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THE WELFARE GAINS OF TRADE INTEGRATION IN THE EUROPEAN MONETARY UNION

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This paper evaluates the welfare gains arising from deeper trade integration in the European Monetary Union. To do this, the European Monetary Union is represented in a realistic way by an intertemporal general equilibrium model with incomplete financial markets, sticky prices, and home bias in production. The model is estimated and not rejected by the data. Two main results emerge: (i) an increase in vertical trade (occuring at the early stage of the production process) implies welfare gains whereas (ii) an increase in horizontal trade (occuring at the late stage of the production process) implies welfare losses.

Keywords: Monetary Union, Trade Integration, Inflation Differentials, Welfare Analysis

1. INTRODUCTION

This article investigates the welfare effects of deeper horizontal or vertical trade integration in the European Monetary Union (EMU). In this article, trade occurs

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along with a three-stage production process: intermediate goods, consumption goods, and retail goods. Vertical trade integration thus refers to the trade of intermediate goods triggered by consumption goods producers and horizontal trade integration refers to the trade of consumption goods triggered by retail goods producers. Independent of its long–run consequences, welfare gains of trade integration usually rest upon the increased correlation of business cycles and the improved overall adequacy of the common monetary policy to national situations. This paper shows that the impact of trade integration is more contrasted when it is assumed that financial markets are incomplete and imperfectly integrated.

We lay out an estimated two-country DSGE model of the EMU that accounts for the imperfect integration of both goods and financial markets. As in Ricci (1997), the model encompasses real and monetary arguments for the costs of conducting a single monetary policy in a monetary union characterized by business cycle asymmetries and inflation differentials. Indeed, the model features home bias in private consumption and production technology, incomplete and imperfectly integrated private financial markets, Calvo-type sticky prices, and i.i.d. productivity and public spending shocks. These assumptions are also set up to be consistent with the current economic situation of the EMU, characterized by persistent asymmetries in business cycles and significant inflation differentials [see Camacho et al. (2006) and Lane (2006) for discussions].

In this tractable framework, productivity and public spending shocks imply asymmetries in business cycles and inflation differentials that cannot be addressed by the central bank of the monetary union. These business-cycle asymmetries and inflation differentials translate into welfare costs, building on two main sources: nominal inertia and imperfect risk sharing combined with a costly access to financial markets. The role of nominal inertia in a monetary union, as well as means to reduce the associated costs, has already been extensively studied in the literature [see among others Benigno (2004); Beetsma and Jensen (2005), and Galí and Monacelli (2008)]. Less attention has been paid to welfare losses related to imperfectly integrated financial markets in a monetary union. In line with Carré and Collard (2003), we show that imperfect risk sharing crucially affects the welfare costs of business-cycle asymmetries and the size, sign, and structure of welfare gains generated by trade integration.

First, we show that an increase of horizontal or vertical trade integration increases the correlation of business cycles through an increase of mutual trade flows. The overall adequacy of the common monetary policy to national situations is thus clearly improved. The volatility of national inflation rates decreases, which significantly increases the aggregate welfare in the monetary union.

Second, vertical and horizontal trade have opposite effects on the pattern of external adjustment to asymmetric shocks. Vertical trade integration reduces the overall need for external adjustment, that is, the volatility of the current account, whereas horizontal trade increases it. Because financial markets are incomplete and imperfectly integrated, a higher (respectively lower) volatility of the current account increases (respectively decreases) the welfare costs related to the imperfect integration of financial markets and imperfect risk sharing.

The result builds on the following mechanism. Under incomplete markets, changes in the current account result in more persistent changes in the net foreign asset position and thus imply wealth transfers between countries, which affects not only the relative supply of labor in countries but also the relative demand for goods and thus relative prices. Wealth transfers implied by larger fluctuations in net foreign assets (or equivalently the current account) thus trigger increased business-cycle asymmetries (private consumption, labor supply) that lead to welfare losses. In our framework, vertical integration affects home bias at a stage of production earlier than the sticky price level, whereas horizontal integration affects home bias at a production stage later than the sticky price level. In the context of incomplete financial markets, vertical and horizontal trade integration thus impact differently on the volatility of the current account, which results in different welfare outcomes. Under complete markets, a similar change in the volatility of the current account does not have the same impact because there is no wealth transfer across countries affecting relative labor supplies and relative prices. To study the role of incomplete financial markets, we solve the model with perfect risk sharing. We show that under complete asset markets, both horizontal and vertical trade integration yield welfare gains. These gains are related to the drop of national inflation rate volatilities. Financial market incompleteness thus appears to be a crucial assumption in determining the welfare effects of horizontal and vertical trade integration.

Quantitatively speaking, we highlight that vertical trade integration leads to important welfare gains for the whole range of possible parameters of the model. In the baseline estimates, we show that a 10% increase in vertical trade implies an average welfare gain equivalent to a 7.67% rise in permanent consumption for constant labor effort.[1] On the other hand, horizontal trade generates welfare losses under incomplete financial markets and welfare gains under complete financial markets. In the baseline estimation under incomplete financial markets, a 10% increase in horizontal trade implies an average welfare loss equivalent to a 2.03% drop in permanent consumption. A sensitivity analysis shows that horizontal trade can lead to welfare gains even under incomplete financial markets. Under complete financial markets, a 10% increase in horizontal trade implies an average welfare gain equivalent to a 6.12% rise in permanent consumption, close to the welfare gains reported when vertical trade integration increases. Finally, the welfare gains caused by a 10% joint increase in both vertical and horizontal trade integration reach 7.45% under incomplete financial markets and 10.50% under complete financial markets.

Two main results emerge, therefore. In a monetary union where financial markets are incomplete, prices are sticky, and there is home bias in production at different production stages, an increase in vertical trade implies welfare gains whereas an increase in horizontal trade implies welfare losses.

The remainder of the paper is organized as follows. Section 2 describes a two-country model of a monetary union. Based on EMU data, Section 3 provides estimates for the structural parameters of the log-linear approximation of the model. The dynamic properties of the model are analyzed in Section 4. Section 5 provides an extensive welfare analysis of an increase in trade integration and presents some sensitivity analysis. A last section offers some concluding remarks.

2. A TWO-COUNTRY MONETARY UNION

The model describes a two-country world with a common currency. Each nation represents half of this monetary union. Each country is populated by a unit continuum of infinitely lived households, a government, and three types of firms producing respectively intermediate, consumption, and retail goods. Monetary policy is delegated to the central bank of the monetary union, which controls the interest rate. The international financial market is incomplete as agents trade a single one-period composite bond.[2]

2.1. Households and National Governments

The representative household $j \in [0, 1]$ of nation $i \in \{h, f\}$ maximizes a welfare index,

$$\sum_{t=0}^{\infty} \beta^t E_0 \left\{ \frac{C_t^i(j)^{1-\sigma}}{1-\sigma} - \frac{N_t^i(j)^{1+\psi}}{1+\psi} \right\}, \quad (1)$$

subject to

$$B_{t+1}^i(j) - R_t B_t^i(j) = W_t^i N_t^i(j) + \Pi_t^i(j) - P_t^i C_t^i(j) - T_t^i(j) - P_{i,t} AC_t^i(j) \quad (2)$$

and the transversality condition

$$\lim_{T\to\infty} \Pi_{s=t}^{T} R_s^{-1} E_t \left\{ B_{T+1}^i(j) \right\} = 0.$$

In equation (1), the subjective discount factor, β, is equal to $(1+\delta)^{-1}$, σ is the intertemporal elasticity of substitution of private consumption, and ψ is the inverse of the Frisch elasticity. The aggregate consumption bundle of agent j in country i is called $C_t^i(j)$ and the quantity of labor that this agent supplies on the labor market, $N_t^i(j)$. Money holdings are not introduced in the utility function because the money market plays no role in the dynamics when the nominal interest rate is the monetary policy instrument [see Beetsma and Jensen (2005)].

In equation (2), $B_t^i(j)$ is the amount of one-period nominal bonds hold by the representative agent of country i at the end of period $t-1$, which pays a gross nominal rate of interest R_t between periods $t-1$ and t. The price index of retail goods (which corresponds to the CPI) in country i is called P_t^i, whereas $P_{i,t}$ is the price of consumption goods (that corresponds to the PPI) in country i. W_t^i is the nominal wage in country i in period t, $\Pi_t^i(j) = \int_0^1 \Pi_t^i(k, j)dk$ is the amount of profits paid by monopolistic consumption goods producers, and $T^i(j)$ is a lump-sum transfer. Finally, in the budget constraint, $AC_t^i(j)$ is a quadratic portfolio adjustment cost that households have to pay to financial intermediaries to access financial markets. The cost is defined according to

$$AC_t^i(j) = \frac{\chi}{2}\left[B_{t+1}^i(j) - B^i(j)\right]^2,$$

where $B^i(j)$ is the steady state level of net foreign assets. The Euler condition that solves equations (1) and (2) is affected by portfolio adjustment costs because

$$\frac{\beta R_{t+1}}{1 + \chi P_{i,t}\big(B_{t+1}^i(j) - B^i(j)\big)} E_t \left\{ \frac{P_t^i C_t^i(j)^\sigma}{P_{t+1}^i C_{t+1}^i(j)^\sigma} \right\} = 1. \qquad (3)$$

The portfolio adjustment cost parameter (χ) affects the sensitivity of net foreign assets/liabilities to variation of the interest rate, as it becomes more or less costly to smooth consumption by accessing financial markets. For instance, when χ decreases, it is less costly for the households to access the financial markets. The labor supply function is based on traditional consumption/leisure arbitrage,

$$N_t^i(j)^\psi C_t^i(j)^\sigma = \frac{W_t^i}{P_t^i}. \qquad (4)$$

2.2. Governments

Governments choose the amount of public spending and balance their budgets using lump-sum transfers. The budget constraint of the government is given by

$$\int_0^1 T^i(j)\,dj + \tau \int_0^1 P_{i,t}(k) Y_t^i(k)\,dk = P_{i,t} G_t^i,$$

where τ is a proportional subsidy to firms. Mixing monopolistic competition and Calvo staggered price contracts on consumption goods markets introduces several distortions with respect to the Pareto-efficient equilibrium. Nominal rigidities imply inefficient fluctuations of both equilibrium inflation and output, whereas the assumption of monopolistic competition affects the steady state. Although monetary and/or fiscal policy may address the first issue, an optimal subsidy τ is able to address the second issue and restores the first-best allocation in the steady state [see Benigno and Woodford (2005)].

National public spending is biased toward national consumption goods; that is,

$$G_t^i = \left[\int_0^1 G_t^i(k)^{\frac{\theta-1}{\theta}}\,dk\right]^{\frac{\theta}{\theta-1}},$$

where the level of aggregate public spending evolves according to

$$G_{t+1}^i = (1-\rho_g)G^i + \rho_g G_t^i + \zeta_{g,t+1}^i,$$

and where $\zeta_{g,t}^i$ is an i.i.d. innovation.

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2.3. Firms

The production of consumption goods is a three-stage process: (i) intermediate goods producers make use of national labor and sell their products on competitive markets, (ii) consumption goods producers combine domestic and foreign intermediate goods and sell their products on monopolistic competition markets while facing Calvo pricing contracts, and (iii) retailers combine domestic and foreign varieties of consumption goods and sell their products on competitive markets.

Intermediate goods producers. First, in each country i, a continuum of identical firms (normalized to one) produce an intermediate good and sell it on a competitive market. The production function of these firms is given by

$$X_t^i = A_t^i L_t^i,$$

where L_t^i is the labor demand and A_t^i is the level of labor productivity, evolving according to

$$A_{t+1}^i = (1-\rho_a)A^i + \rho_a A_t^i + \zeta_{a,t+1}^i,$$

and where $\zeta_{a,t}^i$ is an i.i.d. innovation.

Intermediate goods are sold at their marginal cost W_t^i/A_t^i and intermediate terms of trade are[3]

$$\Sigma_t = \frac{W_t^f/A_t^f}{W_t^h/A_t^h}.$$

Consumption goods producers. Second, intermediate goods are traded within the monetary union and combined by monopolistic consumption goods producers $k \in [0, 1]$. The production function of consumption goods producer k located in country i is

$$Y_t^i(k) = \left[(1-\gamma_i)^{\frac{1}{\phi}} X_{h,t}^i(k)^{\frac{\phi-1}{\phi}} + (\gamma_i)^{\frac{1}{\phi}} X_{f,t}^i(k)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}. \tag{5}$$

In this expression, $X_{h,t}^i(k)$ is the demand for intermediate goods produced in country h of firm k located in country i. The parameter $(1-\gamma_i) \in [0, \frac{1}{2}]$ is the home bias in the production of consumption goods. In the production function (5), ϕ is the elasticity of substitution between intermediate goods. The companion nominal marginal cost of firm k in country i, $MC_t^i(k)$, is given by

$$MC_t^i(k) = MC_t^i = \left[(1-\gamma_i)\left(W_t^h/A_t^h\right)^{1-\phi} + \gamma_i\left(W_t^f/A_t^f\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}.$$

As a consequence, optimal demands for intermediate goods from a consumption goods producer k located in country i are

$$X^i_{h,t}(k) = (1-\gamma_i)\left[\frac{W^h_t/A^h_t}{MC^i_t}\right]^{-\phi} Y^i_t(k),\ X^i_{f,t}(k) = \gamma_i \left[\frac{W^f_t/A^f_t}{MC^i_t}\right]^{-\phi} Y^i_t(k).$$

Consumer goods prices are governed by standard Calvo contracts. Each period, only a fraction $(1-\eta^i)$ of randomly selected firms located in country $i \in \{h, f\}$ are allowed to set new prices. Assuming that firms do not discriminate among markets they address, these firms choose the following optimal price $\overline{P}_{i,t}(k)$ according to

$$\overline{P}_{i,t}(k) = \frac{\theta}{(\theta-1)(1-\tau)} \frac{\sum_{v=0}^{\infty}(\eta^i\beta)^v E_t\left\{\frac{Y^i_{t+v}(k)MC^i_{t+v}}{P^i_{t+v}C^i_{t+v}(j)^{\sigma}}\right\}}{\sum_{v=0}^{\infty}(\eta^i\beta)^v E_t\left\{\frac{Y^i_{t+v}(k)}{P^i_{t+v}C^i_{t+v}(j)^{\sigma}}\right\}}.$$

Aggregating among consumption goods producers and assuming behavioral symmetry, the average price level of consumption goods in country $i \in \{h, f\}$ is

$$P_{i,t} = \left[\left(1-\eta^i\right)\overline{P}_{i,t}(k)^{1-\theta} + \eta^i P^{1-\theta}_{i,t-1}\right]^{\frac{1}{1-\theta}}.$$

Finally, consumption goods terms of trade in the monetary union are defined as[4]

$$S_t = \frac{P_{f,t}}{P_{h,t}}.$$

Retail goods producers. Third, in each country i, a continuum of identical firms (normalized to one) produce retail goods using domestic and foreign consumption goods according to the production function

$$Z^i_t = \left\{(1-\alpha_i)^{\frac{1}{\mu}}\left[\int_0^1 Y^i_{h,t}(k)^{\frac{\theta-1}{\theta}}dk\right]^{\frac{\theta(\mu-1)}{\mu(\theta-1)}} + \alpha_i^{\frac{1}{\mu}}\left[\int_0^1 Y^i_{f,t}(k)^{\frac{\theta-1}{\theta}}dk\right]^{\frac{\theta(\mu-1)}{\mu(\theta-1)}}\right\}^{\frac{\mu}{\mu-1}}$$

and sell them on perfectly competitive markets at the price

$$P^i_t = \left\{(1-\alpha_i)\left[\int_0^1 P_{h,t}(k)^{1-\theta}dk\right]^{\frac{1-\mu}{1-\theta}} + \alpha_i\left[\int_0^1 P_{f,t}(k)^{1-\theta}dk\right]^{\frac{1-\mu}{1-\theta}}\right\}^{\frac{1}{1-\mu}}.$$

In this expression, $Y^i_{h,t}(k)$ is the demand for consumption goods produced in country h by the retail goods producers located in country i. The parameter $(1-\alpha_i) \in [0, \frac{1}{2}]$ is the home bias in the production of retail goods, $\theta \geq 1$ is the elasticity of substitution among national differentiated varieties of consumption goods, and μ is the elasticity of substitution between domestic and foreign consumption goods.

Optimal consumption goods demands from the retail sector located in country i are therefore

$$Y^i_{h,t}(k) = (1-\alpha_i)\left[\frac{P_{h,t}(k)}{P_{h,t}}\right]^{-\theta}\left[\frac{P_{h,t}}{P^i_t}\right]^{-\mu} Z^i_t,$$
$$Y^i_{f,t}(k) = \alpha_i\left[\frac{P_{f,t}(k)}{P_{f,t}}\right]^{-\theta}\left[\frac{P_{f,t}}{P^i_t}\right]^{-\mu} Z^i_t.$$

It has now become standard to consider home bias parameters as relevant measures of goods-market openness. Indeed, in the equilibrium, α_i and γ_i are the shares of imported goods in the production of consumption and retail goods, respectively [see Galí and Monacelli (2005) and Corsetti (2006)]. In the remainder of the paper, we thus consider α_i and γ_i directly as parameters measuring horizontal and vertical trade openness.

2.4. Monetary Policy

A common central bank controls the nominal interest rate within the monetary union,

$$R_{t+1} = (1-\rho_r)R + \rho_r R_t + \varphi\left(\pi^u_t - \pi^u\right),$$

where $\pi^u_t = \frac{1}{2}\pi^h_t + \frac{1}{2}\pi^f_t$ and $\pi^i_t = P^i_t/P^i_{t-1}$. This rule is commonly used in the literature [see among others Taylor (1993), Clarida et al. (1998, 1999), and Rudebusch and Svensson (1998)]. Furthermore, it is a fair approximation of the monetary policy of the European Central Bank with respect to its mission, that is, the stabilization of aggregate inflation in the EMU. Finally, a large empirical literature highlights the smoothness of the nominal interest rate variations in the euro area [see among others Peersman and Smets (1999) and Gerlach and Schnabel (2000)].

2.5. Market Equilibrium

We solve the model assuming that each country is the mirror image of the other on the goods market. Posing $\alpha_h = \alpha$ and $\gamma_h = \gamma$, we simply get $\alpha_f = 1-\alpha$ and $\gamma_f = 1-\gamma$. We also define the aggregate output as

$$Y^i_t = \left[\int_0^1 Y^i_t(k)^{\frac{\theta-1}{\theta}} dk\right]^{\frac{\theta}{\theta-1}}.$$

A competitive equilibrium is defined as a sequence of quantities,

$$\{\mathcal{Q}_t\}_{t=0}^{\infty} = \left\{C^h_t, C^f_t, N^h_t, N^f_t, Y^h_t, Y^f_t, Z^h_t, Z^f_t, L^h_t, L^f_t, B^h_{t+1}, B^f_{t+1}, AC^h_t, AC^f_t\right\},$$

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and a sequence of prices,

$$\{\mathcal{P}_t\}_{t=0}^{\infty} = \left\{\overline{P}_{h,t}(k), \overline{P}_{f,t}(k), P_{h,t}, P_{f,t}, P_t^h, P_t^f, W_t^h, W_t^f, R_{t+1}\right\},$$

such that

(i) For a given sequence of exogenous shocks $\{\mathcal{S}_t\}_{t=0}^{\infty} = \{A_t^h, A_t^f, G_t^h, G_t^f\}$ and prices $\{\mathcal{P}_t\}_{t=0}^{\infty}, \{\mathcal{Q}_t\}_{t=0}^{\infty}$ respects households' first-order conditions and maximizes the profits of intermediate, consumption, and retail goods producers.

(ii) For a given sequence of shocks $\{\mathcal{S}_t\}_{t=0}^{\infty}$ and quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty}$, $\{\mathcal{P}_t\}_{t=0}^{\infty}$ clears intermediate goods markets,

$$X_t^h = (1-\gamma)\left[\frac{W_t^h/A_t^h}{MC_t^h}\right]^{-\phi} Y_t^h + \gamma\left[\frac{W_t^h/A_t^h}{MC_t^f}\right]^{-\phi} Y_t^f,$$

$$X_t^f = (1-\gamma)\left[\frac{W_t^f/A_t^f}{MC_t^f}\right]^{-\phi} Y_t^f + \gamma\left[\frac{W_t^f/A_t^f}{MC_t^h}\right]^{-\phi} Y_t^h,$$

consumption goods markets,

$$Y_t^h = (1-\alpha)\left[\frac{P_{h,t}}{P_t^h}\right]^{-\mu} Z_t^h + \alpha\left[\frac{P_{h,t}}{P_t^f}\right]^{-\mu} Z_t^f + G_t^h,$$

$$Y_t^f = (1-\alpha)\left[\frac{P_{f,t}}{P_t^f}\right]^{-\mu} Z_t^f + \alpha\left[\frac{P_{f,t}}{P_t^h}\right]^{-\mu} Z_t^h + G_t^f,$$

retail goods markets,

$$C_t^i = Z_t^i,$$

labor markets,

$$N_t^i = \int_0^1 N_t^i(j)\,dj = L_t^i,$$

and financial markets,

$$\int_0^1 B_t^h(j)dj + \int_0^1 B_t^f(j)\,dj = 0.$$

In the equilibrium, net foreign assets evolve as follows:

$$B_{t+1}^h - B_t^h = (R_t - 1)\,B_t^h + \alpha\left(P_t^f C_t^f - P_t^h C_t^h\right)$$
$$+\gamma\left(MC_t^f Y_t^f DP_t^f - MC_t^h Y_t^h DP_t^h\right),$$

where DP_t^i is the dispersion of consumption goods production prices in country i.

3. ESTIMATION

We estimate the log-linear version of the model using the simulated method of moments (SMM) of Hansen (1982).[5] In the symmetric competitive flexible price

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TABLE 1. Nominal rigidities in the EMU

	Region	% goods in the CPI changing prices every month	% of country's GDP in the EMU GDP
Germany	h	13.5	29.1
France	f	23.9	21.6
Italy	h	10.0	17.7
Spain	h	13.3	11.0
Netherlands	f	16.2	6.4
Belgium	f	17.6	3.7
Luxembourg	f	23.0	—
Finland	f	20.3	2.0
Portugal	f	21.1	1.8

steady state, we assume that $A^i = A = 1$ and that $\tau = (1-\theta)^{-1}$. Other steady state relations are given by

$$Y = (1-\kappa)^{-\frac{\sigma}{\psi+\sigma}},\; C = (1-\kappa)^{\frac{\psi}{\psi+\sigma}},\; G = \kappa\,(1-\kappa)^{-\frac{\sigma}{\psi+\sigma}},$$
$$N = (1-\kappa)^{-\frac{\sigma}{\psi+\sigma}},\; W/P = 1,\; \text{and } R = \beta^{-1}.$$

We use quarterly data from EMU countries (OECD Economic Outlook quarterly database) subsequent to the German reunification, that is, from 1992 to 2006. Aggregates are converted in the same currency and we focus on the following seasonally adjusted series: GDP (without investment), private consumption, employment, GDP deflator, trade balance, and current account balance (as a percentage of GDP). We also take into account the evolution of the average nominal short-term interest rate in the EMU.

We build two regions based on the levels of nominal rigidities of EMU countries [see Benigno (2004)]. Table 1 indicates the percentage of goods prices in the consumer price index changing every month in EMU countries [data are borrowed from Álvarez et al. (2006)]. We assume that countries in which less than 15% of CPI goods prices change every month belong to the group with high nominal rigidities and countries in which more than 15% of CPI goods prices change every month belong to the group with low nominal rigidities. Consequently, in the first group (region h in the model), we have Germany, Spain, and Italy, and in the second group (region f in the model), we have all remaining countries.[6]

Once the two regions of the monetary union are defined, we aggregate series given the relative time-varying weights of countries in terms of GDP in the region. Inflation rates are computed using GDP deflators. Finally, we take the logs of GDP, private consumption, and employment and detrend all series using the HP filter. We estimate the model using a large sample of second-order moments. We focus on three types of moments: standard deviations (absolute or relative to standard deviation of output), first-order autocorrelations, and cross correlations. Standard deviations and autocorrelations concern all variables and cross–correlations are

TABLE 2. Estimated parameters

ψ	σ	α	γ	χ	η^h	η^f	μ
7.0776*	1.8111*	0.2675*	0.0509*	0.0009*	0.5023*	0.5024*	2.0344*
ρ_a	ρ_g	std($\zeta_{a,t}$)	std($\zeta_{g,t}$)		J-stat	Ov. Id. Stat.	p-value
0.9525*	0.8862*	0.0079*	0.0099*		10. 0875	$\chi^2(16)$	0.8035

*: 99% significant.

those of output with private consumption, output with employment, and private consumption with employment.

A first set of parameters of the model is not estimated. In particular, we set $\beta = 0.988$, which corresponds to an annual real interest rate of 4.7%, consistent with the average real interest rate over the corresponding period in the EMU. Following Rotemberg and Woodford (1997), the elasticity of substitution between varieties is $\theta = 7$, implying an average 16–17% steady state markup (compensated for at equilibrium by the optimal subsidy). The average share of public spending in the GDP is set to $\kappa = 0.25$ [see Galí and Monacelli (2008)]. The elasticity of substitution between intermediate goods is $\phi = 1.5$ [see Hairault (2002)]. Finally, we calibrate parameters of the nominal interest rate rule using standard values for the smoothing parameter $\rho_r = 0.7$ and for the feedback coefficient on aggregate inflation $\varphi = 1.5$ [see Gerlach and Schnabel (2000)].

Other parameters are estimated. The results of the estimation are reported in Table 2.

The test allowed by overidentifying conditions implies a 0.8035% p–value, which indicates that the model is not rejected by the data. Parameter values are consistent with most estimates or calibrations reported in the literature and are significant. The inverse of the Frisch elasticity ψ is equal to 7.08 and lies on the upper bound of the range put forth by Canzoneri et al. (2007). This value is consistent with a sluggish response of labor supply to various shocks in the EMU. The intertemporal elasticity of substitution of private consumption is $\sigma = 1.81$, close to standard values [see Benigno (2004)]. This parameter governs both the intensity of the transmission of monetary policy through the sensitivity of consumption to the real interest rate and the arbitrage between leisure and consumption. Home bias parameters are $\gamma = 0.051$ and $\alpha = 0.27$ and determine the degree of trade openness of intermediate and consumption goods markets. These values are consistent with those found in Faia (2007) and with standard openness measures calculated using EMU data. The estimate of $\chi = 0.0009$ is not far from that of Schmitt-Grohé and Uribe (2003). It implies that households have to pay an average annual 0.36% interest rate premium to access financial markets. Nominal rigidity parameters are very close because $\eta^h = 0.5024$ and $\eta^f = 0.5023$. Our estimate is lower than usual estimate but matches the values put forth by Alvarez et al. (2006). Finally, parameters governing shocks' processes

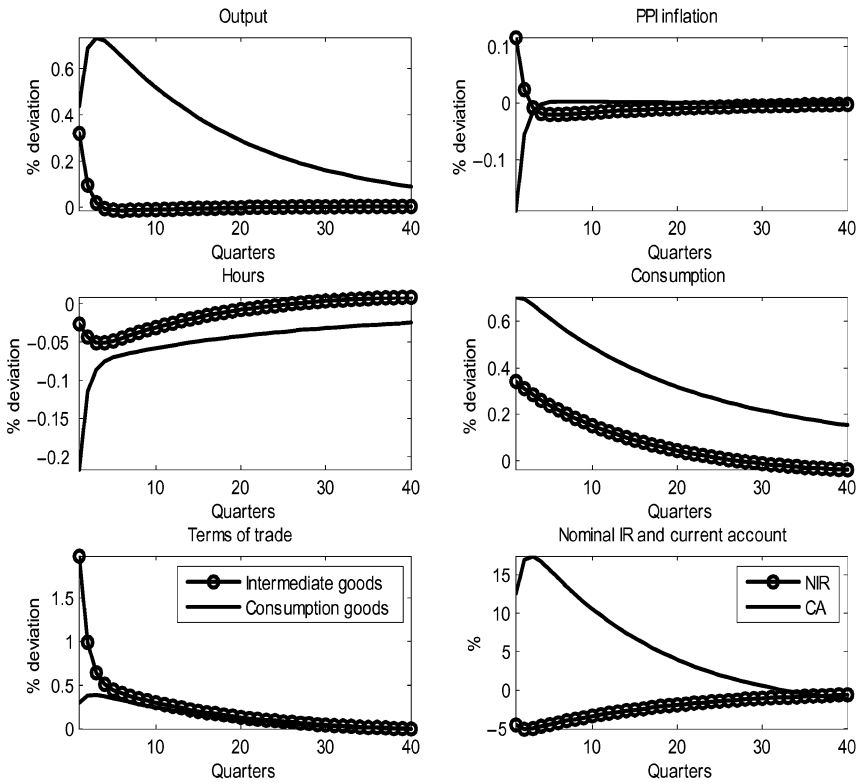

FIGURE 1. IRFs to a unit productivity shock in country h.

are $\rho_a = 0.9525$, $\rho_g = 0.8862$, std$(\zeta_{a,t}) = 0.79\%$, and std$(\zeta_{g,t}) = 0.99\%$. These estimations are consistent with most values found in the RBC literature.

4. DYNAMIC PROPERTIES

In this section we study the dynamic properties of the economy when facing asymmetric productivity and public spending shocks. Figure 1 plots the impulse response functions (IRFs) to a positive unit productivity shock in the home country. Output rises in both countries, although more substantially in country h, peaking at 0.7% for a 1% productivity shock. In country h, the remaining productivity gains are used to reduce the labor effort, about 0.25% on impact. This effect arises because the wealth effect dominates in models with separable utility functions and without physical capital. The wealth effect is reinforced by the 0.17% drop of PPI inflation in the home country.

The transmission of the shock in country f draws both on trade flows and monetary policy. Although agents in country h sustain higher production and consumption levels, they generate intermediate and consumption goods trade

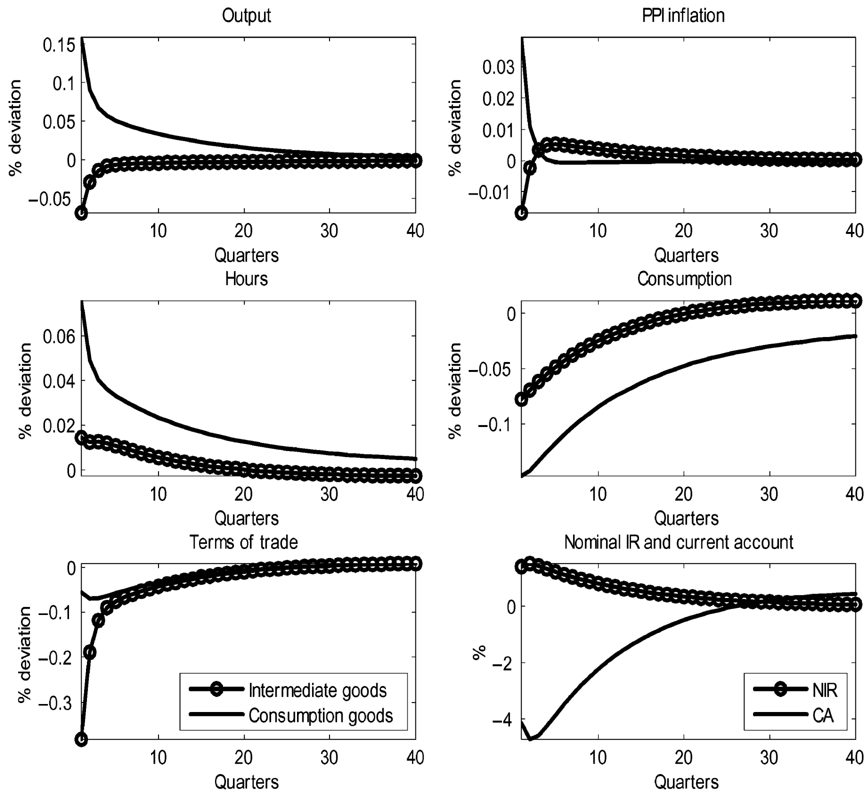

FIGURE 2. IRFs to a unit public spending shock in country h.

flows within the monetary union, which induces a positive reaction of the output in country f by about 0.3% on impact. The common monetary policy also favors a positive transmission. Reacting to aggregate inflation, the central bank lowers its nominal interest rate, which induces an increase of aggregate consumption and output in country f. The supply shock in country h thus translates into a positive demand shock in country f, which generates some PPI inflation, peaking at 0.12% on impact and returning quickly to the steady state.

Because marginal costs and production prices drop in country h and rise in country f, the reaction of both intermediate and consumption goods terms of trade is positive (terms of trade decrease in country h and increase in country f). Consumer goods prices are sluggish, which implies an undershooting of consumption goods terms of trade with respect to the response of intermediate goods terms of trade. Finally, agents in country h accumulate net foreign assets, reflecting an important wealth transfer and implying an increase of the current account roughly peaking at 15% of the quarterly steady state consumption on impact. Figure 2 plots the IRFs to a positive unit public spending shock in the home country.

Output increases by 0.15% on impact in country h, implying a rise in both home and foreign labor supply, required to sustain the quantity of consumption goods demanded in country h. Private consumption drops steadily in both countries. The drop reaches 0.13% in country h, because of the crowding-out effect. The drop is more gentle in country f, reaching 0.08% on impact. Because global demand drops in country f, output clearly falls by 0.07% on impact, although it returns very quickly to the steady state. Mechanisms behind the negative transmission of a public spending shocks in country h are twofold. First, the traditional beggar-thy-neighbor effect—reinforced by home bias in public spending—favors a negative transmission. Second, the transmission also relies (i) on the fall of private consumption in country h, implying a drop in country h imports from country f, and (ii) on the increase of the nominal interest rate implied by the reaction of the central bank to the aggregate inflation. The positive demand shock in country h thus translates into a negative demand shock in country f.

External adjustment implies a negative response of consumption goods and intermediate terms of trade (terms of trade increase in country h and decrease in country f) and an accumulation of net foreign liabilities in country h. The corresponding deficit of the current account peaks at 4.5%–5% of the quarterly steady state consumption on impact.

The IRFs based on our estimations both qualitatively and quantitatively match those obtained by Smets and Wouters (2003), based on area-wide Bayesian estimations. The productivity shock implies an increase in both output and private consumption, associated with a drop of aggregate inflation and the nominal interest rate. Interestingly and in line with Galí (1999), Smets and Wouters (2003) find that both employment and labor fall after a productivity shock. Our estimation confirms their result both in terms of sign and magnitude (about -0.25%). Finally, just as according to Smets and Wouters (2003), our IRFs after public spending shocks display a moderate increase of output, a drop of private consumption, and a weekly persistent increase of the aggregate inflation, which triggers an increase in the nominal interest rate.

5. THE WELFARE GAINS OF TRADE INTEGRATION

In this section, we measure the welfare gains arising from a deeper horizontal or vertical trade integration in the monetary union.

5.1. Welfare Indicators

We built an explicit welfare indicator on a second-order approximation of the aggregate utility function. The welfare measure can be expressed as a discounted sum of utility flows,

$$\omega = -\frac{q}{2}\sum_{t=0}^{\infty}\beta^t E_0\{\ell_t\} + \text{t.i.p.} + O(\|\xi^3\|),$$

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where $q = (1-\kappa)^{-\sigma(1-\psi)/(\psi+\sigma)}$, t.i.p. gathers terms independent of the problem, and $O(\|\xi^3\|)$ are terms of order three or higher. In this expression, the instant welfare contribution ℓ_t is a quadratic function of deviations of key economic variables from their natural equilibrium path,[7]

$$\ell_t = \frac{\theta}{2k^h}(\pi_{h,t} - \widetilde{\pi}_{h,t})^2 + \frac{\theta}{2k^f}(\pi_{f,t} - \widetilde{\pi}_{f,t})^2 + \frac{\sigma + \psi(1-\kappa)}{1-\kappa}\left(y_t^u - \widetilde{y}_t^u\right)^2$$
$$+ (1-\kappa)\,\varsigma_\alpha (s_t - \widetilde{s}_t)^2 + \varsigma_\gamma (\sigma_t - \widetilde{\sigma}_t)^2 + \sigma(1-\kappa)\left(c_t^r - \widetilde{c}_t^r\right)^2$$
$$+ \psi\left(n_t^r - \widetilde{n}_t^r\right)^2, \tag{6}$$

where $k^i = (1-\eta^i\beta)(1-\eta^i)/\eta^i$. In equation (6), a tilde denotes the path of variables in the natural equilibrium, defined as the equilibrium under flexible prices and complete and perfectly integrated asset markets. Superscripts u and r respectively stand for aggregate and relative variables.

The welfare measure ω penalizes national PPI inflation rates, the aggregate output gap, the relative consumption gap, the relative hours gap, and terms-of-trade gaps. The weights assigned to national inflation rates are sensitive to the degree of price stickiness through the values of k^i. Parameter k^i depends negatively on the degree of price rigidities, so that higher weights are given to inflation rates when prices are stickier.

Arguments of our loss function directly relate to other microfounded loss functions, such as those derived by Benigno (2004) or Beetsma and Jensen (2005). In particular, consistency with the assumptions made by Benigno (2004) requires setting $\gamma = 0$, $\alpha = \frac{1}{2}$, implying $c_t^h = c_t^f = c_t$ and $\mu = 1$. The equilibrium of consumption goods markets then implies

$$n_t^r - \widetilde{n}_t^r = y_t^r - \widetilde{y}_t^r = -\frac{(1-\kappa)}{2}(s_t - \widetilde{s}_t) \tag{7}$$

and

$$y_t^u - \widetilde{y}_t^u = (1-\kappa)\left(c_t^u - \widetilde{c}_t^u\right). \tag{8}$$

Using (7) and (8), ℓ_t becomes

$$\ell_t^{'} = \frac{\theta}{2k^h}(\pi_{h,t} - \widetilde{\pi}_{h,t})^2 + \frac{\theta}{2k^f}(\pi_{f,t} - \widetilde{\pi}_{f,t})^2 + \ell_c\left(c_t^u - \widetilde{c}_t^u\right)^2 + \ell_s(s_t - \widetilde{s}_t)^2, \tag{9}$$

where

$$\ell_y = (1-\kappa)\,\sigma + \psi(1-\kappa), \quad \ell_s = \frac{(1-\kappa)(1+\psi(1-\kappa))}{4}.$$

Arguments and the value of coefficients of (9) are then exactly those of the loss function of Benigno (2004). We then compute the consumption equivalent welfare

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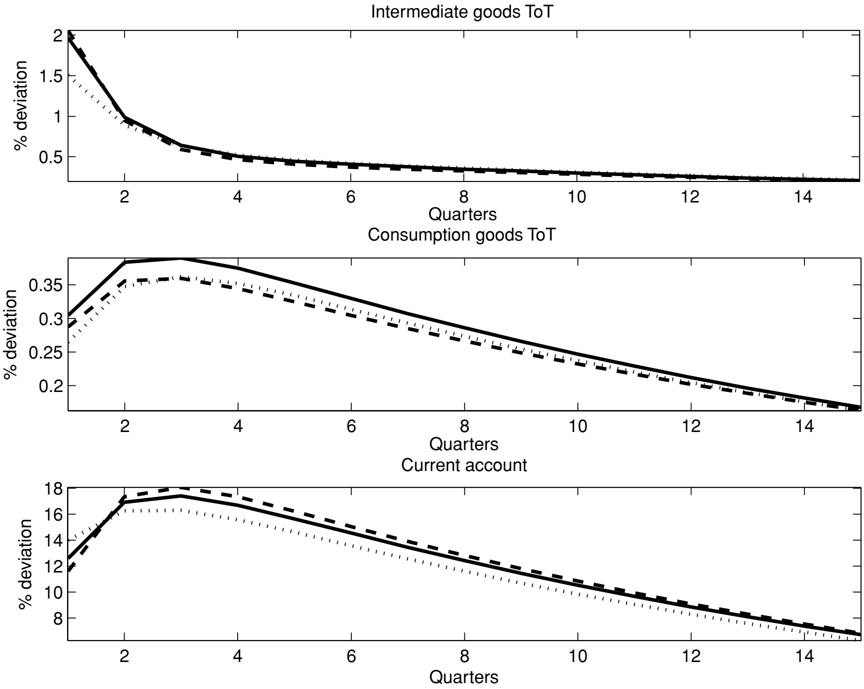

FIGURE 3. IRFs to a unit productivity shock in country *h*—solid line: baseline; dashed line: after horizontal trade integration; dotted line: after vertical trade integration.

loss. As in Beetsma and Jensen (2005), Ψ is defined according to

$$\Psi = 100 \cdot \left\{ \frac{1-\beta}{(1-\kappa)\left[\sigma + \psi(1-\kappa)\right]} (\omega_1 - \omega_0) \right\}^{\frac{1}{2}}, \quad \textbf{(10)}$$

where ω_0 measures the welfare for a given reference situation. Ψ converts the welfare gains associated with a Pareto-superior equilibrium ω_1 into a sizable yardstick in terms of permanent increase of consumption for an unchanged labor effort.

5.2. Baseline Scenario

Before getting more deeply into the results, we first describe the impact of an increase of trade integration on the external adjustment after asymmetric shocks.[8] Basically, Figures 3 and 4 show how trade integration affects the response of intermediate and consumption goods terms of trade, as well as the dynamics of the current account, respectively after a productivity shock and a public spending shock.

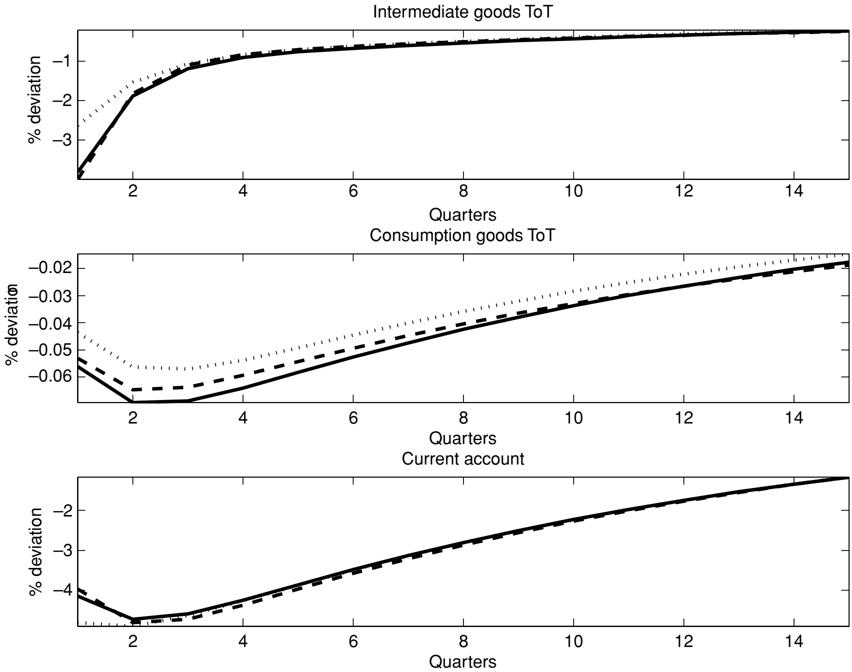

FIGURE 4. IRFs to a unit public spending shock in country h—solid line: baseline; dashed line: after horizontal trade integration; dotted line: after vertical trade integration.

Both figures show that an increase of α and γ reduces the magnitude of terms-of-trade adjustments, because quantities are more responsive to variations of terms of trade [see Warnock (2003) and Coeurdacier (2008) for an extensive analysis]. As a consequence, smaller fluctuations of terms of trade are needed to meet the external equilibrium. Differences appear quite clearly, however, whether the increase of trade integration is vertical or horizontal under incomplete financial markets. An increase of vertical trade integration (γ) triggers a sharp reduction in fluctuations of both intermediate and consumption goods terms of trade, whereas an increase of horizontal trade integration (α) has little or no effect on intermediate terms of trade but clearly reduces fluctuations of consumption goods terms of trade. Another significant difference between trade integration patterns is the impact on current account fluctuations. Whereas vertical trade integration is associated with a reduction (or a very small increase) of current account fluctuations, horizontal trade integration is found to increase the response of the current account, more significantly in the case of productivity shocks. In a nutshell, whereas vertical trade integration reduces the overall need for external adjustment, horizontal trade integration has mixed effects on external adjustment conditions.

These first elements are then complemented by simulation results.[9] Using the baseline estimation and simulating the model, Table 3 contrasts the welfare gains or

TABLE 3. The welfare gains of a 10% deeper horizontal (α) or vertical (γ) trade integration under incomplete financial markets

	Ψ (%)	Standard deviation (%)						
		$\widehat{\pi}_{h,t}$	$\widehat{\pi}_{f,t}$	$\widehat{y}_t^u$	$\widehat{s}_t$	$\widehat{\sigma}_t$	$\widehat{c}_t^r$	$\widehat{n}_t^r$
Baseline	—	0.204	0.201	0.015	1.033	1.161	0.170	0.644
$\alpha_1 = 1.1\alpha_0$ (I)	−2.03	0.201	0.198	0.015	0.990	1.151	0.174	0.653
$\alpha_1 = 1.15\alpha_0$ (II)	−1.10	0.200	0.197	0.015	0.970	1.147	0.175	0.657
$\gamma_1 = 1.1\gamma_0$ (III)	7.67	0.201	0.197	0.015	1.019	1.128	0.170	0.636
$\gamma_1 = 1.15\gamma_0$ (IV)	9.33	0.199	0.195	0.015	1.012	1.113	0.171	0.633
(I) + (III)	7.45	0.198	0.194	0.015	0.977	1.117	0.174	0.645
(II) + (IV)	9.37	0.195	0.191	0.015	0.951	1.096	0.175	0.645

Note: Variables with a circumflex denote deviations from natural equilibrium.

losses (Ψ) arising from a deeper horizontal or vertical trade integration consistent with the evidence documented by Baldwin (2006), that is, a 10% increase of α or γ. An additional scenario where trade integration increases by 15% is also considered. Finally, Table 3 details the evolution of the volatility of variables entering in to the welfare loss function.

On one hand, a 10% increase of γ generates large welfare gains, equivalent to an average 7.67% increase in permanent consumption. The overall volatility of terms entering the loss function is clearly dampened. When vertical trade increases, the composition of consumption goods produced becomes more similar, which implies that shocks affecting the production of intermediate goods asymmetrically have more similar effects on output and marginal costs. This mechanism also contributes to lowering the PPI national inflation rates, as illustrated by the new Keynesian Phillips curves. If marginal costs, the driving force behind the PPI inflation rates, are more correlated, then the PPI inflation rates are affected in the same way. The adequacy of the common monetary policy to national inflation rates and business cycles increases, which enhances its effectiveness and reduces the volatility of national inflation rates. Furthermore, as shown by Figures 3 and 4, the overall need for external adjustment is clearly reduced, which favors a drop in the volatility of terms-of-trade gaps, relative hours gaps, and relative consumption gaps and translates into aggregate welfare gains.

On the other hand, in the baseline scenario, a 10% increase of horizontal trade—measured by a 10% increase of α—implies an average welfare loss equivalent to a 2.03% fall in permanent consumption.

A close examination of volatilities shows that the distance of national inflation rates and consumption goods terms of trade from their natural equilibrium path is clearly reduced. Indeed, the volatility of national inflation rates drops by 1.53% and the volatility of terms-of-trade gaps by 4.38%, which has welfare-improving consequences. Because the composition of the CPI inflation rates and private consumption bundles becomes more similar, for a given monetary policy rule, monetary policy becomes more effective and its ability to stabilize national PPI inflation rates increases. These lower national PPI inflation rates result in a lower pressure on consumption goods terms of trade, which clearly reduces their volatility.

However, although external adjustment relies less on consumption goods terms of trade, the volatility of the current account is enhanced, which leads to welfare losses that more than compensate for the previous welfare gains. These losses are imputable to the increased distance of relative hours and relative private consumptions from their natural level. The fact that agents use the current account more intensively to adjust asymmetric shocks implies important wealth transfers that deeply affect relative labor supplies and private consumptions. Debtor (resp. creditor) households need to increase (resp. decrease) their labor supply and decrease (resp. increase) their consumption level to increase (resp. decrease) their net earnings and repay their debts (resp. lower their savings) in the medium run. The magnitude of the latter effect clearly depends on the level of costs levied by financial intermediaries. Indeed, these costs increase the sensitivity of consumption, labor efforts, and equilibrium wages (and thereby marginal costs) to variations of net foreign assets or liabilities.

Summing up, under incomplete financial markets, horizontal trade integration increases the overall need for external adjustment and thereby the magnitude of wealth transfers. It results in increased business cycle asymmetries and aggregate welfare losses.

Our results match those of other studies that measure the welfare gains associated with the reduction of various distortions in the economy. Canzoneri et al. (2007) estimate that the welfare costs of nominal inertia can reach 4%–5%, mostly depending on the degree of persistence in the economy. In our model, the value of the Frisch elasticity is low, the assumption of imperfect risk sharing adds an important source of persistence, and the estimated persistence of shocks is quite high. The overall persistence is thus important and, consistent with Canzoneri et al. (2007), nominal inertia is quite costly in terms of welfare in our model. Several studies also quantify the welfare gains of financial market integration, building on higher risk sharing and consumption smoothing. For example, Van Wincoop (1999) finds that the welfare gains from risk sharing range from 1% to more than 7% of permanent consumption. Those welfare gains could actually be much higher according to previous studies using alternative methods to measure financial market integration [see Lewis (1996)]. More recently, Demyanyk and Volosovych (2008) document that the welfare gains of financial markets integration range from 1% of permanent consumption for EMU members to more than 8% for new European Union members. In our model, both sources of welfare losses (nominal inertia and imperfect risk sharing) are combined and yield significant welfare losses. As suggested by Dotsey and Ireland (1996), the combination of various frictions may actually result in important welfare losses.

TABLE 4. The welfare gains of a 10% deeper horizontal (α) or vertical (γ) trade integration under complete financial markets

		Standard deviation (%)						
	Ψ (%)	$\widehat{\pi}_{h,t}$	$\widehat{\pi}_{f,t}$	$\widehat{y}_t^u$	$\widehat{s}_t$	$\widehat{\sigma}_t$	$\widehat{c}_t^r$	$\widehat{n}_t^r$
Baseline	—	0.278	0.275	0.015	0.407	1.208	0.025	0.334
$\alpha_1 = 1.1\alpha_0$ (I)	6.12	0.272	0.269	0.015	0.391	1.235	0.022	0.341
$\alpha_1 = 1.15\alpha_0$ (II)	7.32	0.270	0.266	0.015	0.383	1.247	0.021	0.344
$\gamma_1 = 1.1\gamma_0$ (III)	8.70	0.273	0.269	0.015	0.407	1.142	0.025	0.315
$\gamma_1 = 1.15\gamma_0$ (IV)	10.57	0.270	0.266	0.015	0.406	1.111	0.025	0.307
(I) + (III)	10.50	0.267	0.263	0.015	0.390	1.169	0.022	0.323
(II) + (IV)	12.62	0.262	0.258	0.015	0.383	1.149	0.021	0.317

Note: Variables with a circumflex denote deviations from natural equilibrium.

5.3. Complete Financial Markets

In this paragraph, we proceed to the same experiments under complete financial markets. In this case, households have access to a continuum of Arrow–Debreu securities, which allows them to insure against asymmetric shocks. In this case, the marginal utility of private consumption is equal across households, countries, and states of nature. This result is summarized by the following risk-sharing condition:

$$P_t^h C_t^h(j)^\sigma = P_t^f C_t^f(j)^\sigma.$$

As a consequence, the dynamics of the external adjustment relies on terms of trade only and asymmetric shocks do not imply any wealth transfer. Using the baseline parametrization, Table 4 presents the welfare gains of a 10% horizontal and vertical trade integration when financial markets are complete in the monetary union.

The results described in Table 4 shed some additional light on the results under incomplete financial markets. Under complete asset markets, both horizontal and vertical trade integration yield welfare gains, ranging from 6.12% in the case of a 10% increase of horizontal trade integration to 12.62% in the case of a 15% joint increase of horizontal and vertical trade integration. Financial market incompleteness thus appears to be a crucial assumption in determining both the signs and the magnitudes of the welfare gains implied by horizontal and vertical trade integration.

6. SENSITIVITY ANALYSIS

In this section, we investigate the robustness of our results to a wide range of parameter variations. The simulations have been run to evaluate the sensitivity of our results to the asymmetry in the pattern of nominal rigidities. Because these

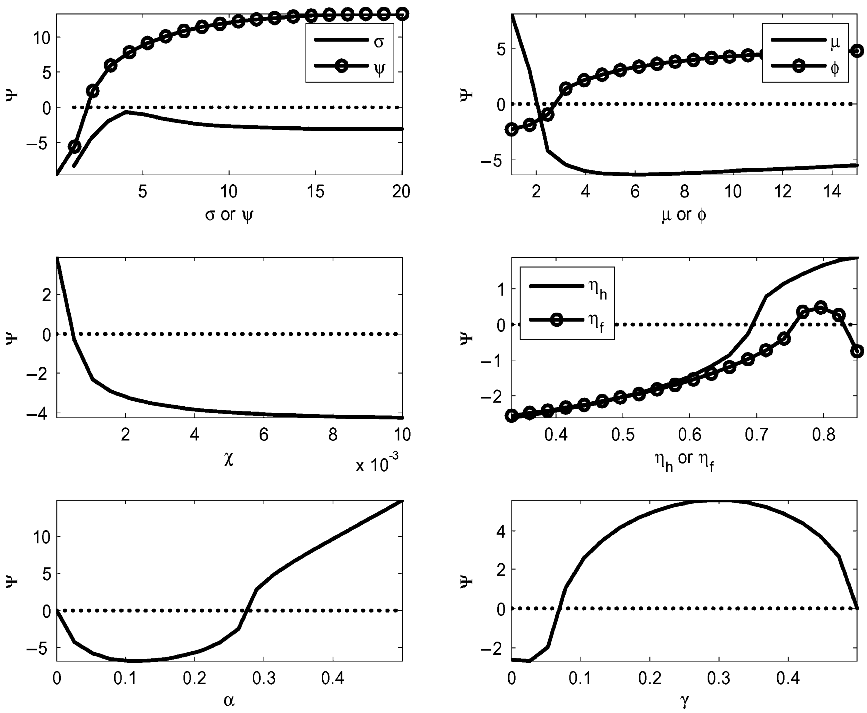

FIGURE 5. Sensitivity of the welfare gains or losses of a 10% increase in horizontal trade: incomplete financial markets.

simulations show that asymmetries in the pattern of nominal rigidities do not play a significant role in generating our results, they are not reported.

Figure 5 reports the sensitivity of welfare gains or losses associated with a 10% increase in horizontal trade to different variations in the set of structural parameters in the case of incomplete financial markets. Figure 5 once more highlights the interaction between two effects when horizontal trade integration increases: (i) welfare gains related to the lower costs of nominal rigidities and (ii) welfare losses caused by the increased volatility of the current account. Depending on parameterization, the overall welfare effect of horizontal trade integration is either positive or negative.

When portfolio management costs (χ) fall below a certain threshold, between 0.09% and 0.1%, or when nominal rigidities are beyond 0.75, horizontal trade integration generates welfare gains. This is the case either because the enhanced volatility of the current account become less costly or because the reduction of national PPI inflation rates generates higher welfare gains. These results clearly show that frictions in financial markets are a key assumption to generate our results. This assumption introduces welfare losses related to imperfect risk sharing among members of the monetary union. The sensitivity analysis reveals that small frictions

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($\chi = 0.09\%$ implies that households have to pay an average 0.36% annual interest rate premium to access financial markets) are sufficient to mitigate the welfare gains from lower inflation rates when horizontal trade increases.

The sensitivity of welfare gains/losses to variations of the elasticity of substitution between intermediate or consumption goods also illustrates the mechanism behind welfare gains or losses. As the elasticity of substitution between consumption goods (μ) increases, changes in the volatility of consumption goods terms of trade implied by enhanced horizontal trade integration are lower. At equilibrium, the volatility of PPI inflation rates is thus reduced, whereas the impact of μ on the volatility of the current account is clearly positive. Welfare gains related to lower national inflation rates are thus dampened, whereas welfare losses caused by the increased volatility of the current account increase. As a consequence, net welfare gains from horizontal trade integration depend negatively on the elasticity of substitution between consumption goods. In contrast, as the elasticity of substitution between intermediate goods (ϕ) increases, intermediate terms of trade are less required to fluctuate to reach the equilibrium on intermediate goods markets, *ceteris paribus*. As a consequence, the rise of the volatility of intermediate terms of trade, relative hours, and relative consumption gaps are reduced when horizontal trade increases, which has a positive impact on welfare gains.

The sensitivity of welfare gains to the inverse of the Frisch elasticity (ψ) and the risk-aversion parameter (σ) is also investigated. When the intertemporal elasticity of labor supply (ψ) increases, the volatility of hours decreases at equilibrium. Because welfare losses relate to the magnitude of wealth effects, and hence to the response of labor supply, lower responses of labor supply imply lower overall welfare losses or higher overall welfare gains when horizontal trade integration increases. The effect of the risk-aversion parameter is somehow surprising. The risk-aversion parameter governs the willingness of households to smooth their consumption over time when undergoing unexpected asymmetric shocks, which is associated with an increased use of financial markets, and should lead to higher welfare losses. However, Figure 5 tells us that these aspects are more than compensated for by the drop of the volatility of terms of trade and of national inflation rates. Risk aversion is thus found to have a (quantitatively small) positive impact on the welfare gains generated by an increase in horizontal trade integration.

Finally, Figure 5 reports the sensitivity of welfare gains/losses to variations in the level of trade openness (α and γ). Clearly, the welfare gains arising in the case of a 10% increase in horizontal trade integration are nonlinear in α and γ. More specifically, the welfare costs undergone because of asymmetries triggered by the increase of the volatility of the current account are clearly surpassed by standard welfare gains when trade openness is high; that is, $\alpha > 0.3$ and $\gamma > 0.1$.

Second, Figure 6 reports the sensitivity of welfare gains or losses associated with a 10% increase in vertical trade to different variations in the set of structural

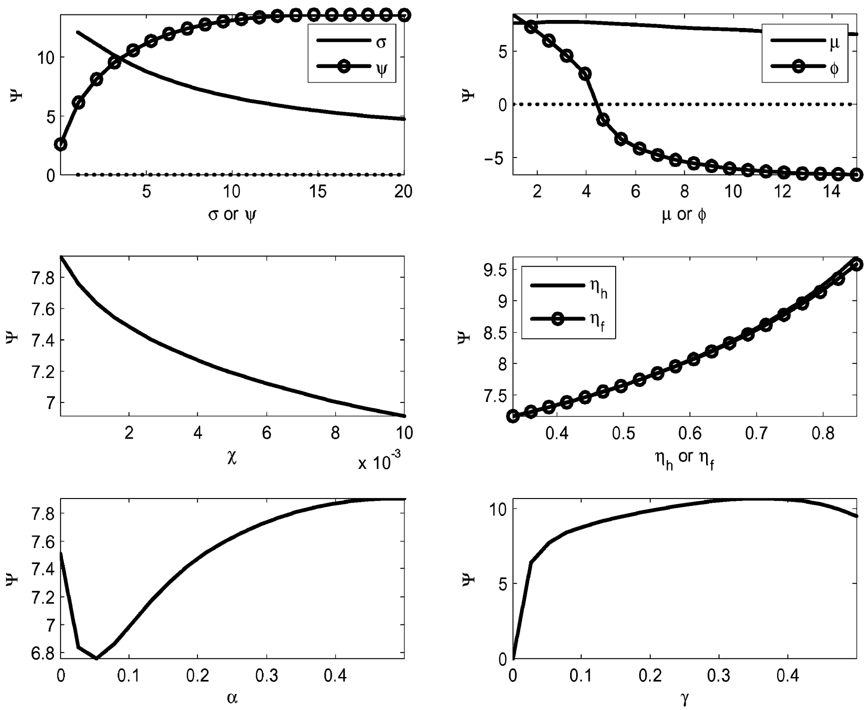

FIGURE 6. Sensitivity of the welfare gains or losses of a 10% increase in vertical trade: incomplete financial markets.

parameters. Welfare gains generated by a 10% deeper vertical trade integration are clearly increasing with the degree of nominal rigidity (η), because the reduction of national inflation rates is both enhanced when vertical trade integration increases and more weighted in the loss function.

These gains are barely sensitive to the level of portfolio management costs (χ), which confirms that financial markets do not play an important role when trade integration is vertical.

The welfare gains of deeper vertical trade integration also clearly decrease with the degree of substitutability between goods. Although the decrease is moderate when the substitutability of consumption goods (μ) increases, welfare gains decline more sharply when the substitutability of intermediate goods (ϕ) increases. In general, higher substitutability reduces the required variations of terms-of-trade volatility when vertical trade increases. As a consequence, as substitutability increases, changes in intermediate and consumption goods terms-of-trade volatility become very small when trade integration increases, which impacts welfare gains negatively. This effect is much stronger for the substitutability between intermediate goods because nominal rigidities bear on consumption goods prices whereas intermediate goods prices are flexible. When the

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substitutability between intermediate goods increases, the volatility of intermediate terms-of-trade gaps tends to become unaffected and the impact of vertical trade integration on welfare vanishes. Because consumption goods terms of trade are staggered, the welfare gains of vertical trade integration do not completely fade away.

As in the case of horizontal trade integration, an increase of the intertemporal elasticity of substitution of labor supply (ψ) has a positive impact on the welfare gains of vertical trade integration. Indeed, the softening effect of an increase of ψ on the volatility of labor supplies affects welfare gains positively.

Finally, the effect of trade openness on the welfare gains arising after a 10% increase in vertical trade integration depends positively on the level of trade openness in both consumption goods markets (α) and intermediate goods markets (γ).

7. CONCLUSIONS

This paper shows that horizontal and vertical trade integration have different outcomes in terms of welfare in a monetary union characterized by business cycle asymmetries and inflation differentials. In both cases, a deeper trade integration reduces inflation differentials by favoring a better diffusion of shocks from one country to another, through increased trade flows. This increased macroeconomic interdependence helps the common monetary policy to be in line with national situations. Equilibrium national inflation rates decrease, and trade integration thus generates welfare gains.

However, under incomplete financial markets, horizontal trade integration increases the volatility of the current account, whereas vertical trade integration reduces the overall need for external adjustment in case of asymmetric shocks. As a consequence, horizontal trade integration implies welfare losses that might exceed the previous welfare gains. For the baseline estimates presented in this paper, horizontal trade integration produces welfare losses equivalent to an average 2.03% drop of permanent consumption and vertical trade integration generates welfare gains that amount to an average 7.67% of permanent consumption. However, an extensive sensitivity analysis indicates that financial market incompleteness and nominal rigidities play a key role in the pattern of welfare gains or losses.

The main conclusion of the paper is that financial frictions, as well as their interactions with nominal rigidities, should be taken carefully into account in analyzing business cycle asymmetries in open economies and/or monetary unions.

NOTES

1. This increase fits the actual consensus concerning the effect of the EMU on intrazone trade [see Baldwin (2006)].

2. Nominal exchange rate issues per se, as well as the analysis of the conditions underlying the adoption of a common currency, are beyond the scope of the paper.

3. The definition of terms of trade is arbitrarily chosen to be consistent with the definition of the real exchange rate, as in Galí and Monacelli (2005). An increase of Σ_t thus implies that intermediate terms of trade actually drop for country h and increase for country f.

4. Here again, the definition of terms of trade is arbitrarily chosen to be consistent with the definition of the real exchange rate. An increase of S_t thus implies that final terms of trade actually drop for country h and increase for country f.

5. The log-linear approximation of the model is presented in Appendix A.1.

6. Austria, Greece, and Ireland are not taken into account because data are unavailable.

7. Appendix A.2 details the derivation.

8. An increase of 50% is assumed here to ease the analysis of the IRFs and make the impact of trade integration clearer.

9. The model is simulated 1,000 times over 120 periods by feeding it with random productivity and public spending innovations each period. The welfare and standard deviations are then averaged over the number of simulations.

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APPENDIX

A.1. LOG–LINEAR APPROXIMATION OF THE MODEL

Euler equations and labor supply

$$\sigma E_t \left\{c_{t+1}^h - c_t^h\right\} = r_{t+1} - E_t\{(1-\alpha)\,\pi_{h,t+1} + \alpha\pi_{f,t+1}\} - \chi(1-\kappa)^{\frac{\psi}{\psi+\sigma}} b_{t+1}^h$$

$$\sigma E_t \left\{c_{t+1}^f - c_t^f\right\} = r_{t+1} - E_t\{(1-\alpha)\,\pi_{f,t+1} + \alpha\pi_{h,t+1}\} + \chi(1-\kappa)^{\frac{\psi}{\psi+\sigma}} b_{t+1}^h$$

$$\psi n_t^h + \sigma c_t^h = w_t^h - p_{h,t} - \alpha s_t \qquad \psi n_t^f + \sigma c_t^f = w_t^f - p_{f,t} + \alpha s_t$$

Inflation and terms of trade

$$\pi_{h,t} = \beta E_t\{\pi_{h,t+1}\} + \frac{(1-\eta^h\beta)(1-\eta^h)}{\eta^h}\left[(1-\gamma)\left(w_t^h - a_t^h\right) + \gamma\left(w_t^f - a_t^f\right) - p_{h,t}\right]$$

$$\pi_{f,t} = \beta E_t\{\pi_{f,t+1}\} + \frac{(1-\eta^f\beta)(1-\eta^f)}{\eta^f}\left[(1-\gamma)\left(w_t^f - a_t^f\right) + \gamma\left(w_t^h - a_t^h\right) - p_{f,t}\right]$$

$$\pi_{h,t} = p_{h,t} - p_{h,t-1} \qquad \pi_{f,t} = p_{f,t} - p_{f,t-1}$$

$$s_t - s_{t-1} = \pi_{f,t} - \pi_{h,t} \qquad \sigma_t = w_t^f - w_t^h + a_t^h - a_t^f$$

Goods-market clearing

$$y_t^h = (1-\kappa)\left[(1-\alpha)\,c_t^h + \alpha c_t^f + 2\alpha\mu(1-\alpha)\,s_t\right] + \kappa g_t^h$$

$$y_t^f = (1-\kappa)\left[(1-\alpha)\,c_t^f + \alpha c_t^h - 2\alpha\mu(1-\alpha)\,s_t\right] + \kappa g_t^f$$

$$a_t^h + n_t^h = (1-\gamma)\,y_t^h + \gamma y_t^f + 2\phi\gamma(1-\gamma)\,\sigma_t$$

$$a_t^f + n_t^f = (1-\gamma)\,y_t^f + \gamma y_t^h - 2\phi\gamma(1-\gamma)\,\sigma_t$$

Current account

$$b_{t+1}^h - b_t^h = \delta b_t^h + \alpha\left\{c_t^f - c_t^h + [2\mu(1-\alpha) - 1]\,s_t\right\}$$
$$+ \frac{\gamma}{1-\kappa}\left\{y_t^f - y_t^h + [2\phi(1-\gamma) - 1]\,\sigma_t\right\}$$

Interest-rate rule

$$r_{t+1} = \rho_r r_t + \beta\varphi\left(\tfrac{1}{2}\pi_{h,t} + \tfrac{1}{2}\pi_{f,t}\right)$$

A.2. THE WELFARE LOSS FUNCTION

The welfare criterion is written

$$\omega_T = \sum_{t=0}^{\infty}\beta^t E_0\left\{\frac{1}{2}\int_0^1\left[\frac{C_t^h(j)^{1-\sigma}}{1-\sigma} - \frac{N_t^h(j)^{1+\psi}}{1+\psi}\right]dj + \frac{1}{2}\int_0^1\left[\frac{C_t^f(j)^{1-\sigma}}{1-\sigma} - \frac{N_t^f(j)^{1+\psi}}{1+\psi}\right]dj\right\}.$$

After using the symmetry among agents, we define

$$u_{c,t}^u = \frac{1}{2(1-\sigma)}\left(C_t^h\right)^{1-\sigma} + \frac{1}{2(1-\sigma)}\left(C_t^f\right)^{1-\sigma},$$
$$u_{n,t}^u = \frac{1}{2(1+\psi)}\left(N_t^h\right)^{1+\psi} + \frac{1}{2(1+\psi)}\left(N_t^f\right)^{1+\psi}.$$

We compute welfare derivations through a second-order approximation of variables to their steady state values and for second-order expressions of shocks equal to zero, that is, $(a_t^i)^2 = (g_t^i)^2 = 0$. Before approximating, we need to state

$$\frac{1}{2}\left[\left(c_t^h\right)^2 + \left(c_t^f\right)^2\right] = \left(c_t^u\right)^2 + \left(c_t^r\right)^2,$$

$$\frac{1}{2}\left[\left(n_t^h\right)^2 + \left(n_t^f\right)^2\right] = \left(n_t^u\right)^2 + \left(n_t^r\right)^2.$$

A second-order approximation to $u_{c,t}^u$ is written

$$U_{C,t}^u \simeq \frac{C^{1-\sigma}}{1-\sigma} + C^{1-\sigma}\left\{c_t^u + \frac{1-\sigma}{2}\left[\left(c_t^u\right)^2 + \left(c_t^r\right)^2\right]\right\} + O(\|\xi^3\|),$$

where $O(\|\xi^3\|)$ gathers terms of higher order.

A second-order approximation to $u_{n,t}^u$ is written

$$U_{N,t}^u \simeq \frac{N^{1+\psi}}{1+\psi} + N^{1+\psi}\left\{n_t^u + \frac{1+\psi}{2}\left[\left(n_t^u\right)^2 + \left(n_t^r\right)^2\right]\right\} + O(\|\xi^3\|). \tag{A.1}$$

Recalling that $(a_t^i)^2 = 0$, a second-order approximation of intermediate goods markets gives

$$n_t^h + \frac{1}{2}\left(a_t^h + n_t^h\right)^2 = (1-\gamma)\left[y_t^h + \phi\gamma\sigma_t + \frac{1}{2}\left(y_t^h + \phi\gamma\sigma_t\right)^2 + dp_t^h\right]$$

$$+\gamma\left\{y_t^f + \phi(1-\gamma)\sigma_t + \frac{1}{2}\left[y_t^f + \phi(1-\gamma)\sigma_t\right]^2 + dp_t^f\right\} + \text{t.i.p.} + O(\|\xi^3\|),$$

$$n_t^f + \frac{1}{2}\left(a_t^f + n_t^f\right)^2 = (1-\gamma)\left[y_t^f - \phi\gamma\sigma_t + \frac{1}{2}\left(y_t^f - \phi\gamma\sigma_t\right)^2 + dp_t^f\right]$$

$$+\gamma\left\{y_t^h - \phi(1-\gamma)\sigma_t + \frac{1}{2}\left[y_t^h - \phi(1-\gamma)\sigma_t\right]^2 + dp_t^h\right\} + \text{t.i.p.} + O(\|\xi^3\|),$$

where t.i.p. stands for terms that are independent of the problem and where

$$dp_t^i = \frac{\theta}{2}\text{var}(p_{i,t}),$$

implying that $(dp_t^i)^2 \in O(\|\xi^3\|)$. Combining the last two expressions, we get

$$n_t^u + \frac{1}{2}\left(n_t^u\right)^2 + \frac{1}{2}\left(n_t^r\right)^2 = y_t^u + \frac{1}{2}\left(y_t^u\right)^2 + \frac{1}{2}\left(y_t^r\right)^2 + \frac{1}{2}\phi\gamma(1-\gamma)(\sigma_t)^2 + \frac{\theta}{4}\text{var}(p_{h,t})$$

$$+ \frac{\theta}{4}\text{var}(p_{f,t}) - \frac{1}{2}a_t^h n_t^h - \frac{1}{2}a_t^f n_t^f + \text{t.i.p.} + O(\|\xi^3\|).$$

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Combining with (A.1), we get

$$u^u_{N,t} \simeq N^{1+\psi}\left\{y^u_t + \frac{1}{2}\left(y^u_t\right)^2 + \frac{1}{2}\left(y^r_t\right)^2 - \frac{1}{2}a^h_t n^h_t - \frac{1}{2}a^f_t n^f_t + \frac{\phi\gamma\,(1-\gamma)}{2}\,(\sigma_t)^2 + \frac{\theta}{4}\text{var}(p_{h,t}) + \frac{\theta}{4}\text{var}(p_{f,t}) + \frac{\psi}{2}\left(n^u_t\right)^2 + \frac{\psi}{2}\left(n^r_t\right)^2\right\} + \text{t.i.p.} + O(\|\xi^3\|).$$

Now turning to $u^u_{c,t}$, we compute a second-order approximation to consumption goods-market conditions, while recalling that $(g^i_t)^2 = 0$, and that the equilibrium of retail goods market implies $z^i_t = c^i_t$,

$$y^h_t + \frac{1}{2}\left(y^h_t\right)^2 = (1-\alpha)(1-\kappa)\left[c^h_t + \mu\alpha s_t + \frac{1}{2}\left(c^h_t + \mu\alpha s_t\right)^2\right] + \alpha(1-\kappa)\left\{c^f_t + \mu(1-\alpha)s_t + \frac{1}{2}\left[c^f_t + \mu(1-\alpha)s_t\right]^2\right\} + \text{t.i.p.} + O(\|\xi^3\|),$$

$$y^f_t + \frac{1}{2}\left(y^f_t\right)^2 = (1-\alpha)(1-\kappa)\left[c^f_t - \mu\alpha s_t + \frac{1}{2}\left(c^f_t - \mu\alpha s_t\right)^2\right] + \alpha(1-\kappa)\left\{c^h_t - \mu(1-\alpha)s_t + \frac{1}{2}\left[c^h_t - \mu(1-\alpha)s_t\right]^2\right\} + \text{t.i.p.} + O(\|\xi^3\|),$$

which implies

$$y^u_t + \frac{1}{2}\left(y^u_t\right)^2 + \frac{1}{2}\left(n^r_t\right)^2 = (1-\kappa)\left[c^u_t + \frac{1}{2}\left(c^u_t\right)^2 + \frac{1}{2}\left(c^r_t\right)^2\right] + \frac{(1-\kappa)\,\mu\alpha\,(1-\alpha)}{2}\,(s_t)^2 + \text{t.i.p.} + O(\|\xi^3\|)$$

or

$$c^u_t + \frac{1}{2}\left(c^u_t\right)^2 + \frac{1}{2}\left(c^r_t\right)^2 = \frac{1}{(1-\kappa)}\left[y^u_t + \frac{1}{2}\left(y^u_t\right)^2 + \frac{1}{2}\left(n^r_t\right)^2\right] - \frac{\mu\alpha\,(1-\alpha)}{2}\,(s_t)^2 + \text{t.i.p.} + O(\|\xi^3\|).$$

$u^u_{c,t}$ now is written

$$u^u_{c,t} \simeq C^{1-\sigma}\left\{\frac{1}{1-\kappa}\left[y^u_t + \frac{1}{2}\left(y^u_t\right)^2 + \frac{1}{2}\left(n^r_t\right)^2\right] - \frac{\mu\alpha\,(1-\alpha)}{2}\,(s_t)^2 - \frac{\sigma}{4}\left(c^h_t\right)^2 - \frac{\sigma}{4}\left(c^f_t\right)^2\right\} + \text{t.i.p.} + O(\|\xi^3\|).$$

Collecting terms, we get

$$u^u_t = u^u_{c,t} - u^u_{n,t} \simeq C^{1-\sigma}\left\{\frac{1}{1-\kappa}\left[y^u_t + \frac{1}{2}\left(y^u_t\right)^2 + \frac{1}{2}\left(n^r_t\right)^2\right] - \frac{\mu\alpha\,(1-\alpha)}{2}\,(s_t)^2 - \frac{\sigma}{4}\left(c^h_t\right)^2 - \frac{\sigma}{4}\left(c^f_t\right)^2\right\}$$

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$$- N^{1+\psi}\left[y_t^u + \frac{1}{2}\left(y_t^u\right)^2 + \frac{1}{2}\left(n_t^r\right)^2 + \frac{\phi\gamma\left(1-\gamma\right)}{2}\left(\sigma_t\right)^2 + \frac{\theta}{4}\mathrm{var}(p_{h,t}) + \frac{\theta}{4}\mathrm{var}(p_{f,t}) - \frac{1}{2}a_t^h n_t^h - \frac{1}{2}a_t^f n_t^f + \frac{\psi}{2}\left(n_t^u\right)^2 + \frac{\psi}{2}\left(n_t^r\right)^2\right] + \text{t.i.p.} + O(\|\xi^3\|).$$

Using the fact that

$$N^{1+\psi} = \frac{Y}{A}N^{\psi} = YC^{-\sigma} = \frac{C^{1-\sigma}}{1-\kappa},$$

the approximation simplifies to

$$u_t^u \simeq \frac{C^{1-\sigma}}{1-\kappa}\left\{-\frac{(1-\kappa)\,\varsigma_\alpha}{2}\left(s_t\right)^2 - \frac{\varsigma_\gamma}{2}\left(\sigma_t\right)^2 + \frac{1}{2}a_t^h n_t^h + \frac{1}{2}a_t^f n_t^f - \frac{\sigma\left(1-\kappa\right)}{2}\left[\left(c_t^u\right)^2 + \left(c_t^r\right)^2\right] - \frac{\psi}{2}\left[\left(n_t^u\right)^2 + \left(n_t^r\right)^2\right] - \frac{\theta}{4}\mathrm{var}(p_{h,t}) - \frac{\theta}{4}\mathrm{var}(p_{f,t})\right\} + \text{t.i.p.} + O(\|\xi^3\|),$$

where $\varsigma_\gamma = \phi\gamma\left(1-\gamma\right) \geq 0$ and $\varsigma_\alpha = \mu\alpha\left(1-\alpha\right) \geq 0$. Recalling that

$$n_t^u = y_t^u - a_t^u,$$
$$c_t^u = \frac{y_t^u - \kappa g_t^u}{1-\kappa},$$

we get

$$u_t^u \simeq \frac{C^{1-\sigma}}{1-\kappa}\left\{-\frac{(1-\kappa)\,\varsigma_\alpha}{2}\left(s_t\right)^2 - \frac{\varsigma_\gamma}{2}\left(\sigma_t\right)^2 - \frac{\theta}{4}\mathrm{var}(p_{h,t}) - \frac{\theta}{4}\mathrm{var}(p_{f,t}) - \frac{\sigma}{2\left(1-\kappa\right)}\left[\left(y_t^u\right)^2 - 2\kappa y_t^u g_t^u\right] - \frac{\sigma\left(1-\kappa\right)}{2}\left(c_t^r\right)^2 - \frac{\psi}{2}\left[\left(y_t^u\right)^2 - 2y_t^u a_t^u\right] - \frac{\psi}{2}\left(n_t^r\right)^2 + \frac{1}{2}a_t^h n_t^h + \frac{1}{2}a_t^f n_t^f\right\} + \text{t.i.p.} + O(\|\xi^3\|).$$

Recalling that

$$\widetilde{y}_t^u = \frac{(1-\kappa)(\psi+1)}{\psi\left(1-\kappa\right)+\sigma}a_t^u + \frac{\kappa\sigma}{\psi\left(1-\kappa\right)+\sigma}g_t^u,$$

the welfare simplifies to

$$u_t^u \simeq \frac{C^{1-\sigma}}{1-\kappa}\left[-\frac{(1-\kappa)\,\varsigma_\alpha}{2}\left(s_t\right)^2 - \frac{\varsigma_\gamma}{2}\left(\sigma_t\right)^2 - \frac{\theta}{4}\mathrm{var}(p_{h,t}) - \frac{\theta}{4}\mathrm{var}(p_{f,t}) - \frac{\sigma+\psi\left(1-\kappa\right)}{2\left(1-\kappa\right)}\left(y_t^u - \widetilde{y}_t^u\right)^2 - y_t^u a_t^u + \frac{1}{2}a_t^h n_t^h + \frac{1}{2}a_t^f n_t^f - \frac{\sigma\left(1-\kappa\right)}{2}\left(c_t^r\right)^2 - \frac{\psi}{2}\left(n_t^r\right)^2\right] + \text{t.i.p.} + O(\|\xi^3\|).$$

Simplifying cross products according to

$$\frac{1}{2}a_t^h n_t^h + \frac{1}{2}a_t^f n_t^f = n_t^u a_t^u + n_t^r a_t^r = y_t^u a_t^u + n_t^r a_t^r + \text{t.i.p.} + O(\|\xi^3\|),$$

we get

$$\begin{aligned} u_t^u \simeq \frac{C^{1-\sigma}}{1-\kappa}\Bigg[& -\frac{(1-\kappa)\,\varsigma_\alpha}{2}(s_t)^2 - \frac{\varsigma_\gamma}{2}(\sigma_t)^2 - \frac{\theta}{4}\text{var}(p_{h,t}) - \frac{\theta}{4}\text{var}(p_{f,t}) \\ & - \frac{\sigma+\psi(1-\kappa)}{2(1-\kappa)}\left(y_t^u - \widetilde{y}_t^u\right)^2 + n_t^r a_t^r - \frac{\sigma(1-\kappa)}{2}\left(c_t^r\right)^2 - \frac{\psi}{2}\left(n_t^r\right)^2\Bigg] \\ & + \text{t.i.p.} + O(\|\xi^3\|). \end{aligned}$$

Using

$$\begin{aligned} \widetilde{\sigma}_t &= \frac{2\kappa\psi(1-2\gamma)}{\varpi_\gamma}g_t^r - \frac{2(1+\psi)}{\varpi_\gamma}a_t^r, \\ \widetilde{s}_t &= \frac{2\kappa\psi(1-2\gamma)^2}{\varpi_\gamma}g_t^r - \frac{2(1+\psi)(1-2\gamma)}{\varpi_\gamma}a_t^r, \\ \widetilde{c}_t^r &= \frac{2(1+\psi)(1-2\gamma)(1-2\alpha)}{2\sigma\varpi_\gamma}a_t^r - \frac{2\kappa\psi(1-2\gamma)^2(1-2\alpha)}{2\sigma\varpi_\gamma}g_t^r, \\ \widetilde{n}_t^r &= \frac{2\left(\varpi_\alpha(1-2\gamma)^2 + 2\varsigma_\gamma\right) - 1}{\varpi_\gamma}a_t^r + \frac{\kappa(1-2\gamma)}{\varpi_\gamma}g_t^r, \end{aligned}$$

where $\varpi_\gamma = 1 + 2\psi(\varpi_\alpha(1-2\gamma)^2 + 2\varsigma_\gamma)$, $n_t^r a_t^r$ decomposes according to

$$n_t^r a_t^r = \frac{(1+\psi)}{\varpi_\gamma}n_t^r a_t^r - \frac{\psi\kappa(1-2\gamma)}{\varpi_\gamma}g_t^r n_t^r + g_t^r \underbrace{\left[\frac{\varpi_\gamma - (1+\psi)}{\varpi_\gamma}a_t^r + \frac{\psi\kappa(1-2\gamma)}{\varpi_\gamma}\right]}_{\psi\widetilde{n}_t^r} n_t^r.$$

Using the expressions of n_t^r and y_t^r yields

$$\begin{aligned} n_t^r a_t^r = &\, (1-\kappa)\,c_t^r \underbrace{\left[\frac{(1-2\gamma)(1-2\alpha)(1+\psi)}{\varpi_\gamma}a_t^r - \frac{\psi\kappa(1-2\gamma)^2(1-2\alpha)}{\varpi_\gamma}g_t^r\right]}_{\sigma\widetilde{c}_t^r} \\ & - (1-\kappa)\,\varsigma_\alpha s_t \underbrace{\left[\frac{2(1+\psi)(1-2\gamma)}{\varpi_\gamma}a_t^r - \frac{2\psi\kappa(1-2\gamma)^2}{\varpi_\gamma}g_t^r\right]}_{-\widetilde{s}_t} \\ & - \varsigma_\gamma\sigma_t \underbrace{\left[\frac{2(1+\psi)}{\varpi_\gamma}a_t^r - \frac{2\kappa\psi(1-2\gamma)}{\varpi_\gamma}g_t^r\right]}_{-\widetilde{\sigma}_t} + \psi\widetilde{n}_t^r n_t^r. \end{aligned}$$

Simplifying,

$$n_t^r a_t^r = \sigma(1-\kappa)\,\widetilde{c}_t^r c_t^r + \psi\widetilde{n}_t^r n_t^r + \varsigma_\gamma\widetilde{\sigma}_t\sigma_t + (1-\kappa)\,\varsigma_\alpha\widetilde{s}_t s_t$$

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and plugging into the approximated aggregate utility function yields

$$u_t^u \simeq \frac{C^{1-\sigma}}{1-\kappa}\Bigg[-\frac{(1-\kappa)\,\varsigma_\alpha}{2}\,(s_t-\widetilde{s}_t)^2-\frac{\varsigma_\gamma}{2}\,(\sigma_t-\widetilde{\sigma}_t)^2-\frac{\theta}{4}\mathrm{var}(p_{h,t})-\frac{\theta}{4}\mathrm{var}(p_{f,t})$$
$$-\frac{\sigma+\psi\,(1-\kappa)}{2\,(1-\kappa)}\left(y_t^u-\widetilde{y}_t^u\right)^2-\frac{\sigma\,(1-\kappa)}{2}\left(c_t^r-\widetilde{c}_t^r\right)^2-\frac{\psi}{2}\left(n_t^r-\widetilde{n}_t^r\right)^2\Bigg]$$
$$+\ \text{t.i.p.}+O(\|\xi^3\|).$$

Now considering the stream of utility flows, the welfare function is written

$$\omega_T=\sum_{t=0}^{T}\beta^t E_0\left\{u_t^u\right\}.$$

Woodford (2003) shows that

$$\sum_{t=0}^{T}\beta^t\,\mathrm{var}(p_{i,t})=\sum_{t=0}^{T}\beta^t\,\frac{\pi_{i,t}^2}{k^i},$$

where $k^i=\left(1-\eta^i\beta\right)\left(1-\eta^i\right)/\eta^i$, which yields the final form of the welfare function

$$\omega_T=-\frac{C^{1-\sigma}}{2\,(1-\kappa)}\sum_{t=0}^{T}\beta^t E_0\left\{\ell_t\right\}+\text{t.i.p.}+O(\|\xi^3\|),$$

and where

$$\ell_t=\frac{\theta}{2k^h}(\pi_{h,t}-\widetilde{\pi}_{h,t})^2+\frac{\theta}{2k^f}(\pi_{f,t}-\widetilde{\pi}_{f,t})^2+\frac{\sigma+\psi\,(1-\kappa)}{1-\kappa}\left(y_t^u-\widetilde{y}_t^u\right)^2$$
$$+\,(1-\kappa)\,\varsigma_\alpha\,(s_t-\widetilde{s}_t)^2+\varsigma_\gamma\,(\sigma_t-\widetilde{\sigma}_t)^2+\sigma\,(1-\kappa)\left(c_t^r-\widetilde{c}_t^r\right)^2+\psi\left(n_t^r-\widetilde{n}_t^r\right)^2,$$

with $\varsigma_\gamma=\phi\gamma\,(1-\gamma)\geq 0$, $\varsigma_\alpha=\mu\alpha\,(1-\alpha)\geq 0$.