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LONG-RUN RISK AND MONEY MARKET RATES: AN EMPIRICAL ASSESSMENT

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Using postwar U.S. data, I study the implied interest rates in a simple long-run risk (LRR) model. Empirical estimates show that, as in standard consumption-based models with power utility preferences, the movements of the implied risk-free rate are entirely determined by the variations of expected consumption growth. This leads to a negative relationship between LRR Euler equation rates and money market rates. Nevertheless, when the low-frequency movements of consumption growth are accounted for, the long-run component of consumption growth is a key element to partially capture the countercyclical variations of the money market rates.

Keywords: Long-Run Risk, Euler Interest Rates, Monetary Policy

1. INTRODUCTION

The consumption Euler equation has become a central paradigm for several macroeconomic models, in both the asset pricing and the real business cycle literature. In these classes of models, the stochastic discount factor provides a core relation between the money market rate and the implied riskless rate. It follows that the growth expectations of marginal utility in the pricing kernel should reflect the stance of monetary policy so that the implied risk-free return should match the observed level of money market rates.

This work is concerned with the monetary implications of a simple long-run risk (LRR) model [cf. Bansal and Yaron (2004)]. As further discussed in the following, if agents have a preference for early resolution of uncertainty and are sufficiently risk-averse, LRR helps explain the equity premium in consumption-based asset pricing models using a persistent component in the process for the aggregate consumption growth. As in any standard macroeconomic model, the Euler equation implied by a LRR model provides a direct implication for monetary policy. Using postwar U.S. data, this link is used to empirically test the capability

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of the model to capture the historical behavior of U.S. interest rates over the past four decades.

Evidence of the empirical weakness of the consumption-based Euler equation is not new to the asset pricing literature. The "equity premium puzzle" documents the mismatch between the sizeable excess return observed in the U.S. market and that generated by the standard consumption CAPM [see Mehra and Prescott (1985)]. The "risk-free rate puzzle" [see Weil (1989)] highlights the difference between the unconditional mean of the risk-free return implied by the consumption-based Euler equation and the observed Treasury bills rate.

Changing the preference structure or the economic environment is useful in reconciling model-generated returns with the unconditional moments of financial data: habit formation has contributed to the resolution of the equity premium puzzle quite successfully [see Abel (1990); Campbell and Cochrane (1999)]. Further, Tallarini (2000) shows that introducing recursive preferences, as in Epstein and Zin (1989), in a real business cycle model improves the model's asset market predictions. More recently, Gavazzoni (2012) and Li and Palomino (2014) studied the LRR implications of New Keynesian models.

Nonetheless, several studies warn about the empirical inconsistencies that threaten these assumptions. Using U.S. data, Canzoneri et al. (2007) find that the riskless return implied by the Euler equations of the cited specifications and the observed pattern of the federal funds rate do not coincide. Also, the unconditional moments of implied money market rates exceed, in most cases, their empirical counterpart, and this evidence holds true whether the stochastic discount factor is nominal or real.[1]

Abstracting from the preference structure, the core of these inconsistencies stems from the interaction between macroeconomic variables and the interest rates implied by the consumption Euler equation. Atkeson and Kehoe (2009) suggest that the Euler equation misses the real link between the policy instrument and the economic forces that drive the monetary policy. In particular, standard asset pricing models relate the risk-free returns to the short-term fluctuations of expected consumption growth. In this context, risk plays no role and the conditional variance of consumption, which enters the log of the Euler equation, is constant.

A historical analysis of U.S. money market rates reveals instead that the response of the Federal Reserve Bank to the expected changes of consumption growth is smooth, whereas the monetary policy mainly reacts to the business cycle fluctuations in real risk. Moreover, it has been shown empirically that an unexpected monetary tightening raises nominal and real money market rates. This is known as the "liquidity effect" [see Christiano et al. (1999) for a detailed review of this literature], and it also implies that a monetary restriction has a negative effect on consumption and its growth rates that lasts for several quarters. Canzoneri et al. (2007) showed that these two effects cannot be reconciled with models that equate Euler equation rates with money market rates. This happens because a decline in expected consumption growth is associated with a decline in real interest rates in all of the Euler equations they considered, including those based on habit formation or recursive preferences. Thus, to reflect the stance of monetary policy, the Euler equation should be developed in a framework where the risk-free returns react smoothly to the variations of expected consumption growth and the volatility of consumption varies at a business cycle frequency.

The LRR model of Bansal and Yaron (2004) seems to meet these requirements. Differently from the standard asset pricing models, Bansal and Yaron (2004) specify consumption as containing a small and persistent component that accounts for the low-frequency movements of the economic growth. The conditional volatility of consumption is allowed to vary over the business cycle, introducing fluctuating economic uncertainty and time-varying risk. In this framework, the Epstein and Zin (1989) preferences emphasize the role of long-run growth and economic uncertainty as additional sources of risk and smooth the response of the risk-free asset to short-term consumption variations.

I compute the riskless return implied by the LRR model of Bansal and Yaron (2004) and its subsequent modifications. The empirical analysis shows indeed that the long-run characteristics of LRR models contribute to the macroeconomic consistency of the model-generated risk-free rate. In particular, when the low-frequency movements of consumption growth are explicitly included in the Euler equation, the correlation between the real risk-free return and the the money market rates becomes positive and the two series behave similarly.[2] Moreover, when the ex ante risk-free rate is used to recover the low-frequency movements, the results improve drastically. Using the risk-free rate as a predictor of the state variable can raise endogeneity concerns for empirical estimation. Nevertheless, it is worth recalling that this is the estimation procedure adopted by seminal contributions [e.g., Bansal et al. (2007); Bansal and Shaliastovich (2013)] that are able to successfully match the relevant financial figures in the LRR asset pricing literature.

The remainder of the paper is organized as follows. Section 2 will specify the consumption Euler equation for the LRR model of Bansal and Yaron (2004); Section 3 explains in detail the estimation procedure and describes the data; Section 4 presents the results; Section 5 concludes.

2. RISKLESS INTEREST RATES

Consider an endowment economy and a representative household that maximizes the following recursive utility function:[3]

$$V_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \mathcal{R}_t (V_{t+1})^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{1}$$

where C_t is the level of consumption at time t, and the operator $\mathcal{R}_t$ makes a risk adjustment to the date-$(t+1)$ continuation value. The risk adjustment is given by

$$\mathcal{R}_t(V_{t+1}) = \left(Et \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}.$$

The parameter γ is the coefficient of risk aversion (RA) and β denotes the household's subjective discount factor, whereas the elasticity of intertemporal substitution (EIS) is given by ψ.

The representative household is assumed to allocate its disposable income between consumption and a one-period bond: specifically, one bond that pays out one consumption unit (the real asset).

Before moving to the general setup, and in order to have a better understanding of how long-run consumption movements are linked to money markets, it is worth analyzing the most basic LRR model.

A simple way to do so is to focus on the case $\psi = 1$, as in Tallarini (2000), coupled with the Bansal and Yaron (2004) LRR specification.[4] As I discuss in detail in the following, the baseline model in Bansal and Yaron (2004) specifies the consumption growth rate as a combination of its unconditional mean (μ_c) and a slow-moving predictable component (x_t) with a persistence parameter (ρ_x), keeping the volatilities for both processes time-invariant (σ_c and σ_x, respectively).

First, it is worth recalling that the utility function in equation (1) is homogeneous of degree 1 in the level of consumption. Let c_t denote the logarithm of the consumption level and v_t the logarithm of the continuation value normalized by the consumption level. The equation can be rewritten as

$$v_t = \frac{1}{1 - 1/\psi} \log \{(1 - \beta) + \beta \exp[(1 - 1/\psi)\mathcal{Q}_t(v_{t+1} + c_{t+1} - c_t)]\}, \qquad (\mathbf{2})$$

where

$$\mathcal{Q}_t(v_{t+1}) = \frac{1}{1 - \gamma} \log E_t \left[\exp\left[(1 - \gamma)v_{t+1}\right]\right].$$

This, in conjunction with the Gaussian shock processes assumed by Bansal and Yaron (2004), allows simple closed-form solutions of the value function. That is, the $\psi = 1$ limit of equation (2) is[5]

$$v_t = \frac{\beta}{1 - \gamma} \log E \left[\exp[(1 - \gamma)(v_{t+1} + c_{t+1} - c_t)]\right]. \qquad (\mathbf{3})$$

Looking at the case where consumers do not observe x, the only variable in the consumers' information set that changes over time is the current consumption level C_t. It follows that v_t is constant. Denoting its value by $\tilde{v}$ and using equation (3) gives

$$\begin{aligned}\tilde{v} &= \frac{\beta}{1 - \gamma} \log E \left[\exp[(1 - \gamma)(\tilde{v} + c_{t+1} - c_t]\right] \\ &= \frac{\beta}{(1 - \beta)(1 - \gamma)} \log E \left[\exp[(1 - \gamma)(c_{t+1} - c_t]\right].\end{aligned}$$

Unconditionally, $\Delta c_{t+1} \sim N\left(\mu_c, \sigma_c^2 + \sigma_x^2/(1 - \rho_x^2)\right)$. So

$$\tilde{v} = \frac{\beta}{1-\beta}\left(\mu_c + \frac{1-\gamma}{2}\left(\sigma_c^2 + \frac{1}{1-\rho_x^2}\sigma_x^2\right)\right). \quad \mathbf{(4)}$$

The last part of equation (4) is a risk adjustment. Its magnitude depends on the sum of the variance of the white noise consumption shock and the unconditional variance of the trend consumption growth rate. The risk adjustment lowers utility whenever the coefficient of risk aversion, γ, is greater than 1. So a high persistence of the LRR component lowers the current utility of consumption, and by increasing its marginal utility, provides a direct link to the risk-free rate of the economy.

2.1. The Long-Run Risk Model

Abstracting from the special case $\psi = \gamma^{-1}$, which leads to the power utility case, the derivation of the real risk-free return proceeds from the log-linearization of the following Euler equation [see Epstein and Zin (1989)]:

$$R_t^* = E_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)}\right]^{-1}, \quad \mathbf{(5)}$$

from which the log of the stochastic discount factor is

$$m_{t+1} = \theta \ln \beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)(r_{a,t+1}), \quad \mathbf{(6)}$$

where $\theta = \frac{1-\gamma}{1-1/\psi}$ and $r_{a,t+1}$ is the log of the *unobservable* gross return on the aggregate consumption claim.

Bansal and Yaron (2004) specify consumption growth according to the following exogenous law of motion:[6]

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}, \quad \mathbf{(7)}$$

$$x_{t+1} = \rho_x x_t + \varphi_e \sigma_t e_{t+1}, \quad \mathbf{(8)}$$

$$\sigma_{t+1}^2 = \sigma^2 + v_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1}, \quad \mathbf{(9)}$$

$$\eta_{t+1}, e_{t+1}, w_{t+1} \sim \text{i.i.d.} N(0, 1).$$

The conditional expectation of consumption growth (that is, $\mu_c + x_t$) results from the combination of the unconditional average, μ_c, and a slow-moving predictable component, x_t. The state variable x_t characterizes the long-run properties of the consumption growth process, with the parameter ρ_x measuring its persistence. Whereas the shock η_{t+1} represents a standard high-frequency innovation in short-term consumption, the innovation term e_{t+1} captures the LRR of consumption prospects, and φ_e determines its predictability. To account for the economic uncertainty affecting consumption growth, its variance σ_{t+1}^2 is defined as an AR(1) process with a persistence parameter v_1. The error term w_{t+1} is a shock to the economic uncertainty and σ_w captures its unconditional volatility.

Bansal and Yaron (2004) show that the real risk-free return r_t^* satisfies

$$r_t^* = -\theta \log(\beta) + \frac{\theta}{\psi} E_t[\Delta c_{t+1}] + (1-\theta) E_t[r_{a,t+1}] + \frac{1}{2}\mathrm{Var}_t\left[\frac{\theta}{\psi}\Delta c_{t+1} + (1-\theta) r_{a,t+1}\right]. \quad \mathbf{(10)}$$

It is worth noting that, besides the consumption dynamics preceding, the complete characterization of the LRR stochastic discount factor relies on (i) an estimatable solution for $r_{a,t+1}$ and (ii) an estimate of the variance term in equation (10). The return $r_{a,t+1}$ is derived by the approximation

$$r_{a,t+1} = k_0 + k_c z_{t+1} - z_t + \Delta c_{t+1}, \quad \mathbf{(11)}$$

which is a function of the log price-to-consumption ratio z_t, with approximating constants $k_c = \frac{\exp(\tilde{z})}{1+\exp(\tilde{z})}$ and $k_0 = \log[1+\exp(\tilde{z})] - k_c\tilde{z}$ [see Campbell and Shiller (1988)].

Bansal and Yaron (2004) show that the variance term $\mathrm{Var}_t[\frac{\theta}{\psi}\Delta c_{t+1} + (1-\theta) r_{a,t+1}]$ is equal to the conditional variance of the stochastic discount factor, which links the riskless return to the economic structure and the market compensation for consumption risks. That is,

$$\mathrm{Var}_t[m_{t+1}] = (\lambda_{m,\eta}^2 + \lambda_{m,e}^2)\sigma_t^2 + \lambda_{m,w}^2\sigma_w^{2,} \quad \mathbf{(12)}$$

where $\lambda_{m,\eta}$, $\lambda_{m,e}$, and $\lambda_{m,w}$ are respectively the unit prices for the short-run risk, the LRR, and the volatility risk. Expressions for the market prices of risk are derived in terms of the parameters of the consumption dynamics and the preference structure. Namely, $\lambda_{m,\eta} = \gamma$, $\lambda_{m,e} = (\gamma - \frac{1}{\psi})(\frac{k_c\varphi_e}{1-k_c\rho})$, and $\lambda_{m,w} = (\gamma - \frac{1}{\psi})(1-\gamma)[k_c(1+\frac{k_c\varphi_e}{1-k_c\rho})^2\, 2(1-k_c v_1)]$.

3. EMPIRICAL ANALYSIS AND DATA

As a first step in the empirical examination, I calculated the riskless returns implied by the consumption CAPM with standard power utility preferences and related them to the observed money market rates. This preliminary check was done to show further evidence of the mismatch between the two interest rates series documented by previous research on U.S. data. Results confirm that the proportional link, inferred by the model, between risk-free rates and short-term consumption growth is insufficient tor capture the cyclical aspect of monetary policy.[7]

3.1. Empirical Procedure: The Long-Run Risk Model

I study the LRR model of Bansal and Yaron (2004) and its subsequent modifications [Bansal et al. (2007); Bansal and Shaliastovich (2013)]. The purpose

Table 1. Baseline calibration

Consumption dynamics	
ρ_x	0.919
σ	0.0023
φ_e	0.205
μ	0.0016
k_1	0.931
σ_w	6.52×10^{-7}
v_1	0.988
Preferences	
β	0.998
γ	10
ψ	1.5
Inflation dynamics	
μ_π	0.0033
$\phi_{\pi g}$	0.429
$\phi_{\pi x}$	0.520

Notes: The table shows the parameters used for the calibration of the risk-free rate in equation (10) (19). The model in (7)–(9) and the inflation process in (15)–(16) are calibrated at a monthly frequency.

is to investigate whether (i) relaxing the restriction $\gamma = \psi^{-1}$ embedded in a power utility specification, (ii) introducing an explicit formulation for long-term consumption growth into the pricing kernel, and (iii) accounting for economic uncertainty reconciles the model-implied risk-free rate with the data. To fully assess the effects of long-run consumption prospects and economic uncertainty on the behavior of the money market rates, I analyze four concurrent specifications of the LRR model:

1. Baseline calibration

   According to the original specification introduced by Bansal and Yaron (2004), I first proceed with the baseline calibration of the equations (7)–(9), the choice of the preference parameters, and the estimation of the time-varying consumption variance, σ_t^2.

   Table 1 reports the parameters that describe the dynamics of consumption and the investor's preferences. The model is calibrated on a monthly basis in order to fit the monthly decision interval of the representative investor and avoid model misspecification.[8] The parameters are set to match the salient features of consumption and asset pricing dynamics in the United States simultaneously.[9]

   The choice of the preference parameters plays a key role in the long-term configuration of the model. In fact, a well-known feature of Epstein and Zin (1989) preferences is that the magnitude of the RA relative to that of the EIS governs the representative agent's timing of the uncertainty resolution.

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Bansal and Yaron (2004) assume that the representative household prefers an *early* resolution of the uncertainty ($\gamma > \frac{1}{\psi}$) with $\psi > 1$. Under this condition, the representative agent gives higher weight to those consumption risks perceived as longer-lasting. Conforming to this configuration, I closely follow the original calibration of the Bansal and Yaron (2004) model, set the coefficient of risk aversion to 10, and let the EIS parameter be 1.5. The subjective discount factor is set to 0.9979.

I follow Bansal et al. (2007) to complete the calibration of the model. The variance σ_t^2 is predicted using a GARCH(1,1) for the autoregressive process of the consumption growth. The time series of the low-frequency component is obtained from the recursive one-step-ahead forecast of its process, relying entirely on the calibration of the parameters ρ_x and ϕ_e.

2. Recovering long-run consumption growth
More recently, Bansal et al. (2007) have shown that the low-frequency component x_t can be directly extracted from consumption and financial data. Hence, to provide a more concrete measure of long-run consumption, I decided to add to the baseline calibration of the model and recover x_t from the data. Following Bansal et al. (2007), I regress consumption growth on the price–dividend ratio.[10] That is, I calculate

$$\Delta c_{t+1} = \zeta Y_t + \sigma_t \eta_{t+1} \tag{13}$$

and take the conditional expectation of the consumption growth:

$$x_t = \zeta Y_t, \tag{14}$$

where $Y_t = \begin{bmatrix} 1 & z_{pd,t} \end{bmatrix}'$, and $z_{pd,t}$ is the price–dividend ratio. It follows that the conditional expected consumption growth can be explicitly computed as the sum of x_t and μ_c.

Again the parameters ρ_x and φ_e are set according to the first-order autoregression of x_t. At a monthly frequency, ρ_x is equal to 0.799 and φ_e is 0.083.[11]

3. Abstracting from the economic uncertainty
To assess the interaction of the economic uncertainty with the empirical performance of the LRR model, I calculate the risk-free rate in (10), abstracting from the time variations of consumption volatility. This step requires a brief reconfiguration of the preceding equations. Specifically, the conditional variance σ_t^2 is replaced with its unconditional mean σ^2. The purpose is to check whether the results are somehow affected by the heteroskedasticity of consumption growth.

4. Bayesian estimation
A natural extension of this calibration is to estimate the LRR setup using a state-space model.[12] To do so, I use an iterative Kalman filter procedure with Gibbs sampling to infer the relevant parameters of equations (7) and (8), where the consumption growth process is the observation equation, and the AR(1) process for x is the state equation. First, I jointly estimate the four process parameters using an informative prior on the volatility of the hidden state and an uninformative one on the persistence (ρ_x). In particular, following Hansen (2007) closely, the prior on the unconditional volatility of the state variable (σ_x) is an inverse gamma with shape parameter 10 and scale parameter 2.209×10^{-6}, which implies a mode of 0.00047. The prior for ρ_x is normal, conditional on σ_x, with mean 0 and standard deviation $\sigma_x \times 1.41 \times 10^6$.

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TABLE 2. Estimated process parameters

ρ_x	σ_x	μ_c	σ_c
0.2	0.00051	0.00480	0.00461
—	(0.00009)	(0.00034)	(0.00024)
0.7	0.00209	0.00483	0.00276
—	(0.00028)	(0.00053)	(0.00031)
0.9343	0.00106	0.00532	0.00338
(0.03971)	(0.00029)	(0.00188)	(0.00030)
0.979	0.00085	0.00521	0.00353
—	(0.00022)	(0.00230)	(0.00028)

Notes: Reported are the estimated parameters on the U.S. postwar data (Q1:1964–Q4:2011; source: BEA). Estimation is performed with an iterative Kalman filter procedure using a Gibbs sampling, of 50, 000 draws, discarding the first 5, 000. The shaded rows show the results from the unconditional estimation where the prior on σ_x^2 is an inverse gamma with shape parameter 10 and scale parameter 2.209×10^{-06}, which implies a mode of 0.00047 for σ_x. The prior for the persistence parameter is normal, conditional on σ_x, with mean 0 and standard deviation $\sigma_x \times 1.41 \times 10^6$ truncated with support $[-1 \quad 1]$. The other rows show the estimates for fixed values of ρ_x and the same priors for the other parameters.

I use rejection sampling to truncate the support of ρ_x to $[-1 \quad 1]$ [see Gelfand et al. (1992)].

To see the predictions of the state-space model over a larger set of parameter values for the persistence ρ_x, I repeat the estimation procedure, fixing it to different levels and keeping the same prior on σ_x. The means and the standard deviations of the estimates, reported in Table 2, are obtained with 50, 000 draws after discarding the first 5, 000 draws.

The unconditional estimate, reported in the shaded rows in Table 2, shows a relatively high persistence of the hidden state of the economy, with ρ_x equal to 0.943. This feature is confirmed by Figure 1, where the posterior distribution of the ρ_x estimates is plotted along with the given prior. It is also worth noting that, when the estimation is performed for given values of ρ_x, the unconditional volatility estimates seem not to be greatly affected (cf. Figure 2). This is probably due to the rather informative prior chosen.

3.2. Data

The empirical analysis is based on U.S. quarterly postwar data. To maintain continuity with the existing literature, the sample covers the period 1964–2011.[13]

Consumption and disposable income. All data for U.S. consumption of non-durable goods and services and real disposable income are acquired from the NIPA (National Income and Product Accounts) Tables of the Bureau of Economic Analysis (BEA). Per capita figures are worked out using population data from the U.S. Census Bureau.

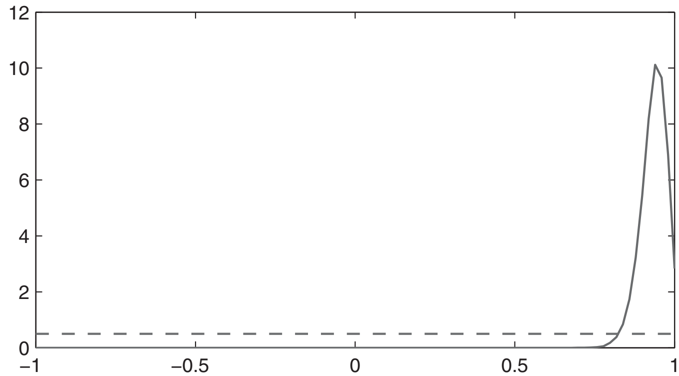

FIGURE 1. Prior and posterior probabilities of the persistence coefficient (ρ_x). Reported are the prior (dashed line) and the posterior (solid line) distributions of the ρ_x estimate. The prior is normal conditional on σ_x with mean 0 and standard deviation $\sigma_x \times 1.41 \times 10^6$ truncated with support $[-1 \quad 1]$. The posterior is obtained via a iterative Kalman filter using aGibbs sampling of 50, 000 draws, discarding the first 5,000.

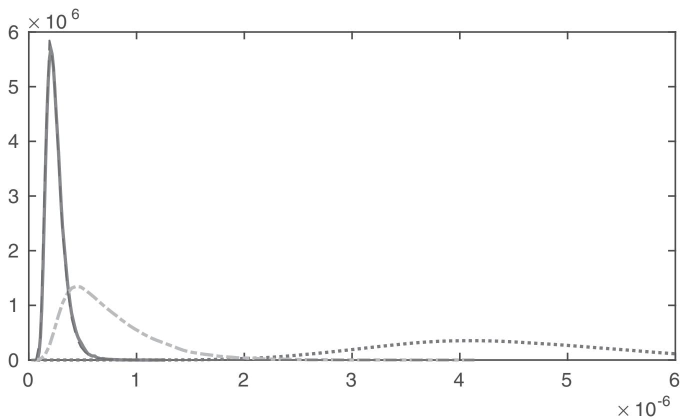

FIGURE 2. Prior and posterior probabilities of the variance of the hidden state (σ_x). This figure reports the prior (dashed line) and the posterior distributions of the σ_x estimates. The posteriors are obtained via an iterative Kalman filter using a Gibbs sampling of 50,000 draws, discarding the first 5,000, and by fixing different values of κ [0.200 (dash–dotted line), 0.700 (dotted line), and 0.979 (solid line)].

Consumption-based inflation. Inflation is computed using a consumption-based price index. This index results from the ratio between the nominal and real consumer spending components. The inflation rate is obtained as the change in the price level over the previous quarter.

Interest rates and the money stock. U.S. data for the money market rate (i.e, the federal funds rate), the 3-month Treasury bills rate, and the monetary aggregate

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TABLE 3. Summary statistics: The LRR model

	Real rates					Nominal rates	
	r_t	r_t^*				i_t	i_t^*
	(Data)	(Baseline)	(Conditional)	(Uncond.)	(Bayesian)	(Data)	(Model)
Mean	1.88	3.31	4.21	4.22	2.97	5.89	5.03
Std. deviation	(2.48)	(1.77)	(1.28)	(1.29)	(2.06)	(3.50)	(1.67)
Min.	−3.18	−3.69	0.83	0.85	−3.18	0.07	1.13
Max.	10.59	9.60	9.17	9.14	7.07	17.78	10.93
Correlation		−0.0261	0.1623**	0.1758**	0.1133*		0.4236***

The left panel of this table compares the federal funds rate (r_t) and the risk-free rate (r_t^*) implied by the LRR model of Bansal and Yaron (2004) and its subsequent modifications. Baseline (second column) collects the results obtained when the equation (10) is calculated according to the baseline calibration in Table 1, where the long-run component x_t is calibrated. Conditional (third column) and Uncond. (fourth column) show results from the procedure introduced by Bansal et al. (2007) and described in Section 3.1. In these cases, x_t is directly recovered from the data. The results in Uncond. abstract from the conditional volatility of consumption, whereas Bayesian shows the results using the iterative Kalman filter estimation. Turning to nominal interest rates, the last two columns report summary statistics for the nominal federal funds rate (i_t) and compare them with the nominal risk-free rate obtained using equation (19).
t-stat: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

M2 are extracted from the Federal Reserve Economic Data (FRED) of the St. Louis Fed.

Stock prices and dividends. U.S. data for the S&P500 Composite index and dividends are available on Robert Shiller's U.S. Stock Price Database.

4. RESULTS

4.1. Riskless Returns and Money Market Rates: The Long-Run Risk Model

Table 3 [column (2)] shows the summary statistics resulting from the baseline calibration of the LRR model.[14] Compared with the power utility results in Canzoneri et al. (2007) (cf. Table 1 in their paper), the LRR results reduce the average difference between the model returns and the money market rates. However, the model still does not succeed in fitting the data, as the interest rate series still diverge. The U.S. risk-free return is on average twice the federal funds rate (3.31% compared with 1.88%) and less volatile.

The short-term movements of the risk-free rate bear no resemblance to the pattern of the federal funds rate (see Figure 3). The correlation coefficient, $\rho = -0.0261$, confirms that the two time series are uncorrelated.

Equation (10) predicts that the movements of the risk-free rate are governed by the long-run fluctuations of the expected consumption growth. This is a prominent difference from the power utility case, where just the high-frequency shocks to consumption are considered. Yet the results show that the baseline calibration of

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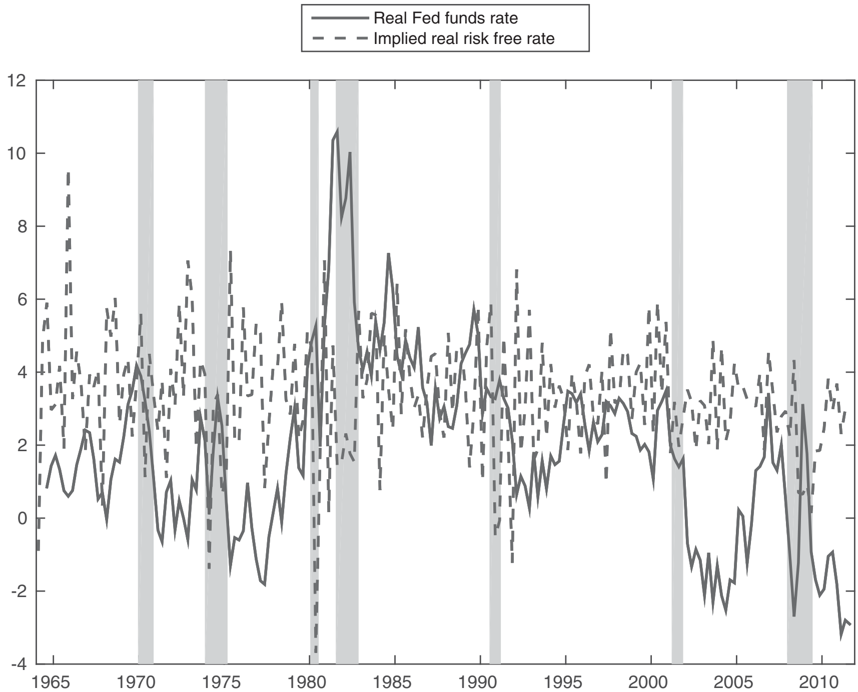

FIGURE 3. Implied real risk-free return vs the ex ante real federal funds rate: LRR model (baseline). This figure plots the real risk-free return (r_t^*) implied by the baseline calibration of the LRR model [see Bansal and Yaron (2004)] for U.S. data and compares it with the real federal funds rates.

the LRR model does not capture the risk-free-rate dynamics. This implies that the short-term variations of consumption still play a dominant role in determining the Euler equation in the baseline LRR calibration.

Interestingly, the third column of Table 3 shows that when the long-run component x_t is explicitly included in the calibration, the results sharply change. The U.S. correlation coefficient turns positive (0.16) and significant. These results suggest that the long-run characteristics of expected consumption growth are crucial for the macroeconomic consistency of the risk-free estimates.

The unconditional moments of the two interest rates still diverge (see Figure 4). The mean of r_t^* rises up to 4.21, so that the difference between the average levels is higher than previous results. Meanwhile, the standard deviation of r_t^* falls to 1.28.

Although these results suggest that the state variable x_t contributes to the macroeconomic consistency of the implied risk free rate, the outcome may be influenced as well by the heteroskedasticity modeled in the volatility of the consumption growth (σ_t^2). This may be at odds with Beeler and Campbell (2012), which found that the effects of economic uncertainty on the consumption–saving

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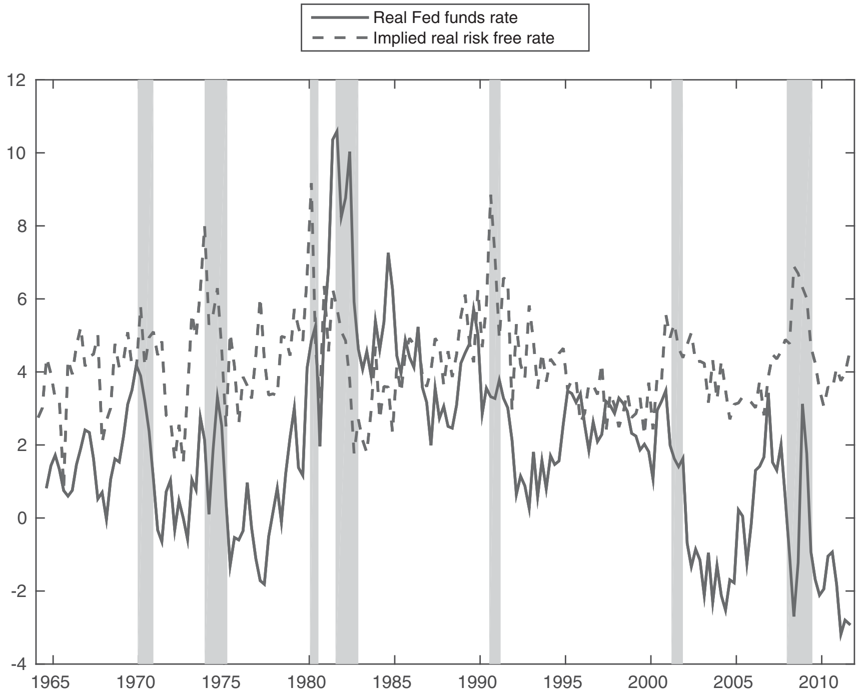

FIGURE 4. Implied real risk-free return (r_t^*) vs. the ex ante real federal funds rate: LRR model (conditional volatility).

decisions of the representative agent should be not significant in a Bansal and Yaron (2004) setting.

To explore the interaction between long–run growth and economic uncertainty in the determination of the risk-free rate, I calibrate the LRR model, abstracting from the time variations of the consumption volatility (see Figure 5). Column (4) of Table 3 reports the values. Results are not affected by this modification. The mean of the risk-free rate is 4.22, with a standard deviation of 1.29. The correlation is slightly higher ($\rho = 0.17$).

Finally, column (5) of Table 3 reports the estimates when the iterative Kalman filter is employed (see Figure 6). Results are not improved by this modification. The mean of the risk-free rate is 2.97 with a standard deviation of 2.06, and a correlation slightly lower ($\rho = 0.11$) and barely significant.

4.2. The Role of Inflation

To improve the comparability of the preceding results with the previous literature, I calibrate the nominal counterpart of the model presented in Section 3.1.

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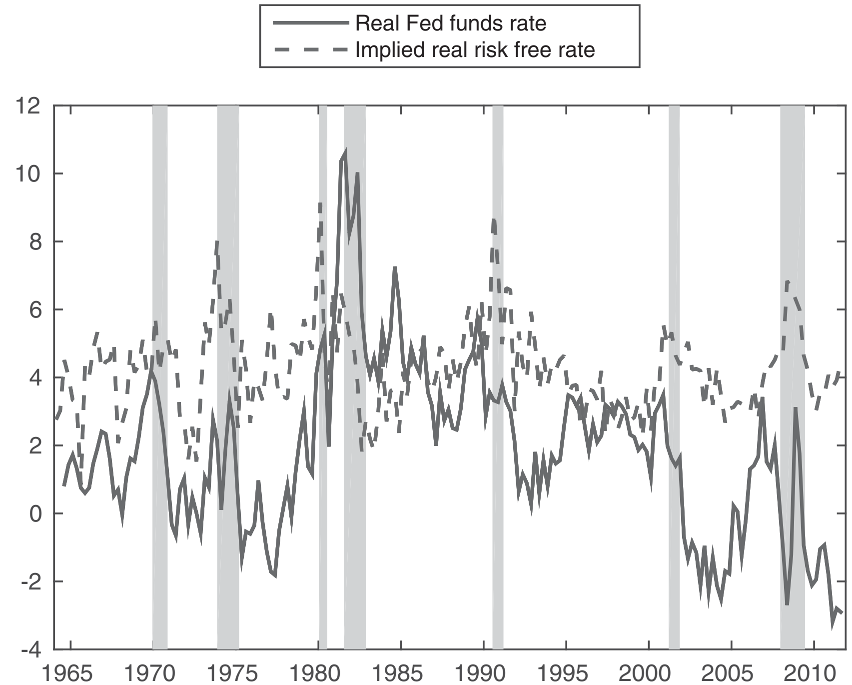

FIGURE 5. Implied real risk-free return (r_t^*) vs. the ex ante real federal funds rate: LRR model (unconditional volatility).

To model nominal figures, I build on Bansal and Shaliastovich (2013) and assume that the inflation process follows

$$\pi_{t+1} = \mu_\pi + z_t + \phi_{\pi g}\sigma_{\pi g}\eta_{t+1} + \phi_{\pi g}\sigma_{xt}e_{t+1} + \sigma_\pi \xi_{t+1}, \tag{15}$$

where μ_π captures the unconditional mean of the inflation and the state variable z_t is defined as

$$z_{t+1} = \alpha_z z_t + \alpha_x x_t + \phi_{zg}\sigma_{gt}\eta_{t+1} + \phi_{xz}\sigma_{xt}e_{t+1} + \sigma_x \xi_{z,t+1}. \tag{16}$$

Using this specification, realized and expected inflation dynamics is linearly affected by both short-term and long-run consumption shocks (η_{t+1} and e_{t+1}), with the parameter pairs $\phi_{\pi g}$; ϕ_{zg}, and $\phi_{\pi x}$; ϕ_{zx} quantifying the sensitivity to consumption innovations. The persistent component of consumption growth, x_t, is assumed to drive long-term inflation behavior by entering equation (16). I complete the inflation process by assuming that its shocks (ξ_{t+1}, $\xi_{z,t+1}$) are independent and identically distributed normal random variables. So the conditional variances of

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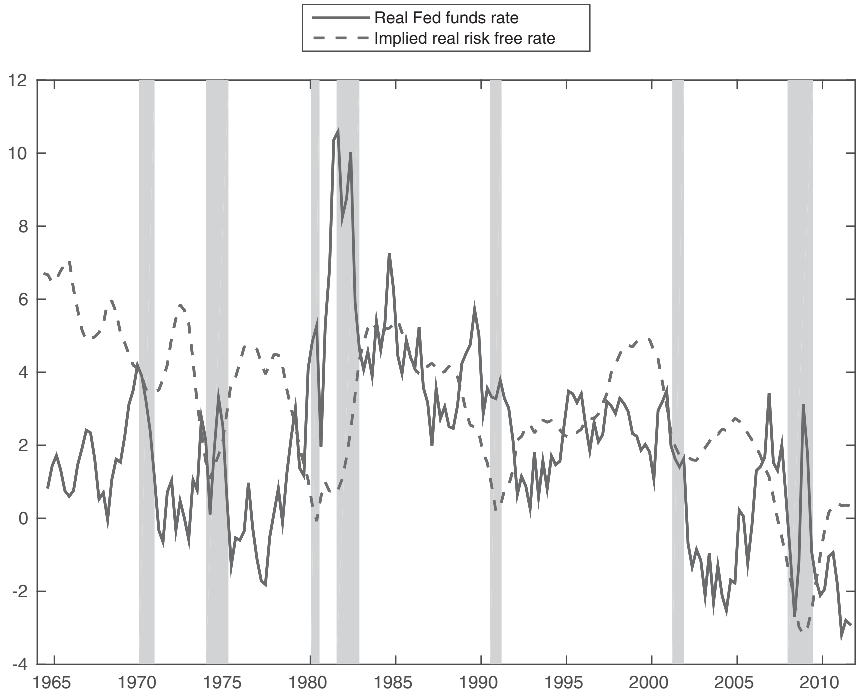

FIGURE 6. Implied real risk-free return (r_t^*) vs. the ex ante real federal funds rate: LRR model (Bayesian estimation).

realized and expected inflation are given by

$$\text{Var}_t \pi_{t+1} = \phi_{\pi g}^2 \sigma_{gt}^2 + \phi_{xz}^2 \sigma_{xt}^2, \tag{17}$$

$$\text{Var}_t z_{t+1} = \phi_{zg}^2 \sigma_{gt}^2 + \phi_{zx}^2 \sigma_{xt}^2. \tag{18}$$

Given the inflation dynamics, it is straightforward to derive the short-term nominal return from the log-linearization of the Euler equation, adjusted for inflation risk:

$$i_t^* = r^* + E_t \pi_{t+1} - \frac{1}{2} \text{Var}_t \pi_{t+1} + \text{Cov}_t(m_{t+1}, \pi_{t+1}), \tag{19}$$

where $\text{Cov}_t(m_{t+1}, \pi_{t+1}) = -\phi_{\pi g} \lambda_\eta \sigma_{gt}^2 - \phi_{\pi x} \lambda_e \sigma_{xt}^2$ denotes the inflation risk premium.

The parameters of inflation are calibrated on a monthly basis in order to match its features in the data. Equation (15) is estimated using a VAR where both short and long-term consumption shocks are allowed to enter the model as additional exogenous variables. The set of parameters measuring the sensitivities of the inflation rate to temporary and persistent consumption innovations are the exogenous

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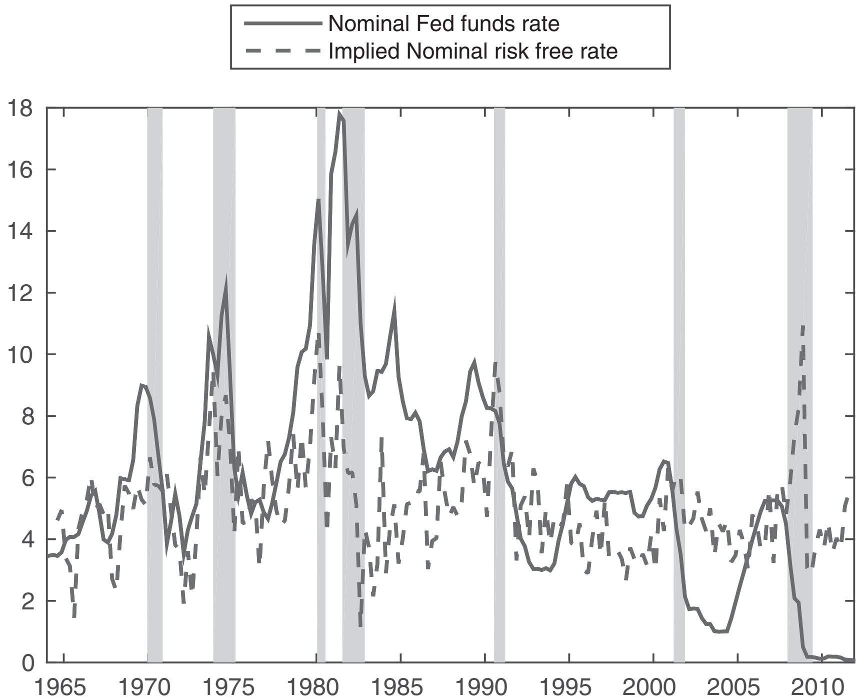

FIGURE 7. Implied nominal risk-free return (i_t^*) vs. the federal funds rate: LRR model.

variables' coefficients derived from a first-order autoregression. The lower panel in Table 1 collects all the parameters.

The difference in the unconditional means disappears when the nominal figures are used and the correlation coefficient increases to 0.42 (cf. the last two columns of Table 3), whereas the volatility is still lower than its empirical counterpart.

It is worth noting that the increase in the correlation coefficient seems not to be due to the influence of price trends on the two nominal interest rates. The comparison, after the series is HP-filtered, of the inflation trend with the trends of the empirical and of the implied nominal rate confirms, as in the power utility case studied in Canzoneri et al. (2007), that the underlying dynamics of the two rates is equally affected by the trend in price movements (see Figure 7). Instead, differently from the power utility case, the cyclical components of i_t and i_t^* follow different behavior: the correlation coefficient is indeed positive ($\rho = 0.47$) and statistically different from zero.

4.3. Recovering the State Variable

As mentioned earlier (see note 10), to recover the low-frequency component x_t, Bansal et al. (2007) use both the ex ante real risk-free rate and the price–dividend ratio as regressors in equation (14). In their contribution, the ex ante real

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TABLE 4. Summary statistics: Bansal et al. (2007) inference

	Real rates			Nominal rates	
	r_t	r_t^*		i_t	i_t^*
	(Data)	(Conditional)	(Unconditional)	(Data)	(Model)
Mean	1.88	4.20	3.87	5.89	5.20
Std. deviation	(2.48)	(1.27)	(1.276)	(3.50)	(1.75)
Min.	−3.18	0.91	0.63	0.07	0.86
Max.	10.59	8.54	8.20	17.78	12.37
Correlation		0.4295***	0.4406***		0.54***

Notes: This table compares the federal funds rate (r_t) and the risk-free rate (r_t^*) using the calibration procedure introduced by Bansal et al. (2007). In this case x_t is directly recovered from financial data using the ex ante risk-free rate as an additional regressor in equation (14). The results in Unconditional abstract from the conditional volatility of consumption. The last two columns report summary statistics for the nominal federal funds rate and compare it with the nominal risk-free rate.
t-stat: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

risk-free rate is obtained by regressing the ex post real short-term rate on its nominal counterpart and the past annual inflation. The ex post real short-term rate is given by the difference between the nominal rate and the inflation rate.[15]

Table 4 shows that, when the long-run component x_t is obtained using the ex ante risk-free rate, the results improve dramatically. First, with complete calibration, the correlation coefficient between the fed funds rate and the implied rate becomes 0.43, and it is strongly significant. Interestingly, the implied unconditional moments are virtually the same as in the case when the price–dividend ratio is the only regressor. This suggests that the improvement is due to better capability in matching the time series behavior of the data. Second, when the model is specified by abstracting from the time variations of the consumption volatility, results are basically unchanged: the correlation is just slightly higher ($\rho = 0.44$), whereas the implied unconditional moments still diverge from their empirical counterparts. Third, when nominal figures are calibrated, the difference in the implied unconditional mean decreases significantly, the correlation coefficient increases to 0.54 (cf. the last two columns of Table 4), and the volatility is still lower than its empirical counterpart. Finally, as for the results obtained using only the price–dividend ratio as a regressor in equation (14), the comparison between HP-filtered nominal and inflation series confirms the positive and significant correlation coefficient ($\rho = 0.48$) for the cyclical components of the empirical and implied nominal rates.

4.4. Business Cycle Properties

Even if the results on the LRR model are encouraging, graphical inspection of the results raises a question about the capability of the implied risk-free rate to mimic the money market rate over the business cycle phases (cf. Figures 3–7).

TABLE 5. Interest rate spread and monetary policy: LRR model

	(Baseline)	(Conditional)	(No conditional)	(Bayesian)	(Nominal)
Coeff. (β)	−0.849***	−0.816***	−0.81***	−1.07***	−0.602***
Std. error	0.072	0.068	0.068	0.027	0.092
Adj. R^2	0.83	0.66	0.66	0.93	0.64
Obs	185	185	185	185	185

The table reports the main results from the regressions

$$(r_t^* - r_t) = \alpha + \beta_{r_t} r_t + \sum_{k=1}^{4} \gamma_k (r_{t-k}^* - r_{t-k}) + u_t,$$

where the real spread between the real implied LRR rate r_t^* and the ex ante real federal funds rate r_t is regressed on its four lags and the federal funds rate r_t, and

$$(i_t^* - i_t) = \delta + \beta_{i_t} i_t + \sum_{k=1}^{4} \zeta_k (i_{t-k}^* - i_{t-k}) + e_t,$$

where the spread between the nominal implied LRR rate i_t^* and the nominal federal funds rate i_t is considered instead. The relevant coefficients β_{r_t} and β_{i_t} for the federal funds rates [column (2)] are reported. To account for the autocorrelation in the error terms u_t and e_t, a Prais and Winsten (1954) procedure is employed.
t-stat: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

As a first piece of analysis, in Figure 8 I plot the smoothed estimates of the hidden state of the economy, derived from the filtering procedure, for three different values of ρ_x: from a very low level of persistence (0.2) to the extreme persistent case (0.979). For all parameter values, the estimates seem to point out fairly well the U.S. postwar recession periods[16] (the gray shaded area in the graph). The estimates with higher levels of ρ_x are smoother and the fluctuations are of larger magnitude. This raises hope for the capability of this specification to capture the behavior of the risk-free rate over the business cycle.

Analyzing the time variations of the spread between the data and the model-generated rates can shed some light on this issue. It is worth noting that in the DSGE literature, the interest rate is usually assumed to be determined by a Taylor-rule-based policy. Thus, a regression of the spread between the data and the model-generated rates on the money market rate can be interpreted as a test of the consistency of the demand side of the economy (through the implied Euler equation) based upon the predicted movements of the monetary side (in terms of the actual real rate).[17]

Table 5, in the first four columns, provides the output obtained from regressing the real spread $(r_t^* - r_t)$ on its four lags and the money market rate. In the real economies, all the coefficients of the money market rates (β_{r_t}) are negative and strongly significant, regardless of the model specification. This suggests that the LRR specification suffers the same flaw as a standard power utility model: the real interest rate implied by the Euler equation responds negatively to a decline in expected consumption growth, and the spread moves in the opposite direction with respect to the monetary policy indicator. Not even iterative Kalman-based

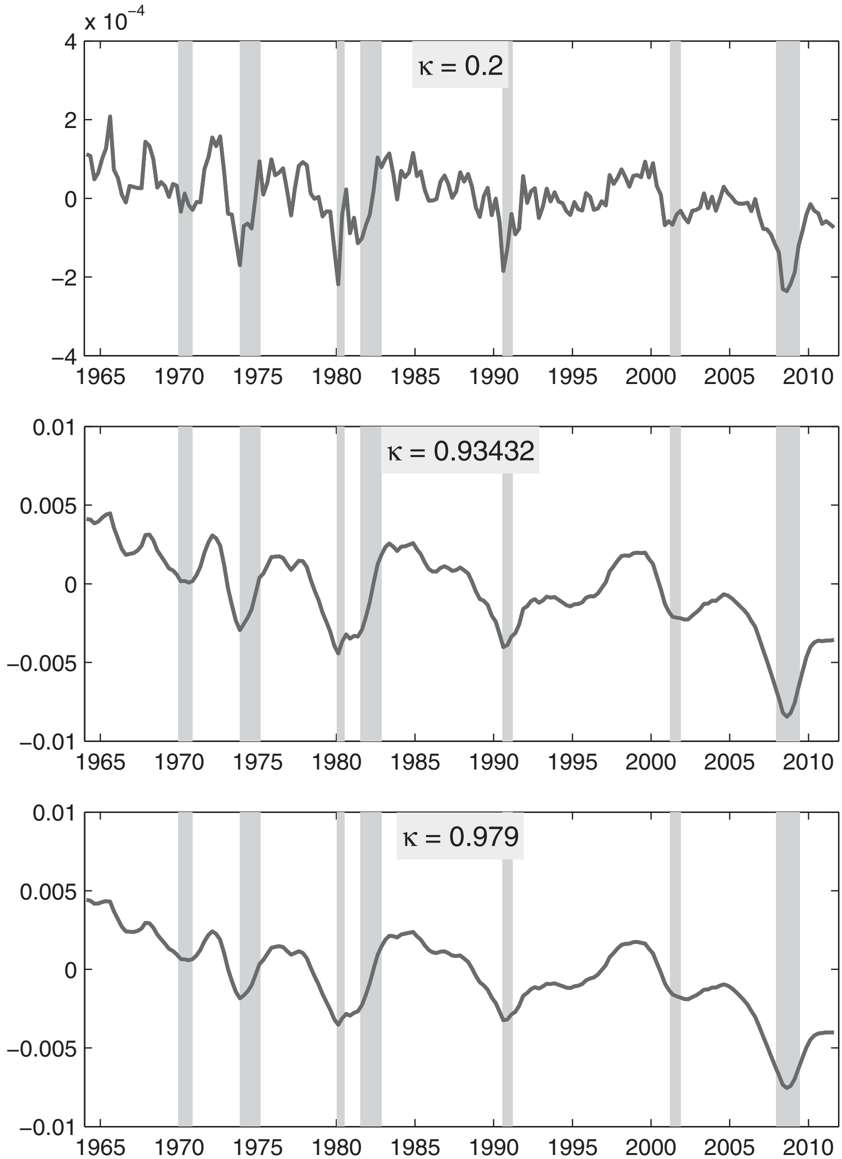

FIGURE 8. Smoothed estimates of the hidden state x for different values of the persistence parameter. The gray areas indicate official NBER recession periods.

estimation helps in solving this issue: the β coefficient of this specification is indeed negative and strongly significant. Results for nominal figures are virtually the same. The last column of Table 4 provides the output obtained from regressing the nominal spread $(i_t^* - i_t)$ on its four lags and the money market rate. Again the

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regression coefficient (β_{i_t}) is negative and strongly significant, implying a wider measured spread when monetary policy eases.[18]

Summarizing, as for the power utility case, the movements of the risk-free rate implied by LRR models are entirely determined by the variations of the expected consumption growth. This result seems to be at odds with the conclusions of Atkeson and Kehoe (2009), who state that time-varying risk is a central indicator for the monetary policy of the Federal Reserve. Nevertheless, differently from the standard power utility specification, the long-run component of consumption growth is a key element to partially capture the behavior of the money market rates.

5. CONCLUDING REMARKS

I studied the macroeconomic implications of a class of consumption-based asset pricing models where a persistent component in the process for the aggregate consumption growth is accounted for. Using postwar U.S. data, I estimated the implied interest rates in the LRR model introduced by Bansal and Yaron (2004) and its subsequent modifications [Bansal et al. (2007); Bansal and Shaliastovich (2013)].

I show that, as in the standard consumption-based CAPM, the movements of the risk-free rate implied by a baseline LRR model are entirely determined by the variations of the expected consumption growth. Nevertheless, differently from the standard CCAPM, the long-run characteristics of the LRR model partially contribute to the macroeconomic consistency of the model-generated risk-free return. In particular, when the low-frequency movements of consumption growth are explicitly included in the Euler equation, the correlation between the implied real risk-free rate and the federal funds rate becomes positive and significant. Moreover, when the ex ante risk-free rate is used to recover the low-frequency state variable of the model, the results improve drastically. Nevertheless, as already noted for other asset pricing specifications [cf. Canzoneri et al. (2007)], I found that the difference between the implied consumption Euler equation rates and the Federal Funds rate is systematically related to monetary policy.

NOTES

1. Ahmad (2005) confirms that this average spread is not an isolated artifact of the United States and that it concerns the other six G7 countries as well.

2. Two notable empirical contributions on the role of LRR for the bond markets are Beeler and Campbell (2012) and Rudebusch and Swanson (2012).

3. In what follows, I implicitly assume that consumption is equal to aggregate dividend, and so the consumption–wealth ratio is the same as the price–dividend ratio. It is known from previous contributions [e.g., Koijen et al. (2010)] that this is not the case in a full-fledged LRR model. Nevertheless, the assumption is kept to make the results comparable with those of Canzoneri et al. (2007).

4. For an analysis in a DSGE setup see Gavazzoni (2012) and Li and Palomino (2014), among others.

5. Equation (18) of Hansen et al. (2008).

6. As mentioned earlier, the baseline model proposed by Bansal and Yaron (2004) does not assume time-varying volatility. The model presented here is their setup for an endowment economy with stochastic volatility.

7. Given the extensive analysis that can be found in the literature [e.g., see Canzoneri et al. (2007), Ahmad (2005), and Reynard and Schabert (2010)], results are not reported here, but are available upon request.

8. Bansal et al. (2007) warn that misalignment between the sampling frequency of consumption data and the agent's decision interval may lead to substantial biases in the economic plausibility of the model and the interpretation of the results.

9. For a detailed description of the calibration procedure applied, see Bansal and Yaron (2004).

10. Differently from Bansal et al. (2007), I decide not to include the "ex ante" real risk free in the regression in order to avoid potential endogeneity biases in the results.

11. The results are in line with Bansal et al. (2007), which found that in this case the low-frequency component of the consumption growth is less persistent than the baseline LRR calibration and slightly more predictable. The role of shocks' frequency in asset pricing models is analyzed in Bandi and Perron (2008), Ortu et al. (2013), and Dew-Becker and Giglio (2013), among others.

12. Relevant contributions on the low-frequency component of consumption are Ortu et al. (2013) and Schorfheide et al. (2014).

13. Ahmad (2005) considers the quarterly time period 1964Q1–2000Q4; Canzoneri et al. (2007) extend it to 2002Q4. To get comparable results, my sample starts in 1964Q1 as well and extends to 2011Q4. I strictly follow Bansal and Yaron (2004) in obtaining the monthly counterpart of the quarterly-based estimations.

14. I ran the calibration for values of the intertemporal elasticity of substitution ranging from 1.1 to 3. As expected, the correlation between the model-implied rate and the real risk-free rate is virtually unaffected by the choice of EIS. This, as noted in Beeler and Campbell (2012) and Brevik and d'Addona (2011), among others, is due to the linear scaling effect that the EIS has on the model implied risk-free. These estimates are available upon request.

15. As already discussed in the Introduction, using the risk-free rate as a regressor raises legitimate endogeneity concerns in the empirical exercise proposed here. Nevertheless, it is worth including this specification, given the capability of LRR models to match the relevant financial figures in the asset pricing literature.

16. Recession dates are from the U.S. Business Cycle Expansions and Contractions by NBER.

17. I am grateful to an anonymous referee who pointed this out.

18. Essentially the same results (available upon request) are obtained if the spread regression is based on the estimates obtained using the procedure introduced by Bansal et al. (2007) (see Subsection 4.3).

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