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# On the generation of magnetic fields due to ponderomotive forces in astrophysical plasmas

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**Abstract.** The generation of magnetic fields due to ponderomotive forces in astrophysical plasma consisting of electrons, ions and positrons is investigated theoretically. It is seen that collisional or non-collisional interactions (between electromagnetic waves and plasma particles) via ponderomotive forces in an inhomogeneous plasma can excite a magnetic field. The growth rate of the magnetic field is illustrated graphically for different values of the temperature and concentration of positrons in the plasma.

## 1. Introduction

The magnetic field is one of the most important factors causing various linear and nonlinear phenomena in a plasma. There are several mechanisms for the generation of magnetic fields in laboratory and space plasmas. In laboratory experiments, magnetic fields are observed that are possibly generated due to anisotropy of an electron plasma [1], thermal instabilities [2], Weibel instabilities [3], the $\nabla n \times \nabla T$ mechanism [4], filamentation [5], the inverse Faraday effect [6, 7] and ponderomotive forces [8], etc. In astrophysical bodies, the dynamo effect is one of the main sources of magnetic fields [9–12]. For magnetic field generation, the dynamo motion should be such as to convert mechanical energy to magnetic energy. In general, the dynamo effect generates an axial magnetic field in the medium [13–15]. In an underdense and moving plasma, a turbulent dynamo effect is possibly a source of magnetic field [16–19]. However, another mechanism is also possible for the generation of magnetic fields in astrophysical bodies. It is also possible to generate a magnetic field in a nonlinear medium, even if its density and temperature do not change [20].

In the present paper, we propose that the ponderomotive force is one of the possible sources of a stellar magnetic field. In an inhomogeneous plasma, the ponderomotive force becomes prominent, and it helps to generate a magnetic field in the medium. We investigated theoretically the role of positrons in a plasma for producing such a magnetic field. It is generally believed that electron–ion–positron plasmas are produced in the early universe [21, 22], and make important contributions to physical processes in active galactic nuclei [23], pulsar magnetospheres [24] and neutron stars [25, 26]. Several authors [27–31] have considered the presence of positrons together with electrons and ions in their studies of the propagation of waves, including solitary waves and double layers, in astrophysical plasmas. However, little attempt has been made to determine the role of positrons in the generation of magnetic fields in cosmic plasmas. Rizzato [32] investigated weak nonlinear electromagnetic wave and low-frequency magnetic field generation in electron–positron–ion plasmas. He showed that the amplitude of the wave becomes a strongly dependent function of the angle between the slow modulations and the fast spatial variations and that there is a possibility of spontaneous generation of a low-frequency magnetic field. In the present paper, we investigated the wave propagation in an electron–ion–positron plasma, with the conclusion that the ponderomotive force is one of the possible mechanisms for generating magnetic fields in cosmic plasmas.

## 2. The origin of the ponderomotive force

Light waves produce radiation pressure, and when an intense laser beam or high-powered microwaves are incident on a plasma, the radiation pressure can become very large and exert a great force on the plasma. This type of force, which is nonlinear in nature and coupled to the charged particles in a subtle way, is called the ponderomotive force. A qualitative discussion on the existence of such a force may be given as follows. In an inhomogeneous high-frequency electric field of an electromagnetic wave, the motion of an electron may be thought to be composed of two parts: (i) a high-frequency oscillation about an effective centre of oscillation and (ii) a relatively slow motion of this oscillation centre. Let us take the electric field as $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \cos \omega t$ and let the amplitude of this field be increasing in the positive $x$ direction. As an electron moves into the stronger field, it will be accelerated more strongly back towards the oscillation centre. On the other hand, when the electron moves towards the weaker field (as $x$ goes negative), it experiences a weaker restoring force. Thus, on average, the electron will experience a slow drift towards the weaker field, as if under the influence of a steady or slowly varying force, while at the same time undergoing a rapid oscillation at the high frequency.

For a quantitative treatment, we consider the equation of motion of a charged particle (of charge $q$ and mass $m$) in an inhomogeneous electromagnetic field:

$$m \frac{d^2\mathbf{r}}{dt^2} = q \left[ \mathbf{E}(\mathbf{r}, t) + \frac{1}{c} \mathbf{v}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}, t) \right]. \tag{1}$$

This equation is exact when $\mathbf{E}$ and $\mathbf{B}$ are evaluated at the instantaneous position of the charged particle. The nonlinearity coming from the first term is due to the fact that $\mathbf{E}$ has to be evaluated at the actual position rather than at the oscillating centre, while the second term is replaced by $\mathbf{v}_1 \times \mathbf{B}_1$, where $\mathbf{v}_1$ and $\mathbf{B}_1$ are the values obtained from the linear theory. Obviously, this term is of second order. At first order, we neglect this term and write the equation of motion as

$$m \frac{d^2\mathbf{r}}{dt^2} = q\mathbf{E}(\mathbf{r}) \cos \omega t. \tag{2}$$

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We separate the motion into a slow motion and a fast motion such that $\mathbf{r} = \mathbf{r}_0 + \delta\mathbf{r}$, where $\delta\mathbf{r} = \langle\delta\mathbf{r}\rangle$, the average over the first scale, or over the period $T = 2\pi/\omega$. Thus $\mathbf{r}_0$ describes the oscillation centre and $\delta\mathbf{r}$ describes the rapidly oscillating motion. $\delta\mathbf{r}$ is determined by

$$m\frac{d^2\delta\mathbf{r}}{dt^2} = q\mathbf{E}_0\cos\omega t, \tag{3}$$

where $\mathbf{E}_0 = \mathbf{E}(\mathbf{r}_0)$. Now solving (3), we get

$$\left.\begin{aligned} \mathbf{v}_1 &= \frac{q}{m\omega}\mathbf{E}_0\sin\omega t, \\ \delta\mathbf{r} &= -\frac{q}{m\omega^2}\cos\omega t. \end{aligned}\right\} \tag{4}$$

For the slow variation, we expand $\mathbf{E}(\mathbf{r})$ about $\mathbf{r}_0$,

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}(\mathbf{r}_0 + \delta\mathbf{r}) = \mathbf{E} + \delta\mathbf{r}\cdot\boldsymbol{\nabla}\mathbf{E}_0,$$

such that the equation of motion becomes

$$m\frac{d^2(\mathbf{r}_0 + \delta\mathbf{r})}{dt^2} = q\mathbf{E}(\mathbf{r}_0 + \delta\mathbf{r})\cos\omega t.$$

Using the expression for $\mathbf{E}(\mathbf{r}_0 + \delta\mathbf{r})$ given above and taking the average over the faster time scale, we get the equation of motion for $\mathbf{r}_0$ as

$$m\frac{d^2\mathbf{r}_0}{dt^2} = q\langle\delta\mathbf{r}\cos\omega t\rangle\cdot\boldsymbol{\nabla}\mathbf{E}_0. \tag{5}$$

Putting the expression for $\delta\mathbf{r}$ given in (4) and noting that the average $\langle\cos^2\omega t\rangle = 1/2$, we get

$$m\frac{d^2\mathbf{r}_0}{dt^2} = -\frac{q^2}{4m\omega^2}\boldsymbol{\nabla}(E_0^2). \tag{6}$$

We now consider the full equation of motion given by (1), containing the $\mathbf{v}_1\times\mathbf{B}_1$ term. The magnetic field $\mathbf{B}_1$ is given by the Maxwell equation

$$\boldsymbol{\nabla}\times\mathbf{E}_1 = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}.$$

Thus, since $\mathbf{E} = \mathbf{E}(\mathbf{r})\cos\omega t$,

$$\mathbf{B}_1 = -\frac{1}{\omega}\boldsymbol{\nabla}\times\mathbf{E}_1|_{r0}\sin\omega t. \tag{7}$$

We now write the complete equation of motion, including $\mathbf{v}_1\times\mathbf{B}_1$ term, as

$$m\frac{dr}{dt} = q\left[(\delta\mathbf{r}\cdot\boldsymbol{\nabla})\mathbf{E}_0 + \frac{1}{c}\mathbf{v}_1\times\mathbf{B}_1\right]. \tag{8}$$

Using the expressions for $\delta\mathbf{r}, \mathbf{E}, \mathbf{v}_1$, and $\mathbf{B}_1$ from (4), (5), and (7), and averaging over the time period $T = 2\pi/\omega$, we get

$$\frac{d\mathbf{r}_0}{dt} = -\frac{q^2}{2m\omega^2}[(\mathbf{E}\cdot\boldsymbol{\nabla})\mathbf{E} + \mathbf{E}\times(\boldsymbol{\nabla}\times\mathbf{E})], \tag{9}$$

where we have used $\langle\sin^2\omega t\rangle = \langle\cos^2\omega t\rangle = 1/2$. The terms within the square brackets can be combined to obtain

$$\frac{d\mathbf{r}_0}{dt} = -\frac{q^2}{2m\omega^2}\boldsymbol{\nabla}(E_0^2) \tag{10}$$

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The right-hand side of (10) is the expression for the ponderomotive force on a single electron. This force can be expressed in terms of the electron plasma frequency $\omega_{pe}$. Although this force acts on the electron, it is ultimately transmitted to the ions. This is due to the fact that this force is a low-frequency effect. A direct effect of this force is the self-focusing of intense electromagnetic waves, such as laser light, in a plasma. This is, in fact, the origin of the ponderomotive force.

## 3. Formulation

We consider a plasma consisting of electrons, positive ions and positrons. The basic equations describing the motion of the plasma interacting with an electromagnetic wave are as follows:

continuity equations

$$\frac{\partial n_s}{\partial t} + \boldsymbol{\nabla} \cdot (\mathbf{n}_s \mathbf{v}_s) = 0; \tag{11}$$

momentum equations

$$m_s n_s \left( \frac{\partial}{\partial t} + \mathbf{v}_s \cdot \boldsymbol{\nabla} \right) \mathbf{v}_s = -k_B \boldsymbol{\nabla}(n_s T_s) + q_s n_s \left( \mathbf{E} + \frac{\mathbf{v}_s \times \mathbf{B}}{c} \right) - m_s n_s \sum_{\substack{k=i,e,p \\ k \neq s}} \nu_{sk} (\mathbf{v}_s - \mathbf{v}_k) \tag{12}$$

and the Maxwell equations

$$\boldsymbol{\nabla} \times \mathbf{E} = \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \tag{13}$$

$$\boldsymbol{\nabla} \times \mathbf{B} = -\frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \sum_s n_s q_s \mathbf{v}_s, \tag{14}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 4\pi \sum_s n_s q_s, \tag{15}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0. \tag{16}$$

Where the subscript $s$ indicates the species of the plasma: $s = e$ for the electrons, $s = i$ for ions, and $s = p$ for positrons. $n_e$, $n_i$, and $n_p$ are the densities; $\mathbf{v}_e$, $\mathbf{v}_i$, and $\mathbf{v}_p$ are the velocities; $m_e$, $m_i$, and $m_p$ are the masses, and $\nu_{ei}$, $\nu_{ep}$, and $\nu_{pi}$ are the collision frequencies; $\mathbf{E}$ is the electric field intensity, $\mathbf{B}$ is the magnetic field intensity, $k_B$ is Boltzmann's constant, and $c$ is the velocity of light; $q_s = -e$ for electrons and $q_s = +e$ for positrons and ions.

We now express all the field variables related to electrons and positrons as sums of slow and fast components in the following manner:

$$\left. \begin{aligned} n_e &= n_{e0} + n_{e1}, & \mathbf{v}_e &= \mathbf{v}_{e0} + \mathbf{v}_{e1}, \\ n_p &= n_{p0} + n_{p1}, & \mathbf{v}_p &= \mathbf{v}_{p0} + \mathbf{v}_{p1}, \\ \mathbf{E} &= \mathbf{E}_0 + \mathbf{E}_1, & \mathbf{B} &= \mathbf{B}_0 + \mathbf{B}_1, \end{aligned} \right\} \tag{17}$$

where suffix '0' stands for the slow and '1' for the fast component. Now, using (17) in (11)–(16), we obtain for the slow components, by averaging over the

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fast components,

$$m_s n_{s0}\left(\frac{\partial}{\partial t}+\mathbf{v}_{s0}\cdot\boldsymbol{\nabla}\right)\mathbf{v}_{s0} = -k_B\boldsymbol{\nabla}(n_{s0}T_s)+q_s n_{s0}\left(\mathbf{E}_0+\frac{\mathbf{v}_{s0}\times\mathbf{B}_0}{c}\right)$$
$$-m_s n_{s0}\sum_{\substack{k=i,e,p\\ k\neq s}}\nu_{sk}(\mathbf{v}_{s0}-\mathbf{v}_{k0})+\boldsymbol{\Pi}_s, \qquad (18)$$

$$\boldsymbol{\nabla}\times\mathbf{E}_0 = -\frac{1}{c}\frac{\partial\mathbf{B}_0}{\partial t}, \qquad (19)$$

where

$$\boldsymbol{\Pi}_{s=e,p} = -m_s n_{s0}\left(\langle(\mathbf{v}_{s1}\cdot\boldsymbol{\nabla})\mathbf{v}_{s1}\rangle+\frac{q_s}{m_s}\left\langle\mathbf{v}_{s1}\times\frac{\mathbf{B}_1}{c}\right\rangle\right)$$
$$-k_B\left\langle\frac{n_{s1}^2}{n_{s0}^2}\right\rangle\boldsymbol{\nabla}(n_{s0}T_s)+k_B\frac{n_{s1}}{n_{s0}}\boldsymbol{\nabla}\langle n_{s1}T_s\rangle,$$
$$\boldsymbol{\Pi}_i = 0.$$

$\boldsymbol{\Pi}_e$ and $\boldsymbol{\Pi}_p$ are the ponderomotive forces, which are steady body forces, acting on electrons and positrons. These nonlinear forces arise due to nonlinear coupling of two fast components, whose average is non-zero; such averages are denoted by $\langle\ \rangle$. Similarly, for the fast components, the following equations are obtained:

$$m_s n_{s0}\left[\left(\frac{\partial}{\partial t}+\mathbf{v}_{s0}\cdot\boldsymbol{\nabla}\right)\mathbf{v}_{s1}+(\mathbf{v}_{s1}\cdot\boldsymbol{\nabla})\mathbf{v}_{s0}\right]$$
$$= -\left(1+\frac{\langle n_{s1}^2\rangle}{n_{s0}}\right)k_B\boldsymbol{\nabla}(n_{s1}T_s)-\frac{n_{s1}}{n_{s0}}k_B\boldsymbol{\nabla}(n_{s0}T_s)$$
$$+q_s n_{s0}\left(\mathbf{E}_1+\frac{\mathbf{v}_{s0}\times\mathbf{B}_1}{c}+\frac{\mathbf{v}_{s1}\times\mathbf{B}_0}{c}\right)$$
$$-m_s n_{s0}\sum_{k\neq s}\nu_{sk}(\mathbf{v}_{s1}-\mathbf{v}_{k1})-m_s n_{s1}\sum_{k\neq s}\nu_{sk}(\mathbf{v}_{s0}-\mathbf{v}_{k0}), \qquad (20)$$

$$\boldsymbol{\nabla}\times\mathbf{E}_1 = -\frac{1}{c}\frac{\partial\mathbf{B}_1}{\partial t}, \qquad (21)$$

$$\boldsymbol{\nabla}\cdot\mathbf{E}_1 = 4\pi\sum_{s=e,p}q_s n_{s1}. \qquad (22)$$

It should be noted that the convection terms$(\mathbf{v}_{s0}\cdot\boldsymbol{\nabla})\mathbf{v}_{s1}$ can be neglected in comparison with $\partial\mathbf{v}_{s1}/\partial t$. The other convective terms $(\mathbf{v}_{s1}\cdot\boldsymbol{\nabla})\mathbf{v}_{s0}$ are also smaller than the scale length of $\mathbf{v}_{s1}$. The terms $q_s n_{s0}(\mathbf{v}_{s1}\times\mathbf{B}_0/c)$ are also ignored compared with the terms $m_s n_{s0}\,\partial\mathbf{v}_{s1}/\partial t$. The other terms $\mathbf{v}_{s0}\times\mathbf{B}_1/c$ are even smaller than $(\mathbf{v}_{s1}\cdot\boldsymbol{\nabla})\mathbf{v}_{s0}$, since $\mathbf{B}_1<\mathbf{B}_0$ and $\mathbf{v}_{s1}$ is generally of the same order as $\mathbf{v}_{s0}$. It should be remembered that the magnitude of the density fluctuations $n_{s1}$ is much less than the equilibrium density $n_{s0}$. It is possible to prove that the linear dispersion relation is

$$\omega^2 = \omega_p^2\left(1+\frac{n_{p0}}{n_{e0}}\right)+k^2c^2, \qquad (23)$$

where $\omega$ and $k$ are the frequency and wavenumber of the electromagnetic wave, and $\omega_p = 4\pi n_{e0}e^2/m_e$ is the electron plasma frequency. Further, the current densities $\mathbf{J}_0$ and $\mathbf{J}_1$ are simplified to

$$\mathbf{J}_0 = \sum_{s=i,e,p} n_{s0} q_s \mathbf{v}_{s0},$$

$$\mathbf{J}_1 = \sum_{s=e,p} q_s n_{s0} \mathbf{v}_{s1}$$

## 4. Growth rate of the magnetic field

From (19), the growth rate of the magnetic field, $\partial \mathbf{B}_0/\partial t$, can be calculated after finding $\mathbf{E}_0$. Neglecting the slowly varying inertial term for electrons and positrons in (18) and using the expressions for the current densities and the quasineutrality condition $n_{i0} + n_{p0} = n_{e0} = n_0$ (say), $\mathbf{E}_0$ is obtained as

$$\mathbf{E}_0 = \Lambda_1 + \Lambda_2 + \Lambda_3 + \Lambda_4, \tag{24}$$

where

$$\Lambda_1 = \frac{k_B}{2} \sum_{s=e,p} \frac{1}{q_s n_{s0}} \boldsymbol{\nabla}(n_{s0} T_s) - \left(\mathbf{v}_{k'0} \times \frac{\mathbf{B}_0}{c}\right) - \frac{1}{2} \sum_{\substack{s=e,p \\ k=e,p \\ k \neq s}} \frac{n_{s0}}{n_{k0}},$$

$$\Lambda_2 = \frac{1}{2}\left(\mathbf{J}_0 \times \frac{\mathbf{B}_0}{c}\right) \sum_{s=e,p} \left(-\frac{1}{q_s n_{s0}}\right) - \frac{1}{2} \sum_{\substack{s=e,p \\ k=e,p \\ k \neq s}} \frac{n_{s0}}{n_{k0}} \left(\mathbf{v}_{s0} \times \frac{\mathbf{B}_0}{c}\right),$$

$$\Lambda_3 = \frac{m_s}{2q_s^2} \sum_{s=e,p} \frac{\nu_{sk'}}{n_{s0}} \mathbf{J}_0 - \frac{1}{2} \sum_{\substack{s=e,p \\ k=e,p \\ k \neq s}} m_s \nu_{kk'} \frac{n_{s0}}{n_{k0}} (\mathbf{v}_{s0} - \mathbf{v}_{k'0}),$$

$$\Lambda_4 = \sum_{s=e,p} \frac{\nu_{sk}}{q_s} m_s \mathbf{v}_{s0} - \frac{1}{2} \sum_{\substack{s=e,p \\ k=e,p \\ k \neq s}} \frac{\Pi_s}{q_s n_{s0}}$$

(with $k' = i$). Now taking the curl of (24) and using (19) and the value of $\Pi_s$, ignoring thermoelectric terms and considering only the ponderomotive force terms, we get

$$\frac{\partial \mathbf{B}_0}{\partial t} = -\frac{1}{2} \sum_{s=e,p} \boldsymbol{\nabla} \times \left(\frac{mc}{q_s} \langle \mathbf{v}_{s1} \cdot \boldsymbol{\nabla} \mathbf{v}_{s1} \rangle \cdot \langle \mathbf{v}_{s1} \times \mathbf{B}_1 \rangle\right), \tag{25}$$

where $m_e = m_p = m$ (say).

It should be mentioned that the density gradient exists only normal to the direction of wave propagation, or the density gradient and temperature gradient lie perpendicular to each other. Let us consider the field variable $f_1 = f_1 \exp(-i\omega t)$ or $f_1 = f_R \cos\omega t + f_I \sin\omega t$, where $f_1$ stands for $v_{e1}$, $v_{p1}$, $\mathbf{E}_1$, or $\mathbf{B}_1$. Then (25) becomes

$$\frac{\partial \mathbf{B}_0}{\partial t} = -\boldsymbol{\nabla} \times \boldsymbol{\epsilon}, \tag{26}$$

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where

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_1 + \boldsymbol{\epsilon}_2, \tag{27}$$

$$\boldsymbol{\epsilon}_1 = \sum_{s=e,p}\left(-\frac{1}{q_s}\right)\mathbf{v}_{sR}\times(\boldsymbol{\nabla}\times\mathbf{v}_{sR}) - \sum_{s=e,p}\frac{1}{m\omega}\mathbf{v}_{sR}\times(\boldsymbol{\nabla}\times\mathbf{E}_I),$$

$$\boldsymbol{\epsilon}_2 = \sum_{s=e,p}\frac{1}{q_s}\mathbf{v}_{sL}\times(\boldsymbol{\nabla}\times\mathbf{v}_{sI}) + \sum_{s=e,p}\frac{1}{m\omega}\mathbf{v}_{sI}\times(\boldsymbol{\nabla}\times\mathbf{E}_R),$$

and we write

$$\mathbf{D}_I = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E}_1), \qquad \mathbf{D}_R = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E}_R), \qquad \gamma = \frac{\alpha_1}{\alpha_2},$$

$$\Omega_1 = \frac{\nu_{ei}}{\omega}, \qquad \Omega_2 = \frac{\nu_{ep}}{\omega}, \qquad \Omega_3 = \frac{\nu_{pi}}{\omega}.$$

Now from (20–22) and using the values of $\mathbf{J}_0$ and $\mathbf{J}_1$, we obtain

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\mathbf{E}_1 = \frac{\omega^2}{c^2}\epsilon_e\mathbf{E}_1 + \beta^2\mathbf{D}_I, \tag{28}$$

where

$$\epsilon_e = 1 - \frac{\omega_p^2}{\omega^2}\{[1+i(\Omega_1+\Omega_2)]^{-1} + \alpha_2[1+i(\Omega_2+\Omega_3)]^{-1}\},$$

$$\beta^2 = \frac{c_s^2}{c^2}(\{1+i[\Omega_1+\Omega_2(1+\alpha_1/\alpha_2)]\}^{-1} + \alpha_1\{1+i[\Omega_3+\Omega_2(1+\alpha_2/\alpha_1)]\}^{-1}),$$

$$c_s^2 = \frac{k_BT_e}{m}, \qquad \alpha_1 = \frac{T_p}{T_e}, \qquad \alpha_2 = \frac{n_{p0}}{n_{e0}}.$$

$\epsilon_e$ is the effective dielectric constant of the medium and $c_s$ is the thermal speed of the electrons. Therefore (26) gives

$$\begin{aligned}\left|\frac{\partial\mathbf{B}_0}{\partial t}\right|_z \frac{ea^2c\nu}{2mc_s^2} &= \left(\frac{1}{1+\alpha_1}\right)^2\left[1-\frac{\omega_p^2}{\omega^2}(1+\alpha_2)\right]\left(\frac{\omega}{\omega_p}\right)^2 \\ &\quad\times\left\{\left[1-\frac{\omega_p^2}{\omega^2}(1+\alpha_2)\right]^2 - \frac{6\nu^2}{\omega^2}\frac{\alpha_1}{\alpha_2}\left(1-\frac{\alpha_1}{\alpha_2}\right)\right\}^{1/2} \\ &\quad\times\sin\left[2\omega\left(\frac{x\epsilon_e^{1/2}}{\beta c}-t\right)\right],\end{aligned} \tag{29}$$

which is also obtained using (28), where $\mathbf{E}_1 = a\exp[i(px-\omega t)]\,\epsilon_k$ is the general solution of (28).

## 5. Results and some concluding remarks

The expression (29) gives the growth rate of the magnetic field that is generated due to the ponderomotive force in an astrophysical plasma consisting of electrons, positive ions, and positrons. In (29), $\epsilon_e$ is the effective dielectric constant, $c$ is the

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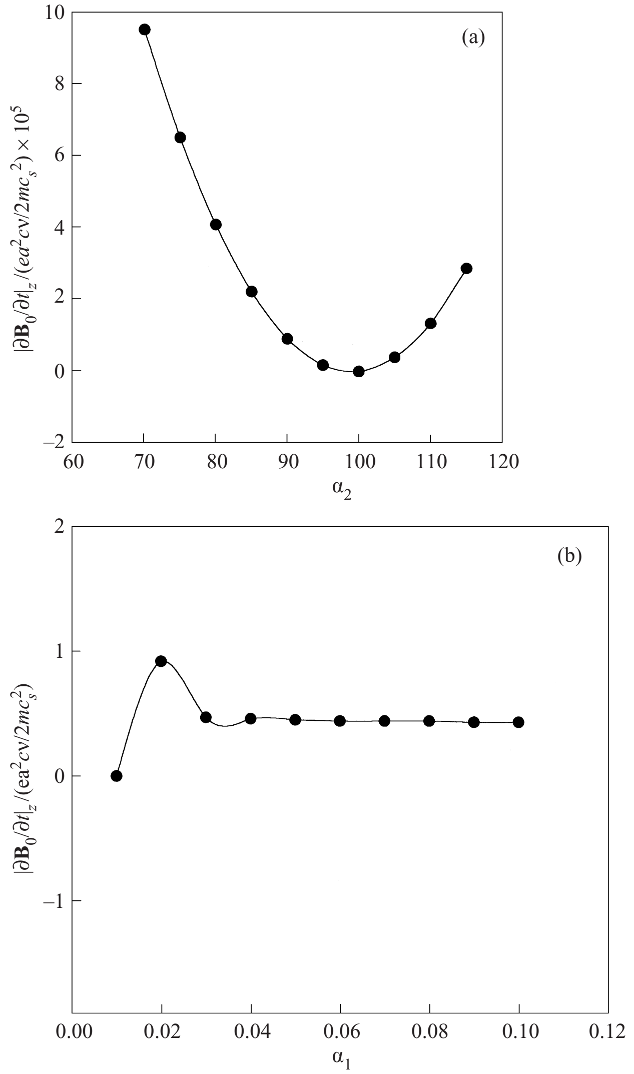


**Figure 1.** Dependence of $|\partial \mathbf{B}_0/\partial t|_z/(ea^2c\nu/2mc_s^2)$: (a) on $\alpha_2$ for fixed $\alpha_1$; (b) on $\alpha_1$ for fixed $\alpha_2$.

speed of light, and $\nu$ is the collision frequency. Thus, it is seen that the growth rate depends on the amplitude of electromagnetic wave, the density and temperature of electrons and positrons, and the collision frequency. To get an idea about the role of positrons in the growth rate of the magnetic field, we consider different values of concentration and temperature of both positrons and electrons (Fig. 1). For a typical astrophysical plasma, we consider different values of the density ratio $\alpha_2$ and the temperature ratio $\alpha_1$. For estimating the numerical result, we have assumed that $a = 10^{-2}$ and $\nu/\omega = 10^{-2}$. From the results, we find that the presence of positrons in an astrophysical plasma plays a key role in the generation of a magnetic field due to the ponderomotive force. From Figs 1(a) and (b), it is seen that the behaviour of the growth rates of the magnetic field for $\alpha_1$ and $\alpha_2$ are very different. Figure 1(b) shows that the growth rate increases up to certain values of

$\alpha_1$, then decreases, and finally becomes constant at higher values of $\alpha_1$. The shape of the growth rate of the magnetic field becomes solitary type for lower values of $\alpha_1$. On the other hand, Fig. 1(a) shows that with increasing value of $\alpha_2$, the growth rate of the magnetic field decreases and then increases. In this case, the shape of the growth rate of the magnetic field is of inverse solitary type. From this, it can be concluded that the shapes of the growth rate of the magnetic field due to the ponderomotive force for various values of $\alpha_1$ and $\alpha_2$ are inverse. It is also important to note that the growth rate is proportional to the square of the amplitude of the electromagnetic wave. So, a high-power electromagnetic wave propagating through an electron–ion–positron plasma would generate a strong magnetic field, which may be one of the sources of various astrophysical phenomena.

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