DIOPHANTINE EQUATIONS FOR POLYNOMIALS
WITH RESTRICTED COEFFICIENTS, I (POWER VALUES)

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Dedicated to the memory of Andrzej Schinzel

Abstract

We give effective finiteness results for the power values of polynomials with coefficients composed of a fixed finite set of primes; in particular, of Littlewood polynomials.

1. Introduction

There is an extensive literature on polynomials with restricted coefficients, in particular, with coefficients belonging to one of the sets $\{-1, 1\}$, $\{0, 1\}$ or $\{-1, 0, 1\}$. In the first case, the polynomials are called Littlewood polynomials, while, in the second case (assuming that the constant term is nonzero), the polynomials are the Newton polynomials. Here we mention only a few papers and directions. The zeros (in particular, the number of real zeros) of polynomials with coefficients belonging to $\{-1, 0, 1\}$ have been studied by Bloch and Pólya [1], Schur [14], Szegő [16], Erdős and Turán [8], and Drungilas and Dubickas [6] (see also papers of Borwein and Erdélyi [2, 3]). A related question concerning the order of vanishing of such polynomials at 1 has been considered by Borwein and Mossinghoff [4]. For similar studies of Littlewood polynomials, see Peled *et al.* [12]. Divisibility properties of such polynomials are also of interest: see, for example, Dubickas and Jankauskas [7] for a question concerning Newton and Littlewood polynomials, and Mossinghoff [11] for the case of described cyclotomic factors.

In this paper, we initiate the study of Diophantine equations involving polynomials with restricted coefficients. As a generalisation of Littlewood polynomials, we consider polynomials whose coefficients are composed of primes coming from a fixed finite set. We are interested in perfect power values of such polynomials, that is, in Schinzel–Tijdeman equations and hyper- and superelliptic equations related to them. We provide effective upper bounds for the solutions of such equations. For this, we need to combine the effective theory of such equations and the theory of S-unit equations with new assertions concerning the root structures of such polynomials. In view of the general interest (indicated above) in polynomials with restricted coefficients, we find the latter results (Lemmas 3.3–3.7) of possible independent interest. This is one of the main reasons why we split our research into parts: in this way, we can present these 'background' results as well. In the continuation of this paper, we take up the general problem of polynomial values of polynomials with restricted coefficients (which requires different techniques and different background knowledge about the root structures and decomposability properties).

2. Notation and main results

Let $S = \{p_1 < p_2 < \cdots < p_k\}$ be a finite set of primes and write $\mathbb{Z}_S$ for the set of integers having no prime divisors outside S. Note that $\pm 1 \in \mathbb{Z}_S$ but $0 \notin \mathbb{Z}_S$ for any S. In particular, $\mathbb{Z}_S = \{-1, 1\}$ for $S = \emptyset$. Write P_S for the set of polynomials in $\mathbb{Z}[x]$ with coefficients belonging to $\mathbb{Z}_S$.

Now we give our main results. The first theorem shows that, under a necessary condition, the polynomials in P_S may only attain power values with bounded exponents.

THEOREM 2.1. *Let $f(x) \in P_S$ of degree d and let b be a nonzero rational number. Then there exist effectively computable constants $C_1 = C_1(p_k)$ and $C_2 = C_2(b, d, p_k)$ depending only on p_k and on b, d, p_k, respectively, such that if $d > C_1$, then the equation*

$$f(x) = by^n \tag{2.1}$$

with $x, y, n \in \mathbb{Z}$ and $|y| > 1$ implies that $n < C_2$.

REMARK 2.2. The condition $d > C_1$ is necessary. Indeed, let d be arbitrary, and choose S such that $\binom{d}{i} \in \mathbb{Z}_S$ for all $i = 0, \ldots, d$. Then, for any $a \in \mathbb{Z}_S$, we have $(x + a)^d \in P_S$. However, (2.1) clearly has infinitely many solutions x, y with any multiple n of d. So we see that we do need a lower bound for d in order for the statement of Theorem 2.1 to be valid.

Now we would like to bound also the solutions x, y of (2.1). However, for this, we need to switch to $S = \emptyset$, that is, to the case of Littlewood polynomials. This is, in fact, necessary; a condition as in Theorem 2.1 saying that the degree of the polynomial should be large enough is not sufficient (for the reason, see Remark 2.4(i) after the statement of the theorem).

THEOREM 2.3. *Let $f(x) \in P_S$ with $S = \emptyset$ (that is, $f(x)$ is a Littlewood polynomial, with all coefficients being ± 1). Assume, further, that $\deg f \geq 3$ and let b be a nonzero rational number. Then all solutions $x, y, n \in \mathbb{Z}$ of the equation*

$$f(x) = by^n \tag{2.2}$$

with $n \geq 2$ satisfy

$$\max(|x|, |y|, n) \leq C_4,$$

except when $n = 2$ and f is one of the forms

$$f(x) = \pm(x^{2k+1} + \cdots + x^{k+1} - x^k - \cdots - 1),$$
$$\pm (x^{2k+1} - x^{2k} + \cdots + (-1)^{k+2} x^{k+1} + (-1)^k x^k + \cdots + 1)$$

with some $k \geq 1$. Here, $C_4 = C_4(b, d)$ is an effectively computable constant depending only on b and the degree d of f.

REMARK 2.4. Here we mention two things.

(i) The statement is not valid for arbitrary S. The restriction $S = \emptyset$ cannot be omitted, even if we restrict to polynomials whose degrees are 'large enough'. To see this, let $f_1(x) \in \mathbb{Z}[x]$ be any not identically zero polynomial. Let S be an arbitrary, finite set of primes, containing 2, such that both $f_1(x)$ and $(f_1(x))^2$ belong to P_S. Then, inductively, define

$$f_{i+1}(x) = ((x^{d_i+1} + 1)f_i(x))^2 \quad \text{(for all } i \geq 1\text{)},$$

where $d_i = \deg(f_i)$ $(i \geq 1)$. Observe that, then, all the polynomials $f_i(x)$ $(i \geq 1)$ are full squares in P_S and, clearly, the sequence $d_1, d_2, d_3, \ldots$ of their degrees is unbounded: that is, for this set S, there exists a polynomial of arbitrarily large degree in P_S that is a square. Hence, clearly, equation (2.2) with $b = 1$ and $n = 2$ has infinitely many solutions in $x, y \in \mathbb{Z}$.

(ii) In the exceptional cases, equation (2.2) (with appropriate choices of b) has infinitely many solutions with $n = 2$ in $x, y \in \mathbb{Z}$. Indeed, for any $k \geq 1$,

$$\pm(x^{2k+1} + \cdots + x^{k+1} - x^k - \cdots - 1) = \pm(x-1)(x^k + \cdots + x + 1)^2,$$

which gives a square value whenever $\pm(x - 1)$ is a square. Similarly, for any $k \geq 1$,

$$\pm (x^{2k+1} - x^{2k} + \cdots + (-1)^{k+2} x^{k+1} + (-1)^k x^k + \cdots + 1)$$
$$= \pm(x + 1)(x^k - x^{k-1} + \cdots + (-1)^k)^2,$$

which gives a square value whenever $\pm(x + 1)$ is a square.

3. Lemmas, auxiliary results and proofs

To prove Theorem 2.1, we need two lemmas. Here and later on, by the height $H(F(x))$ of a polynomial $F(x)$ with integer coefficients we mean the maximum of the absolute values of its coefficients.

LEMMA 3.1. *Let $F(x) \in \mathbb{Z}[x]$ having two distinct (complex) roots of degree D and height H, and let B be a nonzero rational number. Then the equation*

$$F(x) = By^n$$

with $x, y \in \mathbb{Z}$, $|y| > 1$ implies that $n < C_4$, where $C_4 = C_4(B, D, H)$ is an effectively computable constant depending only on B, D and H.

PROOF. The statement immediately follows from the Schinzel–Tijdeman theorem (the main result of [13]; see also [17] and [15, Ch. 9]). □

LEMMA 3.2. *Let S be as above and let A, B be nonzero rational numbers. Then the solutions $x, y \in \mathbb{Z}_S$ of the equation*

$$Ax - By = 1$$

satisfy

$$\max(|x|, |y|) < C_5,$$

where $C_5 = C_5(A, B, p_k)$ is an effectively computable constant depending only on A, B and p_k.

PROOF. The statement is an immediate consequence of a classical result of Győry [10]; see also [9, Ch. 4]. □

PROOF OF THEOREM 2.1. The statement immediately follows by Lemma 3.1 as soon as $f(x)$ has two distinct roots. Therefore, we can assume that $f(x)$ is of the form $f(x) = u(x + v)^d$, with some $u \in \mathbb{Z}$ and $v \in \mathbb{Q}$. Writing $v = v_1/v_2$ in its primitive form, we clearly have $v_2^d \mid u$ and $u, v_1, v_2 \in \mathbb{Z}_S$. Then, checking the coefficients of x^{d-1} and x^{d-2} in f, we easily see that $d, \binom{d}{2} \in \mathbb{Z}_S$. Hence, either $d - 1 \in \mathbb{Z}_S$, or $d - 1$ is even and $(d - 1)/2 \in \mathbb{Z}_S$. In the first case, $d, d - 1 \in \mathbb{Z}_S$ satisfy the equation

$$w_1 - w_2 = 1, \tag{3.1}$$

while, in the second case, $d, (d - 1)/2 \in \mathbb{Z}_S$ satisfy the equation

$$w_1 - 2w_2 = 1 \tag{3.2}$$

with $w_1, w_2 \in \mathbb{Z}_S$. However, by Lemma 3.2, for the solutions of these equations,

$$\max(|w_1|, |w_2|) < C_6,$$

where $C_6 = C_6(p_k)$ is an effectively computable constant depending only on p_k. So, if $d > C_6$, then d cannot come from a solution of either (3.1) or (3.2), which implies that $f(x)$ is not of the form $u(x + v)^d$. Hence, taking $C_1 = C_6$, the statement follows. □

Now we turn to the proof of Theorem 2.3. For this, we shall need five further lemmas. The first four of them are new and of possible independent interest.

LEMMA 3.3. *Let m be a nonnegative integer and let*

$$G(x) = b_0 x^t + b_1 x^{t-1} + \cdots + b_{t-1} x + b_t \tag{3.3}$$

with $b_0, b_1, \ldots, b_t \in \mathbb{Z}$ such that all the coefficients of the polynomial $(x-1)^m G(x)$ belong to $\{-1, 1\}$. Then $b_1 = 0$ implies that $m = 1$.

PROOF. Clearly, without loss of generality, we may assume that $b_0 = 1$. We shall do so during the proof, without further mention. Write B_m $(m \geq 0)$ for the set of coefficients b_1 that occur in some polynomial $G(x)$ (which is monic, of arbitrary degree) satisfying the conditions of the statement. We prove the lemma by describing the sets B_m inductively.

Obviously, we have $B_0 = \{-1, 1\}$. Assume that we have already described the set B_m for some $m \geq 0$, and consider a polynomial $G(x)$ given by (3.3), such that all the coefficients of $(x-1)^{m+1} G(x)$ belong to $\{-1, 1\}$. Then, of course, the same is true for the polynomial

$$(x-1)^{m+1} G(x) = (x-1)^m ((x-1)G(x))$$
$$= (x-1)^m (x^{t+1} + (b_1 - 1)x^t + \cdots + (b_t - b_{t-1})x - b_t).$$

Thus, $b_1 - 1 \in B_m$. It follows immediately that

$$B_{m+1} = B_m + \{1\} \ (= \{h + 1 : h \in B_m\}) \quad \text{for all } m \geq 0,$$

which inductively gives

$$B_m = \{m - 1, m + 1\} \quad \text{for all } m \geq 0.$$

Hence, the statement follows. □

The next lemma describes how the coefficients of a polynomial $G(x)$ can 'spread' if $(x-1)^m G(x)$ is a Littlewood polynomial. In fact, for our present purposes, we only need a specific consequence of this statement (namely, for the coefficient of the second largest power of x), but we find it interesting to describe this phenomenon completely.

LEMMA 3.4. *Let $G(x) \in \mathbb{Z}[x]$ and let m be a nonnegative integer. If all the coefficients of $(x-1)^m G(x)$ belong to $\{-1, 1\}$, then, writing*

$$G(x) = b_0 x^t + b_1 x^{t-1} + \cdots + b_{t-1} x + b_t$$

for all $i = 0, 1, \ldots, t$, we have

$$-\min\left(\binom{m+i}{m}, \binom{m+t-i}{m}\right) \leq b_i \leq \min\left(\binom{m+i}{m}, \binom{m+t-i}{m}\right).$$

Here we use the convention $\binom{0}{0} = 1$.

REMARK 3.5. We note that, as one can easily check by following the proof, the bounds given for the coefficients of $G(x)$ are sharp.

PROOF. First, we show that

$$-\binom{m+i}{m} \leq b_i \leq \binom{m+i}{m} \quad \text{(for } i = 0, 1, \ldots, t), \tag{3.4}$$

by induction on m. For $m = 0$, the statement is obvious. Assume that (3.4) holds for some $m \geq 0$, and assume that all the coefficients of $(x - 1)^{m+1}G(x)$ belong to $\{-1, 1\}$. In view of

$$(x - 1)^{m+1}G(x) = (x - 1)^m((x - 1)G(x))$$
$$= (x - 1)^m(b_0 x^{t+1} + (b_1 - b_0)x^t + \cdots + (b_t - b_{t-1})x - b_t),$$

the induction hypothesis by $b_i = (b_i - b_{i-1}) + b_{i-1}$ $(i \geq 1)$ successively yields

$$-1 \leq b_0 \leq 1, \quad -1 - \binom{m+1}{1} \leq b_1 \leq 1 + \binom{m+1}{1},$$
$$-1 - \binom{m+1}{1} - \binom{m+2}{2} \leq b_2 \leq 1 + \binom{m+1}{1} + \binom{m+2}{2}, \ldots.$$

By a well-known identity,

$$-\binom{m+1+i}{m+1} \leq b_i \leq \binom{m+1+i}{m+1} \quad \text{for } 0 \leq i \leq t,$$

that is, (3.4) is valid also with m replaced by $m + 1$. So, (3.4) holds for all m. Now, observing that $b_t = \pm 1$ and starting the argument at the constant term and going backwards (or, alternatively, working with reciprocal polynomials), a similar argument gives

$$-\binom{m+t-i}{m} \leq b_i \leq \binom{m+t-i}{m} \quad \text{for } i = 0, 1, \ldots, t.$$

Hence, the lemma follows. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

LEMMA 3.6. *Let $n \geq 2$ and let $g(x) \in \mathbb{Z}[x]$ be a nonzero polynomial. If all the coefficients of $(x - 1)^{n-1}g^n(x)$ belong to $\{-1, 1\}$, then $n = 2$.*

PROOF. Write

$$F(x) = (x - 1)^{n-1}g^n(x),$$

and assume that all the coefficients of $F(x)$ belong to $\{-1, 1\}$. Then

$$(x - 1)F(x) = ((x - 1)g(x))^n,$$

and hence

$$H(((x - 1)g(x))^n) = H((x - 1)F(x)) \leq 2. \tag{3.5}$$

Put

$$G(x) = (x - 1)g(x).$$

Obviously, the constant term of $G(x)$ is ± 1. Let ℓ be the smallest positive exponent for which the coefficient of x^ℓ in $G(x)$ is nonzero. (Clearly, such an ℓ exists because $\deg G \geq 1$.) Write u_ℓ for the coefficient of x^ℓ in $G(x)$. Then the coefficient of x^ℓ in

$$((x - 1)g(x))^n = (G(x))^n$$

is $\pm nu_\ell$. Then, by (3.5), we get $| \pm nu_\ell| \le 2$. This implies that $n \le 2$ (in fact, $n = 2$), and the statement follows. $\square$

LEMMA 3.7. *Let $g(x) \in \mathbb{Z}[x]$ be a nonconstant polynomial and let m, n be integers with $0 \le m < n$. If all the coefficients of the polynomial $(x - 1)^m(g(x))^n$ belong to $\{-1, 1\}$, then $n = 2$, $m = 1$ and $g(x)$ is of the form*

$$g(x) = \pm(x^\ell + \cdots + x + 1)$$

with some $\ell \ge 1$.

PROOF. Write

$$g(x) = u_0 x^\ell + u_1 x^{\ell-1} + \cdots + u_{\ell-1}x + u_\ell$$

and

$$G(x) = (g(x))^n = b_0 x^t + b_1 x^{t-1} + \cdots + b_{t-1}x + b_t.$$

Clearly, $u_0 = \pm 1$ and $b_0 = \pm 1$. Further, $b_1 = \pm nu_1$. However, Lemma 3.4 implies that

$$|b_1| \le m + 1.$$

Hence, either $u_1 = b_1 = 0$, or, by $m < n$, we obtain $m = n - 1$. In the former case, $m = 1$ by Lemma 3.3. Then the property that $(x - 1)G(x)$ has only ± 1 coefficients easily implies that $b_2 = \pm 1$. On the other hand (as $t \ge 2$ and $u_1 = 0$), we also have

$$b_2 = \pm nu_2,$$

which is not possible because $n \ge 2$. Hence, $u_1 = 0$ cannot hold, and we are left with the case $m = n - 1$. Then $n = 2$ by Lemma 3.6. Thus, $b_0 = 1$. Further, without loss of generality, we may also assume that $u_0 = 1$. (Then, having described the possible polynomials $g(x)$, we only need to insert a $\pm$ sign in front of them.) So, we can write

$$\begin{aligned}
x^t + b_1 x^{t-1} + \cdots + b_{t-1}x + b_t &= (x^\ell + u_1 x^{\ell-1} + \cdots + u_{\ell-1}x + u_\ell)^2 \\
&= x^{2\ell} + 2u_1 x^{2\ell-1} + (u_1^2 + 2u_2)x^{2\ell-2} + (2u_3 + 2u_1 u_2)x^{2\ell-3} + \cdots.
\end{aligned} \tag{3.6}$$

We show that here we necessarily have $u_i = 1$ $(i = 1, \ldots, \ell)$. For this, recall that all the coefficients of $(x - 1)(g(x))^2$, that is, of

$$(x - 1)(x^t + b_1 x^{t-1} + \cdots + b_{t-1}x + b_t) = (x^{t+1} + (b_1 - 1)x^t + \cdots + (b_t - b_{t-1})x - b_t)$$

are ± 1. That is,

$$b_1 - 1, b_t, b_i - b_{i-1} \in \{-1, 1\} \quad \text{for } 2 \le i \le t. \tag{3.7}$$

In particular, b_i is even if i is odd, and b_i is odd if i is even. Since $b_1 \ne 0$, $b_1 - 1 = 1$ and hence $b_1 = 2$. Comparing the coefficients of x^{t-1} in (3.6) gives $u_1 = 1$. Inductively, assume that $b_i = i + 1$ and $u_i = 1$ for some i with $1 \le i < \ell$. Then (3.7) yields $b_{i+1} \in \{i, i + 2\}$, while (3.6) gives

$$b_{i+1} = u_0 u_{i+1} + u_1 u_i + \cdots + u_{i+1} u_0.$$

This shows that $u_{i+1} \in \{0, 1\}$. However, $u_i = 0$ is impossible. Indeed, otherwise, again by (3.6), $b_{2(i+1)}$ is even, since, in the coefficient of $x^{2(i+1)}$, all products $u_j u_{2(i+1)-j}$ occur twice except for u_i^2, which is assumed to be zero. However, $b_{2(i+1)}$ is known to be odd. Thus, we see that $u_{i+1} = 1$ (and $b_{i+1} = i + 2$). So, the only possibility is given by

$$g(x) = \pm(x^\ell + \cdots + x + 1).$$

Finally, we have to check that the polynomial $g(x)$ given above satisfies the requirements of the lemma (with $n = 2$ and $m = 1$). Indeed,

$$(x - 1)(g(x))^2 = (x - 1)\left(\frac{x^{\ell+1} - 1}{x - 1}\right)^2$$
$$= (x^{\ell+1} - 1)\frac{x^{\ell+1} - 1}{x - 1} = x^{2\ell+1} + \cdots + x^{\ell+1} - x^\ell - \cdots - x - 1.$$

Thus, the lemma is proved.   □

The following lemma is a theorem of Brindza [5]. For its statement, we need some further notation. For any finite set S of primes, write $\mathbb{Q}_S$ for those rationals whose denominators (in their primitive forms) are composed exclusively from the primes in S. By the height $h(s)$ of a rational number s we mean the maximum of the absolute values of the numerator and the denominator of s (written again in primitive form).

LEMMA 3.8. *Let $F(x) \in \mathbb{Z}[x]$ of degree D and height H and write*

$$F(x) = A \prod_{i=1}^{\ell} (x - \gamma_i)^{r_i},$$

where A is the leading coefficient of F, and $\gamma_1, \ldots, \gamma_\ell$ are the distinct complex roots of $F(x)$, with respective multiplicities $r_1, \ldots, r_\ell$. Further, let n be an integer with $n \geq 2$, and put

$$q_i = \frac{n}{(n, r_i)} \quad \text{for } i = 1, \ldots, \ell.$$

Suppose that $(q_1, \ldots, q_\ell)$ is not a permutation of any of the ℓ-tuples

$$(q, 1, \ldots, 1) \quad \text{for all } q \geq 1 \quad \text{and} \quad (2, 2, 1, \ldots, 1).$$

Then, for any finite set S of primes and nonzero rational B, the solutions $x, y \in \mathbb{Q}_S$ of the equation

$$F(x) = By^n$$

satisfy

$$\max(h(x), h(y)) < C_7(B, n, D, H, S),$$

where $C_7(B, n, D, H, S)$ is an effectively computable constant depending only on B, n, D, H, S.

PROOF OF THEOREM 2.3. First, we show that n can be bounded in the required way. Following the lines of the proof of Theorem 2.1, we see that it is sufficient to exclude the case when $f(x)$ is of the form $(x \pm 1)^d$. (In the notation of the proof of Theorem 2.1, we need to have $u = \pm 1$ and $v = \pm 1$.) However, this is clearly impossible.

Hence, from this point on, we may suppose that $n \geq 2$ is fixed. Thus, our statement immediately follows from Lemma 3.8, except in the following two cases:

(i) $n = 2$ and $f(x) = h(x)(g(x))^2$, where $\deg h = 2$ and $h(x), g(x) \in \mathbb{Z}[x]$; and

(ii) n is arbitrary and $f(x) = (h(x))^m (g(x))^n$, where $\deg h \leq 1$, $0 \leq m < n$ and $h(x), g(x) \in \mathbb{Z}[x]$.

Clearly, without loss of generality, we may assume that all the polynomials f, g, h are monic.

In case (i), write $h(x) = x^2 + v_1 x + v_2$ and $g(x) = x^\ell + u_1 x^{\ell-1} + \cdots + u_\ell$. Clearly, $v_2 = \pm 1$. Further,

$$g^2(x) \equiv x^{2\ell} + u_1^2 x^{2\ell-2} + \cdots + u_\ell^2 \pmod 2.$$

Thus, if v_1 is even, then all odd coefficients of $h(x)g^2(x)$ are even, which is a contradiction. So, v_1 must be odd. However, then

$$h(x)g^2(x) \equiv x^{2\ell+2} + u_1^2 x^{2\ell+1} + (u_1^2 + 1)x^{2\ell} + \cdots \pmod 2.$$

Thus, either the coefficient of $x^{2\ell+1}$ or that of $x^{2\ell}$ is even, but this is impossible. So, case (i) cannot hold.

Now, consider case (ii). We can suppose that the polynomials f, g, h are monic and $h(x) = x - 1$. (Indeed, if $h(x) = x + 1$, which is the only other possibility, then, after the substitution $x \to -x$ and multiplying equation (2.2) by an appropriate power of -1, we are easily back to this case.) Thus, the statement follows from Lemma 3.7. (The second possibility for $f(x)$ comes from the case $h(x) = x + 1$.) □

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References

[1] A. Bloch and G. Pólya, 'On the roots of certain algebraic equations', *Proc. Lond. Math. Soc. (3)* **33** (1932), 102–114.

[2] P. Borwein and T. Erdélyi, *Polynomials and Polynomial Inequalities* (Springer-Verlag, New York, 1995).

[3] P. Borwein and T. Erdélyi, 'On the zeros of polynomials with restricted coefficients', *Illinois J. Math.* **41** (1997), 667–675.

[4] P. Borwein and M. J. Mossinghoff, 'Polynomials with height 1 and prescribed vanishing at 1', *Exp. Math.* **9** (2000), 425–433.

[5] B. Brindza, 'On S-integral solutions of the equation $y^m = f(x)$', *Acta Math. Hungar.* **44** (1984), 133–139.

[6] P. Drungilas and A. Dubickas, 'Roots of polynomials of bounded height', *Rocky Mountain J. Math.* **39** (2009), 527–543.

[7]   A. Dubickas and J. Jankauskas, 'On Newman polynomials which divide no Littlewood polynomial', *Math. Comp.* **78** (2009), 327–344.

[8]   P. Erdős and P. Turán, 'On the distribution of roots of polynomials', *Ann. of Math. (2)* **57** (1950), 105–119.

[9]   J.-H. Evertse and K. Győry, *Unit Equations in Diophantine Number Theory* (Cambridge University Press, Cambridge, 2015).

[10] K. Győry, 'Sur l'irréducibilité d'une classe des polynômes, II', *Publ. Math. Debrecen* **19** (1972), 293–326.

[11] M. J. Mossinghoff, 'Polynomials with restricted coefficients and prescribed noncyclotomic factors', *LMS J. Comput. Math.* **6** (2003), 314–325.

[12] R. Peled, A. Sen and O. Zeitouni, 'Double roots of random Littlewood polynomials', *Israel J. Math.* **213** (2016), 55–77.

[13] A. Schinzel and R. Tijdeman, 'On the equation $y^m = P(x)$', *Acta Arith.* **31** (1976), 199–204.

[14] I. Schur, 'Untersuchungen über algebraische Gleichungen', *Sitz. Preuss. Akad. Wiss., Phys.-Math. Kl.* **7–10**(1933), 403–428.

[15] T. N. Shorey and R. Tijdeman, *Exponential Diophantine Equations* (Cambridge University Press, Cambridge, 1986).

[16] G. Szegő, 'Bemerkungen zu einem Satz von E. Schmidt über algebraische Gleichungen', *Sitz. Preuss. Akad. Wiss., Phys.-Math. K1.* **8**(1934), 86–98.

[17] R. Tijdeman, 'Applications of the Gel'fond–Baker method to rational number theory', in: *Topics in Number Theory (Proceedings of the Conference at Debrecen 1974)*, Colloquia Mathematica Societatis János Bolyai, 13 (ed P. Turán) (North-Holland, Amsterdam, 1976), 399–416.

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