Energetics of constant height level bounding in quadruped robots

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SUMMARY
In this paper, we investigate the energetics of constant height level bounding gaits in quadruped robots with asymmetric body-mass distribution along the longitudinal axis. Analytical expressions for mechanical specific resistance for two cases of bounding are derived: bounding with equal front and rear leg step lengths, and bounding with unequal front and rear leg step lengths. Specific resistance is found to be independent of mass distribution in the first case, and dependent in the second case. The quadruped robot has average nonzero acceleration/deceleration due to unsymmetric distribution of mass when front and rear leg step lengths are equal. Results show that lower body lengths, lower step lengths, and higher heights from the ground level give lower specific resistance. The effect of body-mass asymmetry is to accelerate in the first case, and to reduce specific resistance in the second case. This result provides some insight into why certain quadrupedal animals in nature evolved to have body-mass asymmetry.

KEYWORDS: Biomimetic robots; Legged robots; Service robots; Design; Motion planning.

1. Introduction
Legged robots are preferred to wheeled mobile robots for locomotion in environments which generally contain uneven terrain.[1] In such environments, legged robots offer better mobility than their wheeled counterparts.[2] Quadrupedal animals choose a gait based on speed required.[3] Of various possible quadruped gaits, bounding gait is a form of fast running legged locomotion in which a quadruped animal uses front legs as a pair and rear legs as a pair.[4] In this gait, the quadruped lands with both of its front legs and moves the rear legs forward, lands, and swings the front leg pair further to the next step. Unlike other quadrupedal walking gaits, bounding is a highly dynamic gait, which requires proper planning and control based on dynamic considerations.[5,6] In addition to being fast, bounding gait is also energetically more expensive gait for locomotion.[7] Hence, there is a need to study bounding gait in order to choose optimal design and gait parameters.

The minimization of energy consumption plays a major role in the locomotion of legged robots for reducing on-board battery weight or extending the range of a mission. Reduced energy consumption is possible using minimum number of actuators for specific cases such as motion on horizontal straight line.[8] Legged robots usually have repetitive leg motions where large negative work is done by joint actuators.[9] Energetic performance can be improved by minimizing the negative work dissipated in actuators by proper trajectory design or dynamic walking, or by storing energy in compliant elements. Parallel elastic actuators can be used in such cases to improve energy efficiency.[10,11] Our approach to minimize energy expenditure for bounding gaits in this paper is to determine analytical expressions for mechanical cost of transport (CoT) or specific resistance and then choose the design and gait parameters that reduce energy expenditure.

Constant height level bounding gait, which is the focus of this paper, is a type of bounding gait in which the body of a quadruped robot is horizontal (no pitching motion) and at constant height

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from the ground throughout the gait cycle. Assuming the body weight is significant (say, at least 60 to 80 times in quadruped robots) compared with leg weights, the center of mass (CoM) remains essentially at the same height. Although minimizing the vertical motion of center of mass increases the cost of transport,[12] level bounding can be important where there are restrictions on the motion of load being carried by the robot. Specific resistance or cost of transport for quadruped robots reported in literature is based on experiments and/or numerical simulations for bounding gaits with flight phase.[13–15] However, there is a need to determine analytical expressions for cost of transport as a function of design and gait parameters in order to evaluate their influence on energy efficiency. This helps in energy efficient design of quadruped robots. Some recent works on deriving analytical expressions for constant level trot gaits in quadruped robots in 2D and 3D are reported in refs. [16] and [17] respectively.

Studies on the effect of body-mass distribution show that asymmetric loading alters the vertical and horizontal forces generated by fore and hind limbs in level trotting gait of dogs.[18] While this holds true for quadruped robots, the effect of asymmetric body-mass distribution on cost of transport has not been investigated yet in biomechanics and robotics literature. It is important to study the effect of mass asymmetry on energy efficiency in quadruped robots because symmetric distribution of payload on a quadruped robot and the robot mass itself cannot be guaranteed in general. In fact, trying to place the center of mass at the geometric center of robot body places severe restrictions in the designing of a quadruped robot. A recent work reports the effect of asymmetrical body-mass distribution on stability and dynamics in bounding gaits.[19] However, no such work has been reported for energetics.

The objective of this paper is to derive analytical expressions for cost of transport of a specific bounding gait of a quadruped robot with general mass distribution. The constant height level bounding gaits studied in this work will have a duty factor of 0.5 with no double support and flight phases. Two cases of bounding are considered: with equal front and rear leg step length, and with unequal front and rear leg step length. In order to investigate the effect of mass distribution, the quadruped robot being studied is assumed to have asymmetric body-mass distribution along the longitudinal axis. Analytical expressions for specific resistance are derived based on the assumption that total energy expenditure for each gait cycle is equal to the sum of energies consumed by each actuator for the gait cycle when no correcting control is applied and no regeneration. In real world applications, control is required to achieve the prescribed gait due to uncertainty in robot parameters or working conditions, in which case the energy consumed will be higher due to the additional correcting joint torques or forces that try to enforce the given gait trajectory. Therefore, the specific resistance expressions derived in this paper indicate the highest possible energetic efficiency (or the lowest specific resistance) that is ideally achievable.

2. Model of Quadruped Robot

The legs of the quadruped robot studied in this paper consist of two joints each: hip and knee. Hip and knee joints are assumed to be revolute and prismatic respectively. The center of mass is at a distance of a from the body center in the longitudinal direction. Body center is half-way between the front and rear hip joints. Further, the leg masses are assumed to be negligible compared with the body mass. Therefore, energetic cost of swinging the legs will be negligible compared with the energetic cost of moving the body mass.

The net force produced by knee and hip actuators of a leg is such that the ground reaction force is always upward to maintain contact with the ground as shown in the Fig. 1.

Linear velocities of the tip and the joint velocities of the leg are related through Jacobian matrix as

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} = \begin{bmatrix} l\cos\theta & \sin\theta \\ l\sin\theta & -\cos\theta \end{bmatrix} \begin{Bmatrix} \dot{\theta} \\ \dot{l} \end{Bmatrix} = J \begin{Bmatrix} \dot{\theta} \\ \dot{l} \end{Bmatrix}, \tag{1}$$

where J is the Jacobian matrix relating the tip velocity and joint velocities. Equating the power input to the leg through hip and knee joints to the power output at the tip of the leg, one can obtain
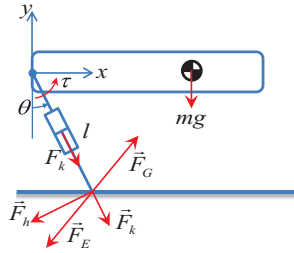
Fig. 1. Tip of the rear leg (which is the end effector) produces reaction force $\boldsymbol{F}_G$ from the ground whose vertical component is upward; the hip joint is revolute with counter clockwise rotation positive; the knee joint is prismatic with outward motion positive.

relationship between joint forces or torques $\boldsymbol{\tau}$ and end effector forces $\boldsymbol{F}_E$ as

$$\boldsymbol{\tau} = J^T \boldsymbol{F}_E \text{ or } \begin{Bmatrix} \tau_h \\ F_k \end{Bmatrix} = J^T \begin{Bmatrix} F_x \\ F_y \end{Bmatrix}. \tag{2}$$

The actuator forces or torques in $\boldsymbol{\tau}$ can be used as nominal control input to a controller that controls the torques produced by the actuators. Forces produced by hip and knee actuators can be independently determined by taking $F_k = 0$ and $\tau_h = 0$ respectively, the sum of which will be equal to $\boldsymbol{F}_E$. The force produced by hip actuator alone is given by

$$\boldsymbol{F}_h = \begin{Bmatrix} \frac{1}{l} \tau_h \cos \theta \\ \frac{1}{l} \tau_h \sin \theta \end{Bmatrix}. \tag{3}$$

Similarly, the force produced by knee actuator alone is given by

$$\boldsymbol{F}_k = \begin{Bmatrix} F_k \sin \theta \\ -F_k \cos \theta \end{Bmatrix}. \tag{4}$$

The following observations can be made on the nature of these forces:

1. The forces produced by hip actuators act perpendicular to the forces produced by knee actuators.
2. The force produced by the knee actuator acts along the line joining the point of contact of the leg with the ground and the hip joint, whereas the force produced by hip actuator acts perpendicular to the leg.

We will use these two observations in the next section where the expressions for specific resistance are derived.

The ground reaction force $\boldsymbol{F}_G$ is equal and opposite to the force developed by the leg at the tip of the leg,

$$\boldsymbol{F}_G = -\boldsymbol{F}_E. \tag{5}$$

For any given ground reaction force $\boldsymbol{F}_G$, the hip and knee actuator forces can be uniquely determined using (2). The work done by leg actuator forces on the ground is equal to the work done by reaction force on the quadruped robot body. During a gait cycle, work done by actuators can be negative. This will reduce the total work done on the quadruped robot body, and hence is not a proper indication of the energetic cost. In order to avoid this, we consider absolute value of work done by each actuator in determining total energetic cost.

For determining specific resistance,[20] we use mechanical cost of transport obtained from

$$c_m = \frac{\frac{1}{T} \int_0^T |P| dt}{mgv}, \tag{6}$$

Table I. Displacement of center of mass for various phases.

Phase	x	ψ
Phase 1	0 to $L_s/4$	$-L_s/4$ to 0
Phase 2	$L_s/4$ to $L_s/2$	0 to $L_s/4$
Phase 3	$L_s/2$ to $3L_s/4$	$-L_s/4$ to 0
Phase 4	$3L_s/4$ to L_s	0 to $L_s/4$

where P is instantaneous power, m is the mass of the quadruped robot, g is the acceleration due to gravity, and v is the average speed. Mechanical cost of transport can also be obtained from

$$c_m = \frac{\sum_i |W_i|}{mgd}, \tag{7}$$

where $|W_i|$ is the absolute value of the work done by ith actuator for one gait cycle, and d is the distance traveled during that gait cycle.

For low speeds and high torques, Joule-thermal losses dominate the energy consumed by actuators.[21] Here we assume that actuators operate in a region where Joule-thermal losses are low and the mechanical cost of transport reasonably approximates actual cost of transport.

3. Energetics of Level Bounding with Equal Front and Rear Leg Step Lengths
Following are the assumptions used in the derivation of specific resistance:

1. Gait cycle consists of only two phases: rear leg support phase and front-leg support phase. Hence, there is no double support phase where both front and rear legs are in contact with the ground. Similarly, there is no flight phase where neither of the legs is in contact with the ground.
2. Acceleration and deceleration of the body are unavoidable during gait cycle. Front legs decelerate the body while rear legs accelerate the body during their respective support phases. This is a consequence of the ground contact being in front of and behind the center of mass during these phases.[22] Initial nonzero forward velocity is assumed at the beginning of the gait cycle. The gait cycle starts with rear leg support phase (accelerating phase) first.
3. Rear foot or front foot does not cross the projection of center of mass in rear leg support phase or front leg support phase. This ensures that the assumption of sign for knee or hip work is valid for the given phase.
4. Friction is sufficiently large to prevent sliding between foot and the ground surface.

Let L_s be the distance traveled by the center of mass for one complete cycle (front and rear leg pair swings). For bound gait, one cycle can be divided into four phases, where phases 1 and 2 are performed with rear legs, and phases 3 and 4 are performed with front legs. Table I shows various phases with respect to the displacement x of the center of mass. During each of these phases, the center of mass moves a distance of $L_s/4$. The variable ψ is the horizontal component of the position of hip joint with respect to the point of contact of leg with the ground. Various phases of the gait cycle are pictorially shown in Fig. 2.

Let L_b be the length of the quadruped robot body and a be the distance of the center of mass of the body from the geometric center. The asymmetry of mass distribution is assumed to be only along the longitudinal axis. In order to respect assumption 3, the following condition on stride length L_s should be satisfied:

$$\frac{L_s}{4} \le \left(\frac{L_b}{2} - a \right). \tag{8}$$

During each of these phases, we determine the energy consumed by each actuator by considering the work done by each actuator separately. Energy consumed by an actuator for a particular phase is taken as the absolute value of the work done by the actuator during that phase.
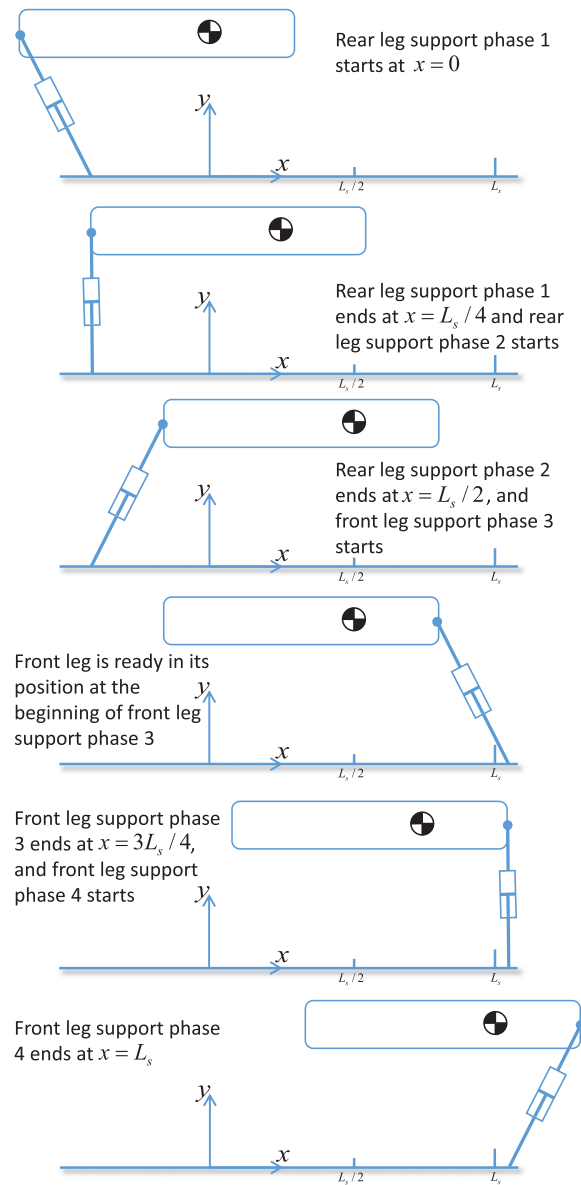
Fig. 2. Locomotion of constant height level bounding gait for one gait cycle for equal and symmetric front and rear leg step lengths. Front and rear legs are not shown in rear and front leg support phases respectively.

3.1. Rear leg support phase

Line of action of the reaction force F_R generated by the rear leg should pass through the center of mass so that there is no unbalanced moment on the body that causes pitching motion. Vertical component of this reaction force should balance the weight, and the horizontal component accelerates the body. Therefore,

$$\text{if } \boldsymbol{F_R} = F_{Rx}\hat{\boldsymbol{i}} + F_{Ry}\hat{\boldsymbol{j}}, \tag{9}$$

$$\text{then } F_{Ry} = mg, \qquad F_{Rx} = \frac{F_{Ry}}{h}(x - x_R), \tag{10}$$

$$\text{where } x_R = -\left(\frac{L_b}{2} + a\right) + \frac{L_s}{4}. \tag{11}$$

Let $\psi = x - \frac{L_s}{4}$, and $c = (\frac{L_b}{2} + a)$. Therefore,

$$F_R = \frac{mg}{h}(\psi + c)\hat{i} + mg\,\hat{j}. \tag{12}$$

Reaction force generated by the rear leg can be written as the resultant of the reaction forces generated by knee and hip actuators:

$$F_R = F_{Rk} + F_{Rh}. \tag{13}$$

Let $\hat{p}$ be the unit vector at the point of contact of the leg with the ground pointing toward the hip joint. Let $\hat{q}$ be the unit vector at the point of contact perpendicular to $\hat{p}$ and whose horizontal component is forward (in the direction of center of mass) as shown in Fig. 1. These unit vectors can be written as:

$$\hat{p} = \frac{\psi\hat{i} + h\,\hat{j}}{\sqrt{\psi^2 + h^2}}, \qquad \hat{q} = \frac{h\hat{i} - \psi\,\hat{j}}{\sqrt{\psi^2 + h^2}}. \tag{14}$$

The force vector F_R can be resolved into two components along $\hat{p}$ and $\hat{q}$. The component along $\hat{p}$ is F_{Rk} since the force generated by knee actuator always acts along the line joining hip and point of contact of foot with the ground. Similarly, the force generated by hip actuator always acts perpendicular to this line along $\hat{q}$. Therefore,

$$F_{Rk} = (F_R \cdot \hat{p})\hat{p}, \qquad F_{Rh} = (F_R \cdot \hat{q})\hat{q}. \tag{15}$$

The resolved forces F_{Rk} and F_{Rh} are

$$F_{Rk} = \frac{\frac{mg}{h}(\psi + c)\psi + mgh}{\psi^2 + h^2}(\psi\hat{i} + h\,\hat{j}), \tag{16}$$

$$F_{Rh} = \frac{mgc}{\psi^2 + h^2}(h\hat{i} - \psi\,\hat{j}). \tag{17}$$

Work done by F_R is equal to the sum of works done by F_{Rk} and F_{Rh},

$$W_R = W_{Rk} + W_{Rh}. \tag{18}$$

If the work done by knee and hip actuators are of opposite signs, the magnitude of total work done will not be equal to the energy consumed because of the cancellation of negative and positive work. Actual energy consumed would be the absolute sum of work done by the actuators. In order to determine energy consumed from the work done by actuators at individual joints, rear leg support phase is divided into two phases: phase 1 and phase 2, during which the signs of work done by knee and hip actuators are known and remain unchanged during these phases. Note that the ground reaction force has both vertical and horizontal components. The vertical component does no mechanical work because of zero vertical displacement. This holds true for the vertical components of forces produced by hip and knee actuators.

3.1.1. Phase 1. During this phase, work done by the ground reaction forces generated by knee and hip actuators are negative and positive respectively which can be inferred from the directions of $\hat{p}$ and $\hat{q}$ as shown in Fig. 3. From now on, we will simply use the phrase "work done by actuator" to actually mean "work done by the ground reaction force component generated by the actuator." Work done by hip actuator alone is given by

$$W_{Rh1} = \int_0^{\frac{L_s}{4}} F_{Rhx}dx = \int_{-\frac{L_s}{4}}^{0} \frac{mgch}{\psi^2 + h^2}d\psi, \tag{19}$$

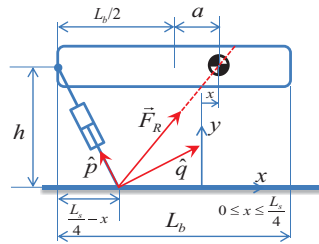$$W_{Rh1} = mgc \arctan\left(\frac{L_s}{4h}\right), \tag{20}$$

where $W_{Rh1} > 0$.

Fig. 3. Rear leg support phase 1.

Work done by knee actuator alone is given by

$$W_{Rk1} = \int_0^{\frac{L_s}{4}} F_{Rkx} dx = \int_{-\frac{L_s}{4}}^0 \frac{\frac{mg}{h}(\psi + c)\psi + mgh}{\psi^2 + h^2} \psi d\psi. \tag{21}$$

Since this integral is difficult to evaluate symbolically, we will indirectly determine it as follows:

Since $W_{R1} = W_{Rk1} + W_{Rh1}$, $W_{Rk1} = W_{R1} - W_{Rh1}$, where W_{R1} is the net work done by hip and knee actuators together given by

$$W_{R1} = \int_0^{\frac{L_s}{4}} F_{Rx} dx = \int_{-\frac{L_s}{4}}^0 \frac{mg}{h}(\psi + c) d\psi, \tag{22}$$

$$W_{R1} = \frac{mgL_s}{4h} \left[\frac{L_b}{2} + a - \frac{L_s}{8} \right]. \tag{23}$$

Therefore,

$$W_{Rk1} = \frac{mgL_s}{4h} \left[\frac{L_b}{2} + a - \frac{L_s}{8} \right] - mgc \arctan\left(\frac{L_s}{4h} \right), \tag{24}$$

where $W_{Rk1} < 0$.

Since W_{Rh1} is positive and W_{Rk1} is negative throughout the phase, energy consumed during the rear leg support phase 1 is

$$E_{R1} = 2mgc \arctan\left(\frac{L_s}{4h} \right) - \frac{mgL_s}{4h} \left[\frac{L_b}{2} + a - \frac{L_s}{8} \right]. \tag{25}$$

3.1.2. Phase 2. During this phase, work done by both knee and hip actuators are positive as shown in Fig. 4. Work done by hip actuator alone is given by

$$W_{Rh2} = \int_{\frac{L_s}{4}}^{\frac{L_s}{2}} F_{Rhx} dx = \int_0^{\frac{L_s}{4}} \frac{mgch}{\psi^2 + h^2} d\psi, \tag{26}$$

$$W_{Rh2} = mgc \arctan\left(\frac{L_s}{4h} \right). \tag{27}$$

Work done by hip and knee actuators together is given by

$$W_{R2} = \int_{\frac{L_s}{4}}^{\frac{L_s}{2}} F_{Rx} dx = \int_0^{\frac{L_s}{4}} \frac{mg}{h}(\psi + c) d\psi, \tag{28}$$

$$W_{R2} = \frac{mgL_s}{4h} \left[\frac{L_b}{2} + a + \frac{L_s}{8} \right]. \tag{29}$$
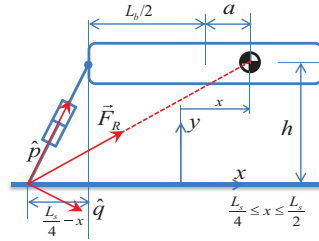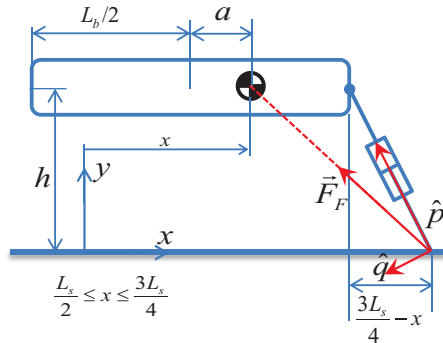
Fig. 4. Rear leg support phase 2.

Fig. 5. Front leg support phase 3.

Work done by knee actuator alone is given by

$$W_{Rk2} = W_{R2} - W_{Rh2}, \tag{30}$$

$$W_{Rk2} = \frac{mgL_s}{4h}\left[\frac{L_b}{2} + a + \frac{L_s}{8}\right] - mgc\arctan\left(\frac{L_s}{4h}\right). \tag{31}$$

Therefore, energy consumed during the rear leg support Phase 2 is

$$E_{R2} = \frac{mgL_s}{4h}\left[\frac{L_b}{2} + a + \frac{L_s}{8}\right]. \tag{32}$$

3.2. Front leg support phase
The line of action of the force generated by the front leg should pass through the center of mass so that there is no unbalanced moment on the body as shown in Fig. 5. Hence,

$$\mathbf{F}_F = F_{Fx}\hat{\mathbf{i}} + F_{Fy}\hat{\mathbf{j}}, \tag{33}$$

$$F_{Fy} = mg, \qquad F_{Fx} = \frac{F_{Ry}}{h}(x - x_F), \tag{34}$$

$$x_F = \left(\frac{L_b}{2} - a\right) + \frac{3L_s}{4}. \tag{35}$$

Let $\psi = x - \frac{3L_s}{4}$, and $b = (\frac{L_b}{2} - a)$. Therefore,

$$\mathbf{F}_F = \frac{mg}{h}(\psi - b)\hat{\mathbf{i}} + mg\,\hat{\mathbf{j}}. \tag{36}$$

Force generated by the front leg shown in Fig. 5 can be written as the resultant of the forces generated by the knee and hip torques:

$$F_F = F_{Fk} + F_{Fh}. \tag{37}$$

The unit vectors $\hat{p}$ and $\hat{q}$ during this phase can be written as:

$$\hat{p} = \frac{\psi \hat{i} + h \hat{j}}{\sqrt{\psi^2 + h^2}}, \qquad \hat{q} = \frac{-h \hat{i} + \psi \hat{j}}{\sqrt{\psi^2 + h^2}}. \tag{38}$$

The force vector F_F can be resolved into two components along $\hat{p}$ and $\hat{q}$. Therefore,

$$F_{Fk} = (F_F \cdot \hat{p})\hat{p}, \qquad F_{Fh} = (F_F \cdot \hat{q})\hat{q}. \tag{39}$$

The resolved forces F_{Fk} and F_{Fh} are

$$F_{Fk} = \frac{\frac{mg}{h}(\psi - b)\psi + mgh}{\psi^2 + h^2}(\psi \hat{i} + h \hat{j}), \tag{40}$$

$$F_{Fh} = \frac{mgb}{\psi^2 + h^2}(-h \hat{i} + \psi \hat{j}). \tag{41}$$

Work done by F_F is equal to the sum of works done by F_{Fk} and F_{Fh},

$$W_F = W_{Fk} + W_{Fh}. \tag{42}$$

The front leg support phase is divided into two phases: phase 3 and phase 4, during which the signs of work done by knee and hip actuators do not change.

3.2.1. Phase 3. During this phase, work done by both knee and hip actuators are negative as shown in Fig. 5. Work done by hip actuator alone is given by

$$W_{Fh1} = \int_{\frac{L_s}{2}}^{\frac{3L_s}{4}} F_{Fhx} dx = -\int_{-\frac{L_s}{4}}^{0} \frac{mgbh}{\psi^2 + h^2} d\psi, \tag{43}$$

$$W_{Fh1} = -mgb \arctan\left(\frac{L_s}{4h}\right). \tag{44}$$

Work done by knee and hip actuators together is given by

$$W_{F1} = \int_{\frac{L_s}{2}}^{\frac{3L_s}{4}} F_{Fx} dx = -\int_{-\frac{L_s}{4}}^{0} \frac{mg}{h}(\psi - b) d\psi, \tag{45}$$

$$W_{F1} = -\frac{mgL_s}{4h}\left[\frac{L_b}{2} - a + \frac{L_s}{8}\right]. \tag{46}$$

Work done by knee actuator alone is given by

$$W_{Fk1} = W_{F1} - W_{Fh1}, \tag{47}$$

$$W_{Fk1} = mgb \arctan\left(\frac{L_s}{4h}\right) - \frac{mgL_s}{4h}\left[\frac{L_b}{2} - a + \frac{L_s}{8}\right]. \tag{48}$$
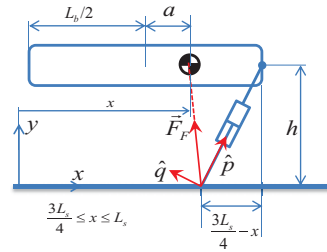
Fig. 6. Front leg support phase 4.

Therefore, after considering the signs of works done, energy consumed during the front leg support phase 3 can be determined as

$$E_{F1} = \frac{mgL_s}{4h} \left[\frac{L_b}{2} - a + \frac{L_s}{8} \right]. \tag{49}$$

3.2.2. Phase 4. During this phase, work done by knee and hip actuators are positive and negative respectively, as shown in Fig. 6. Work done by hip actuator alone is given by

$$W_{Fh2} = \int_{\frac{3L_s}{4}}^{L_s} F_{Fhx} dx = - \int_0^{\frac{L_s}{4}} \frac{mgbh}{\psi^2 + h^2} d\psi, \tag{50}$$

$$W_{Fh2} = -mgb \arctan\left(\frac{L_s}{4h}\right). \tag{51}$$

Work done by knee and hip actuators together is given by

$$W_{F2} = \int_{\frac{3L_s}{4}}^{L_s} F_{Fx} dx = \int_0^{\frac{L_s}{4}} \frac{mg}{h}(\psi - b) d\psi, \tag{52}$$

$$W_{F2} = \frac{mgL_s}{4h} \left[\frac{L_s}{8} - \frac{L_b}{2} + a \right]. \tag{53}$$

Work done by knee actuator alone is

$$W_{Fk2} = W_{F2} - W_{Fh2}, \tag{54}$$

$$W_{Fk2} = \frac{mgL_s}{4h} \left[\frac{L_s}{8} - \frac{L_b}{2} + a \right] + mgb \arctan\left(\frac{L_s}{4h}\right). \tag{55}$$

Therefore, after considering the signs of works done, energy consumed during the rear leg support phase 4 can be determined as

$$E_{F2} = \frac{mgL_s}{4h} \left[\frac{L_s}{8} - \frac{L_b}{2} + a \right] + 2mgb \arctan\left(\frac{L_s}{4h}\right). \tag{56}$$

3.3. Mechanical cost of transport or specific resistance
Total energy consumed is

$$E_{\text{Total}} = E_{R1} + E_{R2} + E_{F1} + E_{F2}, \tag{57}$$

$$E_{\text{Total}} = \frac{mgL_s^2}{8h} + 2mgL_b \arctan\left(\frac{L_s}{4h}\right). \tag{58}$$

The mechanical CoT is given by

$$\text{CoT} = \frac{E_{\text{Total}}}{mgL_s}, \tag{59}$$

$$\text{CoT} = \frac{L_s}{8h} + \frac{2L_b}{L_s} \arctan\left(\frac{L_s}{4h}\right). \tag{60}$$

For small stride lengths, we can write

$$\arctan\left(\frac{L_s}{4h}\right) \approx \frac{L_s}{4h}. \tag{61}$$

Therefore, the approximate cost of transport is

$$\text{CoT} = \frac{L_s}{8h} + \frac{L_b}{2h}. \tag{62}$$

Following observations can be made:

- Specific resistance depends on both stride length and body length. Lower body and stride lengths give lower specific resistance.
- Specific resistance does not depend on unsymmetric distribution of mass when equal front and rear step lengths are used.
- The total work done on the robot body for one gait cycle comprising rear and front leg support phases is nonzero and depends on mass distribution or location of the center of mass,

$$W = W_{R1} + W_{R2} + W_{F1} + W_{F2} = \frac{mgL_s a}{h} \neq 0. \tag{63}$$

Kinetic energy of the quadruped robot at the end of a gait cycle is not the same as that at the beginning of that gait cycle because of the net nonzero work done on it. This means that if $a > 0$, the quadruped robot has average positive acceleration because of the unsymmetric distribution of mass with equal front and rear leg step lengths. Similarly, if $a < 0$, the robot would lose its initial kinetic energy and decelerate for every gait cycle. The front and rear leg step lengths can be made unequal to compensate the effect of unsymmetric mass distribution so as to preserve initial forward velocity of the quadruped robot after each gait cycle.

4. Energetics of Level Bounding with Unequal Front and Rear Step Lengths

In this section, we assume that the front and rear leg step lengths are different. However, we keep the front and rear leg step lengths symmetrical about the vertical lines passing through the front and rear hip joints respectively. Let $L_{sr}/2$ and $L_{sf}/2$ be the rear and front leg step lengths. L_{sr} is the stride length if front leg step length were the same as the rear leg step length $L_{sr}/2$. Similarly, L_{sf} is the stride length if rear leg step length were the same as the front leg step length $L_{sf}/2$. Since the front- and rear-leg step lengths are different, the stride length is

$$L_s = \frac{L_{sr}}{2} + \frac{L_{sf}}{2}. \tag{64}$$

Let us define the rear leg step length in terms of stride length as

$$\frac{L_{sr}}{2} + k = \frac{L_s}{2}. \tag{65}$$

Therefore, the front leg step length becomes

$$\frac{L_{sf}}{2} - k = \frac{L_s}{2}, \tag{66}$$

so that (64) is satisfied. For positive values of a, condition (8) now becomes

$$\frac{L_{sf}}{4} \leq \frac{L_b}{2} - a. \tag{67}$$

   Now the task is to determine the value of k such that the net work done for one gait cycle is zero. Our derivation of mechanical cost of transport for unequal front and rear leg step lengths relies on the results derived for equal front and rear leg step lengths in the previous section.

4.1. Rear leg support phase
In this phase, rear leg step length is $L_{sr}/2$, which is distributed as $L_{sr}/4$ each for rear leg support phases 1 and 2. The work done and energy expressions for phases 1 and 2 can be derived similar to Section 3 with L_s replaced by L_{sr}.

4.1.1. Phase 1. Work done by knee actuator is

$$W_{Rk1} = \frac{mgL_{sr}}{4h}\left[\frac{L_b}{2} + a - \frac{L_{sr}}{8}\right] - mgc \arctan\left(\frac{L_{sr}}{4h}\right). \tag{68}$$

Similarly, work done by hip actuator is given by

$$W_{Rh1} = mgc \arctan\left(\frac{L_{sr}}{4h}\right). \tag{69}$$

Therefore, energy consumed during the rear leg support phase 1 is

$$E_{R1} = 2mgc \arctan\left(\frac{L_{sr}}{4h}\right) - \frac{mgL_{sr}}{4h}\left[\frac{L_b}{2} + a - \frac{L_{sr}}{8}\right]. \tag{70}$$

4.1.2. Phase 2. Work done by knee actuator is

$$W_{Rk2} = \frac{mgL_{sr}}{4h}\left[\frac{L_b}{2} + a + \frac{L_{sr}}{8}\right] - mgc \arctan\left(\frac{L_{sr}}{4h}\right). \tag{71}$$

Similarly, work done by hip actuator is given by

$$W_{Rh2} = mgc \arctan\left(\frac{L_{sr}}{4h}\right). \tag{72}$$

Therefore, energy consumed during the rear leg support phase 2 is

$$E_{R2} = \frac{mgL_{sr}}{4h}\left[\frac{L_b}{2} + a + \frac{L_{sr}}{8}\right]. \tag{73}$$

Total energy consumed for rear leg support phase is given by

$$E_R = E_{R1} + E_{R2} \tag{74}$$

$$= 2mgc \arctan\left(\frac{L_{sr}}{4h}\right) + \frac{mgL_{sr}}{4h}\left[\frac{L_{sr}}{4}\right]. \tag{75}$$

4.2. Front leg support phase

In this phase, front leg step length is $L_{sf}/2$, which is distributed as $L_{sf}/4$ each for front leg support phases 3 and 4. The work done and energy expressions for phases 3 and 4 can be derived similar to Section 3 with L_s replaced by L_{sf}.

4.2.1. Phase 3. Work done by knee actuator is

$$W_{Fk1} = mgb \arctan\left(\frac{L_{sf}}{4h}\right) - \frac{mgL_{sf}}{4h}\left[\frac{L_b}{2} - a + \frac{L_{sf}}{8}\right]. \tag{76}$$

Similarly, work done by hip actuator is given by

$$W_{Fh1} = -mgb \arctan\left(\frac{L_{sf}}{4h}\right). \tag{77}$$

Therefore, energy consumed during the front leg support phase 3 is

$$E_{F1} = \frac{mgL_{sf}}{4h}\left[\frac{L_b}{2} - a + \frac{L_{sf}}{8}\right]. \tag{78}$$

4.2.2. Phase 4. Work done by knee actuator is

$$W_{Fk2} = \frac{mgL_{sf}}{4h}\left[\frac{L_{sf}}{8} - \frac{L_b}{2} + a\right] + mgb \arctan\left(\frac{L_{sf}}{4h}\right). \tag{79}$$

Similarly, work done by hip actuator is given by

$$W_{Fh2} = -mgb \arctan\left(\frac{L_{sf}}{4h}\right). \tag{80}$$

Therefore, energy consumed during the rear leg support phase 4 is

$$E_{F2} = \frac{mgL_{sf}}{4h}\left[\frac{L_{sf}}{8} - \frac{L_b}{2} + a\right] + 2mgb \arctan\left(\frac{L_{sf}}{4h}\right). \tag{81}$$

Total energy consumed during the front leg support phase is given by

$$E_F = E_{F1} + E_{F2} \tag{82}$$

$$= 2mgb \arctan\left(\frac{L_{sf}}{4h}\right) + \frac{mgL_{sf}}{4h}\left[\frac{L_{sf}}{4}\right]. \tag{83}$$

Now W_R and W_F can be calculated in terms of stride length L_s and the unknown k by substituting

$$L_{sr} = L_s - 2k, \qquad L_{sf} = L_s + 2k. \tag{84}$$

The value of k which makes the net work done per gait cycle can be determined by taking

$$W_R + W_F = 0, \tag{85}$$

which gives

$$k = \frac{aL_s}{L_b}. \tag{86}$$

This meansthat if the center of mass is in front of geometric center ($a > 0$), then the rear step length has to be smaller than the front step length to achieve constant average forward velocity.

Substituting L_{sr}, L_{sf}, and k in energy expressions, and calculating total energy consumed for complete gait cycle is given by

$$E = E_R + E_F. \tag{87}$$

4.3. Mechanical cost of transport for steady gait with unequal rear and front leg step lengths
Total energy consumed is

$$E = \frac{mg}{h}\left(\frac{L_s^2}{8} + \frac{a^2 L_s^2}{2L_b^2}\right) + 2mg\left[c \arctan\left(\frac{L_{sr}}{4h}\right) + b \arctan\left(\frac{L_{sf}}{4h}\right)\right], \tag{88}$$

from which the specific resistance or cost of transport is determined as

$$\text{CoT} = \frac{L_s}{h}\left(\frac{1}{8} + \frac{a^2}{2L_b^2}\right) + \frac{2}{L_s}\left[c \arctan\left(\frac{L_{sr}}{4h}\right) + b \arctan\left(\frac{L_{sf}}{4h}\right)\right]. \tag{89}$$

Substituting $a = 0$ in the above equation, one can obtain specific resistance for uniform mass distribution as in (60).

5. Results and Discussion

5.1. For accelerating or decelerating gaits
From (63), it can be seen that equal front and rear leg step lengths lead to accelerating gait if the body center of mass is in front of the body center ($a > 0$). If the robot needs to be accelerated to a different average forward velocity, equal front and rear leg step lengths can be chosen. Generally, the center of mass position is fixed by adjusting the payload either in front of or behind the body center before the robot is started on a mission. If the robot is designed such that the center of mass can be changed during the gait, one can move the center of mass behind the body center in order to decelerate the robot using equal front and rear leg step lengths.

There is an easier way of achieving acceleration or deceleration than by changing the position of center of mass. For level bounding gait studied in this paper, rear leg support phase is accelerating and front leg support phase is decelerating. Hence, when acceleration is desired, front leg step length can be made smaller or rear leg step length can be made larger. Similarly, when deceleration is desired, front leg step length can be made larger or rear leg step length can be made smaller.

5.2. Energetics for uniform or symmetric mass distribution
Figure 7 shows variation of energetic cost with respect to height for stride length $L_s = 0.5$, with body length $L_b = 1$ m. Energetic cost decreases monotonously with increase in body height h.

For smaller stride lengths L_s, energetic cost is directly proportional to L_s as is evident from (62). This is true for $L_s/(4h) < 0.5$ within 10% deviation. For a height of $h = 1$ m, L_s can be as large as 2 m for the linearity assumption to hold (see Fig. 8). As the step length is decreased, energetic cost converges to a limiting value of $L_b/(2h)$ as shown in Fig. 8. Note that, from (8), one-fourth of the maximum stride length cannot exceed half the body length to maintain the assumptions made for various phases, namely, the foot does not cross the line of projection of center of mass on the ground. Hence, in Fig. 8 the maximum stride length is shown to be different for different body lengths.

The mechanical cost of transport of a point mass biped robot undergoing level walking with acceleration and deceleration is $L_s/8h$.[23] This can be obtained from (60) by taking $L_b = 0$, which indicates that quadruped robot in constant height level walking gait with zero body length is a biped robot in constant height walking.

5.3. Energetics for unsymmetric mass distribution
With body-mass asymmetry and equal front and rear leg step lengths, the expression for energetic cost remains the same as without body-mass asymmetry. The effect of asymmetry comes in terms of accelerated gait for the same energetic cost. This indicates that there is a natural tendency to
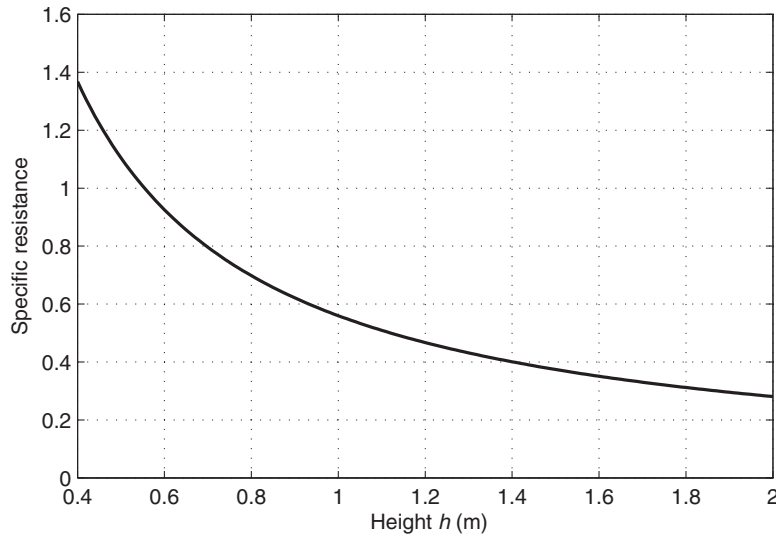
Fig. 7. Variation of energetic cost with height ($L_b = 1$ m, $L_s = 0.5$ m, $a = 0$ m).
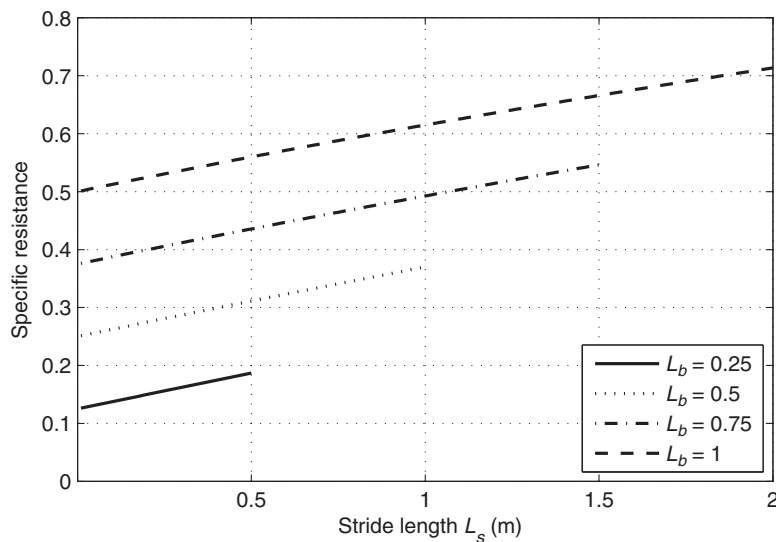

Fig. 8. Variation of energetic cost with stride length for various values of body length ($h = 1$ m, $a = 0$ m).

accelerate in quadruped robots with center of mass in front of the body center. It is a well-known fact in quadrupedal animals that the rear legs (hind limbs) tend to accelerate the body, whereas the front legs (fore limbs) tend to decelerate the body.[22, 24, 25] With the center of mass shifted forward, larger step lengths are possible with rear legs, and smaller with front legs without violating the condition that the front or rear foot does not cross the projection of center of mass on the ground. This will further increase the acceleration due to the increased duration of propulsive effect from rear legs, and with the decreased duration of braking effect from front legs.

The center of mass can be either in front of the body center ($a > 0$) or behind the body center ($a < 0$). Since the specific resistance is an even function of a for steady gait with equal front and rear leg step lengths as described by (89), the energetic cost depends only on the distance from the body center and not on whether a is positive or negative as shown in Fig. 9. For steady forward speed, the energetic cost function is almost quadratic with rapid decrease in energetic cost with increase in the distance of the center of mass from the body center.

In most of the quadrupedal mammals, evolution has shifted the body center forward, thereby increasing their ability to accelerate.[26] Our model indicates that forward shifted center of mass is both energy efficient and acceleration-biased (higher ability to accelerate). A rearward shifted center
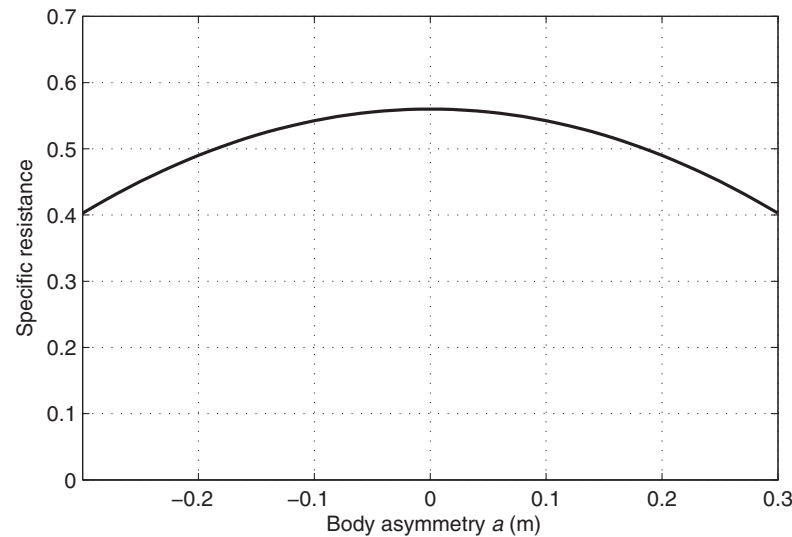
Fig. 9. Energetic cost versus position of center of mass with respect to body center ($h = 1$ m, $L_b = 1$ m, $L_s = 0.5$ m).
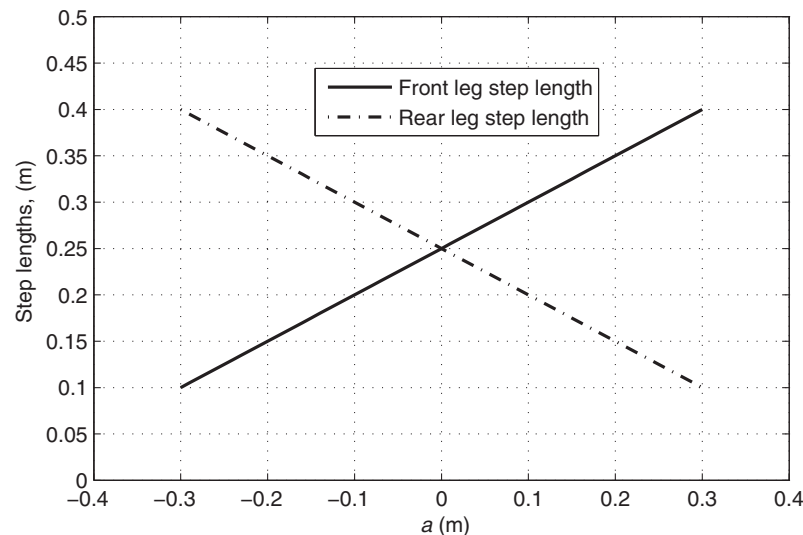

Fig. 10. Front and rear leg step lengths as a function of a for the same stride length of $L_s = 0.5$ m with $L_b = 1$ m.

of mass is deceleration-biased. Fast running quadrupedal animals such as cheetahs and greyhounds, although have their center of mass forward of their body center, have less asymmetry,[26–28] indicating a trade-off between ability to accelerate as well as decelerate. Note that our model's prediction is only indicative because of the massless legs assumption that we make.

In Fig. 9, the energetic cost is plotted with a varying up to 60% of half the body length on either side of the body center. Although further reduction in enegetic cost is possible by increasing $|a|$, it may not be practical to achieve a steady gait (constant average speed) by making front and rear leg step lengths unequal. For steady gait, as a increases, the front and rear leg step lengths for the same stride length of $L_s = 0.5$ m vary as shown in Fig. 10. For this stride length of 0.5 m, $|a|$ cannot be increased further due to violation of condition under which the energetic cost is derived, i.e., foot would cross the projection of center of mass on the ground. For the given body length and the distance of center of mass from the body center, the maximum stride length that is possible can be determined from (67), (84), and (86) as

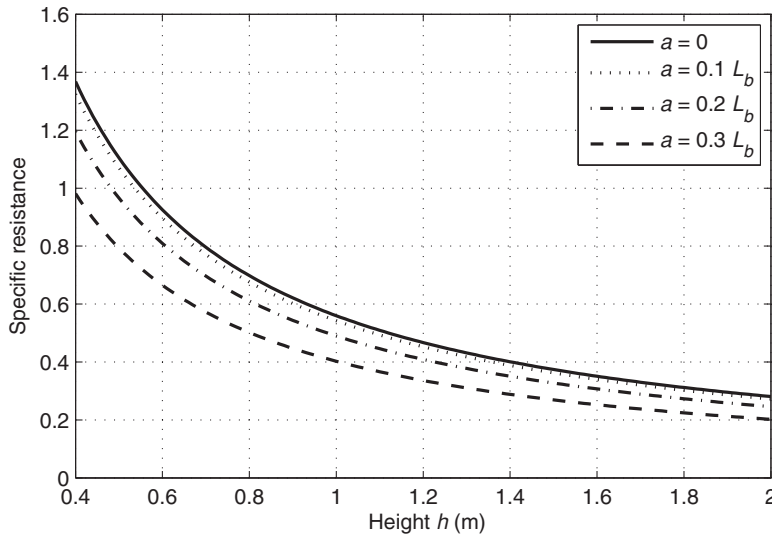$$L_{s\max} = 2L_b \frac{(L_b - 2a)}{(L_b + 2a)}. \tag{90}$$

Fig. 11. Variation in energetic cost with height ($L_b = 1$ m, $L_s = 0.5$).
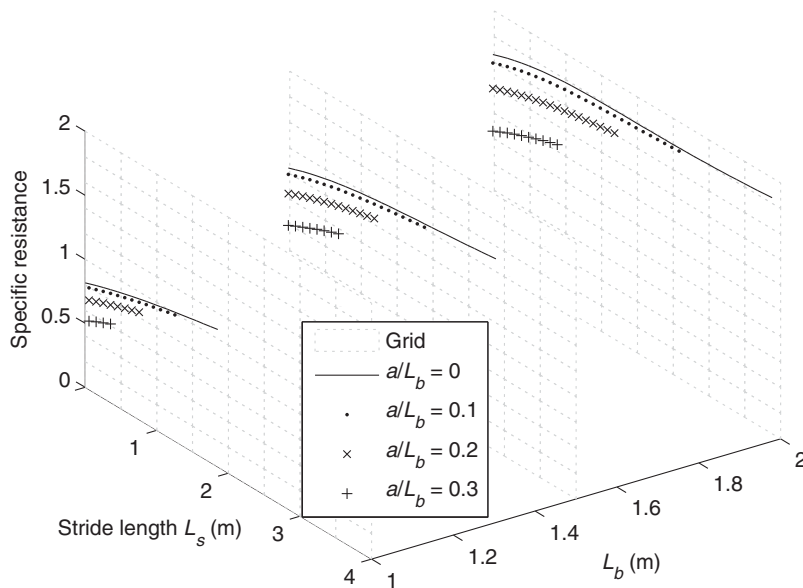

Fig. 12. Variation of energetic cost with stride length for various values of a and L_b with $h = 0.6$ m.

The consequence of allowing lower step lengths for either front leg or rear leg is rise in the energetic cost of swinging the leg forward. When the rear leg step length is lower, the front leg (pair) has to swing forward in the preparation of the next step. If sufficient time is not available, the forward legs have to swing forward rapidly and then brought to rest. This increases the energetic cost of swinging, which has not been considered in our analysis.

With the variation of bounding height, the effect of increased body-mass asymmetry is reduced specific resistance. This effect can be observed even with lower body heights for the same stride and body lengths as shown in Fig. 11.

With the increase of the distance of center of mass from body center, the decrease in energetic cost and also the maximum possible stride lengths for three different values of body length L_b and height h are shown in Figs. 12–14 respectively. With the increase in the value of a, rapid decrease in energetic cost can be observed. At the same time, the maximum stride length possible also decreases. For higher values of body length, there is an increase in the maximum stride length, and at the same time the energetic cost also increases with increase in L_b/h ratio.
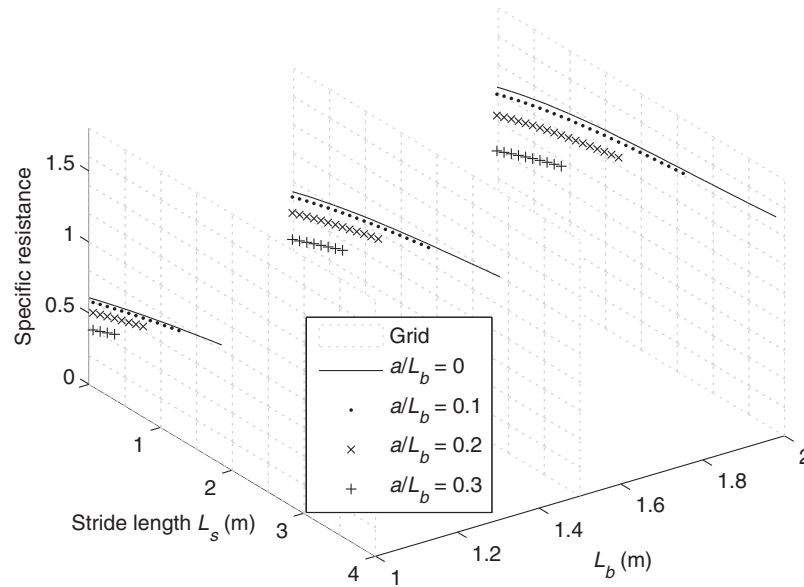
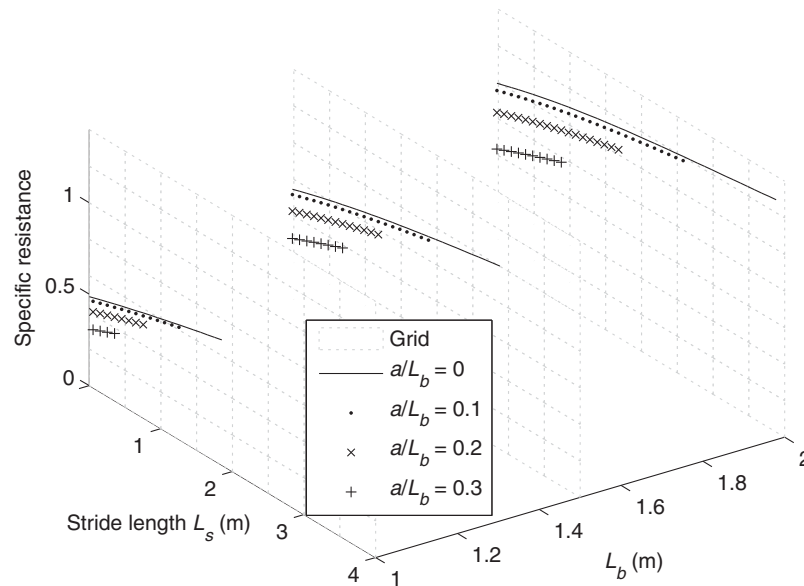Fig. 13. Variation of energetic cost with stride length for various values of a and L_b with $h = 0.8$ m.

Fig. 14. Variation of energetic cost with stride length for various values of a and L_b with $h = 1$ m.

5.4. Actual energetic cost

The energetic cost expressions derived in this paper are based on mechanical work considerations of individual actuators rather than purely mechanical work done on the body by net actuator forces. This is closer to the actual energetic cost determined considering electrical energy expenditure in joint actuators. The actual cost is bound to be higher than the one derived from the expressions given in this paper due to various factors that have not been considered. The actuators produce forces which have vertical components that balance the weight of the robot. Although no mechanical work is done to keep the body at constant height, energy is consumed by actuators to generate these forces. This means that if more time is taken to traverse the same distance, higher energy would be consumed, making the cost of transport a function of stride frequency too as observed in quadrupedal animals.[29] There are other factors such as motor efficiency, gear head efficiency, *etc.* that contribute to increased actual cost. Since the energetic cost obtained by considering the absolute values of mechanical work done by individual actuators is also a significant contributing factor to the actual energetic cost, the

results presented in this work are useful in choosing the optimal design and gait parameters that would reduce the actual energetic cost.

6. Conclusions and Future Work

In this paper, we investigated the energetics of level bounding gaits in quadruped robots with asymmetric body-mass distribution in longitudinal axis. Main results of the paper are the analytical expressions for mechanical specific resistance in constant height level bounding with equal front and rear leg step lengths, and with unequal front and rear leg step lengths for steady gait with body-mass asymmetry. The specific resistance is found to be independent of mass distribution in the first case where the gait is found to be accelerating if mass asymmetry is present. The front and rear leg step lengths are made unequal in the second case in order to obtain a steady gait with constant average speed. The effect of design parameters such as body length and distance of center of mass from the body center, and gait parameters such as gait height and step lengths have been discussed in detail. As future work, we would like to experimentally validate the results derived in this paper.

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