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Efficient Ambiguity Search Technique Using Separated Decision Variables

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In this paper, we propose new separated decision variables that are derived by directly using the normal distribution of each measurement error and allowing substitution of the chi-square variable of the conventional method. In the derivation of the proposed decision variables, we considered not only the related mathematical model, but also the additional unmodelled properties of GPS measurements. Using the sequential pseudo-moving-average technique, we developed a method that easily obtains the combined results of multiple epochs. To verify our proposed algorithm, we analysed its performance using real data and compared the results with those of the conventional method. Our proposed approach performs better than the conventional approach, and effectively reduces computational effort by approximately 60%. Our results demonstrate that our method achieves a solution that is as reliable as the conventional technique, while reducing the time required to only 15% of that required by the conventional technique.

KEY WORDS

1. RTK-GPS. 2. integer cycle ambiguity. 3. ambiguity resolution.

1. INTRODUCTION. For RTK GPS positioning, we must solve the integer ambiguity problem to use precise carrier phase measurements. As we can only measure the accumulated carrier phase that contains integer cycle ambiguity, it is important to determine the appropriate set of ambiguities and convert the carrier phase measurements to precise range measurements. Many researchers have tackled this problem, and many possible solutions have been proposed.

Typical ambiguity search problems can be solved with three sequential steps: search space construction, sequential tests, and verification processes. The first step is the construction of an initial search space. This involves collecting probable candidates for integer ambiguity within the appropriate boundaries, based on the stochastic properties of available measurements, including pseudoranges. This step is critical for the following steps, especially the initial phases of the search process. Too large a number of initial candidates requires excessive computation time. A well-known and very efficient method for this step is the LAMBDA (Least-squares AMBiguity Decorrelation Adjustment) method (Teunissen, 1994) (Jonge de, 1996). We use this method for the first step in the present study. The second step involves sequential tests of candidates. In this phase, we need an appropriate decision variable that efficiently represents the stochastic characteristics of

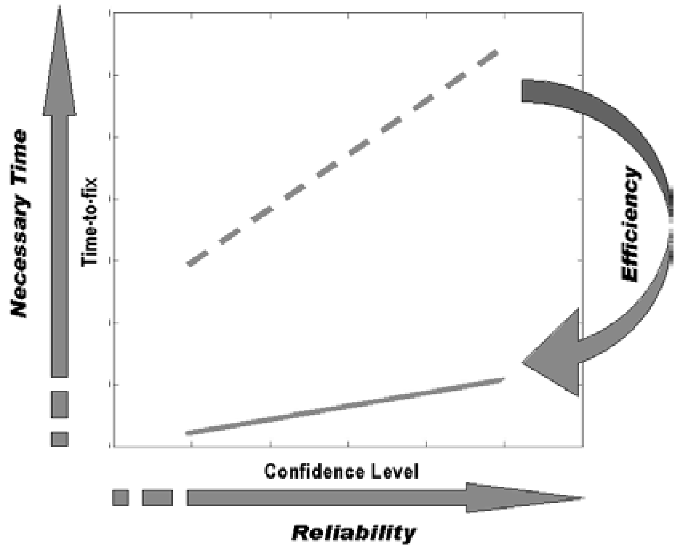

Figure 1. Necessary Time vs Reliability & Efficiency.

candidates. We then use this decision variable and its threshold to test whether each decision variable of the candidate is within the threshold, and eliminate unacceptable candidates on this basis. Conventionally, the chi-square variable has been used for this purpose. The final step is the verification process, using methods such as the "ratio test." This test uses the ratio of minimum and next-to-minimum values of chi-squared variables that have an F-distribution. If a series of ratios exceeds a pre-defined threshold and satisfies the necessary condition, we consider the solution to be true.

Of these three steps, we focus on the second step. This paper proposes new decision variables that enable a more efficient ambiguity search. The key elements of performance indices for an ambiguity search include time-to-fix, computational efficiency, and reliability. In general, more time is required to obtain a more reliable solution. A solution can be determined in just one second, but it would be difficult to guarantee the reliability of such a result. At the same time, we wish to avoid unnecessarily long computations. A good result involves a reasonable and acceptable trade-off between reliability and computation time. In terms of this kind of trade-off, the efficiency of the technique becomes the key to the problem. This concept is described in Figure 1. A more efficient method provides an equally reliable solution with less computation time.

In this study, we derive new decision variables based on a related mathematical model and the additional unmodelled properties of GPS measurements. To verify our proposed algorithm, we process real data and analyse the degree of consistency between our practical results and theoretical predictions. We also analyse the performance of the proposed algorithm and compare it with the conventional method.

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2. ALGORITHMS

2.1. *Problem formulation.* In general, the problem of GPS carrier phase integer ambiguity resolution starts from the following carrier phase measurement model:

$$\begin{aligned}\phi_u^j &= d_u^j + \delta R^j - \delta b^j - I_u^j + T_u^j + B_u + N_u^j \lambda + \varepsilon_u \\ \phi_r^j &= d_r^j + \delta R^j - \delta b^j - I_r^j + T_r^j + B_r + N_r^j \lambda + \varepsilon_r\end{aligned} \tag{1}$$

where ϕ is the carrier phase measurement. The superscript j refers to the j-th GPS satellite and the subscripts u and r refer to the user and the reference station receiver, respectively. d is the computed distance between the receiver and GPS satellite, δR is the residual range error caused by ephemeris error, δb is the residual satellite clock error, I is the ionospheric advance, T is the tropospheric delay, B is the GPS receiver clock offset, N is the integer ambiguity cycle, λ is the wavelength of the carrier phase, and ε is the random error of measurement. Except for the integer N, all terms are expressed in metres.

There are many error sources in this measurement model. To eliminate these errors, we usually double-difference the carrier phase measurements for two different satellites that can be tracked simultaneously by both the user and the reference station. After being double-differenced, the measurement model can be written as:

$${}^i\nabla^j{}_r\Delta_{\mathrm{u}}\phi = {}^i\nabla^j{}_r\Delta_{\mathrm{u}}d - {}^i\nabla^j{}_r\Delta_{\mathrm{u}}I + {}^i\nabla^j{}_r\Delta_{\mathrm{u}}T + \lambda{}^i\nabla^j{}_r\Delta_{\mathrm{u}}N + {}^i\nabla^j{}_r\Delta_{\mathrm{u}}\varepsilon \tag{2}$$

where ${}^i\nabla^j$ represents single-differencing between the i-th and j-th GPS satellites and ${}_r\Delta_{\mathrm{u}}$ represents single-differencing between the reference station and the user receiver. If the distance between the user and the reference station (also referred to as the "baseline") is sufficiently small that we can ignore the spatial decorrelation of atmospheric errors, the double-differenced errors caused by the ionosphere and troposphere can be assumed to be zero. Applying this assumption of the short baseline to the double-differenced distance term, i.e., ${}^i\nabla^j{}_r\Delta_{\mathrm{u}}d$, this term can be converted to the product of the baseline vector $\mathbf{b}$ and the single-differenced line-of-sight vector ${}^i\nabla^j\mathbf{e}_u$. Finally, the equation of concern is simplified as follows:

$${}^i\nabla^j{}_r\Delta_{\mathrm{u}}\phi = {}^{\boldsymbol{i}}\boldsymbol{\nabla}^{\boldsymbol{j}}\boldsymbol{e}_{\mathbf{u}} \cdot \boldsymbol{b} - \lambda{}^i\nabla^j{}_r\Delta_{\mathrm{u}}N + {}^i\nabla^j{}_r\Delta_{\mathrm{u}}\varepsilon. \tag{3}$$

The most important property of the double-differencing process is that the double-differenced integer ambiguity remains an integer. Our main purpose in processing this equation is to determine the most likely integer solution of the ambiguity vector. The solution can be obtained on the basis of the stochastic error model; however, the general integer ambiguity search process does not find and choose the right solution. Instead, it progressively identifies and discards wrong solutions and selects the remaining solution as correct. This is accomplished by calculating the decision variables of the integer candidates within the acceptable boundaries and comparing these to the decision criteria calculated from the probabilistic properties of the error model.

Now we accumulate the double-differenced equations of all measurements from $n+1$ satellites and rewrite them as follows:

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{b} + \lambda \boldsymbol{N} + \boldsymbol{v} \tag{4}$$

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where

$$z = \begin{bmatrix} {}^1\nabla^3{}_r\Delta_u\phi \\ {}^1\nabla^2{}_r\Delta_u\phi \\ \vdots \\ {}^1\nabla^{n+1}{}_r\Delta_u\phi \end{bmatrix},\ \boldsymbol{H} = \begin{bmatrix} {}^1\nabla^2\boldsymbol{e}_u^T \\ {}^1\nabla^3\boldsymbol{e}_u^T \\ \vdots \\ {}^1\nabla^{n+1}\boldsymbol{e}_u^T \end{bmatrix},\ \boldsymbol{N} = \begin{bmatrix} {}^1\nabla^2{}_r\Delta_u N \\ {}^1\nabla^3{}_r\Delta_u N \\ \vdots \\ {}^1\nabla^{n+1}{}_r\Delta_u N \end{bmatrix},\ \boldsymbol{v} = \begin{bmatrix} {}^1\nabla^2{}_r\Delta_u\varepsilon \\ {}^1\nabla^3{}_r\Delta_u\varepsilon \\ \vdots \\ {}^1\nabla^{n+1}{}_r\Delta_u\varepsilon \end{bmatrix} \quad (5)$$

In these equations, we assume that the first GPS satellite is the master satellite in terms of double-differencing the measurements.

2.2. *Sequential ambiguity using decision variables.* The conventional method uses the chi-square test to determine whether a candidate integer set is within the probable boundary of the true integer set. For this purpose, the decision variable γ_{χ^2} that has the properties of the chi-squared distribution is defined as follows:

$$\gamma_{\chi^2} = (\boldsymbol{z} - \boldsymbol{Hb} - \lambda\boldsymbol{N})^T\boldsymbol{Q}^{-1}(\boldsymbol{z} - \boldsymbol{Hb} - \lambda\boldsymbol{N}) \quad (6)$$

where $\boldsymbol{Q}$ is the covariance matrix of $\boldsymbol{v}$. If $\boldsymbol{N}$ is the true ambiguity vector, then γ_{χ^2} theoretically has the chi-squared distribution of *n-3* degrees of freedom for all possible error vectors $\boldsymbol{v}$. On the basis of this property, we can determine that a candidate integer set is wrong if its γ_{χ^2} exceeds the threshold that is calculated numerically for the appropriate confidence level. This can be written as follows:

$$\gamma_{\chi^2} \leqslant \chi^2_{n-3} \text{for true ambiguity vector } \boldsymbol{N} \quad (7)$$

where χ^2_{n-3} is the numerically calculated threshold using the chi-squared cumulative distribution function of *n-3* degrees of freedom. If we do not consider the relationships between the double-differenced measurements, we must then calculate the decision variables for all possible ambiguity combinations. If we assume that the number of candidates equals *m* for each measurement, then the number of possible combinations becomes m^n. To solve this inefficiency problem, Hatch showed that only three of the integer ambiguity elements are independent (Hatch, 1990). We therefore calculate the decision variables only for m^3 candidates.

Mathematically, the chi-squared random variable with *n-3* degrees of freedom is equivalent to the sum of *n-3* squared Gaussian random variables. Based on this idea, we can separate the conventional decision variable γ_{χ^2} into several normal random variables $\gamma_{\sigma_i(i=1,\dots n-3)}$ (Kee, 2003). This can be derived by mathematical equations. As we use separated variables, tests must be performed independently for each measurement.

To derive the new decision variables, we separate equation (4) into two parts: the independent and dependent sets, as follows (Park, 1997):

$$\begin{bmatrix} \boldsymbol{z_I} \\ \boldsymbol{z_D} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H_I} \\ \boldsymbol{H_D} \end{bmatrix}\boldsymbol{b} + \lambda\begin{bmatrix} \boldsymbol{N_I} \\ \boldsymbol{N_D} \end{bmatrix} + \boldsymbol{v} \quad (8)$$

where the subscripts *I* and *D* represent the independent set and the dependent set, respectively. Because only three equations are independent, $\boldsymbol{z_I}$ is the *3x1* vector and $\boldsymbol{z_D}$ is the *(n-3)x1* vector.

If we know the true values for the vector $\boldsymbol{N_I}$, an estimate of the vector $\boldsymbol{N_D}$ can be calculated as follows:

$$\hat{\boldsymbol{b}} = \boldsymbol{H_I}^{-1}(\boldsymbol{z_I} - \lambda\boldsymbol{N_I})\nabla \quad (9)$$

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$$\hat{\mathbf{N}}_D = \frac{1}{\lambda}[\boldsymbol{z_D} - \boldsymbol{H_D}\hat{\boldsymbol{b}}] = \frac{1}{\lambda}[\boldsymbol{z_D} - \boldsymbol{H_D}\boldsymbol{H_I}^{-1}(\boldsymbol{z_I} - \lambda\boldsymbol{N_I})]. \tag{10}$$

If we assume that the error vector $\boldsymbol{v}$ has a normal distribution, then all estimates derived from these measurement equations also have a normal distribution. This property can be applied to the distribution of each element of $\hat{N}_D$ and we can calculate the variances of each element. For the i-th measurement of the dependent set, we can rewrite equation (10) as follows:

$$\hat{N}_{D,i} = \frac{1}{\lambda}[z_{D,i} - \nabla\boldsymbol{e}_{\boldsymbol{D},\boldsymbol{i}}^{\boldsymbol{T}}\hat{\boldsymbol{b}}] = \frac{1}{\lambda}[z_{D,i} - \nabla\boldsymbol{e}_{\boldsymbol{D},\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{H_I}^{-1}(\boldsymbol{z_I} - \lambda\boldsymbol{N_I})]. \tag{11}$$

As $\boldsymbol{N_I}$ is the true integer ambiguity, the mean of $\hat{N}_{D,i}$ must be the true $\boldsymbol{N_{D,i}}$ for all the possible error vectors $\boldsymbol{v}$. Therefore, the variable that we should consider to make a decision is not $\hat{N}_{D,i}$ itself, but its residual $\delta\hat{N}_{D,i} = (\hat{N}_{D,i} - N_{D,i})$, which can be expressed as follows:

$$\begin{aligned}\delta\hat{N}_{D,i} &= \frac{1}{\lambda}[z_{D,i} - \nabla\mathbf{e}_{D,i}^{T}\mathbf{H}_I^{-1}(\mathbf{z}_I - \lambda\mathbf{N}_I)] - \frac{1}{\lambda}[\bar{z}_{D,i} - \nabla\mathbf{e}_{D,i}^{T}\mathbf{b}] \\ &= \frac{1}{\lambda}[(z_{D,i} - \bar{z}_{D,i}) - \nabla\mathbf{e}_{D,i}^{T}\mathbf{H}_I^{-1}(\mathbf{z}_I - \mathbf{z}_I)].\end{aligned} \tag{12}$$

As stated above, $\delta\hat{N}_{D,i}$ has a normal distribution with a zero mean and variance $\sigma^2_{\delta\hat{N}_{D,i}}$. The equation for the variance can be derived from equation (12) as follows:

$$\begin{aligned}\sigma^2_{\delta\hat{N}_{D,i}} &= E\{\delta\hat{N}_{D,i}\delta\hat{N}_{D,i}^{T}\} \\ &= \frac{1}{\lambda^2}[E\{(z_{D,i} - \bar{z}_{D,i})^2\} - 2E\{\nabla\boldsymbol{e}_{\boldsymbol{D},\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{H_I}^{-1}(z_{D,i} - \bar{z}_{D,i})\} \\ &\quad + E\{\nabla\boldsymbol{e}_{\boldsymbol{D},\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{H_I}^{-1}(\boldsymbol{z_I} - \boldsymbol{z_I})(\boldsymbol{z_I} - \boldsymbol{z_I})^{\boldsymbol{T}}\boldsymbol{H_I}^{-\boldsymbol{T}}\nabla\boldsymbol{e}_{\boldsymbol{D},\boldsymbol{i}}\}] \\ &= \frac{1}{\lambda^2}\left[q_{i+3,i+3} - 2\nabla\boldsymbol{e}_{\boldsymbol{D},\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{H_I}^{-1}\begin{bmatrix} q_{1,i+3} \\ q_{2,i+3} \\ q_{3,i+3}\end{bmatrix} + \nabla\boldsymbol{e}_{\boldsymbol{D},\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{H_I}^{-1}\boldsymbol{Q_I}\boldsymbol{H_I}^{-\boldsymbol{T}}\nabla\boldsymbol{e}_{\boldsymbol{D},\boldsymbol{i}}\right]\end{aligned} \tag{13}$$

where $\mathbf{Q_I}$ is the covariance matrix of $\boldsymbol{z_I}$, which in turn is the 3×3 sub-matrix of $\mathbf{Q}$. $q_{i,j}$, is the element of the covariance matrix $\mathbf{Q}$. On the basis of this variance, we define the new decision variable $\gamma_{\sigma_i}(i = 1, \ldots, n-3)$ as follows:

$$\gamma_{\sigma_i} = |\hat{N}_{D,i} - \langle\hat{N}_{D,i}\rangle| \tag{14}$$

where $\langle\hat{N}_{D,i}\rangle$ represents rounding to the nearest integer. Because the residuals for true ambiguities under normal conditions are always less than a half cycle, the decision variable γ_{σ_i} for true ambiguities is equal to the absolute value of $\delta\hat{N}_{D,i}$. The following conditions must therefore be satisfied for all true ambiguities:

$$\gamma_{\sigma_i} \leqslant \kappa\sigma_{\delta\hat{N}_{D,i}} \text{ for the true ambiguity vector } \boldsymbol{N} \tag{15}$$

where κ is the confidence level for the test. For example, if we set κ to 3 then the probability of the true ambiguity satisfying equation (15) is 99·97%; the probability of failure of the test is therefore 0·03%.

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2.3. *Efficiency of the proposed method.* This new decision scheme has two advantages: improvement in the computational efficiency and simplification of obtaining combined results from multiple epochs.

The computational effort required to obtain γ_{χ^2} is generally greater than that required for each γ_{σ_i}. The number of flops for γ_{χ^2} is approximately *n-3* times larger than that for each γ_{σ_i}. This is an obvious result because γ_{χ^2} contains the information for all measurements whereas γ_{σ_i} contains the information for just one measurement. This can be expressed as follows:

$$F_{\gamma_{\sigma_i}} \approx \frac{F_{\gamma_{\chi^2}}}{n-3} \tag{16}$$

where $F_{\gamma_{\chi^2}}$ and $F_{\gamma_{\sigma_i}}$ are the numbers of flops for γ_{χ^2} and γ_{σ_i}, respectively. For the case of a single ambiguity candidate, the computational effort required for both methods is similar, but usually there are many candidates for the initial search process. The savings in computational effort occur mainly during this initial phase. If the total number of candidates is m_t then the total number of flops for γ_{χ^2} is expressed simply as $m_t F_{\gamma_{\chi^2}}$, but it is different for γ_{σ_i}'s case, as follows:

$$\begin{aligned}\sum_{i=1}^{n-3} m_{t,i} F_{\gamma_{\sigma_i}} &= (m_{t,1} F_{\gamma_{\sigma_1}} + m_{t,2} F_{\gamma_{\sigma_2}} + \cdots + m_{t,n-3} F_{\gamma_{\sigma_{n-3}}}) \\ &\approx (m_{t,1} + m_{t,2} + \cdots + m_{t,n-3}) F_{\gamma_{\sigma_i}}\end{aligned} \tag{17}$$

where $m_{t,i}$ represents the number of candidates for the *i*-th test and $m_{t,1}$ equals m_t. Generally, the number of candidates decreases as the number of tests increases. Because the tests using γ_{σ_i} are done on a measurement-by-measurement basis, the following relation is satisfied and the computational efficiency can be improved:

$$(m_{t,1} + m_{t,2} + \cdots + m_{t,n-3}) F_{\gamma_{\sigma_i}} < m_t (n-3) F_{\gamma_{\sigma_i}} \approx m_t F \gamma_{\chi^2}. \tag{18}$$

From equation (18), it is evident that the total number of flops for tests that use γ_{σ_i} is always smaller than that for γ_{χ^2}.

The many conventional methods for conducting an integer ambiguity search obtain a solution from the measurement of several epochs and mainly depend on the property of the change in receiver-satellite geometry. For this reason, measurements sampled at a rate of 1 Hz or higher are meaningless in terms of the epoch-by-epoch approach. As the measurement noise is independent in consecutive samples, it is useful to average the noise to reduce the noise level and improve reliability and efficiency. However, we cannot use the resulting measurements that are solely averaged with respect to time because the receiver-satellite geometry is continuously changing. At this point, we seek a value that indicates the measurements of each epoch and that can be averaged to a theoretically meaningful value. We consider γ_{σ_i} for this purpose. Based on the results derived above, $\frac{\gamma_{\sigma_i}}{\sigma_{\delta \hat{N}_{D,i}}}$ becomes the normal distribution with a zero mean and unit variance. On the basis of basic mathematical relationships, the average of the independent normal random variables once again becomes the normal random variable. Using this property, we can derive the more reliable and efficient decision variable as follows:

$$\gamma_{\sigma_i} \sim N(0, \sigma_{\delta \hat{N}_{D,i}}) \Rightarrow \gamma_{\sigma_i} = \frac{\gamma_{\sigma_i}}{\sigma_{\delta \hat{N}_{D,i}}} \sim N(0, 1^2) \tag{19}$$

$$\begin{aligned}\Gamma_{\sigma_i}(1) &= \tilde{\gamma}_{\sigma_i}(1)\\ \Gamma_{\sigma_i}(k) &= \frac{k-1}{k}\Gamma_{\sigma_i}(k-1)+\frac{1}{k}\tilde{\gamma}_{\sigma_i}(k)\end{aligned} \tag{20}$$

$$\Gamma_{\sigma_i}(k) \leqslant \kappa\frac{1}{\sqrt{k}} \quad \text{for the true ambiguity vector } \boldsymbol{N}. \tag{21}$$

Equation (21) is satisfied when the measurement noise has an ideal normal distribution. However, in reality, there is no such thing as ideal Gaussian noise because of existing unmodelled errors such as the multipath error and the residual errors of atmospheric delay terms. For this reason, $\Gamma_{\sigma_i}(k)$ converge to a value close to zero rather than exactly zero for a range of k when the threshold $\kappa\frac{1}{\sqrt{k}}$ rapidly converges to zero. This resembles the filter sleep of the usual averaging filter. Their possible solutions are therefore also alike in terms of using the concept of the moving-average filter. However, the moving-average filter stores all $\Gamma_{\sigma_i}(k)$ within the averaging window, and this causes the inefficiency. The result of this trade-off is a pseudo-moving-average filter that is formulated with the fixed averaging constant M.

The related expression of this method is as follows:

$$\begin{aligned}\Gamma'_{\sigma_i}(1) &= \frac{1}{M}\tilde{\gamma}_{\sigma_i}(1)\\ \Gamma'_{\sigma_i}(k) &= \frac{M-1}{M}\Gamma'_{\sigma_i}(k-1)+\frac{1}{M}\tilde{\gamma}_{\sigma_i}(k)\end{aligned} \tag{22}$$

$$\Gamma'_{\sigma_i}(k) \leqslant \kappa\sqrt{\frac{1}{2M-1}\left[1-\left(1-\frac{1}{M}\right)^{2k}\right]} \quad \text{for the true ambiguity vector } \boldsymbol{N}. \tag{23}$$

In terms of long-term processing, the pseudo-moving-average filter shows similar function and performance to the moving-average filter. For large k, the threshold converges to constant $\kappa\frac{1}{\sqrt{2M-1}}$, which is equivalent to that of the moving average, of which the window size is $2M-1$. Equation (22) is the final form of the proposed decision variable. With this new decision variable, we can easily obtain the combined results of multiple epochs.

3. EXPERIMENTAL RESULTS.

We tested the validity of the proposed algorithm using real data. Using two Trimble 4000 ssi receivers, we simultaneously logged L1 single frequency data from the reference station and user during approximately 1,000 seconds of unit sampling time. With these data, we performed search processes approximately 800 times as part of a statistical analysis. Because of the short baseline length of 151 metres, we applied the short baseline assumption. Upon double-differencing these measurements, biases caused by ionospheric advance, tropospheric delay, and orbital errors can be ignored, but biases resulting from multipath error cannot be ignored. The details of the processed data are summarized in Table 1.

We first examined the computational effectiveness of the proposed method according to two different procedures: measuring the total number of flops for the first epoch, which occupies the most of the computational effort; and measuring the

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Table 1. Summary of GPS data processing.

	Reference Station	User
Location	Seoul National University	Seoul National University
Latitude	37°26′58.903″N	37°27′03.603″N
Longitude	126°57′09.887″E	126°57′06.006″E
Height	281·925 m	217·704 m
Date	14 June 2004	
Receiver	Trimble 4000 ssi	
# of Satellites	8	
# of Measured Epochs	1,000	
Sampling Time	1 s	
Elevation Mask	5°	

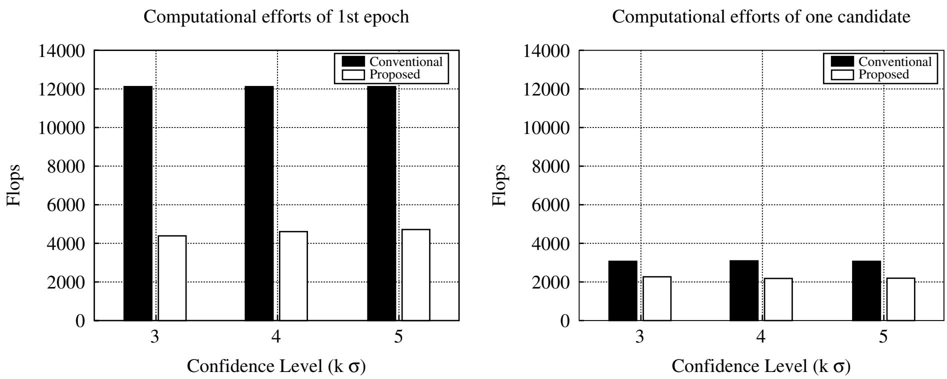

Figure 2. Computational Efforts Required for Proposed and Conventional Search Processes.

number of flops for each individual candidate. The results show that the number of flops of the proposed method for individual candidates is smaller than those of the conventional method. In the case of the total number of flops for the first epoch, the result is even more remarkable. The number of flops of the conventional method is almost three times greater than that of the proposed method. Figure 2 shows these results for each confidence level. Because the numbers of candidates that remain after sequential tests are different for each confidence level, the computational efforts also show slightly varying results.

Performance of the ambiguity resolution is mainly represented by the time required to fix the solution. In this result, the definition of time-to-fix is different from the usual definition. As we did not consider the validation process as a "ratio test," we define the time-to-fix as the time until the number of remaining candidates having passed the sequential decision process becomes one. With this definition, we can measure the efficiency of the algorithm by measuring the absolute value of the time-to-fix. All 800 trials of search processes for each confidence level converge to the true ambiguity vector. The time-to-fix generally increases with higher confidence level. If we focus on the reliability of the method, the slope of the time-to-fix vs confidence level κ trend is a good indicator of the efficiency of the algorithm. For an averaging constant M of 4, the time-to-fix vs κ trend is as shown in Figure 3.

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Table 2. Summary of experimental results.

	Conventional method	Proposed method	Improvement
Computational efforts (4σ case)			
Total flops for first epoch	12,146	4,626	62% reduction
Flops for one candidate	3,091	2,247	27% reduction
Time-to-fix			
3σ	141	12	91% reduction
4σ	244	32	87% reduction
5σ	317	55	83% reduction
Time-to-fix vs κ			
Slope	87·8	21·7	75% reduction

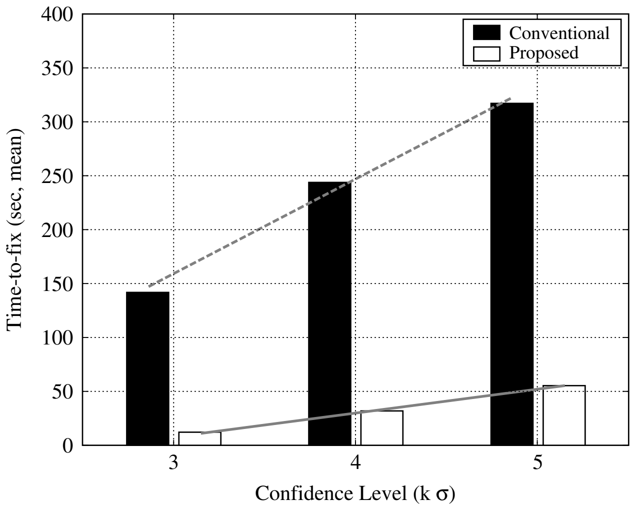

Figure 3. Time-to-fix vs Confidence Level.

As evident in Figure 3, the absolute values of the time-to-fix of the proposed method are much smaller than those of the conventional method for each confidence level. A second point of interest in Figure 3 is the slope of each line. The slope of the time-to-fix vs κ trend for the proposed method is smaller than that of the conventional method. This demonstrates that the proposed method provides a more reliable solution with less additive effort than the conventional method. The experimental results are summarized in Table 2 with explicit values.

The performance of the proposed method is partly dependent on the averaging constant M. With equation (22), we may find that it is not necessary for M to be an integer, as it simply decides the weighting of the current decision variable. In fact, M can take the value of any float number larger than 1, but its range might be limited by its properties in relation to the proposed method. The larger the constant M, the smaller the time-to-fix but the more the method is influenced by measurement biases. Increasing M acts to decrease the weighting of the current decision variable and increase the dependence on past information, as with increasing the window size of

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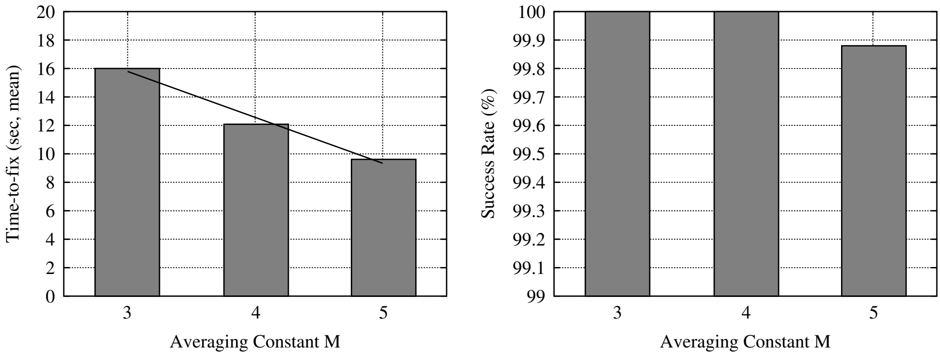

Figure 4. Averaging Constant M vs Time-to-fix and Success Ratio.

the moving average. We fixed the confidence level at 3 and examined the performance of the proposed method with various averaging constants M. The results are shown in Figure 4.

As evident in Figure 4, the time-to-fix decreases for larger M, but the success ratio also falls below 100%. This is because the residual errors of double-differenced measurements are not perfectly Gaussian; the larger M therefore results in the accumulation of bias effects. This is why the constant M is the design parameter of the proposed method. It may be a rather complicated problem to choose an appropriate M, but, as in many other approaches, this may be possible on an empirical basis via the statistical analysis of a very large data set under various conditions. Such an approach must have different values for each receiver and each environmental condition.

4. CONCLUSIONS.

In this study, we derived new decision variables by considering a related mathematical model and the additional unmodelled properties of GPS measurements. To verify our proposed algorithm, we analysed the performance of the algorithm using real data and comparison with the conventional method. The proposed approach showed better performance than the conventional approach. We effectively reduced computational efforts to approximately 60% of that required by the conventional method. In particular, we used a sequential pseudo-moving-average technique to develop a method that can easily obtain the combined results of multiple epochs. As a result, we achieved the efficiency of a sequential test process. Our approach requires only approximately 15% of the time taken by the conventional technique, yet we obtained equally reliable solutions. Our proposed method is a possible solution for a faster and more reliable technique for the ambiguity search. For further practical use of this algorithm, we intend to apply the optimal value of the averaging constant for the pseudo-moving average, considering the performance of the GPS receiver and the environmental conditions. This can be done by statistical analysis of a large data set under varying conditions.

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