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ARTICLES

OPTIMAL GROWTH, DEBT, CORRUPTION, AND R&D

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This paper analyzes optimal paths in a one-sector growth model when the technology is not convex. In such a case, we prove that optimal paths converge to the upper steady state iff the initial wealth is above a critical level. Then, we first show that, thanks to debt and/or R&D, the poverty trap may be avoided. Second, we introduce a distortion: corruption that mostly has dramatic consequences on growth, but may have a beneficial effect if it is not high and if it improves productivity (incentive effect).

Keywords: Optimal Growth, Debt, Corruption, R&D

1. INTRODUCTION

The debate on the effects of corruption on growth is particularly fervent in international organizations, such as the World Bank, the IMF, and the OECD. These organizations claim that, in too many cases, corruption explains, at least in part, poor economic performance either in developing countries or in transitional countries. As a matter of fact, the IMF sees in corruption one of the main causes for the East Asian financial crises as pointed out by Radelet and Sachs (1998).

Similarly, applied studies on economic growth suggest that corruption lowers investment, thereby lowering growth; see, for example, (among many others) Bigsten and Moene (1993) for the case of Kenya, Rose-Ackerman (1999) for Eastern European countries and especially Russia. Moreover, Mauro (1995) finds a significant negative association between corruption and growth for a cross section of 57 countries.

Many economists have spent a great deal of time trying to understand why and how corruption affects economic performance. Basically, for the *reformists*, corruption might raise the economy by improving efficiency of allocation when red tape is cumbersome and secret, whereas, for the *moralists*, corruption always has an adverse effect on the economy [see Bardhan (1997) for a complete survey]. Nevertheless, most of previous studies are static rather than dynamic. In this study, we analyze the effects of corruption in the framework of optimal growth theory.

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Let us recall some stylized facts: In the Ramsey–Cass–Koopmans (see, e.g., Cass (1965)) model of growth, countries converge in the long run; thus, this conclusion follows the hypothesis of diminishing returns in the production function. On the contrary, as in Frankel (1962), when the production function exhibits increasing returns, countries do not converge and in some cases a poverty trap may exist. Similarly, Dechert and Nishimura (1983) prove that when the production function exhibits an initial phase of increasing returns and a second phase of decreasing returns, poor countries are likely to find themselves in a low-development trap [for an optimal growth model with convex-concave technology, see also Amir et al. (1991)]. The same conclusion holds when allowing for indebtedness [see Askenazy and Le Van (1999) for detailed proofs in continuous time]. The common point of these studies is the underlying hypothesis of a benevolent social planner. We depart from this usual assumption.

To pose the problem, the production function is as in Askenazy and Le Van (1999) and the country contracts a debt. However, in a first stage, some of the bureaucrats are corrupted and divert a fraction of international debt. Therefore, we prove (as expected) that corruption has a negative effect on growth. In a second stage, we assume that bureaucrats are social welfare maximizers. However, some of the entrepreneurs divert at each period of time a fraction of national wealth for extra consumption. Therefore, there is an incentive problem. We assume that this extra consumption may (but will not necessarily) become an incentive to work, inducing an increase of productivity. We have in mind the case of countries such as Vietnam and China, where low wages explain embezzlement of wealth by entrepreneurs who seek to enhance their consumption in goods that are not now subject to shortages, thanks to the introduction of market economy, and growth does not decline. To make this point clear, the incentive to manage better follows one of these two processes: First, embezzlement increases the total wealth of the corrupted entrepreneurs. Therefore, we assume an increase in their total consumption. For a fixed fraction of the money that they can divert, the only possible way to enhance this increase (in illegal income/consumption) is for entrepreneurs to increase the total productivity (theirs and others') in the economy, and consequently the total amount of wealth they can divert in this economy. Second, one might assume that only top managers can be corrupted, and thus there is an incentive for other entrepreneurs to increase their productivity in order to became top managers and consequently benefit from corruption. This paper proposes a model of optimal growth that, although simple, broadly fits the pattern of what we know about corruption.

The plan of this paper is as follow: Section 2 and Section 3 introduce our model, Section 4 discusses the implication of debt. In Section 5, we introduce corruption into the previous framework. Finally, in Section 6 we explore optimal paths when international aid is devoted to R&D. The last section concludes.

2. BENCHMARK MODEL

We consider an open developing economy. A household maximizes the discounted, infinite stream of utility:

$$\max_{C_t} \sum_{t=0}^{\infty} \beta^t U(C_t),$$

where C_t denotes the consumption.

Assumption 1. Assume that the utility function U satisfies the following conditions:

(i) U is twice continuously differentiable, $U'(C) > 0$, $U''(C) < 0$, $\forall\ C > 0$ and $U(0) = 0$, $U'(0) = +\infty$.

Concerning the time-discount rate β, we assume that

(ii) $1 > \beta > 0$ and $\beta(1 + r) < 1$, where $r > 0$ is the international interest rate.

As a matter of fact, let r^* be the national interest rate such that $\beta(1 + r^*) = 1$ and, from Assumption 1 (ii), $\beta(1 + r) < 1$; therefore $r^* > r$. In other words, we assume that international lenders offer a lower interest rate. Moreover, a number of studies have indicated that one of the consequences of a development process is a rise in the consumers' subjective time preference rate [see Firoozi (1995)]; for the sake of simplicity, we assume a fixed lower discount rate in developing countries.

A single output is produced with the aid of one factor of production.

Assumption 2. The production function f is assumed to be

(i) continuously differentiable, $f' > 0$, and there exists k_I such that $f''(k) > 0$ for $k < k_I$ and $f''(k) < 0$ for $k > k_I$. In other words, the production function exhibits an initial phase of increasing returns, and a second phase with decreasing returns.

(ii) It is further assumed that $f(0) = 0$, $f'(0) < r + \delta$ (δ is the depreciation rate), and $f'(\infty) = 0$.

Therefore, there exist k_1, k_2 such that $f'(k_1) = f'(k_2) = (r + \delta)$. Furthermore,

(iii) $f(k_2) - (r + \delta)k_2 > 0$.[1]

Then there exist $\bar{k}, \tilde{k}$ verifying $0 < \bar{k} < k_2 < \tilde{k}$, $f(\bar{k}) = (r + \delta)\bar{k}$, and $f(\tilde{k}) = (r + \delta)\tilde{k}$.

At each point in time, the country owns a stock of wealth D_t such that $D_t = S_t + k_t$, where S_t is an asset that returns rS_t, and k_t is the stock of capital. For simplicity, we assume that the country is not allowed to borrow, and then $S_t \geq 0$. This assumption is relaxed in the next section.

We have the following trade balance under free trade and perfect substitutability between asset S_t and capital k_t:

$$M_t = C_t + I_t - f(k_t), \tag{1}$$

where I_t denotes the investment at time t; that is,

$$I_t = k_{t+1} - (1 - \delta)k_t, \tag{2}$$

$$M_t = (1 + r)S_t - S_{t+1}. \tag{3}$$

The dynamics of S_t are as follows: $S_{t+1} = (1+r)S_t - M_t$. By substituting I_t and M_t into (1), and noting that $S_t = D_t - k_t$, one obtains

$$C_t + D_{t+1} = f(k_t) - (r+\delta)k_t + (1+r)D_t. \tag{4}$$

For the open economy, the problem of intertemporal optimization can be written as

$$\begin{cases} \max\limits_{(C_t, D_t, k_t)} \sum\limits_{t=0}^{\infty} \beta^t U(C_t) \\ C_t + D_{t+1} \leq f(k_t) - (r+\delta)k_t + (1+r)D_t \\ C_t \geq 0,\, D_t \geq 0,\, D_0 \text{ given, and } k_t \in [0, D_t]. \end{cases}$$

Since the function U is strictly increasing, if the sequence $\tilde{C} = \{\tilde{C}_t\}$ is a solution, then it must be a solution to the following problem:

$$\begin{cases} \max\limits_{(C_t, D_t, k_t)} \sum\limits_{t=0}^{\infty} \beta^t U(C_t) \\ C_t + D_{t+1} \leq \max\{f(k_t) - (r+\delta)k_t\} + (1+r)D_t \\ C_t \geq 0,\, D_t \geq 0,\, D_0 \text{ given, and } k_t \in [0, D_t]. \end{cases}$$

We define $\varphi(k) = f(k) - (r+\delta)k$, $k \in [0 + \infty[$. From Assumptions 2(i)–(iii), the graph of φ is as in Figure 1. We then define $\Phi(D) = \max_{k \in [0,D]}\{\varphi(k)\}$. The constraints become

$$C_t + D_{t+1} \leq \Phi(D_t) + (1+r)D_t.$$

We can compute the function $\Phi(D)$ as follows:

$$\begin{cases} 0 \leq D \leq \bar{k} \Rightarrow \Phi(D) = 0 \\ \bar{k} \leq D \leq k_2 \Rightarrow \Phi(D) = f(D) - (r+\delta)D \\ k_2 \leq D \Rightarrow \Phi(D) = f(k_2) - (r+\delta)k_2 \overset{\text{def}}{=} \bar{A} > 0. \end{cases}$$

The graph of Φ is as in Figure 1. We note that Φ is continuous and continuously differentiable except at $\bar{k}$, where Φ has left and right derivatives that are different. For simplicity, we define $\Psi(D) = \Phi(D) + (1+r)D$; then,

$$\begin{cases} 0 \leq D \leq \bar{k} \Rightarrow \Psi(D) = (1+r)D \Rightarrow \Psi'(D) = (1+r) \\ \bar{k} \leq D \leq k_2 \Rightarrow \Psi(D) = f(D) + (1-\delta)D \Rightarrow \Psi'(D) = f'(D) + (1-\delta) \\ k_2 \leq D \Rightarrow \Psi(D) = \bar{A} + (1+r)D \Rightarrow \Psi'(D) = (1+r) \end{cases}$$

Remark 1. In $\bar{k}$, we have $\Psi'_+(\bar{k}) = f'(\bar{k}) + 1 - \delta > 1 + r = \Psi'_-(\bar{k})$.

For the open economy, the problem of intertemporal optimization is now written as

$$(\wp) \begin{cases} \max\limits_{(C_t, D_t)} \sum\limits_{t=0}^{\infty} \beta^t U(C_t) \\ C_t + D_{t+1} \leq \Psi(D_t) \\ C_t \geq 0,\, D_t \geq 0,\, D_0 \text{ given.} \end{cases}$$

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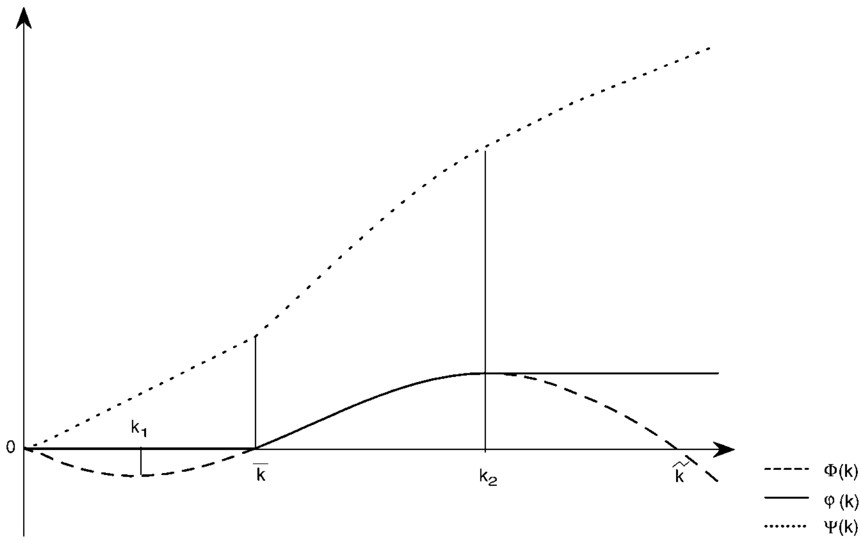

Figure 1. Graphs of functions: Φ, φ, Ψ.

Since U is strictly increasing, it is equivalent to

$$\begin{cases} \max \sum_{t=0}^{\infty} \beta^t U[\Psi(D_t) - D_{t+1}] \\ \forall\, t,\, 0 \le D_{t+1} \le \Psi(D_t) \\ D_0 \ge 0 \text{ is given.} \end{cases}$$

Let W denote the value function; that is,

$$\begin{cases} W(D_0) = \max \sum_{t=0}^{\infty} \beta^t U[\Psi(D_t) - D_{t+1}] \\ \forall\, t,\, D_{t+1} \le \Psi(D_t) \\ D_t \ge 0,\, D_0 \ge 0 \text{ given.} \end{cases}$$

A path $\tilde{D}$ is said to be *feasible* from D_0 if it verifies the above inequalities.

3. PROPERTIES OF OPTIMAL PATHS

The following theorem summarizes the different results on the optimal paths and the value function.

THEOREM 1.

(i) *For any $D_0 \ge 0$, there exists an optimal path $\{D_1^*, D_2^*, \ldots, D_t^*, \ldots\}$. Any optimal path is monotonic.*

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(ii) *The value function verifies the Bellman equation*,

$$\forall D_0 \geq 0, W(D_0) = \max_{0 \leq D_1 \leq \Psi(D_0)} \{U[\Psi(D_0) - D_1] + \beta W(D_1)\}.$$

W is continuous and is the unique solution to the Bellman equation, which is bounded by an affine function. If γ is argmax correspondence of the Bellman equation, then γ is upper semicontinuous. For any optimal path $\{D_1^*, D_2^*, \ldots, D_t^*, \ldots\}$, one has $D_1^* \in \gamma(D_0)$ and $D_{t+1}^* \in \gamma(D_t^*), \forall t$.

(iii) *An optimal path verifies the Euler equation*:

$$\forall t, U'[\Psi(D_t^*) - D_{t+1}^*] = \beta \Psi'(D_{t+1}^*) U'[\Psi(D_{t+1}^*) - D_{t+2}^*].$$

(iv) *There exists an optimal steady state $k^S > 0$; that is, $k^S \in \gamma(k^S)$.*

(v) *Assume, moreover, that*

$$\frac{1}{\beta} + r < \max \frac{f(D)}{D} + 1 - \delta.$$

There exists a poverty trap, that is, a critical value D^c, such that

(a) *if $D_0 < D^c$, then any optimal path from D_0 will converge to zero.*

(b) *if $D_0 > D^c$, then any optimal path from D_0 will converge to k^S.*

Proof.

(i) One can easily check that, for any feasible path $\{D_1, D_2, \ldots, D_t, \ldots\}$, that is, $\forall t, 0 \leq D_{t+1} \leq \Psi(D_t)$, one has $\forall t, 0 \leq D_t \leq A(1+r)^t$, for some $A > 0$ that depends on D_0.

Let $\prod(D_0)$ denote the set of feasible paths from D_0. One can prove that it is a compact set for the product topology.

Let $\tilde{D}$ denote the sequence $\{D_1, D_2, \ldots, D_t, \ldots\}$. Under our assumptions, the function

$$\tilde{U} : \tilde{D} = \{D_0, D_1, \ldots, D_t, \ldots\} \in \prod(D_0) \to \sum_{t=0}^{\infty} \beta^t U[\Psi(D_t) - D_{t+1}]$$

is upper-semi-continuous on $\prod(D_0)$ for the product topology. Since the problem is equivalent to $\max \{U(\tilde{D}) \mid \tilde{D} \in \prod(D_0)\}$, there always exists an optimal solution.

The proof of monotonicity of an optimal path is given by Dechert and Nishimura (1983).

(ii) Obviously, W verifies the Bellman equation. One can easily check that

$$\sup_{D \geq 0} \left\{ \frac{W(D)}{1+D} \right\} < +\infty.$$

Define

$$\|W\| = \sup_{D \geq 0} \frac{|W(D)|}{1+D}.$$

The mapping T,

$$Tf(D_0) = \max_{0 \leq D_1 \leq \Psi(D_0)} \{U[\Psi(D_0) - D_1] + \beta f(D_1)\},$$

is a contraction in the Banach space of functions endowed with the previous norm. W is the unique fixed point of T. Moreover, $W = \lim_{n \to \infty} T^n 0$. By the maximum theorem, $T^n 0$ is continuous for any n. W is the uniform limit of $\{T^n 0\}$ in any compact set of $\mathbf{R}_+$. Hence, W is continuous. Obviously, γ, the argmax correspondence, is upper-semi-continuous [the maximum theorem] and, if $\tilde{D}^*$ is an optimal path, then $D^*_{t+1} \in \gamma(D^*_t), \forall t$.

(iii) We have just to verify that if $\tilde{D}^*$ is an optimal path, then $D^*_t \neq \bar{k}$ for any $t > 0$, but that follows from Askri and Le Van (1998).

(iv, v) Use the proof given by Dechert and Nishimura (1983). ■

Remark 2. There exist two values, k'^s and k^s such that $\Psi'(k'^s) = \Psi'(k^s) = 1/\beta$, $(k'^s < k^s)$. Theorem 1 states that, for $D_0 > k^c$, any optimal path converges to k^s. As in Dechert and Nishimura (1983), we have this result: No optimal path converges to k'^s.

4. OPTIMAL GROWTH AND DEBT

Consider now an economy described as in the preceding model, with the difference that it borrows at date 0 a stock $\bar{E} \geq 0$ from some international organization; in return, it pays a perpetual rent $r\bar{E}$. Let D'_0 denote the initial resource before borrowing. The initial stock of wealth becomes $D_0 = D'_0 + \bar{E}$. The dynamics of the asset S_t now become

$$S_{t+1} = (1+r)S_t - M_t - r\bar{E}.$$

We now deal with the following problem:

$$\begin{cases} \max\limits_{(C_t, D_t, k_t)} \sum\limits_{t=0}^{\infty} \beta^t U(C_t) \\ C_t + D_{t+1} \leq f(k_t) - (r+\delta)k_t + (1+r)D_t - r\bar{E} \\ C_t \geq 0, D_t \geq 0, D_0 = D'_0 + \bar{E} \text{ given}, \ k_t \in [0, D_t]. \end{cases}$$

One can easily check that this problem is equivalent to

$$\begin{cases} \max\limits_{(C_t, D_t)} \sum\limits_{t=0}^{\infty} \beta^t U[\Psi(D_t) - D_{t+1} - r\bar{E}] \\ 0 \leq D_{t+1} \leq \Psi(D_t) - r\bar{E} \\ D_t \geq 0, D_0 = D'_0 + \bar{E} \text{ given}. \end{cases}$$

Let $\hat{D}(\bar{E}) > 0$ verify $\hat{D}(\bar{E}) = \Psi[\hat{D}(\bar{E})] - r\bar{E}$. The proof of the following result is provided by Dimaria and Le Van (1999).

THEOREM 2.

(i) *There exists a unique $\hat{D}(\bar{E})$. $\hat{D}(0) = 0$. $\hat{D}$ increases with $\bar{E}$. The stationary sequence $\{\hat{D}, \hat{D}, \ldots, \hat{D}, \ldots\}$ is feasible from D_0.*

(ii) *There exists a unique feasible path from $\hat{D}(\bar{E})$ that is the stationary sequence $\{\hat{D}(\bar{E}), \hat{D}(\bar{E}), \ldots, \hat{D}(\bar{E}), \ldots\}$.*

(iii) *Let* $\tilde{D}(\bar{E})$ *satisfy* $\Psi[\tilde{D}(\bar{E})] - (1/\beta)\tilde{D}(\bar{E}) - r\bar{E} = 0$. *Then,* $\tilde{D}(\bar{E}) > \hat{D}(\bar{E})$. $\tilde{D}$ *increases with* $\bar{E}$ *and* $\tilde{D}(0) < k^s$. *Assume that* $\Psi'_+[\hat{D}(\bar{E})] < 1/\beta$. *Then there exists* $D_c(\bar{E})$ *such that*
  (a) *if* $D_0 > D_c(\bar{E})$, *then all optimal paths from* D_0 *converge to* k^s;
  (b) *if* $D_0 < D_c(\bar{E})$, *then all optimal paths from* D_0 *converge to* $\hat{D}(\bar{E})$;
  (c) *if* $\hat{D}(\bar{E}) = k'^S$, *then every optimal path from* D_0 *converges to* k^s.

Remark 3. Theorem 2 shows that international aid is helpful for a developing country. The optimal capital stocks of this country either converge to k^S or to $\hat{D} > 0$. Moreover, if $\bar{E}$ is sufficiently large (such that $\hat{D} = k'^S$), then the optimal capital stocks converge to k^S. However, the counterpart is that, at the steady state, the level of consumption is lower than in the benchmark model. This remark points out the ambiguousness of the definition of a developed country based on its GNP per capita. Countries may reach the same high-level steady states of GNP but their consumptions at this steady state may drastically differ. To make this point clear, indebtedness may help the country to reach the upper steady state but it consequently lowers the steady state consumption per capita. Debt has a cost in terms of consumption because of debt burden equal to $r\bar{E}$.

5. CORRUPTION AND OPTIMAL GROWTH

We now analyze the dynamic behavior of an economy in which corruption takes place. In opposition to the main approach of corruption, we are not interested in the way corruption will increase or cease in the future of an economy. Nor will we give any recommendation about how to fight corruption. We take corruption as a datum of the economy. What could be the behavior of optimal paths when they exist? We have in mind a government where some of the bureaucrats use international aid for private unproductive consumption or to settle a slush fund. Next, we look at the dynamics of an economy in which entrepreneurs work to their best only if they can divert a part of national wealth for their own consumption.

5.1. The Case of Corrupted Bureaucrats

We consider the model from the preceding section, in which a developing country borrows a stock of wealth $\bar{E}$ from some international organization and pays in return a perpetual rent $r\bar{E}$. Assume that some bureaucrats divert a fraction $(1-\theta)\bar{E}$ of international aid. We assume that the bureaucrats are also the planners. In some countries, corruption may be found at very high levels of the administration. The effective initial stock of wealth is $D_0 \stackrel{\text{def}}{=} D'_0 + \theta\bar{E}$ (where D'_0 is the initial wealth of the country before borrowing). We consider the case in which there are two stages of corruption. At the first stage, some bureaucrats divert the fraction $(1-\theta)\bar{E}$ for their extra consumption. Therefore, at the first period we have

$$C_0 + I_0 + (1-\theta)\bar{E} \leq f(k_0) + M_0.$$

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In principle, the stock of assets at the first period, S_1, is

$$S_1 = (1+r)S_0 - M_0 - r\bar{E} + (1-\theta)\bar{E} \quad \text{and} \quad S_0 + k_0 = D'_0 + \theta\bar{E}.$$

However, there are some other corrupt officials who divert $\theta'(1-\theta)\bar{E}$, $\theta' > 0$ and transfer it abroad. Hence,

$$S_1 = (1+r)S_0 - M_0 - r\bar{E} + (1-\theta')(1-\theta)\bar{E} \quad \text{and} \quad S_0 + k_0 = D'_0 + \theta\bar{E}.$$

One can verify that the constraints can be written as follows:

$$\begin{aligned}
&D_1 + C_0 + \theta'(1-\theta)\bar{E} \le \Psi(D_0) - r\bar{E},\\
&\forall\, t \ge 1,\ C_t + D_{t+1} \le \Psi(D_t) - r\bar{E}\\
&\text{with } D_0 = D_\theta = D'_0 + \theta\bar{E}.
\end{aligned}$$

PROPOSITION 1. *Assume, moreover,*

$$\max \frac{f(D)}{D} + 1 - \delta > \frac{1}{\beta} + r.$$

Then, there exists an interval $[0, k_0]$ *and* $\bar{E}$ *such that, if* $D'_0 \in [0, k_0]$, *then without corruption* $(\theta = 1)$ *the optimal paths will converge to the upper steady state* k^S; *if the corruption is very high* $(\theta \approx 0)$, *then either the optimal paths will collapse or no optimal solution will exist.*

Proof. If $\bar{E}$ is given, let $\tilde{D}(\bar{E})$ verify that

$$\Psi[\tilde{D}(\bar{E})] - \frac{1}{\beta}\tilde{D}(\bar{E}) = r\bar{E}.$$

We have

$$k^S > x > \tilde{D}(\bar{E}) \Rightarrow \Psi(x) - \frac{1}{\beta}x > r\bar{E}.$$

From Dechert and Nishimura (1983), if $D_0 > \tilde{D}(\bar{E})$, then every optimal path will converge to k^S.

Under the assumption

$$\max \frac{f(D)}{D} + 1 - \delta > \frac{1}{\beta} + r,$$

there exists $\bar{E} > 0$ such that

$$\Psi(\bar{E}) - \frac{1}{\beta}\bar{E} > r\bar{E}.$$

and

$$\tilde{D}(\bar{E}) < \bar{E} < k^S.$$

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Hence, for any D_0' small enough, one has

$$\Psi(D_0' + \bar{E}) - \frac{1}{\beta}(D_0' + \bar{E}) > r\bar{E},$$

and any optimal path from $D_0' + \bar{E}$ will converge to k^S. In other words, without corruption, international aid will help the country take off and thus converge to the upper steady state.

If D_0' is very small and $\theta = 0$, then $D_0 < D_c(\bar{E})$, and the optimal path will collapse or the problem has no solution. In other words, when the corruption is very high (θ is close to zero) the country collapses or there is no optimal solution even if the aid is very important.

5.2. The Case of Entrepreneurs

We assume that the entrepreneurs embezzle a fraction $(1 - \theta)$ of national wealth at each period of time for private consumption. The remaining part of the wealth θD_t is devoted to financial asset S_t and to capital stock k_t. We assume that the entrepreneurs at each period of time, by embezzling the $(1 - \theta)$ of the national wealth, improve the productivity. The production function of each period becomes $A(\theta)f(k)$, where A is decreasing, $A(1) = 1$, $A(0) > 1$.

At period t, we have the following balance:

$$C_t + (1 - \theta)D_t + I_t = A(\theta)f(k_t) + M_t,$$

where I_t is the planned investment and M_t denotes the trade balance. The stock of assets of the whole economy S'_{t+1} will be

$$S'_{t+1} = S_t(1 + r) - M_t - r\bar{E} + (1 - \theta)D_t,$$

with S_t the effective stock of assets after embezzling at period t. Similarly, one has

$$I_t = k'_{t+1} - (1 - \delta)k_t,$$

where k_t denotes the effective stock of capital at date t, k'_{t+1} the planned stock of capital for date $t + 1$. Let $D_t = S'_t + k'_t$. The constraints now become

$$C_t + D_{t+1} \leq A(\theta) \max\left\{ f(k) - \frac{r + \delta}{A(\theta)}k;\ k \in [0, \theta D_t] \right\} + (1 + r)\theta D_t - r\bar{E}$$

and $D_0 = (D_0' + \bar{E})$. ■

PROPOSITION 2. *Assume that* $A(\theta) = 1 + \lambda(1 - \theta)$. *Let* λ *be fixed. Then, for* θ *small enough (high corruption), the optimal path* $\{D_t\}$ *will collapse or there exists no optimal solution.*

Proof. Let k_θ verify

$$f(k_\theta) = \frac{r+\delta}{A(\theta)} k_\theta.$$

It can be checked that, if $\theta D_t \le k_\theta$, then

$$\varphi(\theta, \theta D_t) = \max\left\{ f(k) - \frac{r+\delta}{A(\theta)} k;\, k \in [0, \theta D_t] \right\} = 0.$$

Observe that k_θ decreases with θ. If θ is small enough, one has $\theta(D'_0 + \bar{E}) \le k_{\theta=0} \le k_\theta$. And, in this case, $D_1 \le (1+r)\theta D_0$. So, if θ is sufficiently small, one has $D_1 < D_0$. The optimal sequence $\{D_t\}$ is strictly decreasing. Hence, it must collapse. ∎

We will now prove that if embezzling improves the productivity, then it could be beneficial if corruption is not very high and if the incentive effect is important.

PROPOSITION 3. *Assume that* $f'(0) = 0$. *Assume, as before, that* $A(\theta, \lambda) = 1 + \lambda(1-\theta)$. *Let* D'_0, $\bar{E}$ *be given. There exists* $\bar{\theta}$ *such that, for any* $\theta \ge \bar{\theta}$, *there exists* λ^*_θ, *which verifies* $\forall\, \lambda > \lambda^*_\theta$, *the optimal path from* $D_0 = D'_0 + \bar{E}$ *will converge to a steady state.*

Proof. For the sake of simplicity, the proof of the proposition will be done in two steps.

Step 1. We will prove that there exists $\bar{\theta}$ such that if $\theta \ge \bar{\theta}$ then the feasible set is not empty. Let us recall that the constraints are now as follows:

$$\forall\, t, C_t + D_{t+1} \le \max\{A(\theta, \lambda) f(k) - (r+\delta)k;\, k \in [0, \theta D_t]\} + (1+r)\theta D_t - r\bar{E}.$$

We will exhibit a feasible sequence $\{D_0, \hat{D}, \hat{D}, \ldots\}$ if λ is larger than some $\bar{\lambda}$. We proceed as in Section 1. Let $\varphi(\theta, \lambda, D) = \max\{A(\theta, \lambda) f(k) - (r+\delta)k;\, k \in [0, \theta D]\}$. Since $f'(0) = 0$, for any $\lambda \ge 0$, there exist two points, $k_1(\theta, \lambda)$, $k_2(\theta, \lambda)$, such that

$$f'[k_i(\theta, \lambda)] = \frac{r+\delta}{1 + \lambda(1-\theta)}, \quad i = 1, 2.$$

Recall that $A(\theta, \lambda) = 1 + \lambda(1-\theta)$. Consider the point k_2 of Section 2 [i.e., $f'(k_2) = r + \delta$]. One has $k_2(\theta, \lambda) > k_2$ since $f'[k_2(\theta, \lambda)] < f'(k_2)$, and since f is strictly concave when $k > k_I$. One can also check that

$$f[k_2(\theta, \lambda)] - (r+\delta) k_2(\theta, \lambda) > 0,$$

and hence that

$$f[k_2(\theta, \lambda)] - \frac{(r+\delta)}{1 + \lambda(1-\theta)} k_2(\theta, \lambda) > 0.$$

Thus, there exists $\bar{k}(\lambda, \theta)$, verifying that

$$A(\theta, \lambda) f[\bar{k}(\lambda, \theta)] = (r+\delta)\bar{k}(\lambda, \theta).$$

As in Section 2, one obtains

(i) $0 \leq \theta D \leq \bar{k}(\lambda, \theta) \Rightarrow \varphi(\theta, \lambda, D) = 0$.
(ii) $\bar{k}(\lambda, \theta) \leq \theta D \leq k_2(\theta, \lambda) \Rightarrow \varphi(\theta, \lambda, D) = A(\theta, \lambda) f(\theta D) - (r + \delta)\theta D$.
(iii) $\theta D \geq k_2(\theta, \lambda) \Rightarrow \varphi(\theta, \lambda, D) = A(\theta, \lambda) f[k_2(\theta, \lambda)] - (r + \delta) k_2(\theta, \lambda)$.

Let $\chi(\lambda, \theta, D) = \varphi(\theta, \lambda, D) + (1 + r)\theta D - r\bar{E} - D$.

We claim that if

$$\theta \geq \bar{\theta} = \frac{r\bar{E} + D'_0 + \bar{E}}{(1 + r)(D'_0 + \bar{E})},$$

then

(i) there exists a unique $\hat{D}$ verifying $\chi(\lambda, \theta, \hat{D}) = 0$;
(ii) and $D_0 = D'_0 + \bar{E} \geq \hat{D}$ ($\hat{D}$ depends, of course, on θ and λ).

Therefore,

$$\hat{D} = \varphi(\theta, \lambda, \hat{D}) + (1 + r)\theta\hat{D} - r\bar{E} \leq \varphi(\theta, \lambda, D_0)(1 + r)\theta D_0 - r\bar{E},$$

and,

$$\hat{D} = \varphi(\theta, \lambda, \hat{D}) + (1 + r)\theta\hat{D} - r\bar{E}.$$

In other words, the sequence $\{\hat{D}, \hat{D}, \hat{D}, \ldots\}$ is feasible from D_0. One has $\chi(\lambda, \theta, 0) = -r\bar{E} < 0$. We have, $\chi(\lambda, \theta, \infty) = \lim_{D \to +\infty}[(1 + r)\theta - 1]D = +\infty$ since

$$\theta \geq \frac{r\bar{E} + D'_0 + \bar{E}}{(1 + r)(D'_0 + \bar{E})} > \frac{1}{1 + r}.$$

There exists at least one $\hat{D}$ such that $\chi(\lambda, \theta, \hat{D}) = 0$. To prove that it is unique, it suffices to prove that χ is increasing for $\theta D \in [\bar{k}(\lambda, \theta), k_2(\lambda, \theta)]$. In this interval, $\chi'_D(\lambda, \theta, D) = A(\theta, \lambda)\theta f'(\theta D) + (1 - \delta)\theta - 1$. However in this interval,

$$f'(\theta, \lambda) \geq \frac{r + \delta}{A(\theta, \lambda)},$$

and hence

$$\chi'_D(\lambda, \theta, D) \geq \theta(r + \delta) + (1 - \delta)\theta - 1 = \theta(1 + r) - 1 > 0.$$

We have proved that $\hat{D}$ is unique. We now prove that $D'_0 + \bar{E} \geq \hat{D}$:

(i) If $\theta \leq \theta\hat{D} \leq \bar{k}(\lambda, \theta)$, $\hat{D}$ then satisfies

$$(1 + r)\theta\hat{D} - r\bar{E} = \hat{D} \Leftrightarrow \hat{D} = \frac{r\bar{E}}{\theta(1 + r) - 1}.$$

Then,

$$D'_0 + \bar{E} \geq \hat{D} \Leftrightarrow \theta \geq \frac{r\bar{E} + D'_0 + \bar{E}}{(1 + r)(D'_0 + \bar{E})} = \bar{\theta}.$$

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(ii) If $\bar{k}(\lambda, \theta) \leq \theta\hat{D} \leq k_2(\lambda, \theta)$, then

$$A(\theta, \lambda) f(\theta\hat{D}) + (1 - \delta)\theta\hat{D} - r\bar{E} - \hat{D} = 0.$$

However, in this interval,

$$f(\theta\hat{D}) \geq \frac{r + \delta}{A(\theta, \lambda)}\theta\hat{D}.$$

We then have

$$0 = A(\theta, \lambda) f(\theta\hat{D}) + (1 - \delta)\theta\hat{D} - r\bar{E} - \hat{D} \geq [(1 + r)\theta - 1]\hat{D} - r\bar{E}.$$

Hence,

$$\hat{D} \leq \frac{r\bar{E}}{(1 + r)\theta - 1}.$$

One can check that if $\theta \geq \bar{\theta}$, then $D_0' + \bar{E} \geq \hat{D}$.

(iii) If $k_2(\lambda, \theta) \leq \theta\hat{D}$, then

$$\begin{aligned} 0 &= A(\theta, \lambda) f[k_2(\lambda, \theta)] - (r + \delta)k_2(\lambda, \theta) + (1 + r)\hat{D} - r\bar{E} - \hat{D} \\ &\geq [(1 + r)\theta - 1]\hat{D} - r\bar{E}, \end{aligned}$$

since $A(\theta, \lambda) f[k_2(\lambda, \theta)] - (r + \delta)k_2(\lambda, \theta) \geq 0$. Again,

$$\hat{D} \leq \frac{r\bar{E}}{(1 + r)\theta - 1}.$$

If $\theta \geq \bar{\theta}$, then $D_0' + \bar{E} \geq \hat{D}$.

We have proved that the sequence $\{\hat{D}, \hat{D}, \ldots, \hat{D}, \ldots\}$ is feasible from D_0 if $\theta \geq \bar{\theta}$. We end Step 1.

Step 2. Fix some $\theta \in]\bar{\theta}, 1[$.

Let $\Psi(\lambda, \theta, D) = \varphi(\lambda, \theta, D) + (1 + r)\theta D - r\bar{E}$. We use the trick of Dechert and Nishimura (1983). Consider $H(\lambda, \theta, D) = \Psi(\lambda, \theta, D) - (1/\beta)D$. One has $H(\lambda, \theta, 0) = -r\bar{E}$ and $H_D'(\lambda, \theta, 0) = (1 + r)\theta - 1/\beta < 0$. There are two steady points, D_S^1, D_S^2 ($D_S^1 < D_S^2$):

$$A(\theta, \lambda)\theta f'\left(\theta D_S^i\right) + (1 - \delta)\theta - \frac{1}{\beta} = 0, \qquad i = 1, 2.$$

As in Dechert and Nishimura (1983), if we prove that $H(\lambda, \theta, D_2^S) > 0$, then there exist two points $\tilde{D}_1(\theta, \lambda) < \tilde{D}_2(\theta, \lambda)$, which are the zeros of H. The main point is that Dechert and Nishimura (1983) showed that if $D_0' > \tilde{D}_1(\theta, \lambda)$, every optimal paths will converge to D_S^2. A sufficient condition for $H(\lambda, \theta, D_S^2) > 0$ is

$$\max_D \left\{ \frac{A(\theta, \lambda) f(\theta D) + (1 - \delta)\theta D - r\bar{E}}{D} \right\} > \frac{1}{\beta}.$$

Observe that, since f is convex between 0 and k_I, then $[A(\theta, \lambda) f(\theta D) + (1 - \delta)\theta D - r\bar{E}]/D$ is increasing between 0 and k_I. Hence,

$$\max_D \left\{ \frac{A(\theta, \lambda) f(\theta D) - r\bar{E}}{D} + (1 - \delta)\theta \right\} = \max_{D \geq k_I} \left\{ \frac{A(\theta, \lambda) f(\theta D) - r\bar{E}}{D} + (1 - \delta)\theta \right\}.$$

One has, for $D \geq k_I$,

$$\frac{A(\theta,\lambda) f(\theta D) - r\bar{E}}{D} \geq \frac{A(\theta,\lambda) f(\theta D)}{D} - \frac{r\bar{E}}{k_I},$$

and one has

$$\max_{D \geq k_I} \left\{ \frac{A(\theta,\lambda) f(\theta D) - r\bar{E}}{D} + (1-\delta)\theta \right\} \geq A(\theta,\lambda) \left(\max_{D \geq k_I} \frac{f(\theta D)}{D} \right) - \frac{r\bar{E}}{k_I} + (1-\delta)\theta.$$

If $\lambda \to +\infty$, the last quantity converges to $+\infty$. Hence, there exists $\bar{\lambda}_\theta$ such that, if $\lambda > \bar{\lambda}_\theta$, then one has $H(\lambda, \theta, D_S^2) > 0$, and there exists $\tilde{D}_1(\theta,\lambda)$, $\tilde{D}_2(\theta,\lambda)$, the zeros of $H(\lambda,\theta,D)$ for any $\lambda > \bar{\lambda}_\theta$. Choose λ^* large enough such that if $\lambda > \lambda^*$, then

$$A(\theta,\lambda) f(\theta D_0) - (r+\delta)\theta D_0 + (1+r)\theta D_0 - r\bar{E} > \frac{1}{\beta} D_0,$$

which means $H(\lambda,\theta,D_0) > 0$ for $\lambda > \lambda^*$. From Dechert and Nishimura, one has $D_0 > \tilde{D}_1(\theta,\lambda)$, and any optimal path from D_0 will converge to D_S^2. ■

Remark 4. The upper steady state D_2^S verifies

$$A(\theta)\theta f'\left(\theta D_2^S\right) = \frac{1}{\beta} + \theta(\delta - 1).$$

We can easily check that $1/\beta > \theta(1-\delta)$ since $\beta < 1$. Hence, D_2^S is well defined. One can choose λ large enough such that

$$\frac{1/\beta + \theta(\delta - 1)}{\theta A(\theta,\lambda)} < \frac{1}{\beta} - 1 + \delta,$$

and hence $\theta D_2^S > k^S$ (the steady state without corruption). Summing up, the effect of corruption is quite puzzling. If the incentive parameter λ is fixed, then very high corruption leads the country to a collapse. However, if the "corruption degree" θ is not too high, the country may take off and converge to a steady state that is better than the one corresponding to the case in which no corruption exists, under the condition that the incentive parameter λ is sufficiently high.

6. R&D AND GROWTH

Consider again the model in Section 4. The country receives an international aid $\bar{E}$ Assume that the country, at the first period, spends a fraction of this aid $(1-\theta)\bar{E}$ for training outside the country or, similarly, to import a new technology from abroad. This improves the productivity. The production function at each period becomes $A[(1-\theta)\bar{E}]f(k)$, where $A(x)$ is increasing, $A(0) = 1$, $A(\bar{E}) > 1$. The constraints are

$$\forall t \geq 1,$$
$$C_t + D_{t+1} \leq A[(1-\theta)\bar{E}]\max\left\{ f(k) - \frac{r+\delta}{A[(1-\theta)\bar{E}]}k; k \leq D_t \right\} + (1+r)D_t,$$
$$C_0 + D_1 \leq A[(1-\theta)\bar{E}]\max\left\{ f(k) - \frac{r+\delta}{A[(1-\theta)\bar{E}]}k; k \leq D_0 \right\} + (1+r)D_0,$$

with $D_0 = D_0' + \theta\bar{E}$. Recall that

$$\varphi[(1-\theta)\bar{E}, D] = \max\left\{ f(k) - \frac{r+\delta}{A[(1-\theta)\bar{E}]}k; k \leq D \right\}.$$

The constraints can be rewritten as

$$C_0 + D_1 \leq A[(1-\theta)\bar{E}]\varphi[(1-\theta)\bar{E}, D_0] + (1+r)D_0,$$
$$\forall t, C_t + D_{t+1} \leq A[(1-\theta)\bar{E}]\varphi[(1-\theta)\bar{E}, D_t] + (1+r)D_t.$$

The purpose of the following proposition is to show that if the developing country receives at the first period a sufficiently large amount of aid and if it spends a large amount of this aid to use this technology (and hence a small amount for consumption), then it will converge to a steady state in which capital stock and consumption are both improved compare to the situation without R&D.

Let $\bar{k}$ verify $f(\bar{k}) = (r+\delta)\bar{k}$.

PROPOSITION 4. *Assume that* $D_0' > 0$. *Assume that* $A[(1-\theta)\bar{E}] = 1 + \lambda(1-\theta)\bar{E}$. *There exists* $\bar{E}^*$ *such that if* $\bar{E} > \bar{E}^*$, *then there exists* $\theta(\bar{E}) > 0$ *verifying that for any* $\theta \in]0, \theta(\bar{E})[$, *the optimal paths from* $D_0 = D_0' + \bar{E}$ *will converge to a steady state* D^S. *Moreover,* $D^S > k^S$ (*the steady state of the benchmark model*) *and the associated steady-state consumption* C^S *is larger than the one in the benchmark model.*

Proof. For $\bar{E}$ large enough ($\bar{E} \geq \bar{E}_1$ for some $\bar{E}_1$), one has

$$\varphi(\bar{E}, D_0') = f(D_0') - \frac{r+\delta}{A(\bar{E})}D_0'$$

since

$$\varphi(\bar{E}, D_0') = \max\left\{ f(k) - \frac{r+\delta}{A(\bar{E})}k; k \leq D_0' \right\}.$$

Choose $\bar{E} \geq \bar{E}_1$ such that

$$\lambda f(D_0')\bar{E} > \left(\frac{1}{\beta} + \delta - 1\right)D_0' - f(D_0').$$

We then have $\forall \bar{E} \geq \bar{E}^*$ for some $\bar{E}^*$,

$$A(\bar{E})\varphi(\bar{E}, D_0') + (1+r)D_0' > \frac{1}{\beta}D_0'.$$

For any $\bar{E} \geq \bar{E}^*$, there exists $\theta(\bar{E})$ such that

$$\theta \in]0, \theta(\bar{E})[\Rightarrow A[(1-\theta)\bar{E}]\varphi[(1-\theta)\bar{E}, D_0'] + (1+r)D_0' - \frac{1}{\beta}D_0' > 0.$$

From Dechert and Nishimura (1983), one has $D_0' > \tilde{D}_\theta$ the smallest zero of the function

$$H(\theta, D) = A[(1-\theta)\bar{E}]\varphi[(1-\theta)\bar{E}, D] + (1+r)D - \frac{1}{\beta}D,$$

and any optimal path from $D_0 = D_0' + \bar{E} > \tilde{D}_\theta$ will converge to D^S. D^S verifies $A[(1-\theta)\bar{E}]f'(D^S) = 1/\beta - 1 + \delta = f'(k^S)$. Since $A[(1-\theta)\bar{E}] > 1$, and since f is concave for $k \geq k_I$, we have $D^S > k^S$.

Let $C^S = A[(1-\theta)\bar{E}]f(D^S) - \delta D^S$ denote the steady-state consumption.

Since f is concave when $k \in [k^S, D^S]$, one has $A[(1-\theta)\bar{E}]f'(k) - \delta \geq A[(1-\theta)\bar{E}]f'(D^S) - \delta$ for every $k \in [k^S, D^S]$. Since

$$f'(D^S) = \frac{\frac{1}{\beta} - 1 + \delta}{A[(1-\theta)\bar{E}]} > \frac{r+\delta}{A[(1-\theta)\bar{E}]},$$

the function $A[(1-\theta)\bar{E}]f(k) - \delta k$ is strictly increasing on $[k^S, D^S]$. Hence $A[(1-\theta)\bar{E}]f(D^S) - \delta(D^S) > A[(1-\theta)\bar{E}]f(k^S) - \delta k^S > f(k^S) - \delta k^S$ which is the steady-state consumption of the benchmark model. ■

7. CONCLUSION

The models developed here complete early attempts to emphasize the role of endogenous increasing returns. Once the characterization of solutions of the benchmark model is clear, it is straightforward to add other assumptions to deal with debt, corruption, and R&D. Our key results are: First, a poverty trap can be avoided by debt and/or investment in R&D. Second, R&D could lead to a higher steady state and to a higher level of consumption. Third, if corruption takes place in the bureaucracy, "laissez-faire" could lead to dramatic consequences: collapse or anarchy in the economy (i.e., absence of optimal solutions). We can quote for this result the case of Russia in the late 1990's.

Though some caution is needed, fighting corruption has a cost, and allowing a low level of corruption when it does improve productivity by incentive to work, as we show, could have a positive effect on growth. A number of issues remain unresolved. One can imagine that the level of corruption is linked with the level of national wealth, richer countries being usually less corrupted than poorer. However, the problem turns out to be extremely complicated. Second, it would be interesting to endogenize the incentive parameter λ as a function of the divert fraction θ in order to obtain and optimal value θ, but it is out of the scope of the present paper.

NOTE

1. This assumption is similar to Assumption $P3$ of Dechert and Nishimura (1983). Note that, if Assumption 2(iii) does not hold, one may prove that there is no steady state (from Assumption 1(ii)). In such a case, in k_2 the marginal product is lower than the average product and hence returns are decreasing.

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