Robotica (2002) volume 20, pp. 241–249. © 2002 Cambridge University Press
DOI: 10.1017/S0263574701003976 Printed in the United Kingdom

Stable pinching by a pair of robot fingers with soft tips under the effect of gravity

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(Received in Final Form: November 14, 2001)

SUMMARY
This paper analyses lumped-parameter dynamics of a pair of robot fingers with soft and deformable tips pinching a rigid object under the effect of a gravity force. The dynamics of the system in which area contacts between the finger-tips and the surfaces of the object arise are compared with those of a pair of rigid robot fingers with rigid contacts with an object, with or without effect of the gravity. It is then shown that there exists a sensory feedback from measurement of finger joint angles and the rotational angle of the object to command inputs to joint actuators, and this feedback connection from sensing to action realizes secure grasping of the object in a dynamic sense and regulation of the object posture. It is further shown that there are various types of other feedback connections from sensing to action, which can be used in combination of feedback signals for stable grasping and posture control of the object for realizing sophisticated object manipulation.

KEYWORDS: Robot fingers; Soft tips; Gravity; Stable pinching; Lumped-parameter dynamics.

1. INTRODUCTION
In the early 1980s, "Robotics" was defined by Winston (Professor of MIT) as an interdisciplinary research frontier dedicated to "intelligent connection from perception to action".[1] It was also around the early 1980s that roboticists dreamed enthusiastically about creating "intelligent robots" through looking at the successful employment of a number of robot arms in automation processes and assembly lines in factory. In that decade a variety of multi-fingered robot hands that more or less mimic human hands were designed and manufactured (for example, see Shimoga.[2]) On the other hand, much theoretical works on motion plannings and stability problems of object grasping and manipulation by means of multi-fingered hands has been published in the vast literature, as seen in an extensive survey by Shimoga[2] and others.[3–5] One of noteworthy results obtained was concerned with sufficient conditions guaranteeing secure grasping of an object with a geometric shape.[6,7] In addition, there is a great number of interesting works that pointed out important roles of tactile and/or vision sensings in stable grasping and object manipulation, and carried out actual experiments by using those sensors (see Cutkovsky[8] or more recent papers.[9–12]) Motion control has been also discussed on the basis of a computed torque of complicated nonlinear dynamics of an overall system, including fingers and a rigid object, by assuming pointwise contacts between finger-tips and object surfaces.[13] However, there is a dearth of papers that attempted to analyze sufficient conditions for secure grasping from a dynamic viewpoint, based on full dynamics of motion of such a multi-fingered hand manipulating an object through area contacts between deformable finger-tips and surfaces of the rigid object. Actually, it should be pointed out that a more realistic model of dynamics of a total finger-object system with soft area contacts was neither derived nor analyzed until a recent publication of a series of papers,[14–16] though some important roles of soft area contact were pointed out early in 1980s.[8] More surprisingly, there are very few papers concerned with finding an explicit sensory feedback path connecting from tactile or/and vision sensing to motor control at finger joints, that is crucial in guaranteeing secure grasping in a dynamic sense and dexterous control of object manipulation.

"Sensory-motor coordination" is also one of the most important research subjects in developmental psychology. In fact, much data of observation, concerning the emergence of bimanual coordination from the early patterns observable at birth to the bimanual coordination involved in object manipulation typical of a one-year-old child has been gathered, and theoretical approaches that best account for such development are analyzed.[17,18] In particular, E. Thelen et al.[18] claim that, concerning the development of bimanual coordination, the dynamic point of view postulates that new spaciotemporal orders emerge not from centrally prescribed programs in the infant central vervous system but from the system dynamics. More precisely, E. Thelen et al.[19] concludes that, concerning the emergence of reaching for object grasping, "the infant's CNS does not contain programs that detail hand trajectory, joint coordination and muscle activation patterns. Rather, these patterns are the consequences of the natural dynamics of the system and the active exploration of the match between those dynamics and the task". However, these observations and theories are concerned with reaching, grasping, and bimanual coordination by using a couple of arms, but not concerned with inter-fingers coordination for pinching by using a couple of thumb and index fingers. However, even in the case of sensory-motor coordination for pinching, the intrinsic dynamics must play an important role and, in particular, the

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theories developed by Thelen et al.[18,19] suggest the existence of a certain family of sensory-motor feedback paths.

This paper aims at finding a sensory feedback path from sensing to motor control at finger joints guaranteeing secure grasping in the case that a pair of multijoint fingers with soft and deformable finger-tips are in contact with a parallelepiped object and placed in a vertical plane. In previous papers,[14–16] a full dynamics of the overall finger-object system has been derived in the case that dynamic behaviour of finger-tips is lumped-parametrized by assuming that the soft material is composed of massless springs distributed in all directions, and the finger-tip shape is a hemisphere, motion of the system is confined to a horizontal plane, and therefore the effect of gravity force can be neglected. This paper treats the case that motion of the overall finger-object system is affected directly by the gravity force and, therefore, its dynamics differs from that of the same mechanism whose motion is confined into a horizontal plane. Even in the situation that the system is directly affected by the gravity, it is possible to find an analytic feedback path from joint position sensings (optical encoders at joint actuators) and sensing of the rotation angle of the object (by means of optical range sensors mounted at a finger link) to control commands at joint actuators. It is shown theoretically that feedback control realizes secure grasping in a dynamic sense and eventually regulates the object in the vicinity of the upright position. It is also discussed that a similar but more simplified feedback path in the case of rigid contacts between finger-tips and object surfaces can be found. In a final section, some possible extensions of the theoretical argument developed in the paper are presented, which include an alternative analysis when the mass of the object is unknown or another feedback path for regulating the posture of the object is taken into account.

2. DYNAMICS OF A PAIR OF MULTI-DOF FINGERS WITH SOFT TIPS PINCHING OF A RIGID OBJECT

In previous papers,[14–16] the dynamics of a pair of multi-degrees of freedom robotic fingers with soft and deformable tips pinching of a rigid object, as shown in Figure 1, has been derived. In those papers it is assumed that the motion of the overall finger-object system is confined to a horizontal plane and, thereby, the effect of the gravity force can be ignored. As seen in Figure 1 both finger-tips are soft and deformable and their shape is hemi-spherical. It is also assumed implicitly that the material of soft finger-tips is non-compressible and purely elastic. This naturally induces area contacts between both finger-tips and corresponding object surfaces, and their corresponding holonomic contraints are expressed by the following algebraic equation:

$$\begin{cases} Y_1 = (x_{01} - x)\sin\theta + (y_{01} - y)\cos\theta \\ Y_2 = (x_{02} - x)\sin\theta + (y_{02} - y)\cos\theta \end{cases} \tag{1}$$

where (x, y) denotes the cartesian coordinates of the mass center O of the object. In fact, note that rolling of the last link of a robot finger againt a rigid object generates movement of the center of area contact, as seen in Figure 2, where it is also assumed implicitly that there does not arise any slip between the finger-tip and object surface (this is called in this paper "tight area contact"). The rolling induces equation (1), where Y_i can be defined as

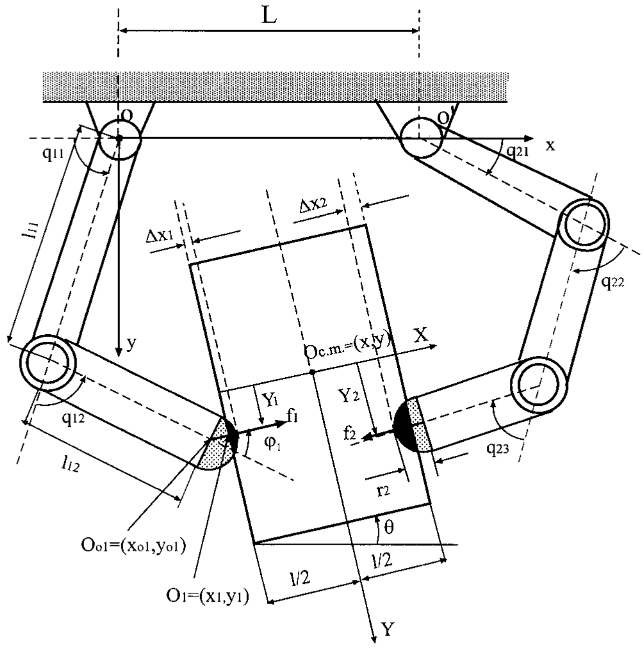

Fig. 1. A pair of 2-DOF and 3-DOF fingers with soft tips.

$$\begin{cases} Y_1 = c_1 - r_1\varphi_1 = c_1 - r_1(-q_{11} - q_{12} + \pi + \theta) \\ Y_2 = c_2 - r_2\varphi_2 = c_2 - r_2(-q_{21} - q_{22} - q_{23} + \pi - \theta) \end{cases} \tag{2}$$

and c_i is some constant (for $i = 1, 2$). The dynamic behaviour of finger-tips can be approximately expressed by concentrated contact forces $f_i(\Delta x_i)$ $(i = 1, 2)$ by means of lumped-parametrization of distributed massless springs, as shown in Figure 3. These contact forces $f_i (i = 1, 2)$ arise in the directions normal to the object surfaces and press the object at both sides of the object from opposite directions. According to the Appendix of the previous paper,[14] the magnitude of the lumped-parametrized reproducing force $f_i(\Delta x_i)$ can be expressed as

$$f_i(\Delta x_i) = K_i(\Delta x_i)^2 \tag{3}$$

where Δx_i denotes the maximum length of deformation of the soft tip of finger i. Then, by introducing Lagrange's multipliers λ_1 and λ_2 corresponding to the equalities

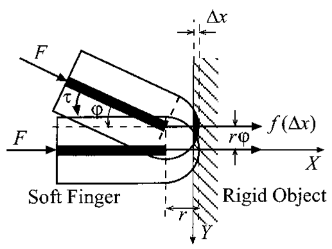

Fig. 2. Geometric constraint induced by tight area contact and rolling.

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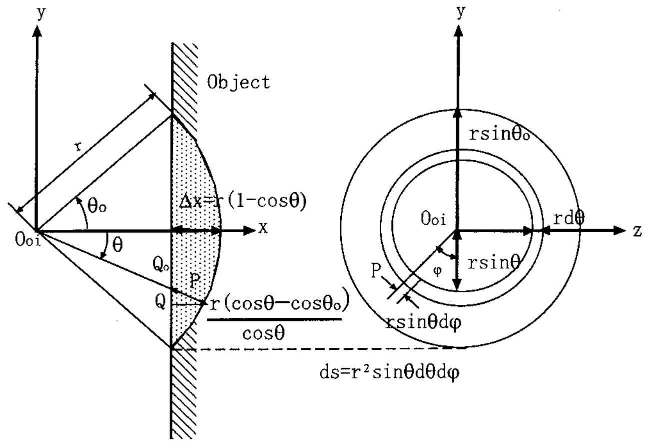

Fig. 3. Lumped-parameterization of the dynamics of a soft material.

$Y_i - c_i + r_i\varphi_i = 0$ (for $i=1, 2$) where Y_i stand for equation (1), and applying the Hamilton's principle to the Lagrangian, we have

$$L = K - P + \sum_{i=1,2} \lambda_i \Phi_i \tag{4}$$

where

$$K = \frac{1}{2}\sum_{i=1,2} \dot{q}_i^{T} H_i(q_i)\dot{q}_i + \frac{1}{2}(M\dot{x}^2 + M\dot{y}^2 + I\dot{\theta}^2) \tag{5}$$

$$P = \sum_{i=1,2} \int_0^{\Delta x_i} f_i(\zeta)\mathrm{d}\zeta \tag{6}$$

$$\Phi_i = Y_i - (c_i - r_i\varphi_i), \quad i=1, 2 \tag{7}$$

It is possible to derive Lagrange's equation of the overall finger-object system:

$$L_i + (-1)^{i-1} f_i \eta_i^{T} + \lambda_i p_i^{T} = u_i, \quad i=1, 2 \tag{8}$$

$$M\ddot{x} - (f_1 + f_2)\cos\theta + (\lambda_1 + \lambda_2)\sin\theta = 0 \tag{9}$$

$$M\ddot{y} + (f_1 + f_2)\sin\theta + (\lambda_1 + \lambda_2)\cos\theta = 0 \tag{10}$$

$$I\ddot{\theta} - (Y_1 f_1 - Y_2 f_2) + \lambda_1(l/2 - \Delta x_1) - \lambda_2(l/2 - \Delta x_2) = 0 \tag{11}$$

where $q_1 = (q_{11}, q_{12})^{T}$, $q_2 = (q_{21}, q_{22}, q_{23})^{T}$,

$$L_i = \left\{ H_i(q_i)\frac{d}{dt} + \frac{1}{2}\dot{H}_i(q_i) \right\}\dot{q}_i + S_i(q_i, \dot{q}_i)\dot{q}_i \tag{12}$$

$$\eta_i^{T} = J_{0i}^{T}\begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}, \quad p_i^{T} = J_{0i}^{T}\begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} - r_i\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{13}$$

and J_{0i} denote the Jacobian matrices of (x_{0i}, y_{0i}) with respect to (q_i) for $i=1, 2$.

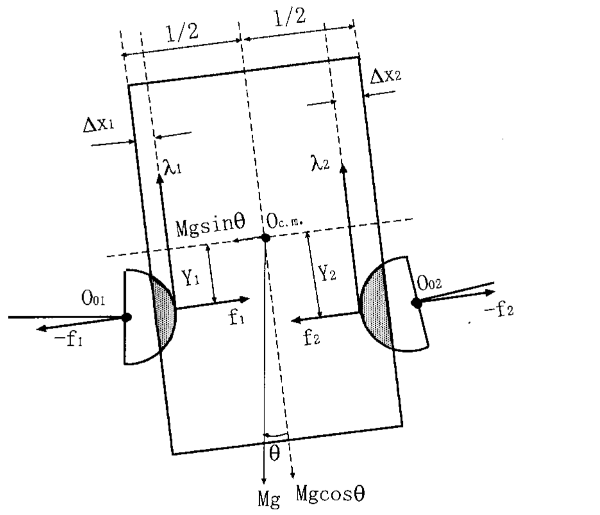

Fig. 4. Grasping of a rigid object.

3. STABLE PINCHING UNDER THE EFFECT OF GRAVITY

When the overall finger-object system is placed in a vertical plane, the motions of fingers and the object are directly affected by the gravity force (see Figure 4). Then, the dynamics of the overall system becomes as follows:

$$L_i + (-1)^{i-1} f_i \eta_i^{T} + \lambda_i p_i^{T} + [g_i(q_i)] = u_i, \quad i=1, 2 \tag{14}$$

$$M\ddot{x} - (f_1 + f_2)\cos\theta + (\lambda_1 + \lambda_2)\sin\theta = 0 \tag{15}$$

$$M\ddot{y} + (f_1 + f_2)\sin\theta + (\lambda_1 + \lambda_2)\cos\theta - [Mg] = 0 \tag{16}$$

$$I\ddot{\theta} - (Y_1 f_1 - Y_2 f_2) + \lambda_1(l/2 - \Delta x_1) - \lambda_2(l/2 - \Delta x_2) = 0 \tag{17}$$

Note that the terms enclosed by the bracket [ ] appear additionally to the equations (8) to (11).

First we assume that all finger link masses, mass M of the object, and length from the mass center of each finger link to its corresponding joint center are known. Then, it is easy to calculate $g_i(q_i)$ for $i=1, 2$ in real-time, which can be included in command inputs, respectively, for $i=1, 2$ in order to compensate the effect of gravity force in equation (14). However, the motion equations (14)–(17) of the object cannot be controlled directly by control commands u_i, that is, either of u_i $(i=1, 2)$ does not enter in equations (15)–(17). Hence, the gravity terms in equation (16) should be compensated indirectly through physical quantities f_i and λ_i which can be controlled by u_i through finger dynamics of equation (14).

Thus, we introduce the control commands u_{gi} $(i=1, 2)$ in the following way:

$$u_{gi} = -c_i\dot{q}_i + g_i(q_i) + (-1)^i r_i f_d (Y_1 - Y_2)/(r_1 + r_2) - (-1)^i \eta_i^{T} f_d + \eta_i^{T}(Mg/2)\sin\theta + p_i^{T}(Mg/2)\{\cos\theta - (-1)^i (Y_1 + Y_2)\zeta^{-1}\}$$

where $\zeta = l - \Delta x_1 - \Delta x_2$. Here, it is important to note that the individual Y_i cannot be calculated from the measurement data of q_i and θ differently from the value of $Y_1 - Y_2$. At this first stage we assume that $Y_1 + Y_2$ is accessible for the construction of the feedback path. However, in the sequel we will use the data on $\dot{Y}_i$ which is composed of $\dot{q}_i$ and $\dot{\theta}$ that can be calculated from the measurement data. Substituting $u_i = u_{gi}$ into equation (14) yields the closed-loop finger dynamics of the following form:

$$\begin{aligned} L_i + c_i \dot{q}_i - (-1)^i \frac{r_i f_d}{r_1 + r_2} (Y_1 - Y_2) \\ - (-1)^i \Delta f_i' \eta_i^T + \lambda_i' p_i^{\mathrm{T}} = 0 \end{aligned} \tag{19}$$

where

$$\Delta f_i' = \Delta f_i + (-1)^i (Mg/2) \sin\theta \tag{20}$$

$$\lambda_i' = \lambda_i - (Mg/2)\{\cos\theta - (-1)^i \zeta^{-1} (Y_1 + Y_2) \sin\theta\} \tag{21}$$

The dynamics of the object are the same as equations (15)–(17) but it is convenient to rewrite them into the forms:

$$M\ddot{x} - (\Delta f_1' - \Delta f_2') \cos\theta + (\lambda_i' + \lambda_2') \sin\theta = 0 \tag{22}$$

$$M\ddot{y} + (\Delta f_1' - \Delta f_2') \sin\theta + (\lambda_1' + \lambda_2') \cos\theta = 0 \tag{23}$$

$$\begin{aligned} I\ddot{\theta} + \Delta f_1' Y_1 - \Delta f_2' Y_2 + \lambda_1' (l/2 - \Delta x_1) - \lambda_2' (l/2 - \Delta x_2) \\ - f_d (Y_1 - Y_2) + (Mg/2)(-\Delta x_1 + \Delta x_2) \cos\theta = 0 \end{aligned} \tag{24}$$

It is important to see that equations (19) to (24) are similar to the closed-loop equations (8) to (11) with $u_i = -c_i \dot{q}_i + (-1)_i f_d r_i (Y_1 + Y_2)/(r_1 + r_2) - (-1)^i \eta_i^{\mathrm{T}} f_d$, except the last term of equation (24) if Δf_i and λ_i are replaced with $\Delta f_i'$ and λ_i' respectively. Bearing this in mind, we apply a similar argument to the proof of the case without the effect of gravity for equations (19) to (24). Firstly, let us take inner products between $\dot{q}_i$ and equation (19), and $(\dot{x}, \dot{y}, \dot{\theta})$ and equations (22), (23) and (24), respectively. This yields

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \{K + \Delta P + f_d (Y_1 - Y_2)^2 / 2(r_1 + r_2)\} \sum_{i=1,2} c_i \|\dot{q}_i\|^2 \\ - (\Delta \dot{x}_1 - \Delta \dot{x}_2)(Mg/2) \sin\theta - \dot{\theta} (\Delta x_1 - \Delta x_2)(Mg/2) \cos\theta = 0 \end{aligned} \tag{25}$$

where

$$\begin{aligned} \Delta P &= A_{f1} + A_{f2} \\ A_{fi} &= \int_0^{\delta x_i} \{f_i(\Delta x_{di} + \xi) - f_d\} \mathrm{d}\xi \end{aligned} \tag{26}$$

and $\Delta x_{di} = f_i^{-1}(f_d)$ (see Figure 5). This is also rewritten in the form

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \left[\{K + f_d (Y_1 - Y_2)^2 / 2(r_1 + r_2)\} + \sum_{i=1,2} \{(-1)^i A_{gi} + A_{fi} + \delta_o\} \right] \\ = - \sum_{i=1,2} c_i \|\dot{q}_i\|^2 \end{aligned} \tag{27}$$

where δ_o is the area defined in Figure 5 and

$$A_{gi} = (Mg/2) \delta x_i \sin\theta \tag{28}$$

Note that if $\pi/4 \geq |\theta| > 0$ and $f_d > Mg$, then $(-1)^i A_{gi} + A_{fi} + \delta_{0i}$ is positive for any $\Delta x_i \geq 0$ for $i = 1, 2$, as shown in Figure 5. Thus, it is possible to state the following result:

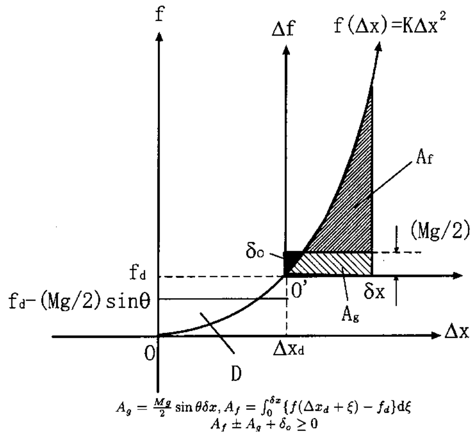

Fig. 5. Quantity of the artificial potential at the initial time is less than $D = \Delta x_d f_d - \frac{1}{2} Mg \sin\theta \Delta x_d - \int_0^{\Delta x_d} f(\xi) \mathrm{d}\xi$.

Stable Grasping under Gravity:

If the state z and $\dot{z}$ at initial time $t = 0$ satisfies

$$0 < E(0) < \min_{i=1,2} \{D_i\} \tag{29}$$

where D_i is the area indicated in Figure 5 and E stands for the total energy-like quantity equivalent to the content of bracket [ ] of the left hand side of equation (27), then grasping is maintained for all $t \geq 0$ and

$$\begin{aligned} &\dot{z} \to 0, f_i + (-1)^i (Mg/2) \sin\theta \to f_d, \\ &Y_1 - Y_2 \to 0, \Delta x_1 - \Delta x_2 \to 0, \\ &\lambda_i - (Mg/2)\{\cos\theta - (-1)^i \zeta^{-1} (Y_1 + Y_2) \sin\theta\} \to 0 \end{aligned} \tag{30}$$

as $t \to \infty$. In particular, if $r_1 = r_2$ and $f_1(\Delta x) = f_2(\Delta x)$ for any $\Delta x \geq 0$, that is, the characteristics of reproducing force of the soft finger-tips are the same, then

$$\theta(t) \to 0 \quad \text{as} \quad t \to \infty \tag{31}$$

which implies that the object posture converges asymptotically to the upright position.

It should be remarked that equation (30) determines the values for 6 physical variables $(\Delta f_1', \Delta f_2', Y_1 - Y_2, \Delta x_1 - \Delta x_2, \lambda_1', \lambda_2')$ at the steady state. Hence at least the composition of a pair of 2-DOF and 3-DOF fingers together with 3-DOF of the object is required, because the two algebraic constraints arising from the tight area contacts should be taken into account.

The proof is composed of many steps as in the following way:

(1) It follows from equation (27) that $E(t) \leq E(0)$ and thereby $\Delta x_i > 0$ $(i = 1, 2)$ for all $t \geq 0$.
(2) Then it follows from equation (27) too that $\dot{q}_i \in L^2(0, \infty)$ for $i = 1, 2$. Further, it is possible to show that $\dot{q}_i \to 0$ as $t \to \infty$ for $i = 1, 2$.
(3) Next, owing to geometric constraints of tight area contacts, $\dot{\theta}(t) \to 0$, $\dot{Y}_1(t) - \dot{Y}_2(t) \to 0$, $\dot{x} \sin\theta + \dot{y} \cos\theta \to 0$ and $\ddot{x} \sin\theta + \ddot{y} \cos\theta \to 0$ as $t \to \infty$. This implies that $\lambda_1 + \lambda_2 - Mg \cos \to 0$ as $t \to \infty$ from equations (22) and 23).
(4) Since it is possible to show that $\ddot{z}$ are bounded, $\dot{z}(t)$ is uniformly continuous. In particular, since $\dot{q}_1$, $\dot{q}_2$ and

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$\dot{\theta}\rightarrow 0$ as $t\rightarrow\infty$, $\ddot{q}_1$, $\ddot{q}_2$ and $\ddot{\theta}$ should converge to zero as $t\rightarrow\infty$. Hence, stationary terms of equations (19) and (24) (that are not related to any component of $\dot{z}$ and $\ddot{z}$) converge to some constant, which can be written in the following way:

$$A'b'\rightarrow\begin{pmatrix} r_1e_1 \\ -r_2e_2 \\ -(r_1+r_2)\end{pmatrix} f_d(Y_1-Y_2)/(r_1+r_2) \rightarrow \text{const.} \tag{32}$$

as $\rightarrow\infty$, where

$$A'=\begin{pmatrix} \eta_1^{\mathrm{T}} & 0 & -p_1^{\mathrm{T}} & 0 \\ 0 & -\eta_2^{\mathrm{T}} & p_2^{\mathrm{T}} & 0 \\ -Y_1 & Y_2 & \zeta & -\dfrac{M_g}{2}\cos\theta \end{pmatrix}, \quad b'=\begin{pmatrix} \Delta f_1' \\ \Delta f_2' \\ \lambda_1' \\ \Delta x_1-\Delta x_2\end{pmatrix} \tag{33}$$

Note that $\Delta x_1+\Delta x_2\rightarrow$ const. as $t\rightarrow\infty$ and thereby $\zeta\rightarrow$ const. as $t\rightarrow\infty$. Hence $\Delta f_i'\rightarrow$ const. as $t\rightarrow\infty$.

(5) Then, it is possible to show that $\ddot{x}$ and $\ddot{y}\rightarrow 0$ as $t\rightarrow\infty$ and $\Delta f_1'-\Delta f_2'\rightarrow 0$ as $t\rightarrow\infty$. Thus, stationary terms of equations (19) and (24) must converge to zero as $t\rightarrow\infty$, which can be written in the following way:

$$Ab\rightarrow 0 \quad \text{as} \quad t\rightarrow\infty \tag{34}$$

where

$$A=\begin{pmatrix} \eta_1^{\mathrm{T}} & -p_1^{\mathrm{T}} & r_1e_1 & 0 \\ -\eta_2^{\mathrm{T}} & p_2^{\mathrm{T}} & -r_2e_2 & 0 \\ -Y_1+Y_2 & \zeta & -(r_1+r_2) & -\dfrac{M_g}{2}\cos\theta \end{pmatrix}, \quad b=\begin{pmatrix} \Delta f_1' \\ \lambda_1' \\ \dfrac{f_d}{r_1+r_2}(Y_1-Y_2) \\ \Delta x_1-\Delta x_2\end{pmatrix} \tag{35}$$

and $e_1=(1,1)^{\mathrm{T}}$, $e_2=(1,1,1)^{\mathrm{T}}$. Since the rank of A is 4, $b\rightarrow 0$ as $t\rightarrow\infty$. Thus, the proof has been completed.

4. FURTHER DISCUSSIONS

In the case of pointwise contacts between rigid finger-tips and a rigid object, the dynamics of the overall system formally become similar to equations (8) and (11) (without the effect of gravity) or equations (14)–(17) (under the effect of gravity), if Δx_i ($i=1,2$) are regarded as zero. However, contact forces f_i ($i=1,2$) must be determined as Lagrange multipliers corresponding to the following algebraic constraints:

$$(x_{01}-x)\cos\theta-(y_{01}-y)\sin\theta+r_1+l/2=0 \tag{36}$$

$$-\{(x_{02}-x)\cos\theta-(y_{02}-y)\sin\theta\}+r_2+l/2=0 \tag{37}$$

Note that the left hand side of equation (36) is equal to Δx_1 and that of equation (37) to Δx_2 in the case of area contacts. Then, it is also possible to consider sensory feedbacks similar to u_{gi} mentioned in previous sections. In the case of a gravity force, a similar property of dynamic stable grasping already treated in previous papers an absence[14–16] can be concluded. However, in the latter case, the situation differs considerably. Firstly, the magnitude of contact forces f_i cannot be evaluated theoretically in a simple form. This means that there arises a certain possibility that contact between finger-tips and the object may be broken, which may lead to certain slipping between finger-tips and the object. Secondly, control of the posture of the object to the vicinity of its upright position cannot be realized by simply using u_{gi} (i=1, 2) even if $r_1=r_2$.

Some extensions of the argument in the previous section are possible to the cases that (1) the mass M of the object is unknown, (2), in addition, all masses of finger links are uncertain, and (3) the signal Y_1+Y_2 in equation (18) is replaced with that of $\int_0^{\mathrm{T}}(\dot{Y}_1+Y_2)\,d\tau$ that can be calculated from $\dot{q}_i$ and $\dot{\theta}$. In the case of (1), it is possible to use the estimate $\hat{M}$ for M defined as

$$\hat{M}(t)=\hat{M}(0)-\sum_{i=1,2}\int_0^t \Gamma^{-1}[\eta_i\sin\theta-p_i \times\{\cos\theta-(-1)^i\zeta^{-1}(Y_1+Y_2)\sin\theta\}]\dot{q}_i\,d\tau \tag{38}$$

where Γ is an appropriate positive constant. Then, instead of u_{gi} in equation (18), we are able to use

$$\bar{u}_{gi}=u_{fi}+g_i(q_i)+\eta_i^{\mathrm{T}}(\hat{M}g/2)\sin\theta-p_i^{\mathrm{T}}(\hat{M}g/2) \times\{\cos\theta-(-1)^i\zeta^{-1}(Y_1+Y_2)\sin\theta\} \tag{39}$$

where

$$u_{fi}=-c_i\dot{q}_i+(-1)^i\left\{\frac{r_if_d}{r_1+r_2}(Y_1-Y_2)\begin{pmatrix}1\\1\end{pmatrix}-f_d\eta_i^{\mathrm{T}}\right\} \tag{40}$$

and to conclude the same statement on dynamic stable grasping under the effect of gravity, provided that the initial guess $\hat{M}(o)$ does not so differ from M. As a matter of course, it is possible to use estimated values for masses of finger links in order to treat the case of (2).

In order to avoid the use of individual $Y_i(t)$ in construction of feedback signals, it is necessary to use the signal

$$Y_1(t)+Y_2(t)-\{Y_1(0)+Y_2(0)\}=\int_0^t\{\dot{Y}_1(\tau)+\dot{Y}_2(\tau)\}\,d\tau \tag{41}$$

which can be evaluated from the measured data on $\dot{\theta}$ and $\dot{q}_i$ ($i=1,2$), as is shown in equation (2). However, it should be remarked that if the quantity of equation (41) is used instead of $Y_1(t)+Y_2(t)$ in the feedback signal of equation (39) then there arises an offset in the closed-loop dynamics which may derive from the introduction of $Y_1(0)+Y_2(0)$, as

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is described in the left hand side of equation (41). In order overcome this problem, we introduce a position feedback defined as

$$k_i\{q_i(t)-q_i(0)\}=k_i\int_0^t \dot{q}_i(\tau)\,\mathrm{d}\tau \tag{42}$$

Thus, we define the feedback command signals in the following way:

$$\bar{u}_{gi}=u_{fi}+g_i(q_i)-k_i\int_0^t \dot{q}_i(\tau)\,\mathrm{d}\tau$$

$$+\frac{Mg}{2}\left[\eta_i^{\mathrm{T}}\sin\theta-p_i^{\mathrm{T}}\left\{\cos\theta-(-1)^i\zeta^{-1}\sin\theta\int_0^t\{\dot{Y}_1+\dot{Y}_2\}\,\mathrm{d}\tau\right\}\right] \tag{43}$$

Then, the closed-loop dynamics of fingers are expressed as

$$L_i+c_i\dot{q}_i+k_i\bar{q}_i-(-1)^i\frac{r_if_d}{r_1+r_2}(Y_1-Y_2)-(-1)^i\Delta f_i'\eta_i^{\mathrm{T}}+\lambda_i'p_i^{\mathrm{T}}$$
$$-p_i^{\mathrm{T}}(-1)^i\zeta^{-1}\{Y_1(0)+Y_2(0)\}\sin\theta=0 \tag{44}$$

where

$$\bar{q}_i=q_i(t)-q_i(0)=\int_0^t \dot{q}_i(\tau)\,\mathrm{d}\tau \tag{45}$$

As discussed in the previous paper,[15] an addition of the last term in the left hand side of equation (44) rewrites the relation of equation (27) into the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[K+\frac{f_d}{2(r_1+r_2)}(Y_1-Y_2)^2+\sum_{i=1,2}((-1)^iA_{gi}+A_{fi})\right.$$
$$\left.+\{Y_1(0)+Y_2(0)\}(\cos\theta+c_0)\right]=-\sum_{i=1,2}c_i\|\dot{q}_i\|^2 \tag{46}$$

Note that the quantity of the last term inside the bracket [ ] arises additionally, where the constant c_0 should be chosen as

$$c_0=\begin{cases}-1 & Y_1(0)+Y_2(0)<0\\ +1 & \text{otherwise}\end{cases} \tag{47}$$

If the magnitude of $|Y_1(0)+Y_2(0)|$ is relatively small in comparison with $\min\{D_i\}$, then it is possible to conclude a similar statement to that of the Stable Grasping under Gravity.

A control of the posture of the object at a given desired angle of rotation (say, $\theta=\theta_d$) is not so easy if θ_d differs from zero, contrarily to the case when there is no gravity effect. Nevertheless, it is meaningful to introduce an additional feedback path constructed as

$$u_{\theta i}=(-1)^i\{\zeta^{-1}\beta\Delta\theta+\alpha\dot{\theta}\}p_i^{\mathrm{T}} \tag{48}$$

if $|\theta_d|$ is relatively small. Then, the overall feedback signals

$$u_i=\bar{u}_{gi}+u_{\theta i} \tag{49}$$

for the dynamics of equation (19) yields the closed-loop dynamics

$$L_i+c_i\dot{q}_i+k_i\bar{q}_i-(-1)^i\frac{r_if_d}{r_1+r_2}(Y_1+Y_2)-(-1)^i\Delta f_i'\eta_i^{\mathrm{T}}$$
$$+\lambda_i'p_i^{\mathrm{T}}-p_i^{\mathrm{T}}(-1)^i\zeta^{-1}[\{Y_1(0)+Y_2(0)\}\sin\theta \tag{50}$$
$$+\beta\Delta\theta+\zeta\alpha\dot{\theta}]=0$$

This dynamics together with dynamics of the object yields the passivity relation in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[K+\frac{f_d}{2(r_1+r_2)}(Y_1-Y_2)^2+\sum_{i=1,2}((-1)^iA_{gi}+A_{fi})\right.$$
$$\left.+\{Y_1(0)+Y_2(0)\}(\cos\theta-\cos\theta_d)+\frac{1}{2}\beta\Delta\theta^2\right]$$
$$=-\sum_{i=1,2}c_i\|\dot{q}_i\|^2-\zeta\alpha\dot{\theta}^2 \tag{51}$$

When $\theta_d=0$, it is possible to choose $\beta>0$ sufficiently large so that

$$\frac{1}{2}\beta\Delta\theta^2+\{Y_1(0)+Y_2(0)\}(\cos\theta-1)\geq 0 \tag{52}$$

Then, it is possible to show that the feedback signals defined by equation (49) can regulate the posture of the object to attain an upright position.

5. SIMULATION RESULTS

In order to verify the theoretical findings, we carry out simulation work in which the proposed control input (43) has been applied to the dynamic equations of dual 2-d.o.f. and 3-d.o.f. fingers. Physical parameters of the fingers and object are presented in Tables I and II. Figures 6 and 7 show

Table I. Parameters of links.

	Mass [kg]	Length [m]	I [kgm^2]	s [m]
$link_{11}$	0.3	0.08	0.00016	0.04
$link_{12}$	0.25	0.07	0.0000102	0.035
$link_{20}$	0.163	0.05	0.0000034	0.025
$link_{21}$	0.163	0.05	0.0000034	0.025
$link_{22}$	0.163	0.05	0.0000034	0.025

Table II. Parameters of object.

Mass [kg]	Width [m]	I [kgm^2]
0.05	0.05	0.0000104

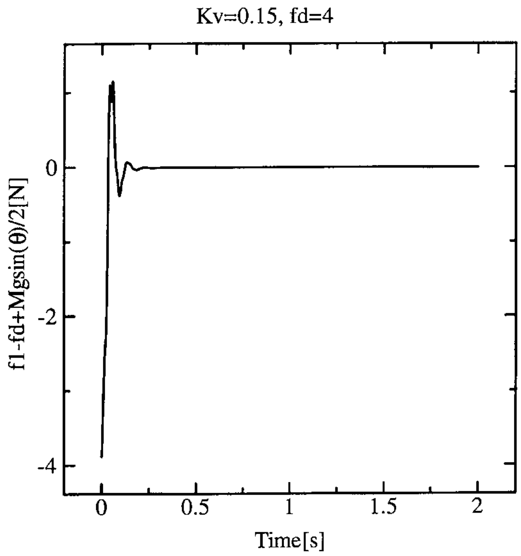

Fig. 6. $f_1 - f_d - Mg/2 \sin \theta$.

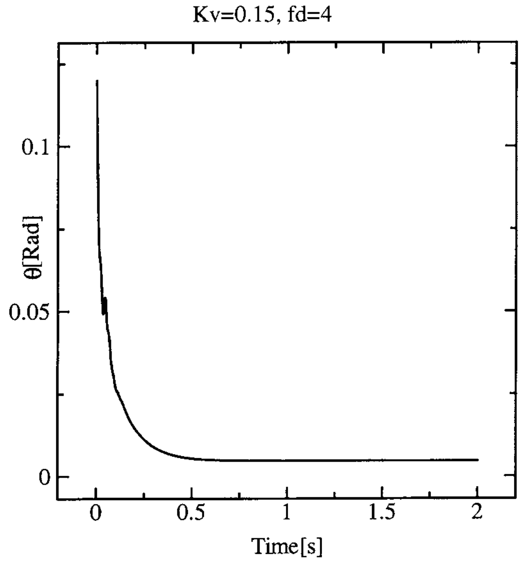

Fig. 8. Rotational angle of the object.

that $f_i - f_d + (-1)^i (Mg/2) \sin \theta$ for $i = 1, 2$ converge to zero in 0.2 second. Figure 8 shows the convergence of rotational angle of the object to a constant value, and Figure 9 demonstrates that the balance of two rotational moments at two sides of the object is attained also very fast. Figures 10 and 11 point out that

$$\Delta\lambda_1 = \lambda_1 - (Mg/2)(\cos \theta - (Y_1 + Y_2)\zeta^{-1} \sin \theta) \to 0$$

and

$$\Delta\lambda_2 = \lambda_2 - (Mg/2)(\cos \theta + (Y_1 + Y_2)\zeta^{-1} \sin \theta) \to 0$$

respectively. However, θ does not converge to zero, but to a nonzero constant value. Next, we add the sensory feedback input $u_{\theta i}$ of equation (48) to equation (43) so that the control input in this case has been designed in equation (49) with $\alpha = 0$. The desired rotational angle of the object at this case is chosen as $\theta_d = 0$[Rad] and it is assumed that $r_1 = r_2$ and $f_1(\Delta x) = f_2(\Delta x)$ for all $\Delta x \geq 0$. It has been shown that the $f_i - f_d + (-1)^i (Mg/2) \sin \theta$ for $i = 1, 2$ converge to zero very quickly, as shown in Figures 12 and 13, respectively, and

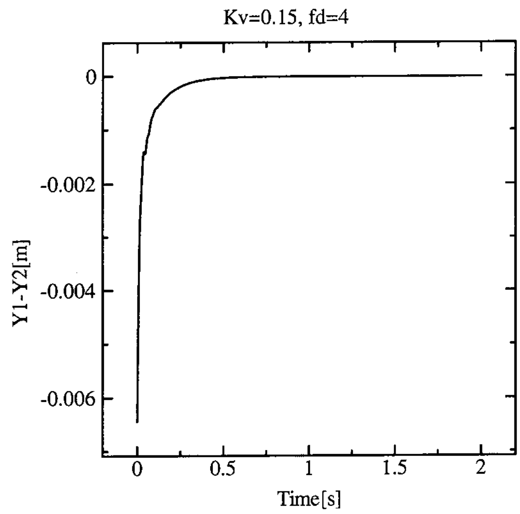

Fig. 9. Difference between Y_1 and Y_2.

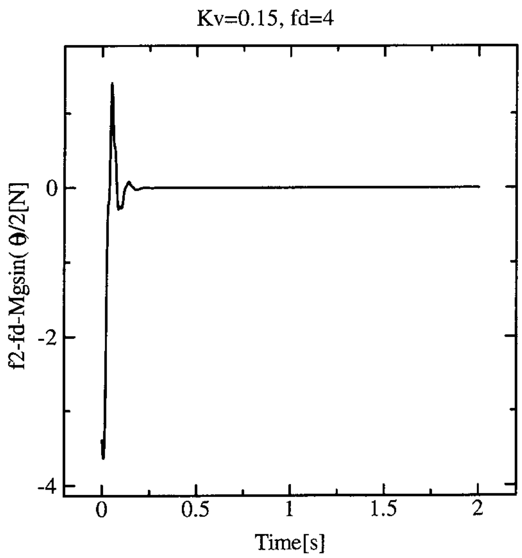

Fig. 7. $f_2 - f_d + Mg/2 \sin \theta$.

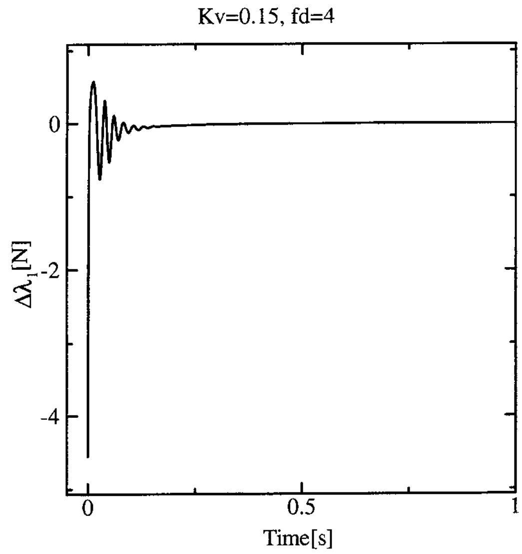

Fig. 10. $\lambda_1 - Mg/2(\cos \theta - \xi^{-1}(Y_1 + Y_2) \sin \theta)$.

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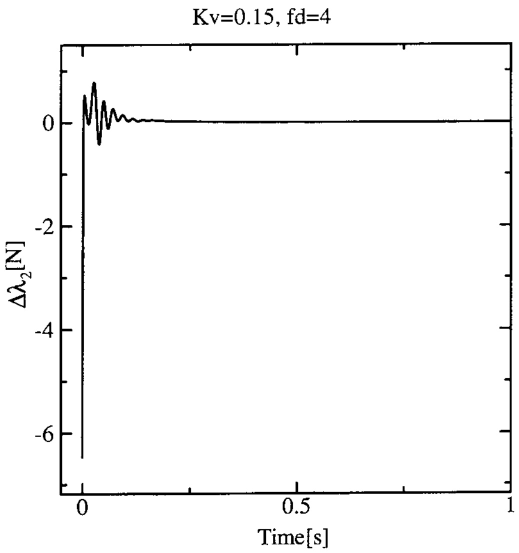

Fig. 11. $\lambda_2 - Mg/2(\cos\theta + \xi^{-1}(Y_1+Y_2)\sin\theta)$.

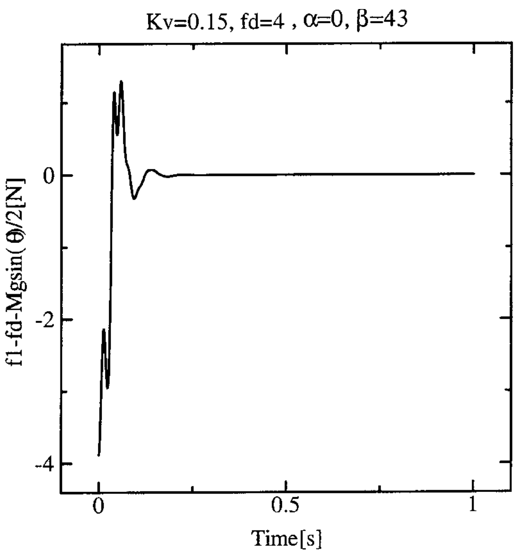

Fig. 12. $f_1 - f_d - Mg/2 \sin\theta$ (*Case* 2: $u_i = \bar{u}_{fgi} + u_{\theta i}$).

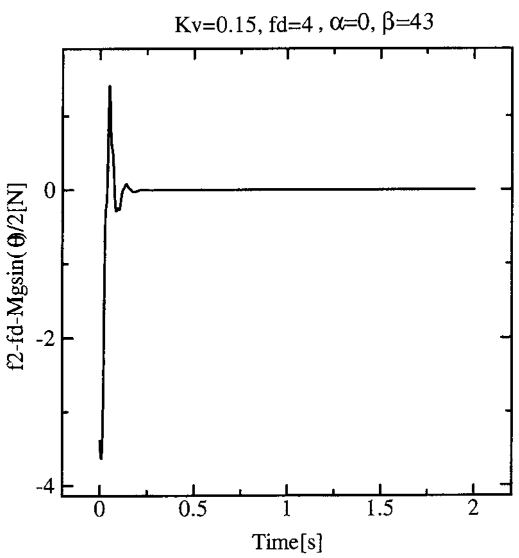

Fig. 13. $f_2 - f_d + Mg/2 \sin\theta$ (*Case* 2).

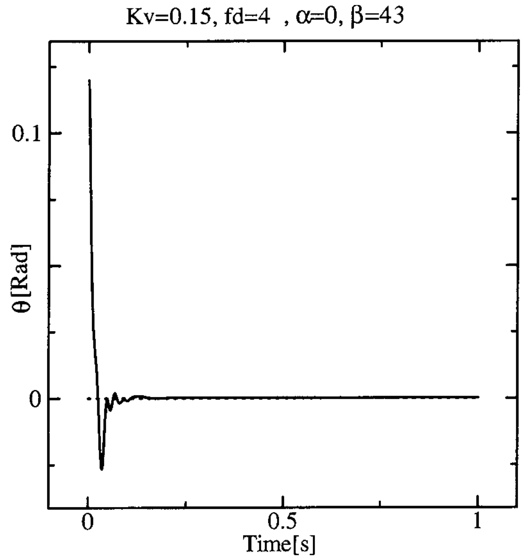

Fig. 14. Rotational angle of the object (*Case* 2).

$\theta \to \theta_d = 0$ in Figure 14. The convergence of $\theta \to \theta_d = 0$ as $t \to \infty$ implies convergences of $f_i \to f_d$ and $\Delta x_1 - \Delta x_2 \to 0$ as $t \to \infty$. Thus, the simulation results reconfirm the correctness of the theoretical findings.

6. CONCLUSIONS

This paper has shown the existence of a sensory-motor coordination for dynamic stable grasping of a rigid object by means of a pair of multi-DOF fingers with soft and deformable finger-tips under the effect of gravity. Computer simulation has verified that the sensory feedback realizes not only stable grasping in a dynamic sense but also controls an object posture in the vicinity of its upright position. Preliminary experimental results in the case that a setup of a pair of fingers and a rigid object is placed in a horizontal plane will be presented in a future paper.[20] Some new results regarding the experiment of the case that the setup is placed in a vertical plane will be presented in a future paper.

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