Econometric Theory, **16**, 2000, 459–461. Printed in the United States of America.

CORRIGENDA

Solution to Problem 98.5.3[1]—Seiji Nabeya has pointed out that in the solution, Paulo M.M. Rodrigues claimed that the process

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + e_t \qquad (t = 1, \ldots, n),$$

with $y_{-1} = y_0 = 0$ and $e_t \sim$ i.i.d. $(0, \sigma^2)$, if the true values are $\varphi_1 = 0$ and $\varphi_2 = -1$, then the least squares estimators $\hat{\varphi}_1$ and $\hat{\varphi}_2$ of φ_1 and φ_2 are asymptotically independent, whereas if $\varphi_1 = 0$ and $\varphi_2 = 1$, then they are not. I will show in both cases that $\hat{\varphi}_1$ and $\hat{\varphi}_2$ are not asymptotically independent.

Using similar notations as those in Nabeya (1999b), define

$$U_{1i} = \int_0^1 W_{1i}(r)\, \mathrm{d}W_{1i}(r), \qquad V_{1i} = \int_0^1 W_{1i}(r)^2\, \mathrm{d}r \qquad (i = 1,2),$$

where $W_{1i}(r)$ $(i = 1,2)$ are independent Brownian motions, and

$$U_2 = \int_0^1 W_1(r)\, \mathrm{d}W_1(r) + \int_0^1 W_2(r)\, \mathrm{d}W_2(r),$$

$$U_3 \int_0^1 W_1(r)\, \mathrm{d}W_2(r) - \int_0^1 W_2(r)\, \mathrm{d}W_1(r),$$

and

$$V_2 = \int_0^1 [W_1(r)^2 + W_2(r)^2]\, \mathrm{d}r,$$

where $W_1(r)$ and $W_2(r)$ are also independent Brownian motions.

According to Chan and Wei (1988), we have in the case $\varphi_1 = 0$ and $\varphi_2 = 1$,

$$n(\hat{\varphi}_1, \hat{\varphi}_2 - 1) \Rightarrow \left(\frac{U_{11}}{V_{11}} - \frac{U_{12}}{V_{12}}, \frac{U_{11}}{V_{11}} + \frac{U_{12}}{V_{12}} \right),$$

denoting by $\Rightarrow$ convergence in distribution.

To prove the asymptotic nonindependence of $\hat{\varphi}_1$ and $\hat{\varphi}_2$, it is sufficient to show

$$E\left[\left(\frac{U_{11}}{V_{11}} - \frac{U_{12}}{V_{12}} \right)^2 \left(\frac{U_{11}}{V_{11}} + \frac{U_{12}}{V_{12}} \right) \right] \neq E\left[\left(\frac{U_{11}}{V_{11}} - \frac{U_{12}}{V_{12}} \right)^2 \right] E\left(\frac{U_{11}}{V_{11}} + \frac{U_{12}}{V_{12}} \right). \tag{1}$$

Taking into account the fact that U_{11}/V_{11} and U_{12}/V_{12} are i.i.d., the left-hand side of (1) becomes,

$$2\left\{ E\left[\left(\frac{U_{11}}{V_{11}} \right)^3 \right] - E\left[\left(\frac{U_{11}}{V_{11}} \right)^2 \right] E\left(\frac{U_{11}}{V_{11}} \right) \right\}$$
$$= 2 \times [-132.686 - 13.286 \times (-1.781)] = -218.037, \tag{2}$$

 0266-4666/00 $9.50

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whereas the right-hand side becomes,

$$4\left\{E\left[\left(\frac{U_{11}}{V_{11}}\right)^2\right]-\left[E\left(\frac{U_{11}}{V_{11}}\right)\right]^2\right\}E\left(\frac{U_{11}}{V_{11}}\right)$$
$$= 4\times[13.286-(-1.781)^2]\times(-1.781) = -72.057, \tag{3}$$

thus establishing (1). The numerical values in these two equations can be found in Nabeya (1999a). Note that the inequality (1) implies that the third central moment of U_{11}/V_{11} is not equal to 0.

Remark 1. I conducted a simulation using uniform random numbers $\{e_t\}$ with the sample length $n = 1{,}200$ and the number of replications $N = 100{,}000$. By averaging N values of $n^3\hat{\varphi}_1^2(\hat{\varphi}_2 - 1)$, $n^2\hat{\varphi}_1^2$, and $n(\hat{\varphi}_2 - 1)$, the estimates for the three expectations in (1) were obtained as -218.753, 20.109, and -3.548, respectively. The first estimate is close to (2), and the product -71.353 of the other two estimates is close to (3).

Chan and Wei (1988) showed in the case $\varphi_1 = 0$ and $\varphi_2 = -1$ that

$$n(\hat{\varphi}_1, \hat{\varphi}_2 + 1) \Rightarrow \left(\frac{2U_3}{V_2}, -\frac{2U_2}{V_2}\right).$$

To prove the asymptotic nonindependence of $\hat{\varphi}_1$ and $\hat{\varphi}_2$, it is sufficient to show

$$E\left[\left(\frac{2U_3}{V_2}\right)^2\left(-\frac{2U_2}{V_2}\right)\right] \neq E\left[\left(\frac{2U_3}{V_2}\right)^2\right]E\left(-\frac{2U_2}{V_2}\right). \tag{4}$$

The joint moment-generating functions for (U_2, U_3, V_2), (U_2, V_2), and (U_3, V_2) were given by Nabeya (1999b). By applying Sawa's (1972) formula or its extension given by Nabeya (1999b) to these joint moment-generating functions, we obtain

$$E\left[\left(\frac{2U_3}{V_2}\right)^2\left(-\frac{2U_2}{V_2}\right)\right] = 32.814, \tag{5}$$

and

$$E\left[\left(\frac{2U_3}{V_2}\right)^2\right]E\left(-\frac{2U_2}{V_2}\right) = 7.328\times 1.664 = 12.192, \tag{6}$$

thus establishing (2).

Remark 2. I conducted a simulation under the same conditions as those in Remark 1. By averaging N values of $n^3\hat{\varphi}_1^2(\hat{\varphi}_2 + 1)$, $n^2\hat{\varphi}_1^2$, and $n(\hat{\varphi}_2 + 1)$, the estimates for the three expectations in (4) were obtained as 31.725, 7.279, and 1.653, respectively. The first estimate is close to (5) and the product 12.029 of the other two estimates is close to (6).

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Remark 3. The independence of $U_2\,/V_2^{1/2}$ and $U_3/V_2^{1/2}$ was proved by Nabeya (1999b).

NOTE

1. Paulo M.M. Rodrigues has pointed out that the contrast results from the asymptotic properties of the least squares estimates in symmetric seasonal processes orthogonality of the regressors and not from the independence of the distribution of the LS estimates.

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There is an error in the results reported in Theorem 7. The definitions of $\overline{V}(s)$ and $\widetilde{V}(s)$ above Theorem 7 on p. 541 should read:

$$\overline{V}(s) = U_1(s) - \int dW_1\,\overline{W}_2'\left(\int \overline{W}_2\,\overline{W}_2'\right)^{-1}\int_0^s \overline{W}_2(r)\,dr,$$

$$\widetilde{V}(s) = U_2(s) - \int dW_1\,\widetilde{W}_2'\left(\int \widetilde{W}_2\,\widetilde{W}_2'\right)^{-1}\int_0^s \widetilde{W}_2(r)\,dr,$$

where

$$U_1(s) = W_1(s) - sW_1(1),$$

$$U_2(s) = W_1(s) + (2s - 3s^2)W_1(1) - 6(r - r^2)\int W_1.$$

In the proof of Theorem 7(ii) and (iii) on p. 557, the sentence should read:

> (ii) and (iii) These parts follow in the same way with B_w, W_1, W_2, and V replaced by $\bar{B}_w$, U_1, $\overline{W}_2$, and $\overline{V}$, respectively, in part (ii) and $\tilde{B}_w$, U_2, $\widetilde{W}_2$, and $\widetilde{V}$ in part (iii).

The critical values in Tables 1–3 are correct as reported. I am grateful to Johan Lyhagen for drawing my attention to the error.

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