Journal of British Studies 56 (July 2017): 453–482. doi:10.1017/jbr.2017.59

"Set Them to the Cyphering Schoole": Reading, Writing, and Arithmetical Education, circa 1540–1700

Jessica Otis

Abstract During the late sixteenth and seventeenth centuries, English men and women replaced their existing oral and object-based arithmetical practices with literate practices based on Arabic numerals. While the adoption of Arabic numerals was incentivized by continental commercial developments, this article argues that England's increasing literacy rates and the development of vernacular arithmetic textbooks enabled changing arithmetical practices. By exploring the qualities of printed books, analyzing marginalia in arithmetic textbooks, and examining changing educational advertisements and curricula over time, we can demonstrate the importance of literacy and literature to early modern arithmetical education.

In late 1631, a fifteen-year-old boy named John Wallis came home from grammar school and "there found that a younger Brother of mine (in Order to a Trade) had for about 3 months, been learning (as they call'd it) to *Write and Cipher*, or *Cast account*" with Arabic numerals. Being curious, he convinced his brother to spend the next two weeks teaching him "the *Practical* part of *Common Arithmetick*," which he "shewed me by steps, in the same method that he had learned them: and I had wrought over all the *Examples* which he before had done in his book." After leaving for university, Wallis continued his education through textbooks, studying "at spare hours; as books of *Arithmetick*, or others *Mathematical* fel occasionally in my way." This education was self-directed because, as he later recalled in his autobiography, "Mathematicks (at that time, with us) were scarce looked upon as *Accademical* studies, but rather *Mechanical*; as the business of *Traders, Merchants, Seamen, Carpenters, Surveyors of Lands*, or the like … [and were] more cultivated in *London* than in the Universities."[1]

Jessica Otis is digital humanities specialist in the University Libraries and assistant professor of history at Carnegie Mellon University. She would like to thank Paul D. Halliday, the Junior Faculty Writing Group in Virginia Tech's Department of History, and the anonymous reviewers of the *Journal of British Studies* for their generous feedback. She would also like to thank all the librarians and archivists who made this research possible.

[1] John Fauvel and Jeremy Gray, eds., *The History of Mathematics: A Reader* (New York, 1987), 316–17; Domenico Bertoloni Meli, "Wallis, John (1616–1703), *mathematician and cryptographer*," *Oxford Dictionary of National Biography* (hereafter *ODNB*) (Oxford, 2004), http://www.oxforddnb.com/view/article/28572. The Wallis family's choice to send the elder son to university and to set up younger sons to earn their livings through trade was common among early modern gentry. Patrick Wallis and Cliff Webb, "The Education and Training of Gentry Sons in Early Modern England," *Social History* 36, no. 1 (February 2011): 20–46.

Wallis intended his autobiographical statements both to castigate the grammar schools and universities for the lack of mathematics in their humanist curricula as well as to extol his own virtues as a self-directed learner.[2] In doing so, however, he inadvertently opened a window to the educational practices of a wider, less elite segment of the English population: those merchants, craftsmen, and other account-keepers learning outside England's Latin-based schools. Far from being extraordinary, Wallis's arithmetical education reflects a typical seventeenth-century learning experience: commercial interests inspired the introduction of Arabic-numeral arithmetic to the Wallis household through the medium of vernacular, printed books, which the Wallis brothers both read out of and wrote back in to. Significantly, for both the Wallis brothers, learning Arabic-numeral arithmetic was a fundamentally literate experience.

Until the sixteenth century, literacy and numeracy were independent skills in England, where men and women could use a variety of symbolic systems to record numerical information but primarily employed material objects to perform arithmetic. Although Arabic numerals had been known in Christian Europe since the tenth century, and in England since the twelfth century, they had not been immediately adopted in part because interpreting and calculating with them required the same skills as interpreting and reproducing alphabetic symbols—that is, a form of literacy. However, European commercial expansion incentivized literate merchants, tradesmen, and account keepers to adopt more sophisticated Arabic-numeral-based accounting practices, beginning with Italian abacists and merchants in the thirteenth and fourteenth centuries.[3] English men and women's adoption of Arabic numerals in the late sixteenth and seventeenth centuries was similarly motivated by commercial pressures and enabled by rising literacy rates.[4] This shift from oral and object-based to literate technologies of knowledge marked a major transformation not only in how people worked with numbers but also in what they imagined they might do with them.[5]

[2] For an analysis of the hyperbole in Wallis's autobiography, see Mordechai Feingold, *The Mathematicians' Apprenticeship: Science, Universities and Society in England, 1560–1640* (Cambridge, 1984), 86–88.

[3] Warren Van Egmond, *Practical Mathematics in the Italian Renaissance: A Catalog of Italian Abbacus Manuscripts and Printed Books to 1600* (Florence, 1980), 9–12. The canonical work on late medieval commercial expansion is Robert S. Lopez, *The Commercial Revolution of the Middle Ages, 950–1350* (Englewood Cliffs, 1971). For more recent sources, see, for example, David Ormrod, *The Rise of Commercial Empires: England and the Netherlands in the Age of Mercantilism, 1650–1770* (Cambridge, 2003); and Sheilagh Ogilvie, *Institutions and European Trade: Merchant Guilds, 1000–1800* (Cambridge, 2011). For more on late medieval Italian accounting practices, see Robert H. Parker and Basil S. Yamey's collection of post-Second World War British essays, especially Geoffrey A. Lee, "The Oldest European Account Book: A Florentine Bank Ledger of 1211"; W. T. Baxter, "Early Accounting: The Tally and the Checker-board"; Christopher W. Nobes, "The Gallerani Account Book of 1305–1308"; and Basil S. Yamey, "Balancing and Closing the Ledger: Italian Practice, 1300–1600," in *Accounting History: Some British Contributions*, ed. Robert H. Parker and Basil S. Yamey (Oxford, 1994), 160–267. On accounting history more generally, see also Ananias Charles Littleton and Basil S. Yamey, *Studies in the History of Accounting* (New York, 1978); and David Oldroyd and Alisdair Dobie, "Bookkeeping," in *The Routledge Companion to Accounting History*, ed. John Richard Edwards and Stephen P. Walker (New York, 2009), 95–119.

[4] Deborah E. Harkness, *The Jewel House: Elizabethan London and the Scientific Revolution* (New Haven, 2007), 101.

[5] The general shift from oral to literate technologies of knowledge has been well explored by scholars such as Peter Burke, *Popular Culture in Early Modern Europe* (New York, 1978); Walter Ong, *Orality and Literacy: The Technologizing of the Word* (New York, 1982); and Adam Fox, *Oral and Literate*

In this article, I argue that rising literacy rates and the growth of the English book trade—in particular, the introduction of vernacular arithmetic textbooks—facilitated these changing arithmetical practices.[6] Textbooks were well suited to teaching written, rather than object-based, methods of arithmetic and enabled individual educators to reach wider audiences, at more affordable prices, than face-to-face methods of instruction. By exploring the qualities of printed books, analyzing marginalia, and examining curricular shifts over the course of the seventeenth century, I demonstrate the importance of literacy and didactic literature to early modern arithmetical education.

EARLY MODERN LITERACY AND NUMERACY

The term *numeracy* is still enough of a neologism that it is worth beginning with definitions, which can vary depending on the context in which the term is deployed. The *Oxford English Dictionary* (*OED*) defines *numeracy* as "the quality or state of being numerate; ability with or knowledge of numbers," which is in some sense a trivial definition. Every human is born with an innate number sense and the number words within the English language guaranteed that there were no innumerates in early modern England. However, the *OED* also defines *numerate* as being "competent in the basic principles of mathematics, esp. arithmetic; able to understand and work with numbers."[7] This shift from an unspecified "knowledge" of number to competency in arithmetic provides a nontrivial, examinable behavioral marker for numeracy that would have also been recognizable in the early modern period.

Prior to the wholesale adoption of Arabic numerals, most English men and women relied on counting boards—checkered cloths or tables, from which the exchequer famously derived its name—to perform arithmetical calculations.[8] The act of calculation was predicated upon the manipulation of physical objects, specifically circular

Culture in England, 1500–1700 (Oxford, 2000). For more on the specific shift from oral and object-based to literate forms of numeracy—and the consequences of these changes—see David Glimp and Michelle R. Warren, *Arts of Calculation: Quantifying Thought in Early Modern Europe* (New York, 2004); and Jessica Otis, "By the Numbers: Understanding the World in Early Modern England" (PhD diss., University of Virginia, 2013). For an analysis of numeracy and systemic changes more generally, see Stephen Chrisomalis, *Numerical Notation: A Comparative History* (Cambridge, 2010) as well as the canonical study of George Francis Hill, *The Development of Arabic Numerals in Europe Exhibited in Sixty-Four Tables* (Oxford, 1915).

[6] For more on print and the dissemination of knowledge in England, see, for example, Elizabeth Eisenstein, *The Printing Press as an Agent of Change* (Cambridge, 1979); Adrian Johns, *The Nature of the Book: Print and Knowledge in the Making* (Chicago, 1998); and Jason Peacey, *Print and Public Politics in the English Revolution* (Cambridge, 2013).

[7] *Oxford English Dictionary* (hereafter *OED*), 2nd ed., s.v.v., "numeracy, *n.*" and "numerate, *adj.2*"; Brian Butterworth, *What Counts: How Every Brain Is Hardwired for Math* (New York, 1999); Stanislas Dehaene, *The Number Sense: How the Mind Creates Mathematics* (New York, 1997); Keith Thomas, "Numeracy in Early Modern England: The Prothero Lecture, Read 2 July 1986," in *Transactions of the Royal Historical Society*, 5th ser., no. 37 (London, 1987), 104–5.

[8] Thomas, "Numeracy," 120. For more on the popular numerical practices of early modern England, see also Patricia Cline Cohen, *A Calculating People: The Spread of Numeracy in Early America* (Chicago, 1982); Geoffrey Howard, *A History of Mathematics Education in England* (Cambridge, 1982); John Denniss, "Learning Arithmetic: Textbooks and Their Users in England 1500–1900" in *The Oxford Handbook of*

https://doi.org/10.1017/jbr.2017.59 Published online by Cambridge University Press

objects called counters or jetons, across these boards. It was also conceptually and practically distinct from the act of recording, which could be done through a variety of other symbolic systems. People who had no need to record the results of their calculations permanently, or who chose to use object-based symbolic systems such as the ubiquitous wooden tally sticks, could thus be simultaneously numerate and illiterate. Even people who used literate symbolic systems to record numerical information were still reliant on the counting board for their calculations, as neither number words nor Roman numerals enabled anything other than the most trivial of addition and subtraction. While it was possible for a single individual to minimize his or her contact with counting boards by consulting pre-made arithmetical tables and "ready reckoner" books, someone still had to perform the original calculations.[9] Thus, at the beginning of the early modern period in England, there was no inherent connection between literacy and numeracy, which instead relied on one's facility with material objects.

However, the ability to write—whether with pen and paper, dust boards, wax tablets, carving, or even needle and thread—was a prerequisite skill to using Arabic numerals.[10] This symbolic system enabled the simultaneous recording of numerical information and performing of arithmetical calculations, and it became increasingly widespread in the late sixteenth and seventeenth centuries. Peter Wardley and Pauline White's collaborative study of probate inventories, which focused on the adoption of Arabic numerals solely for calculation, found that most people in their sample adopted Arabic-numeral arithmetic between 1590 and 1650, with the main transition period beginning in the 1620s and 1630s and continuing through to 1650. The Norfolk probate inventories showed some of the earliest uses of Arabic numeral sums, in 1584, while most of the places surveyed had earliest adoption dates between 1607 and 1612. This corresponded well with Wardley's earlier research on Bristol and West Cornwell, where he found a slightly earlier adoption period of 1570 to 1630.[11] This timing is not unique to England; across the English Channel, Antwerp merchants also began using Arabic numerals in their

the History of Mathematics, ed. Eleanor Robson and Jacqueline Stedall (Oxford, 2009): 448–67; Harkness, *Jewel House*; and Otis, "By the Numbers," 45.

9 For an example of a multiplication table included in a book with wax tablets, see Peter Stallybrass et al., "Hamlet's Tables and the Technologies of Writing in Renaissance England," *Shakespeare Quarterly* 55, no. 4 (Winter 2004): 379–419, at 397.

10 Arabic and Italian mathematicians used the dust board for Arabic-numeral arithmetic until pen and paper began to supplant it in the fourteenth century. Warren Van Egmond, "The Commercial Revolution and the Beginnings of Western Mathematics" (PhD diss., Indiana University, 1976), 343. Wax tablets continued to be produced and used for a variety of purposes, including calculation, throughout the early modern period. Stallybrass et al., "Hamlet's Tables," 402–3. While women did not perform calculations with their embroidery, they did reproduce letters and numbers on their samplers, and many surviving instances of women's handwriting show the clear influence of embroidered letters. Eleanor Hubbard, "Reading, Writing, and Initialing: Female Literacy in Early Modern London," *Journal of British Studies* 54, no. 3 (July 2015): 553–77, at 565.

11 Peter Wardley and Pauline White, "The Arithmeticke Project: A Collaborative Research Study of the Diffusion of Hindu-Arabic Numerals," *Family and Community History* 6, no. 1 (May 2003): 5–17, at 6. More extensive data and reports from the Arithmeticke Project are available online at http://www.rw007a7896.pwp.blueyonder.co.uk/

accounts during the late 1580s.[12] Overall, these dates are also consistent with broader trends in Arabic-numeral adoption for both recording and calculation, which indicated that account keepers began adopting Arabic numerals to record information about dates in the early to mid-sixteenth century; to record information about other quantities in the mid- to late sixteenth century; and to perform calculations at the turn of the seventeenth century.[13]

At the end of the seventeenth century, Arabic numerals were still only one symbolic system among many, but they had also become the predominant symbolic system in England.[14] Indeed, they had become so synonymous with the concept of calculation—particularly calculations associated with commerce and account keeping—that the physician John Arbuthnot could conflate the two while declaring that it "would go near to ruine the Trade of the Nation, were the easy practice of *Arithmetick* abolished: for example, were the Merchants and Tradesmen oblig'd to make use of no other than the *Roman* way of notation by Letters" in conjunction with nonliterate methods of calculation. He even went so far as to equate the use of Arabic numerals with civilization itself: "the Nations, that want it, are altogether barbarous, as some *Americans*, who can hardly reckon above twenty."[15] While older forms of numeration and calculation continued, Arabic numerals were the new standard by which eighteenth-century numeracy would be judged—a standard that had become inherently literate.[16]

Although some scholars write as if the widespread adoption of Arabic numeral arithmetic was inevitable, delayed only by the obtuseness of the English population, the actual timing of Arabic-numeral adoption in England suggests that the literacy prerequisite created a prohibitively high barrier during the thirteenth through the fifteenth centuries.[17] The sixteenth and seventeenth centuries, by contrast, were a period of rising literacy rates throughout English society. David Cressy used signature counting—a method that conflates the technically distinct abilities of reading and writing, withholding the status of "literate" from those who could read but not write—to estimate overall male literacy rates of only 10 percent in 1500. However, this rose to 30 percent by 1600, the same period during which people increasingly began adopting Arabic numerals for calculation, and reached 50 percent by 1700, when Arbuthnot hyperbolically associated Arabic-numeral

[12] Ad Meskins, "Mathematics Education in Late Sixteenth-Century Antwerp," *Annals of Science* 53, no. 2 (1996): 137–155, at 154.

[13] Otis, "By the Numbers," 57–66.

[14] High-profile examples of the survival of other symbolic systems include the use of tallies in the British exchequer until the beginning of the nineteenth century as well as the continuing use of Roman numerals in regnal titles and to paginate the prefaces of printed books.

[15] John Arbuthnot, *An essay on the usefulness of mathematical learning, in a letter from a gentleman in the city to his friend in Oxford* (Oxford, 1701), 27.

[16] Thomas, "Numeracy," 119. For more on eighteenth-century popular numeracy, see Benjamin Wardhaugh, *Poor Robin's Prophecies: A Curious Almanac, and the Everyday Mathematics of Georgian Britain* (Oxford, 2012).

[17] Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, trans. Paul Bronner (Cambridge, MA, 1969), 422; Georges Ifrah, *From One to Zero: A Universal History of Numbers*, trans. Lowell Bair (New York, 1985), 481; Eugene Smith and Louis Charles Karpinski, *The Hindu-Arabic Numerals* (Boston, 1911), 132.

arithmetic with civilization.[18] Urban areas, where merchants and tradesmen congregated, tended to be more literate than rural areas, with London and Bristol both having closer to 65 percent literacy in the mid-seventeenth century.[19] But in Wales, less than 20 percent of the population could sign their names in the late 1640s, leading to the beginning of a concentrated popular literacy drive in 1650. In Scotland, by contrast, seventeenth-century literacy rates were comparable to those in northern England, with a positive correlation between literacy and high social or economic status.[20]

While signature counting is a generally recognized method of judging literacy rates, scholars have raised significant concerns about its use in the early modern period, when reading and writing were taught sequentially rather than concurrently. Thus, these numbers must dramatically underestimate the number of people who had the ability to read, particularly those who had the ability to read printed black letter—the "type for the common people," which was given to convicted criminals attempting to prove their literacy in order to claim benefit of clergy—as opposed to roman type or various styles of handwriting.[21] Furthermore, judging literacy based on the ability to sign one's full name, rather than initials or a partial name, discounts as illiterate those people who had the ability to write but not fluently. The extent of the possible underestimate can be seen by looking at Cressy's calculations for female literacy, which persistently lagged 10 to 20 percent behind their male counterparts' rates.[22] By contrast, Eleanor Hubbard's investigation of female literacy, which allowed both signatures and initials to stand as proxies for literacy, suggested that between 1570 and 1640 London's native female population had a 36 percent literacy rate and that a further 22 percent of its female immigrants were also literate. Significantly, Hubbard's calculations also show the same rising trajectory as Cressy's, with women born before 1560 having a 16 percent literacy rate, rising to 29 percent for those born in 1600 and beyond.[23] Therefore, it is important to acknowledge that any specific set of literacy-rate calculations likely underestimates those with some ability to read, even as they provide a realistic approximation for the increasing percentage of the population with the prerequisite writing skills to even consider adopting Arabic-numeral arithmetic.

VERNACULAR ARITHMETIC TEXTBOOKS

European knowledge of Arabic-numeral arithmetic originated in Italy's late medieval abacus schools and manuscript instructional books, called abbaci or *libri d'abbaco*.

[18] David Cressy, *Literacy and the Social Order: Reading and Writing in Tudor and Stuart England* (Cambridge, 1980), 142–56.

[19] Jonathan Barry, "Popular Culture in Seventeenth-Century Bristol," in *Popular Culture in Seventeenth-Century England*, ed. Barry Reay (London, 1985), 62; Fox, *Oral and Literate Culture*, 18.

[20] Heidi Brayman Hackel, "Popular Literacy and Society," in *The Oxford History of Popular Print Culture*, vol. 1, *Cheap Print in Britain and Ireland to 1660*, ed. Joad Raymond (Oxford, 2011), 88–100, at 93.

[21] For more on the multiplicity of early modern literacies, see Keith Thomas, "The Meaning of Literacy in Early Modern England," in *The Written Word: Literacy in Transition, Wolfson College Lectures 1985*, ed. Gerd Baumann (Oxford, 1986), 97–131, at 99.

[22] Cressy, *Literacy*, 142–56.

[23] Hubbard, "Reading, Writing, and Initialing," 568–69.

Thirteenth- and early fourteenth-century Italian mathematicians modified the high medieval rules of the Arabic-numeral system to better suit the writing materials favored by Italy's increasingly sedentary merchants and then began to teach the system to merchants through classroom lessons, which were soon also gathered into manuscripts for later reference. Arabic-numeral arithmetic proved particularly well suited to being summarized in written media, as the calculations were intended to be written in the first place. While classroom instruction predated manuscript production, the one closely followed the other; the earliest record of an abacus school comes from 1284, while the oldest of the vernacular *libri d'abbaco* still extant was written around 1290.[24] By the fifteenth century, Italians almost universally used Arabic numerals for their commercial transactions. While merchants' dealings with Arabic merchants and the increasing complexity of the late medieval Italian economy must have provided the incentive for learning the new numbers, it was the abacus schoolmasters and *libri d'abbaco* that provided urban Italians with the means to do so.[25]

As printing presses began to spread through Europe, these new arithmetical rules and pedagogical structures moved from manuscript to print with the Treviso Arithmetic of 1478 and Luca Pacioli's more famous *Summa de Arithmetica, Geometria, Proportioni et Proportionalita* in 1494. It was at this point that *libri d'abbaco* appear to have shifted from reference books to teaching books, primarily for those who were self-educating rather than those who were paying for classroom instruction.[26] Arithmetic textbooks also began to be published throughout the rest of Europe; the first German arithmetic textbook (1482) predated the *Summa*, while the first French and Spanish arithmetic textbooks were published in 1512, and the first Portuguese arithmetic textbook appeared in 1519.[27] These printed textbooks must have constituted a dramatic increase in the number of students that a schoolmaster-turned-author could potentially reach. Some contemporary sources claimed that Italian schoolmasters taught up to two hundred students at a time, while Dutch sources cite schools of up to four hundred students. However, other sources estimate concurrent enrollments of twenty-five to forty students in Italy and fifty students in Antwerp, with one schoolmaster teaching about nine hundred students during his forty-five-year career.[28] Therefore, with even a single print run of one thousand books, the schoolmaster could effectively double his students or, if he chose to use them in conjunction with his classroom activities, produce more textbooks than his face-to-face students could use in his lifetime. Well-written textbooks could exponentially expand a schoolmaster's audience, profiting the schoolmaster and providing the self-guided student instruction at a fraction of the classroom cost.

In England, the sixteenth and seventeenth centuries saw the increasing production of printed books as a widespread, literate method of communication. More than

[24] Van Egmond, *Practical Mathematics*, 6–7.

[25] Van Egmond, "Commercial Revolution," 72, 320–22, 341, 596; John V. Tucker, "Data, Computation and the Tudor Knowledge Economy," in *Robert Recorde: The Life and Times of a Tudor Mathematician*, ed. Gareth Roberts and Fenny Smith (Cardiff, 2012), 165–88, at 171–72.

[26] Van Egmond, *Practical Mathematics*, 30–31.

[27] Frank J. Swetz, *Capitalism and Arithmetic: The New Math of the 15th Century Including the Full Text of the Treviso Arithmetic of 1478*, trans. David Eugene Smith (La Salle, 1987), 24.

[28] Van Egmond, "Commercial Revolution," 105–6; Meskins, "Mathematics Education," 140.

https://doi.org/10.1017/jbr.2017.59 Published online by Cambridge University Press

125,000 titles survive from the early modern period, with annual publication rising from at least 475 titles in the first decade of the sixteenth century to more than 3,935 titles in the first decade of the seventeenth century, to 18,247 titles during the chaos of the 1640s.[29] Due to questions about the survival of texts, these figures should be seen as a lower bound for the print production in each of these time spans, rather than an accurate accounting. D. F. McKenzie argues that we must assume a significantly greater rate of loss of texts during the late sixteenth and early seventeenth centuries, based on his quantitative analysis of London printing presses and the labor force attached to them.[30] The rate of loss for cheap print and basic educational works, such as almanacs and ABCs, is particularly high, with hundreds of thousands of copies known to have been printed during the early modern period but only a few books still extant.[31]

Most of the printing industry was concentrated in London, the home of the Stationers' Company and the center of English book production. London booksellers abounded and they carried large inventories of diverse titles. By the 1660s, George Thomason had collected 14,942 pamphlets and 7,216 newspapers, while Charles Tias had over 90,000 octavo and quarto books in his shop, house, and warehouse.[32] There was a considerable trade in books outside of London as well. In 1585, Roger Ward of Shrewsbury had an inventory of 2,500 books, including 546 different titles; in 1615, Michael Harte of Exeter died with a stock of more than 4,500 items; and in 1644, John Awdley of Hull had 832 different titles for sale.[33] Peddlers also carried smaller stocks of printed broadsides and books with them to sell at inns throughout the kingdom.[34]

The increasing availability of printed books—and the increasing percentage of the population able to read them—enabled the early modern production of more than sixty unique titles related to basic arithmetic education. Approximately half of these titles were reprinted at least once, and together they went through nearly three hundred still-extant editions in the sixteenth and seventeenth centuries. Only three different arithmetic textbooks, in twenty editions, survive from the first three quarters of the sixteenth century, but their rate of production began to increase in the 1570s and continued to grow throughout the seventeenth century. Six of these titles were bestsellers reprinted twenty or more times, including four

[29] John Barnard and Maureen Bell, "Appendix 1: Statistical Tables," in *The Cambridge History of the Book in Britain*, vol. 4, *1557–1695*, ed. John Barnard and D. F. McKenzie (Cambridge, 2002), 779–85.

[30] D. F. McKenzie, "Printing and Publishing, 1557–1700: Constraints on the London Book Trade," in Barnard and McKenzie, eds., *Cambridge History*, 4:553–67, at 556–58.

[31] R. C. Simmons, "ABCs, Almanacs, Ballads, Chapbooks, Popular Piety and Textbooks," in Barnard and McKenzie, eds., *Cambridge History*, 4:504–13, at 504. For more on early modern almanacs, see Bernard Capp, *Astrology and the Popular Press: English Almanacs, 1500–1800* (Ithaca, 1979); Timothy Feist, *The Stationers' Voice: The English Almanac Trade in the Early Eighteenth Century* (Philadelphia, 2005); Alison A. Chapman, "Marking Time: Astrology, Almanacs, and English Protestantism," *Renaissance Quarterly* 60, no. 4 (Winter 2007): 1257–90, at 1269–70; and Louise Hill Curth, *English Almanacs, Astrology and Popular Medicine, 1550–1700* (Manchester, 2007).

[32] Fox, *Oral and Literate Culture*, 16; Margaret Spufford, *Small Books and Pleasant Histories: Popular Fiction and Its Readership in Seventeenth Century England* (London, 1981), 101.

[33] Thomas, "Meaning of Literacy," 112; Peacey, *Print and Public Politics*, 59; Fox, *Oral and Literate Culture*, 15.

[34] Tessa Watt, *Cheap Print and Popular Piety, 1550–1640* (Cambridge, 1991), 6; Spufford, *Small Books*, 66.

seventeenth-century titles that continued to be reprinted into the eighteenth century.[35] Judging by reprint rates, demand for arithmetic books far outstripped demand for more advanced mathematical books on geometry, astronomy, navigation, and surveying; a similar pattern held for the Antwerp book market.[36] Contemporary complaints about "learned bookes [that] can not bee vnderstoode of the common sorte" generally referred to these more advanced books, such as the geometry books of Euclid, Robert Recorde, and Thomas Digges, as well as Leonard Digges's book on surveying.[37] It is likely that these specialized skills appealed to only a small subset of arithmetic students or were more often acquired through hands-on training and observation than textbooks.[38]

The first English arithmetic textbook, Cuthbert Tunstall's *De Arte Supputandi*, was written in Latin, but it was soon followed by vernacular competitors, beginning with the anonymously authored *An Introduction for to Learn to Reckon with the Pen & with the Counters*, which went through at least nine early modern editions. Unlike Italian *libri d'abbaco*, which focused exclusively on teaching Arabic numeral arithmetic, *An Introduction for to Learn to Reckon* taught both the new Arabic numeral arithmetic—"with the Pen"—alongside the established counting board arithmetic—"with the Counters"—despite the unwieldiness of needing to use multiple large printed images to portray even simple counting-board operations.[39] This textbook was thus aimed at beginners with no experience in either method of arithmetic or knowledge of the potential need for both. A few Dutch arithmetic textbooks also show this multiplicity, and it is possible that *An Introduction for to Learn to Reckon* was a direct translation of a Continental arithmetic textbook, as many early modern English schoolbooks drew from Continental sources.[40] Historian Jean Vanes claims that the printer John Herford produced this work from French and Dutch originals; however, while the use of French and Dutch mathematical terms, money, and geography suggests that the tract did have a Continental origin, Herford only printed the 1546 edition.[41] A fragment from 1526—which is possibly

[35] *English Short Title Catalog* (hereafter *ESTC*), http://estc.bl.uk. Exact numbers are not possible to calculate due to the loss of sources over time and occasional difficulties distinguishing between editions printed in the same year. At least an additional seventy editions of basic accounting books were also produced during this period. Basil S. Yamey, H. C. Edey, and Hugh W. Thomson, *Accounting in England and Scotland, 1543–1800: Double Entry in Exposition and Practice* (London, 1963), 202–8. For more on mathematical publishing in general, see Harkness, *Jewel House*, 104. For the canonical work on early modern English mathematicians and their publications, see E. G. R. Taylor, *The Mathematical Practitioners of Tudor and Stuart England* (Cambridge, 1954).

[36] Harkness, *Jewel House*, 104–5; Meskins, "Mathematics Education," 152; Simon Schaffer, "Science," in Raymond, ed., *Oxford History*, 1:398–416, at 399.

[37] Edward Worsop, *A Discouerie of sundrie errours and faults daily committed by Lande-Meaters, ignorant of Arithmetike and Geometrie* (London, 1582), A2v. While Keith Thomas's reference to this passage carries with it a strong implication that arithmetic is too difficult to learn, the context of the full list makes it clear that Worsop is referring to Recorde's far less heralded geometry textbook, *The Pathway to Knowledge*. Thomas, "Numeracy," 118.

[38] Schaffer, "Science," 399–400.

[39] Van Egmond, *Practical Mathematics*, 6; *An introduction for to lerne to recken with the pen or with the counters* (London, 1539), A1r.

[40] Meskins, "Mathematics Education," 152; Simmons, "ABCs, Almanacs, Ballads," 505.

[41] Jean Vanes, *Education and Apprenticeship in Sixteenth-Century Bristol* (Bristol, 1982), 21–22; Travis D. Williams, "The Earliest English Printed Arithmetic Books," *The Library: The Transactions of the Bibliographical Society*, 7th ser., 13, no. 2 (2012): 164–84, at 175.

but not definitively the same work—was printed by Rychard Fakes and claimed to have been "Translated out of Fre*n*she in to Englyshe not without grete labour," while Nycolas Bourman printed a 1539 edition.[42] On the frontispiece of a 1581 edition, held by the library of Worcester College, Oxford, the annotator William Clarke notes that the authorship is "ascribed to W. Awdley" but that this is most probably a mistaken reference to the printer of two earlier editions, John Awdley.[43]

These early translations were swiftly overtaken in popularity by two vernacular textbooks with definite English origins: Robert Recorde's *The Ground of Artes*, first published in 1543, and Humfrey Baker's *The Wellspring of Sciences*, of 1562. Together, these two textbooks went through about seventy editions and only went out of print in the late seventeenth century, eclipsed by a new style of arithmetic textbooks that also included sections on decimal fractions, logarithms, and algebra.[44] However, marginalia in surviving editions of *The Ground of Artes* and *The Wellspring of Sciences* show that they were often resold or passed down within families.[45] For example, one 1623 edition of Recorde's *The Ground of Artes* passed through the hands of at least a half dozen annotators, including "John Griffiths" and "Mary Griffiths: his Daaghter."[46] Thus, these textbooks continued to be used throughout the eighteenth century, particularly by female students who had less access to institution-based education and who were more likely to be taught arithmetic at home—if they were taught at all.[47] Recorde's books were so popular that his name became a byword for arithmetic textbooks in the seventeenth century. As grammar schoolmaster John Brinsley explained to his readers, if any of their students wished to learn more than numeration with Arabic numerals, "you must seeke *Records* Arithmetique, or other like Authors, and set them to the Cyphering schoole."[48]

As on the Continent, English textbooks emerged in the wake of face-to-face instructional activities. Recorde and Baker had both previously worked as mathematical tutors and drew on their personal teaching experiences to create a literary equivalent to the tutor—the arithmetical textbook.[49] This debt is most obvious in Recorde's *The Ground of Artes*, which was organized in the form of a dialogue between a master mathematician and a scholar, his pupil. The interactions between

[42] *The arte and science of arismetique* (London, 1526), 1r.

[43] For further analysis of the influences, structures, and authorships of these early arithmetic textbooks, see Williams, "Earliest English Printed," 164–84.

[44] These new arithmetics included bestsellers by James Hodder, Edmund Wingate, and especially Edward Cocker.

[45] Didactic books, in general, often had long "afterlives" in the early modern period. Natasha Glaisyer, "Popular Didactic Literature," in Raymond, ed., *Oxford History*, 1:510–19, at 514.

[46] 21793, fol. 2v, Huntington Library, San Marino (hereafter HEH).

[47] Harkness, *Jewel House*, 118.

[48] John Brinsley, *Ludus Literarius: or, the Grammar Schoole; shewing how to proceede from the first entrance into learning, to the highest perfection required in the Grammar Schooles* (London, 1627), 26.

[49] Swetz, *Capitalism and Arithmetic*, 24. For more on Recorde's career as a physician and royal administrator, see Jack Williams, *Robert Recorde: Tudor Polymath, Expositor and Practitioner of Computation* (London, 2011); idem, "The Lives and Works of Robert Recorde," in Roberts and Smith, eds., *Robert Recorde*, 7–24; Taylor, *Mathematical Practitioners*, 15, 167, 313; and Stephen Johnston, "Recorde, Robert (c. 1512–1558), *mathematician*," *ODNB*, http://www.oxforddnb.com/view/article/23241. For more on Baker, see Taylor, *Mathematical Practitioners*, 172, 318; and Anita McConnell, "Baker, Humphrey (*fl*. 1557–1574), *writer on astrology and arithmetic*," *ODNB*, http://www.oxforddnb.com/view/article/1123.

them deliberately mimicked an oral tutoring session, blurring the line between the oral and the literate.[50] The scholar challenged the master's authority, made calculation errors that the master had to correct, and asked questions about complicated ideas such as the Arabic-numeral place-value system. In the original dedication of *The Ground of Artes*—made into a preface in later editions—Recorde explicitly explained his hope that the book could replace professional, face-to-face instruction for a widespread readership, particularly

> suche as shall lacke enstructers, for whose sake I haue so playnely set forthe the examples, as no boke (that I haue sene) hath done hetherto, which thyng shall be great ease to y^{e} rude reader. Therfore good M. Whalley, though this booke can be vnto your selfe but small ayde, yet shall it be some help unto your young chyldren, whose furtheraunce you desyre no lesse than your owne.[51]

In 1582, a new edition of the book justified itself by harkening back to Recorde's preface, claiming that "it is a Booke [that] hath done many a thousand good" but that now had accumulated errors so that "when a young beginner commeth to a confused or mistaken figure, it bringeth him into a wonderful discoragement and maze."[52] The assumed lack of an external instructor to help the young beginner at such moments of confusion was part of the justification for a new, corrected edition of the text.

After Recorde's death, *The Ground of Artes* continued to be published under the editorship of mathematicians with previous practical teaching experience, who used their work editing *The Ground of Artes* as a textual method to promote their skills to an audience of merchants, tradesmen, and other literate men and women. John Dee taught mathematics in London during the 1550s, while John Mellis and Robert Hartwell both advertised themselves as mathematical tutors. Mellis styled himself a "schoolmaster" and through the 1582 to 1610 editions he

> giueth intelligence: That if any bee minded to haue their children or seruants instructed or taught in this noble Arte of Arithmetick, or any briefe practise thereof. [His] method is such by long custome of teaching, that (God to friend) he will bring them (if their capacitie be any thing) to their desire therein in a short time.[53]

[50] For more on the use of dialogue in pedagogical texts, see Peter Burke, "The Renaissance Dialogue," *Renaissance Studies* 3, no. 1 (March 1989): 1–12; Ian Green, *The Christian's ABC: Catechisms and Catechizing in England, c. 1530–1740* (Oxford, 1996), 17–21; Schaffer, "Science," 401; and Glaisyer, "Popular Didactic Literature," 513. On the blurred line between oral and literate knowledge in early modern England more generally, see Fox, *Oral and Literate Culture*.

[51] Robert Recorde, *The Ground of Artes Teachyng the Worke and Practise of Arithmetike* (London, 1543), 8r–v. The work was officially dedicated to a landowner and royal official named Richard Whalley, who had at least five children of an age to be learning arithmetic at that time. Alan Bryson, "Whalley, Richard (1498/9–1583), *administrator*," *ODNB*, http://www.oxforddnb.com/view/article/29161.

[52] Robert Recorde, *The Grounde of Artes*, ed. John Mellis (London, 1582), A2v.

[53] Robert Recorde, *The Grounde of Artes*, ed. John Mellis and John Wade (London, 1610), Mm8r. For more on Mellis, see Thompson Cooper, "Mellis, John (*fl. c.* 1564–1588), *writer on arithmetic and bookkeeping*," *ODNB*, http://www.oxforddnb.com/view/article/18529.

After they gained this understanding of arithmetic, they might also learn accounts of debtor and creditor, a subject on which he also published an introductory textbook.[54] Hartwell styled himself a "Philomathematicus" and "Practitioner in the Mathematicks" who taught students in his school "In Fleetestreete, neere the Cundite, within Hanging Sword Court," which by 1632 had moved to "Great Saint Bartholomewes in the new street."[55] He advertised his arithmetical offerings in far more detail than Mellis: whole numbers and fractions, the extraction of roots, astronomical fractions, proportions, the rules of equation with algebra, and accounting. He also offered his students a variety of advanced lessons in the practical, real-world applications of mathematics that built upon basic arithmetical skills, including geometry, trigonometry, logarithms, navigation, dialing, and the use of mathematical instruments.[56]

Like Recorde, Humfrey Baker never explicitly advertised his tutoring services in his textbook, *The Wellspring of Sciences*. However, a surviving broadside that includes a detailed list of mathematical subjects Baker was able to teach, along with an example of his ability to reconcile merchant accounts, demonstrates the close correlation between his two instructional activities. The content and lesson order for both were largely similar: they began with numeration, addition, subtraction, multiplication, division, and progression in whole numbers before covering the same ground with fractions. They then continued with commerce-based applications of these operations, including the various rules of three, the rule of gain and loss, the rules of fellowship and partnership, the rules of interest, the rule of allegation, and the rule of suppositions or false positions. Both Baker's in-person arithmetical curriculum and the literate approximation of this curriculum provided by his textbook were geared toward commercial applications.

However, toward the end of his textbook and his tutoring advertisement, Baker showed his awareness of the instructional benefits and limits of the material forms of textbooks by expanding upon his standard catalog of arithmetical skills, choosing the subjects most appropriate to each informational medium. In a book that could be carried around in a pocket for reference, he included practical information on trading geared toward a merchant who might be traveling abroad and working with other merchants from across Europe. This consisted of rules about the trade of merchandise with tare and allowances, rules relating to bartering, examples of how to exchange money from one place to another, and information about weights and measures throughout Europe. By contrast, in person he taught more advanced mathematical subjects, which required more expert assistance to learn, as well as subjects geared toward tradesmen and those requiring manipulation of material instruments.[57] This included algebra; measurement of land and solid objects; the

[54] Mellis's accounting textbook was also a newly revised edition of an earlier book, *Profitable Treatyce* by Hugh Oldcastle, but in this case the original is no longer extant. Yamey, Edey, and Thomason, eds., *Accounting*, 155–59; Hugh Oldcastle, *A Briefe Instruction and maner how to keepe bookes of Accompts after the order of Debitor and Creditor, & as well for proper Accompts particle, &c.*, ed. John Mellis (London, 1588).

[55] Robert Recorde, *The Grounde of Artes*, ed. John Dee, John Mellis, and Robert Hartwell (London, 1623), 596. Robert Recorde, *The Ground of Artes*, ed. John Dee, John Mellis, and Robert Hartwell (London, 1632), 611.

[56] Recorde, *Grounde of Artes* (1623), Rr8v.

[57] Harkness, *Jewel House*, 133. For more on early modern instruments, see Jim Bennett, "Early Modern Mathematical Instruments," *Isis* 102, no. 4 (December 2011): 697–705.

"principles of geometry, to be applied to the ayde of all Mechanicall worke-men"; the use of the quadrant, geometrical square, cross-staff, astronomer's staff, astrolabe, and Ptolemy's ruler; and double-entry bookkeeping. While it was possible to teach instruments through books using paper cutouts, these were flimsy and harder to learn on than their more durable wooden or metal counterparts.[58]

The arithmetic textbooks of men like Recorde and Baker found a steady market during the late sixteenth and seventeenth centuries. Ownership marks in surviving textbooks indicate that these books were generally purchased within one to two years of publication. These textbooks were usually printed in the more portable and cheaper octavo or duodecimo formats. While scholars such as Joseph Dane and Alexandra Gillespie have pointed out that there is no material reason for smaller books to be cheaper than larger ones, extant prices indicate that duodecimo arithmetic textbooks were sold at lower prices than octavos, which were in turn cheaper than quartos.[59] Prices for seventeenth-century arithmetic textbook ranged from as little as sixpence to 4*s*, putting them roughly in line with the cost of other educational books (threepence to 3*s* 6*d*); the slightly higher cost likely derives from the unusual typography required to print mathematical books, which tended to be produced by a specialized subset of London printers.[60] Many new arithmetic textbooks in octavo were priced about 2*s*6*d* in the 1630s and 1640s, rising to 3*s* in the 1650s, 3*s*6*d* in the 1660s, and 4*s* in the 1670s and 1680s.[61] Octavo textbooks that were reprints of older texts tended to be priced slightly lower than those by newer authors. Robert Clavell's 1672 survey of London books priced relative newcomer *Wingate's Arithmetick* at 4*s*, while *Record's Arithmetick* and *Baker's Arithmetick* were 3*s* and 2*s*6*d* respectively.[62] Textbooks in duodecimo format and used copies of octavo textbooks could be found for around one to two shillings, although one lucky student bragged of an even better deal and secured a witness to prove that he had come by the book honestly: "Six pence was y^e price of it. Ralph Sheldon his booke witnes to it John Peeke anno dominum 1646."[63] While even this deal was more expensive that what scholars term "cheap" print—such as two-pence annual almanacs—it was a bargain compared to the twenty shillings an already experienced accountant paid to have an expert teach him Arabic-numeral arithmetic in 1607.[64]

Successful titles were reprinted whenever the previous edition sold out, as Thomas Rooks—the stationer who reprinted *Hodder's Arithmetick*—proudly explained in 1667:

[58] Cab Lib g, Society of Antiquaries, London.

[59] Joseph A. Dane and Alexandra Gillespie, "The Myth of the Cheap Quarto," in *Tudor Books and Readers: Materiality and the Construction of Meaning*, ed. John N. King (Cambridge, 2010), 25–45; Robert Clavell, *A Catalogue of All the Books Printed in England since the Dreadful Fire of London in 1666, to the End of Michaelmas Term, 1672* (England, 1673), 43.

[60] Simmons, "ABCs, Almanacs, Ballads," 506; Adrian Johns, "Science and the Book," in Barnard and McKenzie, *Cambridge History*, 4:289.

[61] X513 W72p 1630, University of Illinois, Urbana-Champaign Special Collections, 3r; C.175.d.34, British Library (hereafter BL), 2v; 1607/500, BL, 1r; 8532.aa.24, BL, 2r; Adams.8.65.35, Cambridge University Library (hereafter CUL), 1r; Vet.A3, fol.1247, Bodleian Library, Oxford University, 2r; and 313383, HEH, 1r.

[62] Clavell, *Catalogue,* 43.

[63] 646/A, Wellcome Library, London; 8506.aa.34, BL; M.6.58, CUL; and C.115.n.43, fol. A2r, BL.

[64] Watt, *Cheap Print,* 263; Thomas, "Numeracy," 120.

https://doi.org/10.1017/jbr.2017.59 Published online by Cambridge University Press

> [I]n this bad time of trade of Books, in less than ten months, I sold of them 1550. There being very few of this kind yet set forth by an Teacher of this Art; and as I am informed, those which are extant, of very little use to the *Learner*, without the help of an expert *Tutor*. … Now I desire your candid ingenuity further to observe, that these Books of the third Edition are sold and out of print, and now I present you with a 4th Edition.[65]

First appearing in 1661, *Hodder's Arithmetick* was thus reprinted three times in six years—a rate at which it continued to be reprinted for the rest of the century and into the early 1700s, despite James Hodder's death sometime in the mid-1670s.[66] The longevity of these titles, most of which continued to be reprinted long after their original authors died, indicates that popular demand for vernacular arithmetic textbooks remained high throughout the seventeenth century.

Rooks's denigration of arithmetic textbooks that required the "help of an expert *Tutor*" may have been a marketing tactic to boost *Hodder's Arithmetick*'s sales, but it also might indicate that some students had genuine difficulty learning from textbooks. While many authors attempted "to render the Rules of those excellent Arts … so plain and obvious, as that they may be easily apprehended without the Assistance of a living Master," others thought of their textbooks as aids to classroom instruction, such as R. B., author of a one-off textbook whose title page declared it to have been "*Designed for the use of the Free Schoole at Thurlow in Suffolk*."[67] Others expected their textbooks to be used in a range of situations, especially toward the end of the seventeenth century, as they became part of a wider landscape of educational opportunities. Edward Cocker claimed to have addressed a fourfold audience in the 1678 edition of his *Cocker's Arithmetick*: already educated merchants, overworked schoolmasters, self-educating children, and—apparently—mathematical frauds. His description of how he expected his textbook to be used in classrooms was both bluntly honest and an intriguing window into how even the paid student might be expected to partially self-educate. He expected his textbook to serve "most excellent Professors … of this noble Science … as a monitor to instruct your young *Tyroes*, and thereby take occasion to reserve your precious moments, which might be exhausted that way, for your important affairs"—instructing their students in elementary arithmetic apparently no longer being one of them. Given the high rate at which *Cocker's Arithmetick* was reprinted for the next 150 years, Cocker must have been correct in predicting at least some of the uses for his text, although "the pretended Numerists of this vapouring age" were probably not as receptive to his text as schoolmasters and self-educators.[68]

ANALYZING ARITHMETICAL MARGINALIA

While arithmetic textbooks were published—and purchased—extensively during the late sixteenth and seventeenth centuries, the extent to which these textbooks aided in the learning process cannot be determined entirely from the fact of book

[65] James Hodder, *Hodder's Arithmetick: or, That Necessary Art Made Most Easie* (London, 1667), a4v–5r.

[66] Ruth Wallis, "Hodder, James (*fl.* 1659–1673), *arithmetician*," *ODNB*, http://www.oxforddnb.com/view/article/13416.

[67] John Mayne, *Arithmetick: Vulgar, Decimal, & Algebraical* (London, 1675), A3r; R. B., *An Idea of Arithmetick* (London, 1655), A1r.

[68] Edward Cocker, *Cocker's Arithmetick* (London, 1678), A2v–A3r.

https://doi.org/10.1017/jbr.2017.59 Published online by Cambridge University Press

ownership.[69] However, as mentioned above, both John Wallis and his younger brother learned arithmetic out of some sort of book: either a printed arithmetic textbook or a handwritten book of arithmetical notes, problems, and solutions that served the same purpose.[70] In the process, Wallis "wrought over all the *Examples* which he before had done in his book"; in other words, the boys left behind textual traces of their learning process in the form of marginalia. Indeed, early modern students were taught in school to mark up their books with personal notes, a process that leaves traces of students with access to additional paper for scratchwork.[71] By examining marginalia, we can draw certain inferences as to how arithmetic textbooks were used.

William Sherman has surveyed the sixteenth- and seventeenth-century books held by the Huntington Library and has concluded that over 50 percent of surviving sixteenth-century books have substantive marginalia. For certain subjects—such as practical guides to law, medicine, and estate management—the percentage of marginalia remains over 50 percent for the seventeenth century as well, although the overall marginalia rate for sixteenth- and seventeenth-century books combined is just over 20 percent. However, Sherman argues, the number of surviving books with marginalia is only a fraction of those books that were annotated during the sixteenth and seventeenth centuries. The more heavily used books would have been more vulnerable to decay. One arithmetic textbook held in the Senate House Library contains a handwritten series of laments—possibly inserted by the nineteenth-century mathematician and historian Augustus De Morgan—over the difficulty of finding a copy of the same, with similar speculations that editions have vanished because they were used until they fell apart.[72] Moreover, many book owners had no compunction about effacing the marks of previous users, particularly by cropping off marginalia or by bleaching the pages.[73]

In my own research, I consulted 365 copies of arithmetic textbooks that survived from the sixteenth and seventeenth centuries—a little over 30 percent of the 1,165 individual copies currently listed in the *English Short Title Catalog*. The books in this sample set are not random but are a total population sample of arithmetic textbooks from libraries that I have been able to visit. Of the 365 textbook copies examined, 53 percent contain marginalia that clearly indicate arithmetical knowledge and engagement with text, including manicules, underlining, corrections to the text, marginal glosses, and scratchwork. Arithmetical scratchwork often duplicated or expanded upon the examples in the text, but it also encompassed a wide variety of other calculations, from personal accounting to determining the current age of the textbook. This reader engagement even extended to commentary on previous annotators, such as one annoyed writer who complained about the strike-outs in the text:

[69] For an example of book ownership versus book reading, see Owen Gingerich, *The Book Nobody Read: Chasing the Revolutions of Nicolaus Copernicus* (New York, 2004).

[70] For an example of a manuscript arithmetic textbook, see MS HA School Exercises Box 5, Folder 1, HEH, which is an educational commonplace book dating to 1623. The book also contains extensive notes on geometry and rules of measurement, as presented by the London mathematical tutor, John Speidell.

[71] William Sherman, *Used Books: Marking Readers in the Renaissance England* (Philadelphia, 2008), 3.

[72] [DeM] L.1 [Cocker] SSR.1700, fol. 3r, Senate House Library, University of London (hereafter SHL).

[73] Ibid., 5–6, 9.

"The book is right, and needs not this blottinge."[74] Another 23 percent of the textbooks contain nonarithmetical marginalia consisting mostly of ownership marks—such as owners' names, years of ownership, purchase dates, and purchase prices—or handwriting practice and doodles, which suggests that many books were used simultaneously to practice arithmetical and literary skills. The remaining 24 percent of textbooks have no markings datable to before 1800.[75]

These findings were relatively consistent across holding library, publication decade, and textbook series, and the averages do not appear to have been skewed by outliers. Of the libraries consulted, the three with the largest collections of arithmetic textbooks—the British Library, the Huntington Library, and the Bodleian Library—had overall marginalia rates of 72 percent, 66 percent, and 58 percent, respectively. It is worth noting that the Huntington Library, which now includes the Burndy Library collection and whose holdings Sherman studied extensively, has marginalia rates in line with its two nearest-size neighbors. The next three largest collections—held by the University of Illinois Urbana-Champaign, University of London's Senate House Library, and Cambridge University Library—show somewhat higher marginalia rates at 81 percent, 85 percent, and 84 percent, respectively. This higher rate of marginalia also holds for libraries with even smaller collection sizes, where sampling size makes percentages less useful. It is probable that a collecting bias toward "clean" copies of books has affected the numbers for the largest libraries, as opposed to the smallest libraries, where copies were acquired on a more ad hoc basis. Looking at marginalia by decade of publication reveals marginalia in between 50 and 100 percent of books, but the lower survival rate of textbooks prior to the 1570s is probably responsible for the most extreme percentages. After the 1570s, marginalia rates by publication decade range between 60 percent and 90 percent.

The most significant divergences appear among the different textbook series: the textbooks of the popular Robert Recorde, Humfrey Baker, and James Hodder have marginalia rates of 87 percent, 80 percent, and 82 percent, respectively, while the textbooks of Edmund Wingate show marginalia rates of just 53 percent.[76] While Recorde's surviving textbooks far outnumber those of these other authors, his longevity does not distort the results; controlling for the difference in publication run length, by limiting the numbers only to those copies of Recorde's arithmetic published after 1630, when Wingate's first edition was published, gives Recorde the even higher marginalia rate of 95 percent.

Wingate's comparatively low marginalia rates probably result from his original focus on the use of newfangled logarithms to avoid the "tedious and obscure" methods of "*Multiplication*, and *Diuision*, which so confound, and perplex the new *Practitioner*."[77] In his first edition, Wingate raced through the first five parts of arithmetic, spending longer on numeration in whole numbers, fractions, weights, and measures than on the multiplication and division that students found so confusing,

[74] 512 K47e, fol. K1v, American Philosophical Society Library, Philadelphia.

[75] They do have library markings, including notes on acquisition and rebinding, which generally date to the twentieth century. For example, the subset of arithmetic textbooks from the Senate House Library were largely acquired in the nineteenth century by Augustus De Morgan, and most contain notes in his hand.

[76] For more on the breakdown of textbooks by libraries and first author, see the appendix.

[77] Edmund Wingate, *Arithmetique Made easie, In Two Bookes* (London, 1630), 4v–A1r.

which barely rated sixteen pages between them in a book of nearly five hundred pages. In second and third editions published decades later, editor John Kersey spent considerable effort bringing the text into better alignment with other arithmetic textbooks, expanding the early parts of the book so that "Learners, as desire only so much skill in Arithmetick, as is useful in Accompts, Trade, and such like ordinary employments" could find nine chapters solely on arithmetic in whole numbers "before any entrance be made into the craggy paths of Fractions, at the sight whereof some Learners are so discouraged." He also "plainly and fully delivered the Doctrine of *Fractions*, both in *Vulgar* and *Decimal*," making the book "now supplied with all things necessary to the full knowledge of *Common Arithmetick*" as well as logarithms.[78] With these changes, *Wingate's Arithmetick* rated a dozen further editions before the end of the century. It is likely that readers seeking an elementary arithmetic education found these changes an improvement, as Kersey's additions increased the marginalia rate on Wingate's arithmetic considerably—only 27 percent of the pre-1658 editions are annotated, compared to 71 percent of post-1658 editions. Looking at only these post-1658 editions thus brings his marginalia rate in line with those of authors such as Edward Cocker, at 72 percent, as well as Thomas Blundeville and Thomas Masterson, both at 70 percent.

While it would be dangerous to read too much into these numbers—the sample size for Masterson is a mere ten books—it does suggest that there is a positive correlation between marginalia rates and the quality of the textbook's instruction as perceived by students. While experienced arithmeticians might purchase texts for reference or instruction in advanced techniques like logarithms, students who were still learning basic arithmetic actively marked up their textbooks. Assuming these marginalia percentages are either representative of, or form a lower bound for, books that did not survive, then a significant number of students must have learned arithmetic from printed textbooks. It was to these students that authors and their subsequent editors particularly needed to appeal when determining a textbook's content.

EVOLVING CONTENT STANDARDS

The importance of these textbooks to the educational process can be seen in the way that their writers modified their content in response to the perceived needs of student audiences. When nonconformist minister, mathematician, and schoolteacher Adam Martindale wrote his autobiography at the end of the seventeenth century, he described his 1642 attempt to write

> a booke of Arithmetick for whole numbers and fractions in the old method of Record, Hill, Baker, &c. (for then I knew nothing of Decimals, Logarithms or Algebra) but

[78] Edmund Wingate, *Mr. Wingate's Arithmetick*, ed. John Kersey (London, 1658), A4r–A5r. These changes were likely made possible by Edmund Wingate's death in 1656. Taylor, *Mathematical Practitioners*, 205; Bertha Porter, "Wingate, Edmund (*bap*. 1596, *d*. 1656), *mathematician and legal writer*," rev. H. K. Higton, *ODNB*, http://www.oxforddnb.com/view/article/29732.

> somewhat more contracted by with an appendix of mine owne invention touching extracting the rootes of fractions.[79]

For Martindale, it was not enough to merely describe the textbook manuscript that he had lost during the civil wars as being for whole numbers and fractions; he also had to excuse the lack of subjects that by 1692 had become an expected part of the standard arithmetical textbook—the "artificial" arithmetic that consisted of decimals, logarithms, and "symbolic" arithmetic or algebra.[80]

Both Recorde and Baker's textbooks are typical of Martindale's "old method" of arithmetical instruction, which focused on introducing Arabic-numeral calculations to an English population more used to Roman numerals and counting boards. This method always began with a presentation of the first five "parts" of arithmetic: numeration, addition, subtraction, multiplication, and division. It then usually continued with progression—the construction of number sequences through addition or multiplication—and some variant of the rule of three—a memorized set of steps that enabled users to solve ratios and that was particularly useful for the conversion of currencies or the unequal division of assets. Beyond that, arithmetic textbooks' content varied, particularly with respect to the tables, practice problems, and other content that was meant to be of use in readers' daily lives. While Roman numerals occasionally appeared in these textbooks, they usually did so in the section on numeration, where the student first learned Arabic-numeral symbols. By the seventeenth century, most textbooks omitted Roman numerals entirely and related Arabic numerals back to English number words instead. The exception to this is Recorde's textbook, which, until the 1699 edition, also always included a second section on the counting boards used to perform arithmetical operations in tandem with written Roman numerals. However, neither Baker nor any subsequent authors offered instructions on counting boards, and counting boards were not mentioned in any of the printed advertisements. For the vast majority of authors, the only system they promoted and provided instruction for was Arabic numeral arithmetic.

Although the main structure of these textbooks remained the same, authors and editors constantly updated their textbooks' contents and touted the most significant changes on their title pages in hopes of convincing potential buyers that their textbooks were more useful than anyone else's. Many of these additions were printed charts and tables that would have remained useful to the commercially inclined reader even after he or she mastered the basics of arithmetic. Humfrey Baker edited several editions of *The Wellspring of Sciences* before his death, and he included a variety of "most necessary Rules and Questions" aimed at an audience of merchants and artificers.[81] Starting in the 1591 edition, he also included tables of "measures and

[79] BL Add. MSS. 4239, fol. 18r. The inclusion of Hill in this list is curious, as the *ESTC* only records one edition of his arithmetic textbook, as opposed to Recorde and Baker's frequently reprinted textbooks. It is possible that other editions have been lost to the historical record or that Martindale had personal experience with Hill's arithmetic that made him highly value the book despite its failure to be reprinted. Thomas Hylles, *The arte of vulgar arithmeticke* (London, 1600).

[80] Adam Martindale, *The Countrey-Survey-Book: or Land-Meters Vade Mecum* (London, 1692), M3r. Martindale was not the only one to begin writing an arithmetic textbook, but he never make it to publication. For example, BL Add. MSS. 4473, fols. 24–27, contains the partially completed textbook of "William Senior professor of the Mathematiques 1641," who taught mathematics out of his house.

[81] Humfrey Baker, *The Wellspring of Sciences* (London, 1564), A1r.

https://doi.org/10.1017/jbr.2017.59 Published online by Cambridge University Press

waights of diuers places of Europe" that would also be of use to merchants who traded on the Continent.[82] John Mellis added "sundry new rules" including "a third parte of rules of practice" to his 1607 edition of *The Ground of Artes*. He similarly appealed directly to merchants and traders by adding "diuerse such necessarie rules as are incident to the trade of merchandise" and "diuerse Tables and instructions that will bring great profit and delight vnto Merchants, Gentlemen, and others."[83] Of all his tables, the ones that set forth the current value of various coins must have been the most useful to readers because he specifically highlighted these tables in the title of the 1610 and of all subsequent editions. Robert Norton added "the art and application of Decimall arithmetic" in 1615, but this failed to appeal to readers and was dropped from the next edition.[84] However, Norton's tables of board and timber measures survived and were mentioned prominently on the title pages of subsequent editions. Norton's table for 10 percent interest found enough of an audience that Robert Hartwell replaced it with more extensive interest tables in 1631. Hartwell's new tables were "of Interest vpon interest, after 10 and 8 per 100" as well as "the true value of annuities to be bought or sold present, respited, or in reuersion."[85]

The second half of the seventeenth century saw a striking shift in these title-page advertisements, with commercially based tables replaced by subjects that would have previously been considered advanced mathematics. At the same time, the very words used to describe Arabic-numeral arithmetic in whole numbers changed from *cyphering*—a term meant to differentiate it from counting-board arithmetic—to *vulgar arithmetic*—a phrase meant to differentiate it from more advanced forms of Arabic-numeral arithmetic such as decimal arithmetic or even algebra. These content changes were significant enough for Martindale to call them a whole new "method" of arithmetic textbook, which now included arithmetic in whole number and fractions alongside decimal fractions, logarithms, and algebra.

The same changes could be seen both in textbooks and in teaching methods more generally, indicating that textual and verbal instruction remained closely linked. Robert Norton's 1615 attempt to include decimal arithmetic into *The Ground of Artes* might have failed, but Henry Phillips, editing the 1670 edition of *The Well-spring of Sciences*, proudly announced the inclusion—once again—of the "Art of Decimal Fractions, intermixed with Common Fractions, for the better Understanding thereof."[86] This time, subsequent editions continued to include Phillip's decimal arithmetic. In 1650, Jonas Moore published the first of several editions of his arithmetic, which included "ordinary operations in numbers, whole and broken" along with decimals, the "new practice and use of the logarithmes, Nepayres bones," algebra, and the mathematics of "the art military."[87] William Leybourn published his *Arithmetick: Vulgar, Decimal, Instrumental, Algebraical* in 1659, while in 1685

[82] Humfrey Baker, *The Wel Spring of Sciences* (London, 1591), A1r.

[83] Robert Recorde, *The Grounde of Artes*, ed. John Dee and John Mellis (London, 1607), A1r.

[84] Robert Norton had previously translated a Dutch treatise on decimal arithmetic and published it in 1608. Simon Stevin, *Disme: the Art of Tenths, or, Decimall Arithmetike*, trans. Robert Norton (London, 1608); Robert Recorde, *The Grounde of Artes*, ed. John Dee, John Mellis, and Robert Norton (London, 1615), A1r.

[85] Robert Recorde, *The Grounde of Artes*, ed. John Dee et al. (London, 1631), A1r.

[86] Humfrey Baker, *Baker's Arithmetick*, ed. Henry Phillippes (London, 1670), A1r.

[87] Jonas Moore, *Moore's Arithmetick* (London, 1650), A1r.

https://doi.org/10.1017/jbr.2017.59 Published online by Cambridge University Press

Edward Cocker first published his decimal arithmetic as a separate companion volume to his textbook on vulgar arithmetic. Cocker's arithmetic textbooks are particularly significant because they emerged as the new standard for English arithmetic textbooks in the eighteenth century. They were so prominent that they even became the basis for a figure of speech: "correct according to Cocker."[88]

Similar changes also occurred in late seventeenth-century tutoring advertisements, indicating that the downplaying of commercial interests in favor of a broader spectrum of arithmetical skills was part of a global pedagogical shift on the part of specialty mathematics teachers rather than a publishing trend. While tutors had offered lessons on decimals, logarithms, and algebra before, these subjects—especially decimals—began to take on new prominence in advertisements. In 1650, John Kersey, an editor of Edmund Wingate's arithmetic textbook, advertised his ability to teach arithmetic in whole numbers as well as arithmetic in three different types of "fractions"—vulgar, decimal, and astronomical. He also would teach logarithms but rated this skill less highly, burying it at the bottom of his advertisement in a note on his ability to teach the construction and use of mathematical instruments.[89] By 1683, Henry Mose placed decimals on par with whole numbers and fractions when he simply stated his ability to teach "arithmetick in whole numbers and fractions, vulgar and decimal, and merchants accompts."[90] Also by the 1680s, Adam Martindale promised to instruct students in all the parts of arithmetic: vulgar arithmetic—being whole numbers, fractions, and balancing accounts—and artificial arithmetic—being decimals, logarithms, instruments, and algebra.[91]

These changes in the standard curriculum for arithmetic textbooks and specialty mathematical tutors suggests that, by the end of the seventeenth century, fewer students were seeking an initial grounding in arithmetic with a commercial emphasis. While these skills were still taught, basic lessons were insufficient to make a book or a private mathematical school profitable and had to be supplemented with lessons on more advanced and difficult subjects. It is highly unlikely that there was a sudden decrease in the need for Arabic numeral education at a time when people were increasingly adopting Arabic numerals for calculation; instead, these changes reflected the increasing diversity of student opportunities for learning arithmetic from both formal and informal sources. In a crowded market, textbooks and tutors could stand out by teaching advanced skills as well as basic arithmetic.

PETTY-SCHOOL ARITHMETIC

Much has been written about formal education in the early modern period, generally with a focus on the changes brought about by the Reformation, humanist learning,

[88] Ruth Wallis, "Cocker, Edward (1631/2–1676), *calligrapher and arithmetician*," *ODNB*, http://www.oxforddnb.com/view/article/5779. See also the auction advertisement pasted inside the back cover of [D.-L.L]L2[Cocker]SR, SHL.

[89] Edmund Wingate, *Arithmetique Made Easie*, ed. John Kersey (London, 1650), 462–65; Ruth Wallis, "Kersey, John, the elder (*bap.* 1616, *d.* 1677), *mathematician*" and "John Kersey the younger (b. c. 1660, d. in or after 1721)," *ODNB*, http://www.oxforddnb.com/view/article/15474.

[90] James Hodder, *Hodder's Arithmetick*, ed. Henry Mose (London, 1683), A8v.

[91] Martindale, *Countrey-Survey-Book*, M3r.

https://doi.org/10.1017/jbr.2017.59 Published online by Cambridge University Press

and growing literacy rates.[92] The mathematical arts and sciences do not fare well in these narratives, as grammar schools concentrated on teaching boys Latin and Greek, while scholars considered the Universities of Cambridge and Oxford to be reactionary and antithetical to the "new sciences," including mathematics.[93] This portrayal of early modern mathematical education has not gone completely unchallenged; in particular, Mordechai Feingold definitively rebutted the case against university mathematics.[94] However, the overall view remains pessimistic, emphasizing the lack of a universal mathematical education rather than noting the variety of paths to mathematical training for the children who wanted or needed it, beginning at the petty-school level.

Early modern children usually were first exposed to formal education at around the age of five or six, in what were generally called petty schools.[95] A 1406 statute enabled children of both sexes to attend petty schools, where they were supposed to acquire at least a rudimentary knowledge of reading and writing in English.[96] These schools varied widely in form, ranging from "dame schools" run by women, often informally in their homes, to "song schools" attached to great cathedrals and intended to educate the boys of the choir. Falling somewhere in between were petty schools attached to grammar schools, which were intended to prepare boys for entrance to those grammar schools. These petty schools could be run by ushers out of grammar school or by more informally by masters in their private homes.[97] Beginning in the 1580s, petty schools increasingly became a source of education in numeracy, arithmetic, and elementary accounting.

Examples of ideal petty school curricula can be found in printed books on elementary education. Schoolmaster and author Francis Clement first wrote *The Petie Schole* in the 1570s—his preface is dated 21 July 1576, and the work was entered into the stationer's register on 20 July 1580.[98] The original edition was focused solely on English orthography, promising "to enable both a childe to reade perfectly within one moneth, & also the vnperfect to write English aright." However, by the time of its republishing in 1587, he felt the need to add—and advertise the addition of—patterns for writing secretary and roman hands along with instructions on how "to number by letters, and figures" and "to cast accomptes, &c."[99] In this second edition he introduced his students to both Roman and Arabic numerals—including explaining the arithmetical origins of the common proverb, "but stand

[92] For an excellent, recent discussion of these issues, see Ian Green, *Humanism and Protestantism in Early Modern English Education* (Burlington, 2009).

[93] For more on the universities' resistance to any new institutions of learning—regardless of proposed curriculum—that might challenge their supremacy, see Mordechai Feingold, "Tradition versus Novelty: Universities and Scientific Studies in the Early Modern Period," in *Revolution and Continuity: Essays in the History and Philosophy of Early Modern Science*, ed. Peter Barker and Roger Ariew (Washington, 1991), 45–59.

[94] Feingold, *Mathematicians' Apprenticeship*.

[95] Charles Hoole, *The Petty-Schoole, Shewing a Way to Teach Little Children to Read English with Delight and Profit, (especially) According to the New Primar* (London, 1659), 2.

[96] Helen M. Jewell, *Education in Early Modern England* (New York, 1998), 17.

[97] Richard DeMolen, *Richard Mulcaster and Educational Reform in the Renaissance* (Nieuwkoop, 1991), xviii.

[98] *ESTC*, record no. 006176804.

[99] Francis Clement, *The Petie Schole with an English Orthographie* (London, 1587), A1r.

(we say) like a cypher in Algorisme"—but only alluded to the possibility of calculating with the latter. He felt that it would be sufficient to teach counting-board accounting, including "the due placyng, laying downe, and tykyng vp of counters" and therefore limited his arithmetical instruction to the conversion of monetary units, as well as addition and subtraction through the use of counters on a counting board.[100] Nor was Clement the only author to advocate teaching arithmetic to petty-school students in the late sixteenth and seventeenth centuries. Charles Hoole, in his 1659 *The Petty-School*, desired teachers to have "good skil in Arithmetick" so that students could be taught "to read English very well, and afterwards to write and cast accounts."[101] Like Clement, Hoole prioritized the ability to read but was unclear whether he expected students to receive serial or simultaneous instruction in writing and arithmetic.

Arithmetic similarly began to appear in schoolmasters' licenses in the 1580s. Of the eleven licenses reproduced by David Cressy in his education sourcebook, three included arithmetic alongside reading and writing.[102] In 1583, a "literatus" named Will Bradley was licensed to "teach boys the art of writing, reading, arithmetic and suchlike at Bury St Edmunds." Four years later, Thomas Cullyer of Norwich was licensed "to teach boys and infants the abc, art of reading, writing, arithmetic and suchlike." Other advertisements placed writing and arithmetic into closer proximity, grouping them together in such a way as to imply that the skills could be taught simultaneously. One 1599 license, which survived in full, authorized a fishmonger, William Swetnam of the parish of St. Margaret Pattens in London, "to teach and instruct children in the principles of reading and introduction into the accidence, and also to write and cast accounts" within the city of London.[103] At Maidenhead, Berkshire, the local chaplain "demanded but 3d a week for every scholar that learned English only, and for such as learned to write and read or to cypher or learn grammar 4d weekly."[104] There were also countless unlicensed and informal teachers, most of whom are lost to the historical record, but brief surviving mentions of their curricula often included arithmetic alongside reading and writing.[105] While it is not possible to determine how often—or how well—any of these schoolmasters taught arithmetic, the subject was becoming part of the constellation of possibilities for their students.

In the seventeenth century, a growing number of schools were founded explicitly to teach poor children literacy and arithmetic together, with the expectation that these lessons would prepare them for future apprenticeships or other honest careers. A thorough command of reading and writing in English was a prerequisite for many trades, and a "youth brought up at school will be taken Apprentice with less

[100] Ibid., 65, 71–85.

[101] Hoole, *Petty-Schoole*, 30.

[102] Schoolmasters' licenses rarely survived in full, as they were kept by individual schoolmasters in their private records. Instead, most instances of licenses come from ecclesiastical visitations, where the contents of licenses were summarized for the visitation record. David Cressy, *Education in Tudor and Stuart England* (New York, 1975), 32.

[103] In this usage, *accidence* signifies the "branch of grammar which deals with the inflection of words, grammatical morphology." *OED*, s.v., "accidence, n^2"; Cressy, *Education*, 33–34.

[104] Cressy, *Literacy*, 36.

[105] Ibid., 35–41.

https://doi.org/10.1017/jbr.2017.59 Published online by Cambridge University Press

mony then one illiterate."[106] In an early example from 1586, George Whately of Stratford-upon-Avon set up a school in Henley-in-Arden where 30 children between the ages of 8 and 13 could learn reading, writing, and arithmetic.[107] In 1624, Sir William Borlase founded a petty school at Marlow to teach twenty-four poor children to read, write, and cast accounts; this course of instruction was expected to take approximately two years, after which the boys would have acquired the skills prerequisite to being bound as apprentices.[108] Similar schools to teach poor children to read, write, and cast accounts were founded in Beccles, Suffolk, in 1631; in Cheshunt in 1642; in Greenwich in 1643; and in Westhallam, Derbyshire in 1662.[109] In 1694, Simon, Lord Digby, bequeathed £4 per annum to teach boys reading, writing, and accounting to prepare them for a range of future careers such as bailiffs, gentlemen's servants, or honest tradesmen.[110] The Great Yarmouth Children's Hospital, in 1696, also aimed to prepare children for apprenticeship and rewarded the schoolmaster "for teaching every child, viz., twenty shillings when it can read well in the Bible, twenty shillings more when it can write well, twenty shillings when it can cypher well to the rule of three inclusive, and twenty shillings when each girl can sew plain work well."[111]

While arithmetic gradually became an enduring component of children's early education, it is important not to overestimate the quality or universality of early modern petty-school instruction. Edmund Coote—author of *The English Schoolmaster*, which was reprinted 48 times over the course of the seventeenth century—focused on teaching reading and writing, with special attention to orthography for those who would go on to grammar schools. Although he felt obliged to include instruction on "the first part of Arithmetick, to know or write any number," he refused to spend more than a page on it, "my Book growing greater than I purposed."[112] Like Coote, many schoolmasters must not have taught anything beyond the most basic introduction to numeration by Roman and Arabic numerals, of the kind that could also be had from an ABC book.[113] Even those who taught addition or subtraction did not necessarily have to be skilled arithmeticians. As late as 1701, John White—the master of Mr. Chilcot's English-Free-School in Tiverton with "near Forty Years Practice in Teaching"—expounded the benefits of rote learning in his *The Country-Man's Conductor*:

[106] Christopher Wase, *Considerations Concerning Free-Schools as Settled in England* (London, 1678), 33.

[107] Kenneth Charlton, *Women, Religion and Education in Early Modern England* (London, 1999), 146.

[108] Jewell, *Education*, 95.

[109] John Lawson and Harold Silver, *A Social History of Education in England* (London, 1973), 107; Robert Ashton, "Leman, Sir John (1544–1632), *merchant and mayor of London*," *ODNB*, http://www.oxforddnb.com/view/article/16420; Charlton, *Women, Religion and Education*, 151, 148; N. Plumley, "The Royal Mathematical School within Christ's Hospital: The Early Years—Its Aims and Achievements," *Vistas in Astronomy* 20 (1976): 51–59, at 58.

[110] Charlton, *Women, Religion and Education*, 147.

[111] Cressy, *Literacy*, 30; Charlton, *Women, Religion and Education*, 149.

[112] Edmund Coote, *The English School-Master* (London, 1651), A2r, H2r.

[113] See, for example, *The ABC with The Catechisme: That is to say, an Instruction to bee taught and learned of euery Child, before he be brought to be confirmed by the Bishop* (London, 1633), which was reprinted at various times throughout the seventeenth century.

> As to the Arithmetical Part (When your Children have gotten some Perfection in their English) let them learn it by heart, and if neither Teacher nor Learner understand the Use of the Rules, yet when they come to learn Arithmetick in earnest, it will be a great help to them and ease to their Master.[114]

Understanding the rules of arithmetic took second place to memorization, and White argued that even the teacher did not need to understand what he taught. White further recommended that children be taught arithmetic "before or as soon as they are put to writing."[115] This is a logical progression, given the reliance of Arabic numeral arithmetic on writing skills. However, a significant number of children, particularly in rural villages, likely dropped out of petty school after learning to read but before learning to write. Thus even those students who had access to arithmetical instruction might not have been able to take advantage of the opportunity.[116]

GRAMMAR SCHOOLS, TUTORS, AND APPRENTICESHIPS

After boys, in theory, learned reading, writing, and at least basic enumeration at a petty school, they had several different options for continuing their educations, all of which allowed for further arithmetic instruction as needed. Boys could attend grammar schools, obtain apprenticeships, or seek out other, specialized instruction such as public lectures and mathematical schools. These options were not mutually exclusive; many first attended grammar schools or mathematical schools and afterward were bound apprentices or gained admission to universities. Other boys traveled even more complicated educational paths. Sixteen-year-old Robert Ellison, a student at the prestigious grammar school of Eton, was supposed to begin an apprenticeship but was told that he "cannot come from thence [Eton] into a merchants' compting house without being some months at school in London to learn to write and also accounts."[117] He thus required a combination of a formal grammar school, a specialty writing and arithmetic school, and an indentured apprenticeship to prepare for his future career in trade.

As the case of Robert Ellison implies, arithmetic was not a substantial component of the continuing education of boys who attended grammar schools. The humanist curriculum of grammar schools focused on Latin, Greek, and reading the classics, none of which required a great knowledge of arithmetic much less that of more complicated mathematics. As argued above, it was possible for a boy to enter grammar school having already obtained a rudimentary understanding of numbers and arithmetic through his petty school. However, schoolmaster John Brinsley was probably only exaggerating slightly when he complained that innumeracy was "a verie ordinarie defect" and that he had seen "Schollers, almost readie to go to the Vniuersitie, who yet can hardly tell you the number of Pages, Sections, Chapters, or other diuisions in their bookes" nor "helpe themselues by the Indices, or Tables of such

[114] John White, *The Country-Man's Conductor in reading and writing true English … and some arithmetical rules to be learnt by children, before or as soon as they are put to Writing* (Exeter, 1701), A1r, A5v.
[115] White, *Country-Man's Conductor*, A1r.
[116] Thomas, "Meaning of Literacy," 102–3.
[117] Jewell, *Education*, 85–86.

books."[118] While students were expected to be fluent readers, their later introduction to writing and arithmetic meant those were often less practiced skills.

Many grammar schools sought to remedy the defects in their students' petty-school educations by arranging optional extra lessons on holidays and half days. The grammar school at Rotherham offered writing and accounting lessons as early as the fifteenth century, while Bristol grammar-school students were released early on Thursdays and Saturdays for lessons with the local scrivener.[119] Statutes written by the trustees of the Blackburn Grammar School in 1597—and confirmed again in 1600—made provision for associated "petties" to be instructed in arithmetic, and schoolmasters could force grammar school students to take remedial, petty-level lessons at any time in which they were not actively engaged in their primary curriculum:

> Uppon dayes and tymes excepted from teachinge, the Scollars may be caused by the Schoole Master and the Usher to larne to write, cipher, cast accounts, singe or such licke, and allso upon holidayes, and other convenient tymes.[120]

This section of the statutes was probably enforced in practice, as the trustees also assert an unusual commitment to mathematical education in their statutes: "The principles of Arithmeticke, Geometrie, and Cosmographie with some introduction into the sphere, are proffitable."[121] A set of 1629 statutes written by Samuel Harsnet, future archbishop of York, for the Chigwell school even required one of its schoolmasters to be proficient in both writing and arithmetic in addition to Latin:

> I ordain that the second schoolmaster, touching his years and conversation, be in all points endowed and qualified as is above expressed touching the Latin schoolmaster; that he write fair secretary and Roman hands; that he be skilful in cyphering and casting of accounts and teach his scholars the same faculty.[122]

The trustees thus wanted a schoolmaster who could actively teach his students writing alongside several different mathematical skills, including the use of Arabic numerals for calculations, counters for "casting" accounts, and the bookkeeping skills necessary to record their accounts. But most grammar schools probably relied on outside tutors—ideally those who could teach both writing and arithmetic—to teach their remedial students.[123]

The common need for tutoring in writing and arithmetic increased the inherent pedagogical link between them, leading to the rise of writing-cum-arithmetic tutors who advertised their skills in both capacities. As early as 1582 to 1610,

[118] Brinsley, *Ludus Literarius*, 25.

[119] Jewell, *Education*, 84.

[120] Although the grammar school was teaching Arabic numerals and ciphering as early as 1597, the school's various accountants used Roman numerals to record monetary entries and sums until 1669/70. George Alfred Stocks, ed., *The Records of Blackburn Grammar School, Remains, Historical and Literary, connected with the Palatine Counties of Lancashire and Chester*, n.s., 66 (Manchester, 1909), 1:73.

[121] Ibid., 1:74.

[122] Cressy, *Education*, 65.

[123] William Lilly, *Merlini Anglici Ephemeris: Or, Astrological Judgments for the year 1677* (London, 1677), F8v.

John Mellis, the editor of an arithmetic textbook, ran a school "within the Mayes-gate in short Southwarke nigh Battle bridge" where "children or seruants" could be taught arithmetic, accounting, algebra, and "any manner of hand vsuall within this Realme of England."[124] The Restoration, in particular, saw a significant expansion in the number of these tutors who formed their own private schools in and around London. During the 1660s and 1670s, James Hodder taught both writing and arithmetic in a house "next dore to the Sunne in Tokenhouse Yard, Lothbury, City of London"—aside from a 1666–1671 interlude in Bromley by Bow—and his school was continued by Henry Mose, "late servant and successor to" Hodder, through 1720.[125] Similarly, Edward Cocker taught writing and arithmetic from 1657 to 1676, holding classes in St. Paul's churchyard, Northampton, and lastly Southwark. John Hawkins took over the Southwark school after his death, styling himself a "writing master," until his own death in 1692.[126] Hawkins's conflation of writing and arithmetic continued in his advertising for a 1680 edition of Cocker's arithmetic book, where he noted that it had been commended by "many eminent mathematician and writing-masters in and near London," implying that the opinion of a writing master should have similar value to that of a mathematician when it came to teaching arithmetic.[127] While there were fewer of these writing-cum-arithmetic tutors outside London, Cocker's Northampton school was not the only one. In 1677, Peter Perkins "taught Writing and Arithmetick, with any or all parts of the *Mathematicks* at easie Rates" near the grammar school at Guildford in Surrey.[128] Nor were all of these tutors and students male; a woman named Elizabeth Beane—who also conflated writing and arithmetic, being referred to variously a "mistress in the Art of Writing" and tutor in "The Art of Writing and Arithmetick"—taught female students in the 1680s.[129]

As with arithmetic lessons at the petty-school level, the availability of extracurricular arithmetic lessons from grammar schools and outside tutors did not mean that all students took advantage of those lessons. Local clerks, scriveners, and mathematical teachers charged fees to cover the costs of these lessons, and parents would have been most likely to pay for such lessons—particularly those focused on learning to cast accounts—in the case of sons who would eventually be bound as apprentices. However, the majority of early modern boys did not continue formal schooling by attending universities but instead left school to pursue vocational education as apprentices to agriculture or some trade.[130] Patrick Wallis estimates that, by the late seventeenth century, over 9 percent of England's teenaged males were serving

[124] Mellis advertised his school in the versions of Recorde's *The Ground of Artes* that he edited, from 1582 until 1607. His advertisement also appeared in the 1610 edition, "now lastly corrected by John Wade," but was replaced by N. Physhe in the 1615 edition. Recorde, *Ground of Artes* (1607), Mm8r; Recorde, *Ground of Artes* (1610), A1r; Recorde, *Records Arithmeticke* (1615), Oo3v.

[125] Ruth Wallis, "Hodder, James (*fl*. 1659–1673), *arithmetician*," *ODNB*, http://www.oxforddnb.com/view/article/13416.

[126] Ruth Wallis, "Cocker, Edward (1631/2–1676), *calligrapher and arithmetician*," *ODNB*, http://www.oxforddnb.com/view/article/5779.

[127] Edward Cocker, *Cocker's Arithmetick*, ed. John Hawkins (London, 1680).

[128] Lilly, *Merlini Anglici Ephemeris*, F8v.

[129] Sarah Powell and Paul Dingman, "Arithmetic Is the Art of Computation," http://collation.folger.edu/2015/09/arithmetic-is-the-art-of-computation.

[130] Green, *Humanism*, 310.

apprenticeships in London alone, where two thirds of adult males had been apprentices in their youth.[131] It was to these sorts of apprenticeships that the poor recipients of charity education aspired, children of tradesmen flocked, and younger sons of the gentry defaulted in order to make a living.

Some form of arithmetic must have been necessary for any tradesman who expected to be paid by his customers and even more so for boys who pursued careers in carpentry, surveying, and navigation. However, mathematical instruction was not commonly specified in apprentice indentures. The relative silence on the subject of arithmetic in indentures was probably due to an assumption that accounting would necessarily be included in any apprentice's instruction; at least a rudimentary ability to calculate was necessary to trade. The seventeenth-century Southampton apprenticeship registers only record one instance in which an indenture explicitly included arithmetic: in January 1630/1, the orphaned Giles New of Southampton was apprenticed to a clothier who promised to instruct him "in the trade of clothier and to write and cipher."[132] In most indentures, it must have been understood that clauses such as "all other trades of sciences as the said [master] shall use" included the keeping of accounts.[133] Many apprentices began their apprenticeships in their masters' counting-houses, observing counting-house clerks perform calculations. This was called "learning the lines," and it would enable the apprentice to eventually calculate accounts on his own. Merchants, vintners, drapers, and haberdashers were especially likely to follow this practice.[134]

The Southampton apprenticeship register is similarly silent on the subject of other advanced mathematics that would be necessary for the practice of specific trades. For example, no explicit mention is made of mathematical training for John Jolliffee, who was learning to be a seaman in January of 1654/5. Given that his father was a weaver, he probably did not have extensive training in the use of mathematical instruments. Thus additional mathematical education would have been a necessary part of his master's "instruct[ing] him in y^e^ art of navigacon &c."[135] In January of 1648/9, another boy, David Jenvy, was to be instructed "in the art of merchandizing beyond the seas … . Master to permitt ye apprentice to trade and trafficque for himselfe with a stocke of 50 li when he goes to sea, which is to be in ye two last yeares."[136] To trade overseas successfully, Jenvy needed to have training in advanced arithmetical subjects such as the rules for commuting and exchanging money. However, this training was understood to be part of his general education, and there was no need to list it separately in his indenture. Whether he already had this skill before beginning his apprenticeship or whether he obtained it by shadowing

[131] Chris Minns and Patrick Wallis, "Rules and Reality: Quantifying the Practice of Apprenticeship in Early Modern England," *Economic History Review* 65, no. 2 (May 2012): 556–79, at 559; Patrick Wallis, "Apprenticeship and Training in Premodern England," *Journal of Economic History* 68, no. 3 (September 2008): 832–61, at 836.

[132] Arthur J. Willis and A. L. Merson, eds., *A Calendar of Southampton Apprenticeship Registers, 1609–1740* (Southampton, 1968), 19.

[133] John Rigges, apprenticed in 1611 to his uncle, was to be instructed in his uncle's trade and "alsoe to be enxtructed in all other trades or sciences as the said Frauncis Rigges shall use during the said terme." Ibid., 2.

[134] Vanes, *Education and Apprenticeship*, 21.

[135] Willis and Merson, *Calendar,* 86.

[136] Ibid., 38.

his master, reading an arithmetic textbook, or attending lessons with a local tutor is unknown, but all were possible routes to acquiring the arithmetic he needed for his future career.

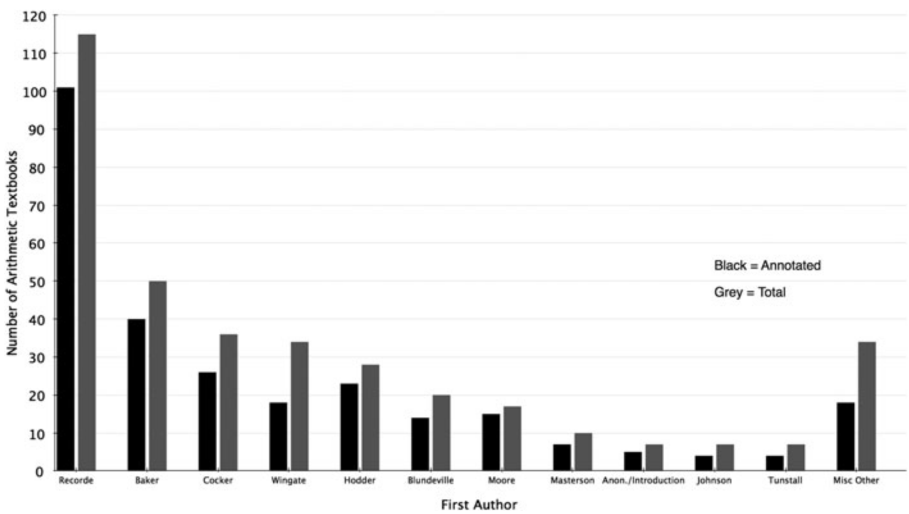
In the early modern period, the previously separate skills of writing and arithmetic were linked in English mathematical and pedagogical practices. This connection began with the commercial impetus to adopt Arabic numerals, which made at least some proficiency with writing a prerequisite skill to performing arithmetic and which lent themselves well to both face-to-face and text-based instruction. The sixteenth century's rising literacy rates and the creation of a new genre of vernacular arithmetic textbooks made instruction in Arabic-numeral arithmetic available to any literate person—male or female—with a few shillings to spare for the purchase of a new or used book. Early textbooks sold steadily, if slowly, until a surge of interest in the 1570s and 1580s led to the publication of a host of new textbooks and a drive to incorporate basic Arabic-numeral arithmetic into the petty-school curriculum.

Over the next fifty years, account books and probate inventories show the overwhelming adoption of Arabic numerals for calculation among the literate part of the English population. The successful introduction of Arabic-numeral arithmetic to England is also reflected in textbooks and tutors' changing content standards in the mid-seventeenth century, as they began to cede more introductory lessons in Arabic numerals to a variety of formal and informal educational alternatives in order to focus on more advanced topics like decimals, logarithms, and algebra. Seventeenth-century petty schools and charity schools increasingly incorporated arithmetic into their curricula while writing-cum-arithmetic tutors—many of whom were themselves textbook authors—reinforced the pedagogical connection between writing and the pen-and-paper arithmetic of Arabic numerals. While the use of counting boards persisted in some places throughout the eighteenth century, the pen-and-paper arithmetic of Arabic numerals had become the predominant symbolic system for calculation, generating a wealth of literary traces that scholars can use to study the changing nature of English numeracy.

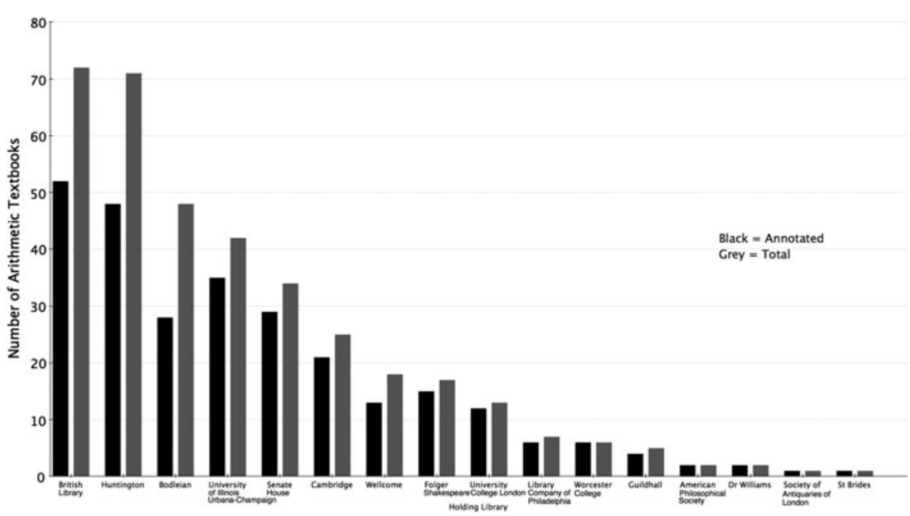
APPENDIX: ARITHMETIC TEXTBOOK MARGINALIA BY FIRST AUTHOR AND HOLDING LIBRARY

The breakdown of arithmetic textbook marginalia rates by first author—that is, the original author who gave his name to each textbook series—indicates some differences in the marginalia rates for each series (appendix figure 1). The authors are arranged by the number of surviving textbooks, with the grey bar indicating the total number of textbooks consulted per author and the black bar indicating the total number of annotated textbooks. Robert Recorde's textbooks are particularly highly annotated (101 out of 115 examined) and have the largest survival rate in the sample. The only author who equals this annotation rate is Moore (15 of 17), although Baker and Hodder come close. Baker's textbooks have the second-highest survival rate (40 out of 50 annotated), while the high number of surviving editions of Cocker's textbooks, published in the late seventeenth century, hint at his burgeoning popularity that continued into the eighteenth century. Wingate's arithmetics are outliers, with significantly less annotation (18 out of 34) than others.

The breakdown of arithmetic-textbook marginalia rates by holding library shows a clear disparity in the number of books held by larger vs. smaller libraries consulted (appendix figure 2). The libraries are arranged by the number of surviving textbooks, with the grey bar indicating the total number of textbooks consulted per library and the black bar indicating the total number of annotated textbooks. The Huntington Library in California, subject of William Sherman's in-depth study, has 66 percent annotated books in line with its nearest neighbors in size, the British Library at 72



Appendix Figure 1—Marginalia by First Author. Sample Size: 365.



Appendix Figure 2—Marginalia by Holding Library. Sample Size: 365.

https://doi.org/10.1017/jbr.2017.59 Published online by Cambridge University Press

percent and the Bodleian Library at the University of Oxford at 58 percent. Other libraries with smaller collections have even higher percentages of annotated books, which aligns with the tendency of larger libraries to have historically prioritized collecting "clean," unannotated copies of books as opposed to smaller libraries that obtained these books on a more ad hoc basis.