J. Plasma Physics (2011), *vol.* 77, *part* 5, *pp.* 653–662.
doi:10.1017/S0022377811000043

Dispersion relation of transverse oscillation in relativistic plasmas with non-extensive distribution

SAN-QIU LIU and XIAO-CHANG CHEN

Department of Physics, Nanchang University, Nanchang 330031, P.R. China
(sqlgroup@ncu.edu.cn)

(Received 27 May 2010, revised 13 December 2010 and accepted 18 January 2011;
first published online 15 February 2011)

Abstract. The generalized dispersion equation for superluminal transverse oscillation in an unmagnetized, collisionless, isotropic and relativistic plasma with non-extensive q-distribution is derived. The analytical dispersion relation is obtained in an ultra-relativistic regime, which is related to q-parameter and temperature. In the limit $q \to 1$, the result based on the relativistic Maxwellian distribution is recovered. Using the numerical method, we obtain the full dispersion curve that cannot be given by an analytic method. It is shown that the numerical solution is in good agreement with the analytical result in the long-wavelength and short-wavelength region for ultra-relativistic plasmas.

1. Introduction

Non-extensive statistical mechanics (NSM) has been developed recently as a very useful tool to describe the complex systems whose properties cannot be exactly described by Boltzmann–Gibbs (B-G) statistical mechanics [1]. It is described by a non-extensive parameter q: $q \neq 1$, gives power-law distribution functions, and only when the parameter $q \to 1$ Maxwellian distribution is recovered [2]. It is thought to be a useful generalization of B-G statistics and to be appropriate for the statistical description of the long-range interaction systems. NSM has been successfully applied to stellar polytropes [3], two-dimensional Euler and drift turbulence in a pure electron plasma column [4], as well as to the peculiar velocity function of galaxy clusters [5]. In particular, Liu et al. [6] showed a reasonable indication for the non-Maxwellian velocity distribution from plasma experiments.

Dispersion relations are fundamental for studying the wave in the plasma. According to the dispersion relations, we can study the problem of instability, propagation, refraction and absorption of the plasma wave. Recently, there has been a great deal of interest in studying the dispersion property in plasmas in the context of the non-extensive statistics. Lima et al. [7] have studied the non-relativistic dispersion relation of longitudinal oscillation when the plasma is described by the q-distribution, and the result is found to fit experimental data better than if the distribution is given by a Maxwellian. Liu et al. [8] have studied the dispersion relation of ion acoustic waves based on q-distribution in a non-relativistic plasma. These analyses were restricted to the non-relativistic regime. However, relativistic plasmas are present in many physical systems, like pulsar magnetospheres [9, 10], AGN jets [11, 12] and experiments in laser plasma interactions [13]. Munoz [14] has studied the dispersion relation of longitudinal oscillation based on one-dimensional q-distribution in a relativistic plasma. Furthermore, many authors have studied the dispersion property based on Kappa distribution in a relativistic plasma, e.g. Podesta [15] has studied the dispersion relation and Landau damping of longitudinal oscillation in relativistic plasmas with power-law distribution of Kappa-type; Roman et al. [16] have discussed the dispersion of waves in relativistic plasmas with Kappa distribution; Zhou et al. [17] have studied the growth of whistler mode by a relativistic Kappa-type distribution. As such empirical distribution functions emerge naturally from a non-extensive statistical description, we will show the consistency of Kappa distribution with Tsallis statistical mechanics in Sec. 2. In this paper, we will study the dispersion relation of transverse oscillation based on q-distribution in a relativistic plasma, which has not been obtained as yet.

In the present paper, the dispersion property for superluminal transverse oscillation in an unmagnetized, collisionless, isotropic and relativistic plasma is discussed in the context of non-extensive q-distribution. The analytical dispersion law for ultra-relativistic plasmas is obtained under the long-wavelength and short-wavelength approximation. Because the analytical dispersion curve is discontinuous and restricted to ultra-relativistic temperature ($\alpha \ll 1$), the full dispersion curve, ranging from weakly to ultra-relativistic case, is obtained by numerically solving the generalized dispersion equation in Sec. 5.

This paper is organized as follows. In Sec. 2, we briefly introduce the non-extensive q-distribution function. The generalized dispersion equation for transverse oscillations is obtained in Sec. 3. In Sec. 4, we obtain the analytical dispersion laws in an ultra-relativistic regime. In Sec. 5, we present numerical calculations and discussion. Conclusion is provided in Sec. 6.

2. Relativistic non-extensive *q*-distribution function

First, let us recall some basic facts about Tsallis statistics. In Tsallis statistics, the entropy has the form [2] of

$$S_q = k_{\mathrm{B}} \frac{1 - \sum_i p_i^q}{q-1}, \tag{2.1}$$

where k_{B} is the Boltzmann constant, q is a parameter quantifying the degree of non-extensivity and p_i is the probability of the ith microstate. The B-G entropy is recovered in the limit $q \to 1$. The basic property of Tsallis entropy is the non-additivity or non-extensivity for $q \neq 1$. For example, for two systems A and B, the rule of composition [2] reads

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q) S_q(A) S_q(B). \tag{2.2}$$

In the non-extensive description, the equilibrium distribution function for a relativistic plasma can be written as [18]

$$f_q(\mathbf{p}) = A_q \left[1 - (q-1) \frac{\varepsilon}{k_{\mathrm{B}} T} \right]^{\frac{1}{q-1}}, \tag{2.3}$$

where the normalization constant reads

$$A_q = \frac{n_0}{4\pi (mc)^3 \lambda}, \qquad 2/3 < q \leqslant 1 + 1/\alpha, \tag{2.4}$$

in which

$$\lambda = \int_0^{u_{\max}} [1-(q-1)\alpha\sqrt{(1+u^2)}]^{1/(q-1)} u^2 du, \tag{2.5}$$

where the reduced velocity $u = p/mc$, the temperature parameter $\alpha = mc^2/k_B T$, $\varepsilon = mc^2\sqrt{(1+u^2)}$ is the energy, T is the physical temperature, which is defined as $T = (\partial S_q/\partial U_q)^{-1}[1+(1-q)\cdot S_q/k_B]$ for such plasmas [19], $\mathbf{p}$, c, m, k_B, U_q and n_0 denote, respectively, the momentum of particles, speed of light in vacuum, rest mass of particles, Boltzmann constant, internal energy and number of electrons per unit volume of the plasma. As one may check, for $q < 2/3$, the q-distribution is unnormalizable. For $2/3 < q \leqslant 1$, the momentum of the particles can take any value ($p_{\max} = \infty$, i.e. $u_{\max} = \infty$). For $q \geqslant 1$, the distribution function (2.3) exhibits a cutoff on the maximum value allowed for the momentum of the particles, which is given by

$$p_{\max} = mc\sqrt{[1/\alpha(q-1)]^2 - 1}, \tag{2.6}$$

namely,

$$u_{\max} = \sqrt{[1/\alpha(q-1)]^2 - 1}, \tag{2.7}$$

and hence a real value for the cutoff in (2.7) exists only if

$$q \leqslant 1 + 1/\alpha. \tag{2.8}$$

Finally, then, unlike the non-relativistic case [7], where $1/3 < q < \infty$, in the relativistic regime $2/3 < q \leqslant 1 + 1/\alpha$. We see that in the limit $q \to 1$, $p_{\max}$ goes to infinity and (2.3) reduces to the Jüttner distribution function

$$f_{q=1}(\mathbf{p}) = \frac{n_0}{4\pi m^2 c k_B T}\frac{1}{K_2(\alpha)}\exp(-\varepsilon/k_B T), \tag{2.9}$$

where $K_2(\alpha)$ is the MacDonald function.

Recently, empirically derived kappa distributions are becoming increasingly widespread in space plasma physics. The origin of the kappa distribution in Tsallis statistical mechanics has already been examined by several authors [19–24]. In the Tsallis framework, the phenomenologically introduced kappa distribution and the Tsallis-like distribution of velocities are accidentally of the same form, using the transformation of indices: $q = 1 + 1/\kappa$. As we shall see in [19], the first and second kind of kappa distributions, which are widely used in space physics, coincide with the ordinary and escort Tsallis probability distributions, respectively. Therefore, the connection is complete between Tsallis statistical mechanics and empirical kappa distributions, and the capability of Tsallis statistics is available to the space plasma physics community for analysing and understanding the kappa-like properties of the various particles and energy distributions observed in space.

In order to illustrate the effects of q-parameter on system energy in relativistic plasmas, the curve of the average energy of particle as a function of the q-parameter is plotted in Fig. 1, where temperature parameter $\alpha = 0.1$. From Fig. 1, we see that the average energy of particle decreases with the increase of q-parameter. In other words, the bigger the q-parameter, the lower the system energy will be, and hence the weaker the relativistic effect.

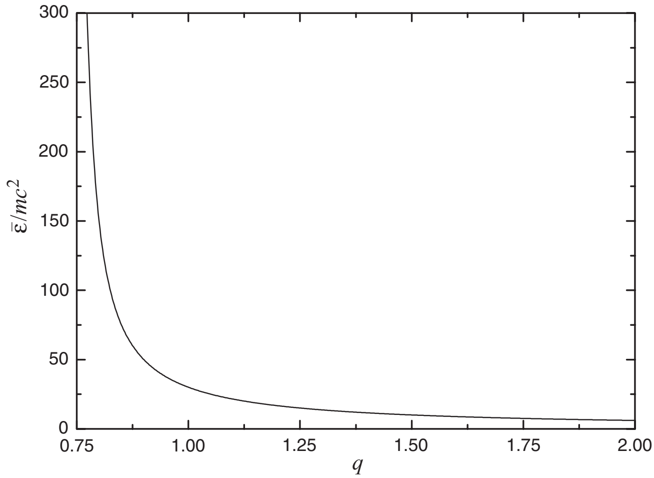

Figure 1. Distribution curve of average energy of particle for different values of q-parameter.

3. Generalized dispersion equation

Let us consider an unmagnetized, collisionless, isotropic and relativistic proton–electron plasma. Assuming that the protons remain at rest, these protons are treated as a background. According to kinetic theory, the transverse dielectric function of electron can be be written as [25]

$$\varepsilon_k^t = 1 + \frac{2\pi e^2}{\omega k^2}\int \mathrm{d}\mathbf{p}\frac{\partial f_q}{\partial \varepsilon}\frac{k^2 v_\perp^2}{\omega - kv_\parallel + i\delta}, \tag{3.1}$$

where $v_\perp$ and $v_\parallel$ represent the electron velocities in directions perpendicular and along the wave vector $\mathbf{k}$, respectively, e is the electronic charge and $i\delta$ comes from Landau rules [26]. Introducing the reduced velocity $u = p/m_e c$, we have $\gamma = (1+u^2)^{1/2}$. Here, $(\mathbf{u}, \theta, \varphi)$ represents the spherical coordinate system, and hence $v_\perp = cu\sin\theta/\gamma$, $v_\parallel = cu\cos\theta/\gamma$, $\mathrm{d}\mathbf{p} = (m_e c)^3 u^2 \mathrm{d}u(-\mathrm{d}\cos\theta)\mathrm{d}\varphi$. Hence (3.1) becomes

$$\varepsilon_k^t = 1 - \frac{4\pi^2 e^2 c}{\omega k}(m_e c)^3\int\frac{\partial f_q}{\partial \varepsilon}\frac{u^3}{\gamma}\,\mathrm{d}u\int_{-1}^{1}\frac{1-x^2}{x-\gamma\bar{v}_p/u - i\delta}\,\mathrm{d}x, \tag{3.2}$$

where $\bar{v}_p = \omega/kc$.

Using the Plemelj formula [25]

$$\frac{1}{z \pm i0} = \wp\frac{1}{z} \mp i\pi\delta(z), \tag{3.3}$$

where $\wp$ denotes the principal value, and hence the x integral in (3.2) becomes

$$\int_{-1}^{1}\frac{1-x^2}{x-\gamma\bar{v}_p/u - i\delta}\,\mathrm{d}x = -2\frac{\gamma\bar{v}_p}{u}\left\{1 + \frac{u}{\gamma\bar{v}_p}[1-(\gamma\bar{v}_p/u)^2]\right.$$
$$\left.\times\left[\frac{1}{4}\ln\left(\frac{\bar{v}_p + u/\gamma}{\bar{v}_p - u/\gamma}\right)^2 - \frac{i\pi}{2}\theta\left(1-\frac{\gamma\bar{v}_p}{u}\right)\right]\right\}, \tag{3.4}$$

where $\theta(\xi)$ is the step function that indicates that $\theta(\xi) = 1$ at $\xi > 0$ and $\theta(\xi) = 0$ at $\xi < 0$.

https://doi.org/10.1017/S0022377811000043 Published online by Cambridge University Press

According to the transverse dispersion equation

$$\varepsilon_k^t(\omega, k) = \frac{k^2c^2}{\omega^2}, \tag{3.5}$$

and substituting (2.3) and (3.4) into (3.2), we obtain the generalized dispersion equation of transverse oscillation

$$1 - \frac{\alpha\omega_{pe}^2}{2\lambda k^2c^2}\int_0^{u_{\max}} [1-(q-1)\,\gamma\alpha]^{\frac{1}{q-1}-1}\, Hu^2 \mathrm{d}u = \frac{k^2c^2}{\omega^2}, \tag{3.6}$$

where $\omega_{pe} = \sqrt{4\pi n_0 e^2/m_e}$ is the plasma frequency, $u_{\max}$ is the maximum reduced velocity allowed by (2.7) if $1 \leqslant q \leqslant 1+1/\alpha$, or ∞ if $2/3 < q \leqslant 1$, and

$$H = 1 + \frac{u}{\gamma\bar{v}_p}[1-(\gamma\bar{v}_p/u)^2]\left[\frac{1}{4}\ln\left(\frac{\bar{v}_p+u/\gamma}{\bar{v}_p-u/\gamma}\right)^2 - \frac{i\pi}{2}\theta\left(1-\frac{\gamma\bar{v}_p}{u}\right)\right]. \tag{3.7}$$

4. Dispersion relation for ultra-relativistic plasma

We restrict our discussion here to superluminal waves ($\bar{v}_p > 1$), so the imaginary parts of the dispersion relation disappear [27]. When $\bar{v}_p < 1$, the waves becomes subluminous, and then the oscillation frequency is found to be complex. In fact, these subluminal waves are strongly damped, and the wave may no longer exist. If temperatures are ultra-relativistic, namely $\alpha \ll 1$, we can take $\varepsilon \approx pc$ and $u/\gamma \approx 1$ approximately, and then the generalized dispersion equation (3.6) can be obtained analytically.

When $\bar{v}_p \gg 1$, (3.7) becomes

$$H \approx \frac{2}{3}\frac{k^2c^2}{\omega^2}\frac{u^2}{\gamma^2}\left(1 + \frac{1}{5}\frac{k^2c^2}{\omega^2}\frac{u^2}{\gamma^2}\right), \tag{4.1}$$

and substituting (4.1) into (3.6), we obtain the dispersion relation for long-wavelength branch

$$\omega^2 = \omega_p^2 + \frac{6}{5}k^2c^2, \quad 2/3 < q \leqslant 1 + 1/\alpha, \tag{4.2}$$

in which

$$\omega_p = \sqrt{\frac{(3q-2)\alpha}{3}}\omega_{pe}, \tag{4.3}$$

is the corrected plasma frequency.

When $\bar{v}_p > 1$ and $\bar{v}_p \approx 1$, (3.7) becomes

$$H \approx 1 - \frac{\ln(\gamma + u)}{\gamma u}, \tag{4.4}$$

and substituting (4.4) into (3.6) yields the dispersion relation for short-wavelength branch

$$\omega^2 = \frac{3}{2}\omega_p^2 + k^2c^2, \quad 2/3 < q \leqslant 1 + 1/\alpha. \tag{4.5}$$

As expected, in the limit $q \to 1$, (4.2) and (4.5) reduce to

$$\omega^2 = \frac{\alpha}{3}\omega_{pe}^2 + \frac{6}{5}k^2c^2, \quad \bar{v}_p \gg 1, \tag{4.6}$$

https://doi.org/10.1017/S0022377811000043 Published online by Cambridge University Press

$$\omega^2 = \frac{\alpha}{2}\omega_{pe}^2 + k^2c^2, \quad \bar{v}_p > 1 \text{ and } \quad \bar{v}_p \approx 1, \tag{4.7}$$

and they are the results based on the relativistic Maxwellian distribution in B-G statistics.

The analytic solution of the transverse dispersion relation is related to q-parameter and temperature in the context of the relativistic non-extensive statistics. In the limiting case ($q \to 1$), the dispersion relation based on the relativistic Maxwellian distribution is recovered [28]. Because the analytical dispersion curve is discontinuous and restricted to ultra-relativistic temperature ($\alpha \leqslant 1$), using the numerical method, we obtain the full dispersion curve in the following section which cannot otherwise be obtained by an analytic method.

5. Numerical calculation and analysis

In this section, we will numerically evaluate the dispersion equations (3.6), (4.2) and (4.5). It is convenient to introduce the dimensionless variables

$$\Omega = \omega/\omega_{pe}, \quad \mathrm{K} = kc/\omega_{pe}, \quad \bar{v}_p = \Omega/\mathrm{K} = \omega/kc, \tag{5.1}$$

and hence, (3.6), (4.2) and (4.5) can be expressed as

$$1 - \frac{\alpha}{2\lambda K^2}\int_0^{u_{\max}} [1-(q-1)\gamma\alpha]^{\frac{1}{q-1}-1} Hu^2\,\mathrm{d}u = \frac{K^2}{\Omega^2}, \quad 2/3 \leqslant q \leqslant 1+1/\alpha, \tag{5.2}$$

$$\Omega^2 = (3q-2)\frac{\alpha}{3} + \frac{6}{5}K^2, \quad \Omega \geqslant K, \quad 2/3 < q \leqslant 1+1/\alpha, \tag{5.3}$$

$$\Omega^2 = (3q-2)\frac{\alpha}{2} + K^2, \quad \Omega > K \text{ and } \quad \Omega \approx K, \quad 2/3 < q \leqslant 1+1/\alpha. \tag{5.4}$$

Equation (5.2), from which the full dispersion curve can be obtained, is the dimensionless generalized transverse dispersion equation. Equations (5.3) and (5.4) are the dimensionless dispersion relations for the ultra-relativistic case under the long-wavelength and short-wavelength approximation.

Figure 2 exhibits the analytical and numerical dispersion curves for different values of α, where the non-extensive parameter $q = 0.85$. The dot line, solid line and dash line denote, respectively numerical dispersion curves, analytical dispersion curves for the long-wavelength and short-wavelength branches.

It can be seen from Fig. 2 that, when $\alpha = 0.05$, the analytical dispersion curves for the long-wavelength and short-wavelength branches are in good agreement with the numerical result under the condition $\Omega \geqslant K$ and $\Omega \approx K$, respectively. The difference between the analytical and numerical result increases with increasing α. Since the analytical dispersion relation is derived under the condition of ultra-relativistic temperature ($\alpha \leqslant 1$), it is not suitable for the moderate and weakly relativistic case.

In order to illustrate the effects of non-extensive parameter q and temperature parameter α on the dispersion relation, we have computed the dimensionless generalized transverse dispersion equation (5.2) for different value of q and α. Figures 3 and 4 show the dispersion curves for different values of q and fixed value of α. On the contrary, Figs 5 and 6 show the dispersion curves for different values of α and fixed value of q.

From Figs 3 and 4, we note that, for the fixed value of α, if q increases the oscillation frequency increases. Figures 5 and 6 show that, for the fixed value of q,

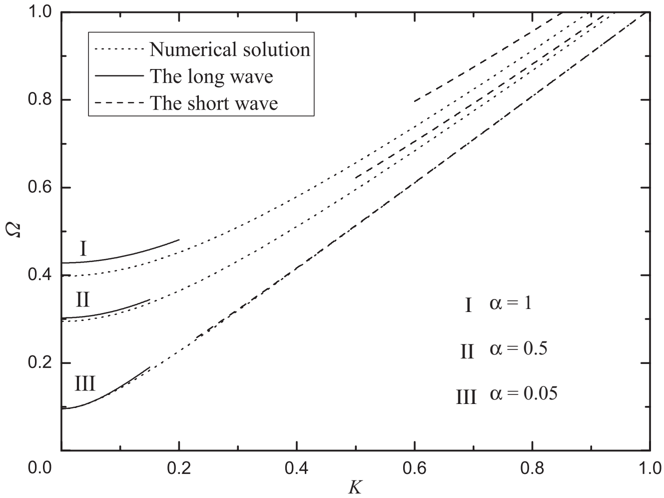

Figure 2. The numerical and analytical dispersion curves when $q = 0.85$.

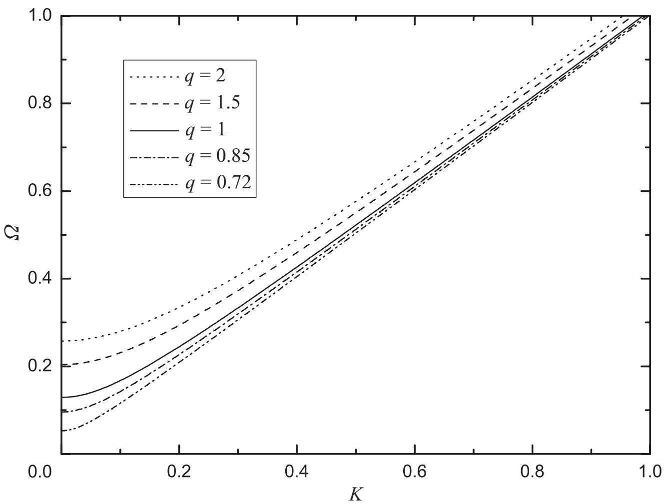

Figure 3. The dispersion curves for different values of q when $\alpha = 0.05$.

if α increases the oscillation frequency also increases, which is due to the plasma frequency being corrected because of the relativistic effect. It is evident from Fig. 1 that the average energy of the particle decreases with increasing q-parameter, namely the bigger the q-parameter, the weaker the relativistic effect, and hence the smaller the mass of the particle, the higher the corrected plasma frequency. Similarly, the bigger the α, the weaker the relativistic effect, and hence the higher the corrected plasma frequency.

https://doi.org/10.1017/S0022377811000043 Published online by Cambridge University Press

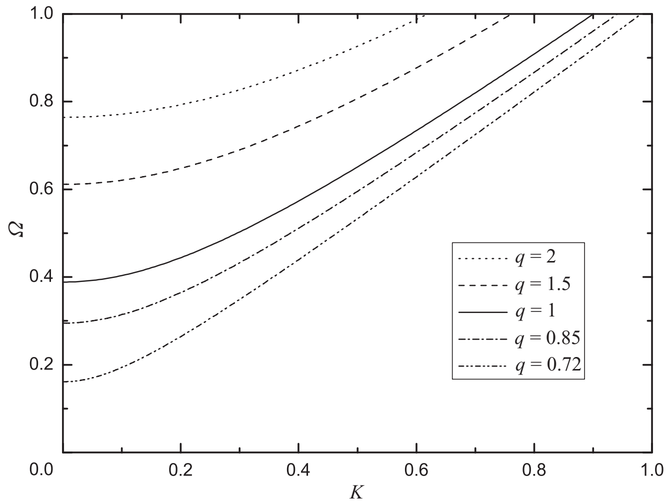

Figure 4. The dispersion curves for different values of q when $\alpha = 0.5$.

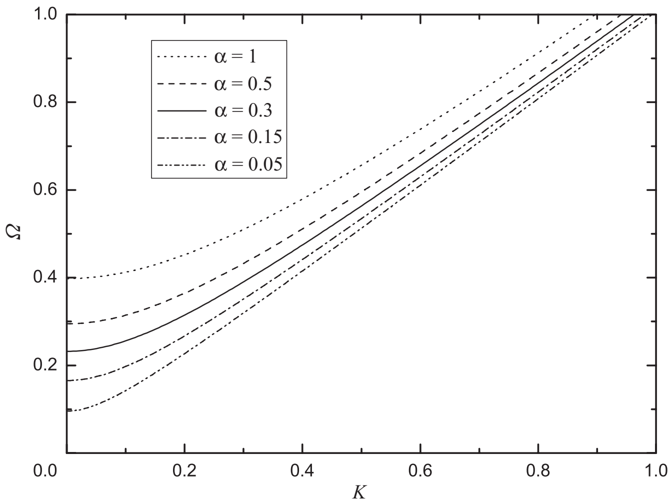

Figure 5. The dispersion curves for different values of α when $q = 0.85$.

6. Conclusions

In this paper, we have discussed the dispersion property for superluminal transverse oscillation in an unmagnetized, collisionless, isotropic and relativistic plasma in the context of non-extensive distribution. The analytical dispersion relation is obtained under the long-wavelength and short-wavelength approximation in the ultra-relativistic regime. In the limiting case ($q \to 1$), the result based on the relativistic Maxwellian distribution is recovered. Because the analytical dispersion curve is discontinuous and restricted to ultra-relativistic temperature ($\alpha \ll 1$), the full dispersion curve, ranging from weakly to ultra-relativistic case, is obtained by numerically solving the generalized dispersion equation. It is shown that the

https://doi.org/10.1017/S0022377811000043 Published online by Cambridge University Press

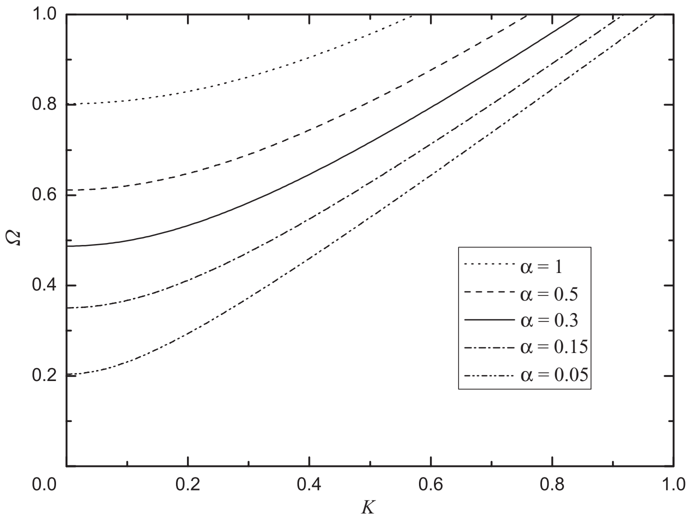

Figure 6. The dispersion curves for different values of α when $q = 1.5$.

analytical dispersion curves for the long-wavelength and short-wavelength branches are in good agreement with the numerical result when $\alpha \ll 1$. The corrected plasma frequency increases if the value of either q or α increases, which is due to the relativistic mass variation.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant no. 10963002) and International S&T Cooperation Program of China (Grant no. 2009DFA02320) and Jiangxi Province, Program for Innovative Research Team in Jiangxi Province and Nanchang University.

References

[1] Gell-Mann, M. and Tsallis, C. 2004 *Nonextensive Entropy – Interdisciplinary Applications*. New York: Oxford University Press.

[2] Tsallis, C. 1988 Possible generalization of Boltzmann–Gibbs statistics. *J. Stat. Phys.* **52**, 479–487.

[3] Plastino, A. R. and Plastino, A. 1993 Tsallis entropy, Ehrenfest theorem and information theory. *Phys. Lett.* A **177**, 177–179.

[4] Boghosian, B. M. 1996 Thermodynamic description of the relaxation of two-dimensional turbulence using Tsallis statistics. *Phys. Rev.* E **53**, 4754–4763.

[5] Lavagno, A., Kaniadakis, G., Rego-Monteiro, M., Quarati, P. and Tsallis, C. 1998 Nonextensive thermo-statistical approach of the peculiar velocity function of galaxy clusters. *Astrophys. Lett.* **35**, 449–451.

[6] Liu, J. M., De Groop, J. S., Matte, J. P., Johnston, T. W. and Drake, R. P. 1994 Measurements of inverse bremsstrahlung absorption and non-Maxwellian electron velocity distributions. *Phys. Rev. Lett.* **72**, 2717–2720.

[7] Lima, J. A. S., Silva, R. Jr. and Santos, J. 2000 Plasma oscillations and nonextensive statistics. *Phys. Rev.* E **61**, 3260–3263.

[8] Liyan, L. and Jiulin, D. 2008 Ion acoustic waves in the plasma with the power-law q-distribution in nonextensive statistics. *Physica* A **387**, 4821–4827.

[9] Curtis, M. F. 1991 *The Theory of Neutron Stars Magnetospheres*. Chicago, IL: University of Chicago Press.

[10] Cheng, A. F. and Ruderman, M. A. 1980 Particle acceleration and radio emission above pulsar polar caps. *Astrophys. J.* **235**, 576–586.

[11] Hirotani, K., Iguchi, S., Kimura, M. and Wajima, K. 2000 Pair plasma dominance in the parsec-scale relativistic jet of 3C 345. *Astrophys. J.* **545**, 100–106.

[12] Wardle, J. F. C., Homan, D. C., Ojha, R. and Roberts, D. H. 1998 Electron–Positron jets associated with the quasar 3C 279. *Nature* **395**, 457–461.

[13] Mourou, G. and Umstadter. D. 1992 Development and applications of compact high-intensity lasers. *Phys. Fluids* B **4**, 2315–2325.

[14] Munoz, V. 2006 A nonextensive statistics approach for Langmuir waves in relativistic plasmas. *Nonlinear Processes Geophys.* **13**, 237–241.

[15] Podesta, J. J. 2008 Landau damping in relativistic plasmas with power-law distributions and applications to solar wind electrons. *Phys. Plasmas* **15**, 122902.

[16] Shcherbakov R. V. 2009 Dispersion of waves in relativistic plasmas with isotropic particle distributions. *Phys. Plasmas* **16**, 032104.

[17] Zhou, Q.-H., Jiang, B., Shi, X.-H. and Li, J.-Q. 2009 Whistler-mode waves growth by a generalized relativistic kappa-type distribution. *Chin. Phys. Lett.* **26**, 025201.

[18] Lavagno, A. 2002 Relativistic nonextensive Thermodynamics. *Phys. Lett.* A **301**, 13–18.

[19] Livadiotis, G. and McComas, D. J. 2009 Beyond kappa distributions: exploiting Tsallis statistical mechanics in space plasmas. *J. Geophys. Res.* **114**, A11105.

[20] Leubner, M. P. 2002 A nonextensive entropy approach to kappa distribution. *Astrophys. Space Sci.* **282**, 573–579.

[21] Leubner, M. P. 2004 Core-Halo distribution function: a natural equilibrium state in generalized thermostatistics. *Astrophys. J.* **11**, 469–478.

[22] Milovanov, A. V. and Zelenyi, L. M. 2000 Functional background of the Tsallis entropy: "Coarse-grained" systems and "kappa" distribution functions. *Nonlinear Processes Geophys.* **7**, 211–221.

[23] Nieves-Chinchilla, T. and Vinas, A. F. 2008 Kappa-like distribution functions inside magnetic clouds. *Geofis. Int.* **47**, 245–249.

[24] Shizgal, B. D. 2007 Suprathermal particle distributions in space physics: kappa distributions and entropy. *Astrophys. Space Sci.* **312**, 227–237.

[25] Li, X. Q. 2004 *Collapsing Dynamics of Plasmons*. Beijing: Chinese Science and Technology Press.

[26] Lifshitz, E. M. and Pitaevskii, L. P. 1981 *Physical Kinetics*. Oxford: Pergamon.

[27] Schlickeiser, R. 1994 Longitudinal oscillations in hot isotropic Maxwellian plasmas. *Phys. Plasmas* **1**, 2119–2124.

[28] Mikhailovskii, A. B. 1980 Oscillations of an isotropic relativistic plasma. *Plasma Phys.* **22**, 133–149.