Dynamic biped robot locomotion on less structured surfaces M.-Y. Cheng[†] and C.-S Lin[‡]

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SUMMARY

Most studies in the past on the control of biped locomotion considered only level surfaces. However, in the real world the ground is rarely completely flat. More research on locomotion on less structured surfaces is needed. In this study, we investigated a control design method that searches for suitable control and trajectory parameters using a Genetic Algorithm (GA). Many sets of parameters are generated through the search and the best set is selected based on a robustness measure developed from the linearized Poincaré map. This technique reduces tedious analysis and is favorably applicable to the design for locomotion on unstructured surfaces, for which analytical approaches are less appropriate. Simulations have been performed. Control parameters for different slopes were obtained and stored in a database. During the control, the control parameters suitable for the current surface slope were retrieved and the trajectory for a level surface was modified according to the surface slope. The control parameters changed values according to the terrain. Simulation results were promising.

KEYWORDS: Biped robot locomotion; Genetic algorithms; Poincaré map

1. INTRODUCTION

Building a biped robot that can mimic human walking is of interest to many researchers and engineers. One favorable application is to have biped robots work for human beings in hazardous or dangerous circumstances. Research on biped robot locomotion could also create valuable results to help design artificial legs for handicapped people. Compared to mobile robots, a biped robot possesses a better capability of adapting to less structured terrain. However, a biped robot has highly nonlinear dynamics and is statically unstable. Its locomotion control is difficult. Extensive research is needed before biped robots can be widely and practically used. In the past two decades, many studies in this area have been done.¹⁻¹³ Most of the earlier designs were for locomotion on level surfaces. Since in the real world the ground is rarely completely flat, more research should be devoted to locomotion on unstructured surfaces.

Biped locomotion control is a difficult problem. Less structured surfaces make the control even harder. Conventional control techniques are usually based on analysis,

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model approximation and intuitive assumptions. In this paper, a different approach that is based on parameter search and stability evaluation is presented. Many sets of workable control parameters are generated by a genetic algorithm (GA),¹⁴ and the best set is selected based on evaluation of control robustness. This technique eliminates a lot of tedious analysis and is favorably applicable to the design for locomotion on unstructured surfaces. The proposed technique views the ground as a combination of surfaces with different slopes. Using the method, control parameters for different slopes are obtained and stored in a database. During the control, control parameters suitable for the current surface slope are retrieved and the trajectory for a level surface is modified according to the surface slope.

In Section 2, we will briefly introduce a 5-link biped model that was used in simulation. The control structure with control gains as parameters to be determined and two possible ways for generating the nominal trajectory are introduced in Section 3. An introduction to the genetic algorithm and the Poincaré map¹⁵, which are the basis of our method, is given in Section 4. Section 5 presents the method that evolves the controller for biped robot locomotion on less structured surfaces. Simulation results are provided in Section 6 and conclusions are given in Section 7.

2. THE 5-LINK BIPED MODEL USED IN SIMULATION

A biped robot may perform static walking or dynamic walking. A biped robot that performs static walking has large feet and keeps balance by having its center of mass always located within the range of its foot support. This type of walking is usually slow. Dynamic walking can be much faster. The biped robot keeps walking to maintain its stability. The control is harder than that for static walking. In this paper, we focus on dynamic walking and select the 5-link biped structure developed by Furusho and Masubuchi⁴ for illustrating our technique. Rather complete model data for this 5-link biped robot are available in their article. Figure 1 shows the biped model. Only the motion on the sagittal plane is considered in this study. The ground condition is assumed to be rigid and non-slip. At any time instant, only one foot has a contact point with the ground. Each step consists of a single-leg-support phase and a legsupport-exchange event. Since the biped robot only has a single contact point with the ground, ankle torques are not applicable

2.1. Motion equation for single-leg-support phase

Using the Lagrangian formulation, dynamic equations for the 5-link biped robot during the single-leg-support phase



Fig. 1. A 5-link biped robot model.

can be derived. The following is a general representation of the dynamic equations in the matrix-vector form

$$A(\underline{\theta})\underline{\ddot{\theta}} + B(\underline{\theta})\underline{\dot{\theta}} + Cg(\underline{\theta}) = \tau \tag{1}$$

where $\underline{\theta} = [\theta_1, \theta_2, \dots, \theta_5]^T$ is the vector of angular link positions, $g(\underline{\theta})$ is the vector $[\sin \theta_1, \sin \theta_2, \dots, \sin \theta_5]^T$, $\tau = [\tau_1, \tau_2, \dots, \tau_5]^T$ is the vector of generalized coordinate torques, $A(\underline{\theta})$ and $B(\underline{\theta})$ are 5×5 matrices whose elements are functions of positions and link parameters, and *C* is a 5×5 constant matrix. The superscript "*T*" denotes the vector transpose.

2.2. Motion equation for leg-support-exchange

When the swing leg lands on the ground, an impulsive force is generated due to the impact. The impulsive force could lift another foot up immediately; this results in a legsupport-exchange. The Lagrange impulsive equation¹⁶ can be applied to derive the algebraic equation for the legsupport-exchange. The angular velocity of each link right after the impact can be expressed as⁴

$$\underline{\dot{\theta}}^{+} = A^{-1}(\underline{\theta})[A(\underline{\theta}) + U^{T}(\underline{\theta})D(\underline{\theta})]\underline{\dot{\theta}}^{-}$$
(2)

where $\underline{\dot{\theta}}^+$ is the vector of generalized coordinate velocities right after the impact and $\underline{\dot{\theta}}^-$ is the one right before the impact. $D(\underline{\theta})$ and $U(\underline{\theta})$ are 2×5 matrices. Since the time period for the leg-support-exchange is assumed to be zero, the link angular positions after the leg-support-exchange remain unchanged during the impact event. Equations (1) and (2) describe a single step of biped locomotion.

3. THE CONTROL STRUCTURE AND NOMINAL TRAJECTORY FOR BIPED ROBOT LOCOMOTION

The new design method is based on search for trajectory and control parameters. In this section, we will introduce the selected control structure and the used trajectory. The parameters to be determined include the control gains and probably also some joint positions on the path.

3.1 Control structure

Control engineers may suggest different control schemes. A controller usually has parameters that can be adjusted to optimize the locomotion. The proposed method tries to

search for good parameter values. In this paper, for illustration and demonstration, a simple controller with feedforward compensation and feedback control is selected. The vector of generalized torques applied to links is expressed as

$$\tau = A(\underline{\theta})\underline{\ddot{\theta}}_{d} + B(\underline{\theta})\underline{\dot{\theta}} + Cg(\underline{\theta}) + \underline{K}_{d}(\underline{\dot{\theta}}_{d} - \underline{\dot{\theta}}) + \underline{K}_{p}(\underline{\theta}_{d} - \underline{\theta}) \quad (3)$$

where $\underline{\theta}_d$ indicates the nominal trajectory, and \underline{K}_d and \underline{K}_p are diagonal matrices with control gains on the diagonal. The first three terms at the right hand side of (3) are for feedforward compensation and the last two are for feedback control. τ is the 5×1 vector of generalized torques applied to five links. The considered biped model has only four DC motors for four joints (two at the hip and two at knees) and cannot create an ankle torque directly. One solution for control is to apply four joint torques to generate the link torques T_1 , T_2 , T_3 , T_4 and T_5 that are closest to the components of τ determined by the control law in (3). A least mean square method can be used to determine these generalized torques.¹⁷ The generalized torque T_5 at link 5 must be equal to $-(T_1+T_2+T_3+T_4)$. With the goal to minimize error, the cost function can be defined as

$$E = (T_1 - \tau_1)^2 + (T_2 - \tau_2)^2 + (T_3 - \tau_3)^2 + (T_4 - \tau_4)^2 + (-T_1 - T_2 - T_3 - T_4 - \tau_5)^2$$

Partial derivatives of E with respect to T_i 's generate the following equations:

$$2T_1 + T_2 + T_3 + T_4 = \tau_1 - \tau_5$$
$$T_1 + 2T_2 + T_3 + T_4 = \tau_2 - \tau_5$$
$$T_1 + T_2 + 2T_3 + T_4 = \tau_3 - \tau_5$$
$$T_1 + T_2 + T_3 + 2T_4 = \tau_4 - \tau_5$$

which give

$$T_{1}=0.8\tau_{1}-0.2(\tau_{2}+\tau_{3}+\tau_{4}+\tau_{5})$$

$$T_{2}=0.8\tau_{2}-0.2(\tau_{1}+\tau_{3}+\tau_{4}+\tau_{5})$$

$$T_{3}=0.8\tau_{3}-0.2(\tau_{1}+\tau_{2}+\tau_{4}+\tau_{5})$$

$$T_{4}=0.8\tau_{4}-0.2(\tau_{1}+\tau_{2}+\tau_{3}+\tau_{5})$$

$$T_{5}=-(T_{1}+T_{2}+T_{3}+T_{4})$$

$$=0.8\tau_{5}-0.2(\tau_{1}+\tau_{2}+\tau_{3}+\tau_{4})$$
(4)

3.2. Nominal trajectory

The way to generate the nominal trajectory is another choice of the designer. This study uses either link angle profiles or the synthesized gait for specifying the nominal trajectory ($\underline{\theta}_d$ in (3)).

3.2.1. Link angle profile method. The nominal trajectory for each link can be individually specified. Figure 2 illustrates the possible nominal trajectory for a link. The time function of the link angle (*i.e.* $\underline{\theta}_{di}$) is divided into several line segments. A curvy transition from a line segment into another is required in order to generate smooth



Fig. 2. A possible nominal trajectory for a link. The values at t_0 , t_2 , t_5 are set-points.

locomotion. The transitions are specified by quadratic polynomials. The coefficients of the polynomials are determined to have necessary boundary conditions satisfied.

In this study, six set-points are used to specify the desired trajectory for each link. To keep the biped body upright during walking, the desired link angle θ_{d3} is set to zero. Thus there are only four link angle profiles left to be determined. Twenty four (4×6) set-points are needed to uniquely describe the complete biped trajectory. Those are parameters to be selected in trajectory design.

3.2.2 Gait synthesis method. Another way for generating trajectory in our simulations is through gait synthesis,^{7,12} which uses constrained relations to specify the desired gait. Different constrained relations may be used. In this study, the following five are used (see Figure 3):

- (i) The upper body is kept erect.
- (ii) The knee angle σ of the supporting leg remains unchanged during the walking.
- (iii) The motion of the supporting leg is assumed to behave like a linearized inverted pendulum.



Fig. 3. Five constrained relations used to specify a desired biped gait.

- (iv) The tip of the swing leg is assumed to move along a parabolic curve.
- (v) The *x* coordinate of the body is in the middle between the tips of the supporting and swing legs.

The above constrained relations uniquely specify a biped gait. The closed-form solutions of link angles can be obtained by solving equations from these five constraints¹⁸. It is noted that no parameter values are left to be determined; this is different from the case using link angle profiles.

4. INTRODUCTION TO GENETIC ALGORITHM AND POINCARÉ MAP

We have previously applied the genetic algorithm for controller design¹⁷ and the Poincaré map method for stability analysis¹⁹, both for locomotion on level surfaces. In this study, these two methods are integrated together to study the controller design for biped locomotion on less structured surfaces. An introduction to these two methods will be given in this section.

4.1 Genetic algorithms

Genetic algorithms (GAs) are searching algorithms based on some similar mechanics of natural selection and natural genetics. To incorporate GA to solve an optimization problem, the parameters of the problem are coded into a finite-length of string as a chromosome. The genetic algorithm starts with a population of randomly generated chromosomes and evolves toward better solutions by applying genetic operators. Reproduction, crossover, and mutation are three basic GA operators, which can be easily implemented in a computer program. During the search, natural selection and genetic operators are used to repeatedly create the new generation until some pre-established goal is met. An evaluation or fitness function plays an important role during the search process. In each generation, the fitness is used to judge whether a solution is acceptable or not. The fitness function could be the profit, utility, risk or other measurement to be maximized or minimized.

4.1.1. Basic genetic operators. A brief introduction to the basic genetic operators is given below.

Reproduction operator

Reproduction is a process in which individual binary strings are copied according to their fitness values. A string with a higher fitness value has a larger probability to be selected as one or more offspring in the next generation.

Crossover operator

A crossover combines the features of two parent structures to form two offsprings. A crossover provides a means for strings to mix and match their desirable traits through a random process. This can be done by randomly selecting a crossover point (a bit position) and swapping two substrings up to this point. This genetic operator is illustrated below using two binary coded strings X and Y of length eight:

 $\begin{array}{c} X = 1000 \\ Y = 0011 \\ 0010 \end{array}$

After a crossover, two new strings are obtained as

$$X' = 10000010$$

 $Y' = 00110101$

Mutation operator

Mutation refers to randomly changing part of a gene. Using the binary coding, a mutation simply complements the values of one or more bits. The following diagram illustrates this operator.

101	1 0 0 1 1	$ \xrightarrow{\text{MUTATION}} \boxed{1}$	1	1	1	0	0	0	1
Î	Î								

Two bits are changed in this example. The mutation assures that it is possible to recover an important piece of lost information.

4.1.2. A typical genetic algorithm. The following summarizes a typical genetic algorithm:

- Step 1. Select a fitness function (the goal for the optimization process).
- Step 2. Code the parameters to be varied for optimization into genes. These genes compose a chromosome.
- Step 3. Randomly generate a fixed population of chromosomes and evaluate their fitness values.
- Step 4. Apply the reproduction operator to generate a new population of chromosomes (the ones with better fitness values from the previous generation have bigger probabilities of being selected).
- Step 5. Randomly choose the mating gene pair and the crossover site. Apply the crossover operator to generate the new pair of chromosomes. Apply the mutation operator.
- Step 6. Repeat step 5 until a fixed size of population is generated.
- Step 7. Compute the fitness values for the members of the new population. Stop if the result is satisfied. Otherwise, repeat from step 4.

Genetic algorithms can be used to search for the control gain matrices \underline{K}_d and \underline{K}_p , and nominal trajectory $\underline{\theta}_d$ to yield the best control performance of biped locomotion. For more details about the application of genetic algorithm into the controller design of biped locomotion, please refer to reference 17.

4.2. Introduction to the use of the Poincaré map

The Poincaré map method is a powerful tool for investigating periodic motion of dynamic systems. Since biped locomotion is periodic, the Poincaré map method can be applied to analyze the biped locomotion.

Figure 4 illustrates the idea of the Poincaré map. Let (x(t), y(t); t) be the trajectory of a moving object. Assume that without disturbance, x(t), y(t) are periodic functions of time with a period T equal to 2π . One may select the Poincaré section Σ at $t=t_0+2n\pi$, then points u_0 at $t=t_0$ and u_1 at $t=t_0+2\pi$ will have the same x and y values. Point u_0 (or u_1) is called the fixed point of the Poincaré map **P**. To study the stability of this periodic motion, one can study the stability of the fixed point of the Poincaré map instead. What one needs to evaluate is if there is a deviation from the fixed point at the beginning, what will be the deviation on the next map (see Figure 5 for differences between u_0 and u_{0}^{*} , and u_{1} and u_{1}^{*}). This relationship can be described as a Jacobian matrix (the linearized Poincaré map). To have a stable system, the points on the Poincaré section are expected to converge to the fixed point eventually. All the eigenvalues of the Jacobian matrix evaluated at the fixed point must be within the unit circle. The largest modulus of the eigenvalues gives an indication of stability and can be used as a robust measure for design evaluation. A smaller value implies a greater stability. Details on the procedure for evaluating the linearized Poincaré map for biped locomotion can be found in reference 19. The fixed point in biped locomotion can be the biped state right after the leg exchange.

5. CONTROLLER DESIGN FOR BIPED LOCOMOTION ON LESS STRUCTURED SURFACES

The proposed method views the ground as a combination of surfaces with different slopes. A biped robot presumes that it is walking on surfaces with different slopes. The current contact point of the supporting leg and the location of the next expected landing point determine the height change as well as the slope of the presumed surface. Dot lines in Figures 6 and 7 indicate such surfaces.

The nominal trajectory of biped locomotion for the horizontal surface is generated either from the link angle profile or gait synthesis method. For an inclined surface, the



Fig. 4. Poincaré map. x(t), y(t) are periodic functions of time with a period T equal to 2π . Σ is the Poincaré section.

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nominal trajectory is modified by adding an offset to each link angle.

The proposed method is summarized below:

1. (a) If the nominal trajectory is to be specified using the link angle profile method, GA is used to search for the suitable values of the trajectory set-points for a horizontal surface.

(b) If the nominal trajectory is derived from the gait synthesis method, the nominal trajectory is uniquely specified for the horizontal surface.

- 2. The trajectory generated in step 1 is for the horizontal surface. The nominal trajectory to be used for a δ^0 inclined surface is generated by adding $-\delta^0$ (see Figure 8) to each link angle (except for the body, *i.e.* link 3).
- 3. GA is then used to search for control gains using the nominal trajectories for different inclined surfaces. Many sets are obtained.
- 4. The different sets of control parameters for the same inclined surface are compared by using the robustness measurement introduced in Section 4.2 (the maximum modulus of eigenvalue of the linearized Poincaré map). One with a smaller modulus is expected to be more robust.



Fig. 5. Linearized Poincaré map is indeed a Jacobian matrix evaluated at the fixed point.

Explanations: A set of control parameters works well for a range of slopes. For instance, the ones for the 5° inclined surface may work well for the inclined surfaces with an angle between 4° and 6° . Therefore, only one set of control parameters is needed for the range $[4^{\circ}, 6^{\circ}]$. One can search for the control parameters for inclined surfaces with a 2° increment and pick the necessary sets that will cover the complete concerned range. The results will be saved in a database such that the biped robot can choose appropriate control parameters from it as needed.

5. For a control to walk on a δ^0 inclined surface, the nominal trajectory for the horizontal surface is modified by adding an offset $-\delta^0$ (Figure 8) to each link angle (except for the body, *i.e.* link 3) and the control parameters for this inclined surface are used.

6. SIMULATION

Both the gait synthesis and the link angle profile methods are used to generate the nominal trajectories. Tables I and II list the examples of control parameter's databases for biped locomotion on inclined surfaces.

Although each set of control parameters is searched for a specific slope, it should be applicable to a wider range of slope. Tables III and IV list the applicable ranges for those sets given in Tables I and II. Table III gives the results for the case using the gait synthesis. The second column provides the maximum moduli of eigenvalues evaluated at the specific slopes given in the first column. All the maximum moduli are small and the biped locomotion is expected to be robust even with some variation in the tilt angle. The third column verifies this. Each set of control parameters is suitable for a wide range of slopes, and only five sets of control parameters are needed for inclined surfaces with tilt angles between -16° and 12°

Table IV gives the results for the case using link angle profiles. The nominal trajectory of biped locomotion on the horizontal surface (with a maximum modulus of eigenvalue 0.6491) plus an offset was used. The second column indicates that the maximum moduli for these biped locomotions are larger than those in Table III. This suggests that the biped locomotion might be less robust. The results



Fig. 6. During the surface change (steps 2, 4, 6), the biped robot presumes it is walking on a virtual inclined surface (indicated by the dash line).



Fig. 7. During the surface change (steps 2, 4, 6), the biped robot presumes it is walking on a virtual inclined surface (indicated by the dash line).

in the third column affirm this. The applicable ranges of the tilt angle are smaller than those in Table III. Eight different sets of control parameters are needed for tilt angles between -11° and 8.5°

Fig. 8. Offset needs to be added when walking on an inclined surface if the nominal trajectory for the horizontal surface is used. The offset makes the pose of the biped robot remain similar to that for the horizontal surface.

Figures 9–11 show biped locomotion on different ground surfaces with height changes. The nominal trajectory is generated from the gait synthesis. Figures 12–14 are results with the nominal trajectory generated from the link angle profiles. If the control gains for the level surface are used for the less structured surface, the tolerance for the height variation will be only about 1 cm. Different control gains obtained from the GA search do contribute to the improvement of the tolerance to surface variation.

Figures 15 and 16 show the stick diagrams of a biped robot moving onto an inclined surface from a horizontal one and then back to a horizontal surface. The set of control parameters for the 5° or -5° inclined surface in Table I is used for the transition from the horizontal surface to the 10° or -10° inclined surface.

7. CONCLUSIONS

Most of research in the past on biped robot locomotion on unstructured surfaces focused on the development of control structures and improvement of robot mechanism. The development of a controller often involves tedious analysis and model approximation. Very few researchers addressed the issue of control robustness. In this paper, we presented

Table I. Database of control parameters \underline{K}_d and \underline{K}_ρ (in (3)) suitable for biped locomotion on different inclined surfaces. Nominal trajectory is generated using the gait synthesis method. $|\lambda_{max}|$ is the maximum modulus of eigenvalue of the linearized Poincaré map.

Control parameters	-10° slope $ \lambda_{\text{max}} =$	-5° slope $ \lambda_{\text{max}} =$	0° slope $ \lambda_{max} =$	5° slope $ \lambda_{\rm max} =$	10° slope $ \lambda_{\text{max}} =$
	0.3170	0.3857	0.1649	0.3209	0.3090
K _{p1}	112.588	283.412	283.412	142.047	205.765
K_{p2}	555.176	508.588	508.588	807.795	310.588
K_{p3}	966.706	962.824	962.824	647.646	590.118
K_{p4}	838.588	885.176	885.176	905.906	842.471
K_{p5}	892.941	745.412	745.412	555.512	640.588
\mathbf{K}_{d1}	12.598	15.748	15.748	103.000	85.039
K _{d2}	39.37	102.362	102.362	19.000	127.559
K _{d3}	102.362	100.787	100.787	175.000	171.654
K_{d4}	36.22	64.567	64.567	106.000	64.567
K _{d5}	173.228	182.677	182.677	181.000	170.079

Table II. Database of control parameters \underline{K}_d and \underline{K}_p (in (3)) suitable for biped locomotion on different inclined surfaces. Nominal trajectory is generated using the link angle profile method. $|\lambda_{max}|$ is the maximum modulus of eigenvalue of the linearized Poincaré map.

Control parameters	$-10^{\circ} \text{ slope} \\ \lambda_{\max} = \\ 0.6853$	$-7^{\circ} \text{ slope} \\ \lambda_{\max} = \\ 0.7873$	$\begin{array}{c} -6^{\circ} \text{ slope} \\ \lambda_{\max} = \\ 0.6778 \end{array}$	$-1^{\circ} \text{ slope} \\ \lambda_{\max} = \\ 0.6889$	$\begin{array}{l} 0^{\circ} \text{ slope} \\ \lambda_{\max} = \\ 0.6491 \end{array}$	3° slope $ \lambda_{max} =$ 0.3918	6° slope $ \lambda_{max} =$ 0.6762	$\begin{array}{l} 10^{\circ} \text{ slope} \\ \lambda_{\max} = \\ 0.4575 \end{array}$
K _{n1}	586.518	688.757	990.0	241.292	430.941	343.429	282.857	569.589
$K_{n^2}^{p^1}$	554.788	159.933	190.627	276.223	217.412	109.429	319.667	23.249
K_{p3}^{p2}	561.839	459.60	267.49	389.335	714.353	976.714	850.509	780.763
K_{p4}^{p3}	604.145	427.871	790.157	271.233	966.706	826.286	399.10	802.074
\mathbf{K}_{n5}^{P}	501.906	456.075	617.216	550.685	912.353	635.0	970.626	373.914
\mathbf{K}_{d1}^{PD}	104.953	210.945	381.024	118.0	263.529	287.275	141.176	183.529
K_{d2}	166.213	39.685	81.811	119.333	65.882	97.322	249.412	147.451
K _{d3}	218.283	240.63	306.22	186.0	233.725	131.325	105.098	15.686
K_{d4}	319.362	144.724	249.37	274.0	189.804	257.961	197.647	202.353
K _{d5}	328.551	300.00	378.031	279.333	160.0	268.514	225.882	291.765



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Table III. Maximum moduli and applicable angle ranges for the parameters generated for five different slopes. Nominal trajectory is derived from the gait synthesis method.

tilt angle of inclined surface	maximum modulus of eigenvalue (evaluated at the angle in the first column)	the range of tilt angle of (virtual) inclined surface
-10°	0.3176	-16° ~ -7°
-5°	0.3857	-7° ~ -3°
0°	0.1632	- 3°~3.5°
5°	0.5209	3.5°~7.5°
10°	0.3696	7.5°~12°

Table IV. Maximum moduli and applicable angle ranges for the parameters generated for eight different slopes. Nominal trajectory is derived from the link angle profile method.

tilt angle of inclined surface	maximum modulus of eigenvalue	the range of tilt angles of the inclined surface suitable for the same set of control parameters
-10°	0.6853	-11.0° ~ -9.0°
- 7°	0.7873	-9.0° ~ -8.0°
-6°	0.6778	-8.0° ~ -3.5°
-1°	0.6889	-3.5° ~ -1.0°
0°	0.6491	$-1.0^{\circ} \sim 1.0^{\circ}$
3°	0.3918	$1.0^{\circ} \sim 4.7^{\circ}$
6°	0.6762	4.7°~8.5°
10°	0.4575	9.75°~10.25°



Fig. 9. Height differences are -5 cm, -5 cm, 4.5 cm, 4.5 cm at 0.3 m, 1.05 m, 1.9 m and 2.5 m. During the four transition steps, control parameters for -10° , -10° , 10° , 10° inclined surfaces are used. Nominal trajectories are generated from the gait synthesis method.



Fig. 10. Height differences are 4.5 cm, -1.75 cm, 3.5 cm, -4.75 cm at 0.35 m, 1.05 m, 1.8 m and 2.45 m. During the four transition steps, control parameters for 10° , -5° , 5° , -10° inclined surfaces are used. Nominal trajectories are generated from the gait synthesis method.



Fig. 11. Height difference is 4.5 cm at 0.4 m, 0.6 m, 0.8 m, 1.0 m, 1.2 m, 1.4 m and 1.6 m. During the seven transition steps, control parameters for the 10° inclined surface are used. Nominal trajectories are generated from the gait synthesis method.



Fig. 12. Height differences are -5.95 cm, -5.95 cm, 5 cm, 5 cm at 0.55 m, 1.55 m, 2.7 m and 3.65 m. During these four transition steps, control parameters for -10° , -10° , 10° , 10° inclined surfaces are used. Nominal trajectories are generated from the link angle profile method.



Fig. 13. Height differences are -5.95 cm, 3 cm, -3 cm, 5 cm at 0.55 m, 1.6 m, 2.55 m and 3.6 m. During these four transition steps, control parameters for -10° , 5° , -5° , 10° inclined surfaces are used. Nominal trajectories are generated from the link angle profile method.



Fig. 14. Height differences are -5.95 cm at 0.55 m, 1.15 m, 1.95 m and 2.75 m. During these four transition steps, control parameters for the -10° inclined surface are used. Nominal trajectories are generated from the link angle profile method.



Fig. 15. Biped robot walks on surfaces with different slopes. The slope of the inclined surface is 10° . The nominal trajectory is generated by the gait synthesis method.



Fig. 16. Biped robot walks on surfaces with different slopes. The slope of the inclined surface is -10° . The nominal trajectory is generated by the gait synthesis method.

a new method for designing the controller. The method is based on a parameter search using the genetic algorithm and a stability evaluation using the linearized Poincaré map. Many sets of workable control parameters are generated by the genetic algorithm, and the best one is selected according to control robustness. The use of the genetic algorithm eliminates most tedious analysis. Such a technique is well applicable to the design for locomotion on unstructured surfaces, for which analytical approaches are difficult.

Using the method, control parameters for different slopes can be obtained, selected, and stored into a database. During the control, control parameters suitable for the current surface slope are retrieved and the trajectory for a level surface is modified according to the surface slope. A 5-link biped model was used for simulation. Two possible ways for generating the nominal trajectory were used in design and control simulation. Simulation results showed that the biped robot was able to walk on different types of terrain with the height variation up to about 6cm and the slope variation up to 10°. These variations are considered large in biped robot locomotion control.

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