, to electrostatic confinement of the dust grains (e.g. Rosenberg and Krall 1994). It may also be interesting to investigate possible application to an experiment on the effect of negative ions on drift waves recently reported by Knist et al. (2011). In that experiment, parameters appear to be such that the ion collision frequencies can be comparable to the frequency of the drift wave. Thus if the theory presented here applies, we speculate there may be additional factors involved in wave excitation, such as perhaps a parallel electron current. However, as pointed out in Knist et al. (2011) a rigorous theoretical treatment of the drift wave observations in that experiment may require a three-dimensional global analysis, which is beyond the scope of this local, slab model analysis.

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