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Production of sound by unsteady throttling of flow into a resonant cavity, with application to voiced speech

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An analysis is made of the sound generated by the time-dependent throttling of a nominally steady stream of air through a small orifice into a flow-through resonant cavity. This is exemplified by the production of voiced speech, where air from the lungs enters the vocal tract through the glottis at a time-variable volume flow rate $Q(t)$ controlled by oscillations of the glottis cross-section. Voicing theory has hitherto determined Q from a heuristic, reduced complexity 'Fant' differential equation. A new self-consistent, integro-differential form of this equation is derived in this paper using the theory of aerodynamic sound, with full account taken of the back-reaction of the resonant tract on the glottal flux Q. The theory involves an aeroacoustic Green's function (G) for flow–surface interactions in a time-dependent glottis, so making the problem non-self-adjoint. In complex problems of this type, it is not usually possible to obtain G in an explicit analytic form. The principal objective of this paper is to show how the Fant equation can still be derived in such cases from a consideration of the equation of aerodynamic sound and from the adjoint of the equation governing G in the neighbourhood of the 'throttle'. The theory is illustrated by application to the canonical problem of throttled flow into a Helmholtz resonator.

Key words: aeroacoustics, flow–structure interactions, general fluid mechanics

1. Introduction

Omnidirectional 'monopole' sound is generated by the unsteady discharge of air into the atmosphere from a wall aperture or nozzle (Rayleigh 1945; Morse & Ingard 1968; Crighton *et al.* 1992; Howe 1998). When the sound radiates unimpeded in all directions from the source, the velocity potential φ at large distances r is

$$\varphi \approx -\frac{Q(t - r/c_o)}{4\pi r}, \tag{1.1}$$

where $Q(t)$ is the volume velocity of the air leaving the source at time t and c_o is the speed of sound in the ambient air.

The monopole strength Q depends on the type of source flow, such as that produced in the vocal tract by contraction of the lung cavity (Fant 1960; Flanagan 1972; Stevens 1998) or by volumetric expansion produced by unsteady burning in the combustion chamber of a gas turbine or furnace (Strahle 1971, 1978; Crighton *et al.* 1992; Poinsot

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& Veynante 2005). In addition, however, the actual variations of $Q(t)$ depend on the acoustic properties of the system upstream of the exit, on the influence of unsteady 'jetting' at the exit and on possible 'throttling' of the flow by time-dependent variations in the nozzle exit cross-sectional area. Jetting and throttling can also occur within the flow upstream of the exit – in the vocal tract, these occur notably at the glottis whose cross-section continually changes during 'voiced speech' because of vibrations of the vocal folds. Then, the glottis itself behaves as a monopole of the corresponding strength $Q(t)$, which now radiates into a resonant chamber formed by the supraglottal vocal tract.

In complex, confined flows of this type, the source strength Q has been determined in two distinct ways. The first dates back to Fant's (1960) pioneering treatment of the voicing problem, involving a 'lumped parameter' approximation in which Rayleigh's (1945) representation of the inertia of unsteady flow through an aperture is balanced against a constant subglottal overpressure p_{sg}, nonlinear pressure forces (estimated from Bernoulli's equation) associated with turbulence losses, and viscous forces at the walls. This argument yields the equation of motion

$$\mathscr{L}\frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{Q^2}{2A_g} + R'Q = \frac{A_g p_{sg}}{\rho_o}, \tag{1.2}$$

where $\mathscr{L}$, $A_g \equiv A_g(t)$ are respectively the 'inductance' and cross-sectional area of the glottis at its narrowest section, ρ_o is the mean air density, and the coefficient R' represents the effects of viscous losses. A similar empirical model was proposed independently by Cummings (Cummings 1984, 1986; Howe 1998; Luong, Howe & McGowan 2005) for studying the dissipation of sound by vorticity production in small apertures. The solution of (1.2) is used in (1.1) to determine the radiated sound; for confined systems like the vocal tract, the calculated source strength is inserted into an appropriate *source-filter* transmission formula that relates Q to the sound heard by a listener in free space (Fant 1960; Flanagan 1972; Rothenberg 1981; Stevens 1998; Titze 2008).

In reality, of course, there is no actual monopole source. For voiced speech the physical sources of sound are contraction of the lung cavity and the subsequent appearance of aeroacoustic volume and surface sources in and near the glottis, including frictional drag, shed vorticity and oscillatory motions of the vocal folds driven by the unsteady pressure. In this case, the second and more fundamental method of calculating the radiation is therefore by direct numerical simulation of the entire structural and compressible motion (Zhang *et al.* 2002; Zhao *et al.* 2002; Hofmans *et al.* 2003; Thomson, Mongeau & Frankel 2005; Duncan, Zhai & Scherer 2006) or, alternatively, because the flow is low Mach number, by a numerical determination of the aeroacoustic sources using equations for incompressible flow (Zhao *et al.* 2002) followed by their substitution into an acoustic prediction formula derived from Lighthill's *acoustic analogy* theory (Lighthill 1952; Howe 1998; Howe & McGowan 2007). Both of these numerical approaches tend to be computationally intensive and often cannot be run in a timely manner for more than one or two voicing cycles.

For routine purposes, there is a very much greater need for a 'reduced complexity' treatment of the kind provided by the Fant equation (1.2). A systematic derivation of this equation has been formulated by Howe & McGowan (2007). This makes use of two independent predictions of the sound furnished by (i) the linear acoustic theory of sound production by a monopole of strength $Q(t)$, and (ii) Lighthill's acoustic analogy representation of the sound in terms of tissue-surface sources and vortex sources within the flow. A corrected and fully self-consistent form of Fant's approximation (1.2), inclusive of hydroacoustic back-reactions from the supraglottal tract, is then obtained by equating the results of these two calculations. The procedure as described is actually tractable only in the simple case of a nominally infinitely long supraglottal tract (Howe & McGowan 2011), for which reflections from the mouth and surface appendages can be neglected.

All computational fluid dynamical treatments have hitherto ignored supraglottal resonances ('formants'), and the possibly large back-reaction on the glottis of these resonances. The sound generated at the glottis is assumed to radiate towards the mouth with negligible reflections. Many estimates of the back-reaction consist of *ad hoc* approximations of the coupling of the sound and surface tissue in the glottis (e.g. Flanagan & Landgraf 1968; Gupta, Wilson & Beavers 1973; Zanartu, Mongeau & Wodicka 2007; Titze 2008). In the simplest, first approximation, elementary source-filter theory can be used (Titze 1988), but it fails at voicing frequencies approaching the first formant, for example, at which the back-reaction is apparently strong enough to cause the voicing monopole to produce undesirable, involuntary and abrupt changes in frequency (Titze & Story 1995; Austin & Titze 1997; Joliveau, Smith & Wolfe 2004; Titze 2008).

A theory of throttled flow into a resonant chamber that does not depend on a weak interaction approximation is proposed in this paper. This is an extension of the self-consistent method of Howe & McGowan (2007) for radiation into an infinite duct, which made use of an analytical representation of an aeroacoustic Green's function tailored to the time-dependent geometry of the system. However, for the more complicated resonant systems to be considered in this paper, an explicit derivation of the corresponding Green's function is difficult or impracticable. Our objective is to show how, nevertheless, the system of equations defining Green's function can be manipulated along with Lighthill's equation of aerodynamic sound to supply the Fant equation. The principal conclusion of the paper is that Fant's equation is precisely the inhomogeneous *adjoint* equation (driven by all of the aeroacoustic sources) of the equation that governs the behaviour of Green's function within the time-dependent 'throttle'. To fix ideas, the discussion is framed in terms of the problem of voiced speech, although no attempt is made here to model precisely the full mechanics of the vocal tract. However, the theory is applicable more generally, for example, to combustion-generated noise within a resonant furnace and to similar resonant, aeroacoustic systems.

The new approach is discussed (§ 2) for the basic problem of the direct radiation of sound from the glottis into free space, in the absence of a supraglottal tract, i.e. for sound production by throttling of nominally steady flow from a nozzle of time-dependent cross-section. Green's function is known explicitly for this problem, and this permits immediate validation of the adjoint equation procedure. Extension is then made to two cases involving throttled flow into a resonant cavity. The first (§ 3) is the canonical problem of flow into a Helmholtz resonator, which is the simplest possible approximation to a supraglottal tract having one dominant resonant frequency. Feedback from the resonant response of the cavity leads to a modified Fant equation, which becomes an integro-differential equation involving an integration over the entire history of the glottal oscillations. Numerical results discussed in § 4 clarify the importance of these back-reactions to the glottal flow. Finally, an outline derivation is given in the Appendix of the Fant equation governing an improved model of the supraglottal tract, having multiple resonance frequencies.

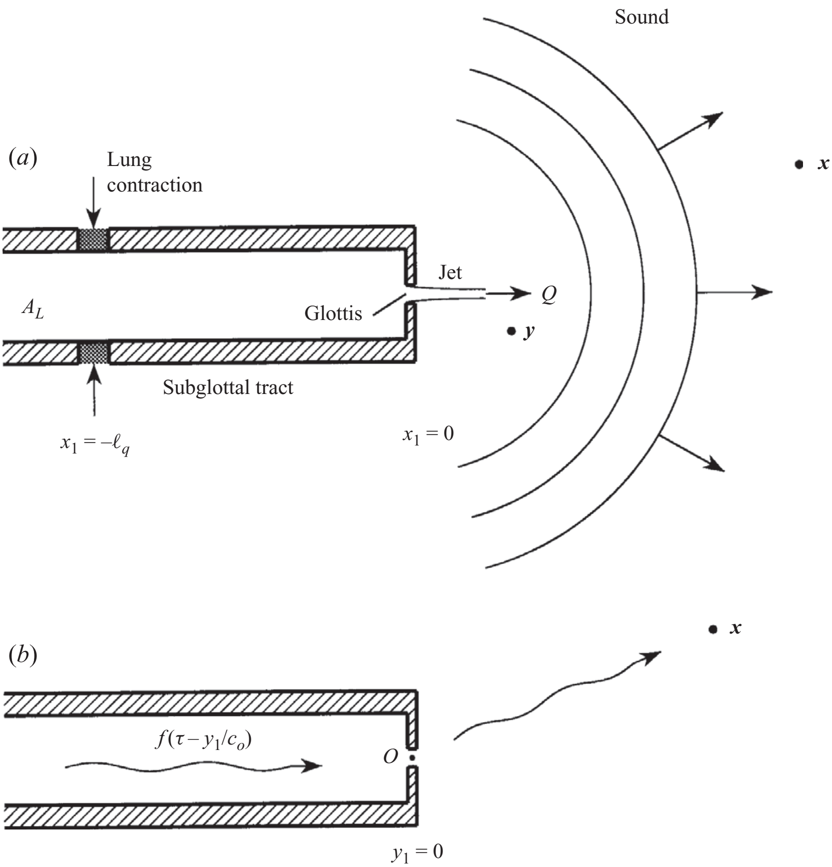

FIGURE 1. (a) Direct monopole radiation into free space produced by the throttling of nominally steady, low-Mach-number flow by a small aperture (the 'glottis') of time-dependent cross-sectional area A_g. (b) Components of the 'advanced potential' Green's function propagating as a function of $(\boldsymbol{y}, \tau)$ towards an observer at $\boldsymbol{x}$ in free space and evanescent for $\tau > t$.

2. Radiation from throttled flow into free space

2.1. *Formal representation of the sound*

Consider sound radiation from the idealised configuration illustrated in figure 1(a). The subglottal tract is modelled by a hard walled, semi-infinite, circular cylindrical duct of cross-sectional area A_L; the supraglottal tract is absent, and communication with atmospheric free space is assumed to occur directly through a small axisymmetric opening (the 'glottis') whose time-dependent cross-sectional area at its narrowest point is A_g. Take the origin of coordinates $\boldsymbol{x} = (x_1, x_2, x_3)$ at the nominal midpoint of the glottis, with the negative x_1 axis extending axially into the duct. Let voicing be initiated by steady contraction of the 'lungs', represented in the model of figure 1(a) by uniform peripheral contraction of a short section of the subglottal tract (centred on $x_1 = -\ell_q$) that behaves as a volume source of constant overall strength q, say. Sound production occurs when the overpressure produced by this contraction reaches the glottis, which opens as the vocal folds are forced apart by the elevated pressure. The subsequent vibration of the folds causes the pulsing of fluid through the glottis forming of an unsteady jet (indicated schematically in the figure). In a first approximation, the acoustic energy that travels towards the lungs is absorbed by the subglottal system. Thus, the sound that radiates to $x_1 = -\infty$ within the subglottal tract is assumed to be absorbed without reflection within the lung complex. This is a simplified version of the problem treated by Howe & McGowan (2009) of the excitation of waves in a finite-length model of the subglottal system.

Vortex sound theory (Howe 1998, 2002, 2008) will be used to obtain a formal representation of the sound for an observer in free space at $\boldsymbol{x}$ at time t. Sound is produced by moving boundaries, by vorticity within the jet and elsewhere, and by the interaction of vorticity with boundaries. In a first approximation, the dominant aspects of speech production occur *homentropically* at low Mach number, with dissipation confined to regions of strong shear at the boundaries, whereas in the body of the fluid, sound propagates as if the fluid were inviscid. In these circumstances, the pressure p can be regarded as a function of the fluid density ρ alone, and the vortex sound equation takes the simplified form (Howe 2002; Howe & McGowan 2007)

$$\left(\frac{1}{c_o^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_j^2}\right) B = \operatorname{div}(\boldsymbol{\omega} \wedge \boldsymbol{v}), \tag{2.1}$$

where $\boldsymbol{v}$ is the velocity, $\boldsymbol{\omega} = \operatorname{curl} \boldsymbol{v}$ is the vorticity, $B = \int (\mathrm{d}p/\rho) + (v^2/2)$ is the total enthalpy, c_o is the speed of sound (assumed to be uniform throughout the fluid), and the repeated subscript j implies summation over $j = 1, 2, 3$. In acoustic regions of the flow (where $\boldsymbol{\omega} = 0$), $B = -\partial\varphi/\partial t$ and the pressure fluctuation $p = -\rho_o \partial\varphi/\partial t$, where φ is an appropriate velocity potential.

The solution $B(\boldsymbol{x}, t)$ of (2.1) subject to appropriate boundary conditions will be expressed in terms of an 'advanced potential' Green's function $G(\boldsymbol{x}, \boldsymbol{y}, t, \tau)$, which satisfies

$$\left(\frac{1}{c_o^2}\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial y_j^2}\right) G = \delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - \tau), \quad G = 0 \text{ for } \tau > t, \tag{2.2}$$

where $\boldsymbol{y} = (y_1, y_2, y_3)$. Here G represents 'incoming' waves as a function of $(\boldsymbol{y}, \tau)$ that vanish after convergence onto the source $\delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - \tau)$ at $\tau = t$. The specific functional form of G is chosen to satisfy $\partial G/\partial y_n = 0$ on the moving solid boundary $S(\tau)$ at time τ, where y_n is a local normal coordinate on $S(\tau)$ directed into the fluid.

Equations (2.1) and (2.2) are combined in the usual way (Morse & Feshbach 1953; Howe & McGowan 2007; Howe 2008) using Green's theorem, the radiation condition and the momentum equation, to supply the causal solution of (2.1) at low Mach numbers in the form

$$B(\boldsymbol{x}, t) = \int_{-\infty}^{\infty} \oint_{S(\tau)} \left[G\frac{\partial \boldsymbol{v}}{\partial \tau} - \nu \frac{\partial G}{\partial \boldsymbol{y}} \wedge \boldsymbol{\omega} \right] \cdot \mathrm{d}\boldsymbol{S}(\boldsymbol{y})\, \mathrm{d}\tau - \int_{-\infty}^{\infty} \int_{V(\tau)} \frac{\partial G}{\partial \boldsymbol{y}} \cdot \boldsymbol{\omega} \wedge \boldsymbol{v}\, \mathrm{d}^3\boldsymbol{y}\, \mathrm{d}\tau, \tag{2.3}$$

where the vector surface element $\mathrm{d}\boldsymbol{S}$ on $S(\tau)$ is directed into the fluid, $V(\tau)$ denotes the spatial region occupied by the fluid at time τ, and ν is the kinematic viscosity of the fluid. Viscous dissipation is absent from the vortex sound equation (2.1), which describes propagation within the body of the fluid. However, frictional effects can be important at solid boundaries, and this accounts for the appearance of viscosity (via the momentum equation) in the surface integral of (2.3).

The first term in the surface integral (involving $\partial\boldsymbol{v}/\partial\tau$) corresponds to the direct 'monopole' radiation produced by lung contraction and by similar sources associated with volumetric changes of the vibrating vocal folds. The viscous term is a surface drag dipole that is concentrated principally within the glottis. It is typically small compared with the sound generated by the vortex source $\boldsymbol{\omega} \wedge \boldsymbol{v}$ of the volume integral. This is also of dipole type, and its main contribution is from the jet produced downstream of the glottis by the lung overpressure; its strength is modulated by the variations in the glottis area A_g produced by fold vibration. In all cases the oscillation frequencies are such that the relevant acoustic wavelengths are much larger than the duct diameter and the axial length of the glottis. The sound can therefore be calculated by use of the *compact* approximation to the Green's function (Howe 1998, 2002).

2.2. *The lung overpressure*

Without loss of generality, it may be assumed that the axial extent of the constant strength source q is very much smaller than the acoustic wavelength and that lung contraction begins at time $t = -\ell_q/c_o$. By putting $q = 2A_L p_{\mathrm{I}}/\rho_o c_o$, where p_{I} denotes a constant pressure, the effect of this source is equivalent to a distribution of normal velocity v_n on the duct wall (directed into the fluid), where

$$v_n = \frac{2A_L p_{\mathrm{I}}}{\ell_p \rho_o c_o} H\left(t + \frac{\ell_q}{c_o}\right) \delta(x_1 + \ell_q), \tag{2.4}$$

$H(\cdot)$ is the Heaviside step function and ℓ_p is the duct perimeter.

The pressure wave produced by this source is given by the first term in the square brackets of the surface integral of (2.3), involving $\partial \boldsymbol{v}/\partial\tau$. In particular, the form of this wave in the subglottal tract prior to its arrival at the glottis is calculated by taking for G the compact approximation for radiation in an infinite duct of cross-section A_L (Howe 1998, 2002):

$$G(\boldsymbol{x}, \boldsymbol{y}, t, \tau) = \frac{c_o}{2A_L} H\left(t - \tau - \frac{|x_1 - y_1|}{c_o}\right). \tag{2.5}$$

Using this to calculate the pressure wave $p(t - x_1/c_o)$ propagating towards the glottis (where $x_1 > -\ell_q$), we find

$$p = p_{\mathrm{I}} H\left(t - \frac{x_1}{c_o}\right). \tag{2.6}$$

The pressure p_{I} is therefore just equal to the amplitude of the overpressure incident on the glottis (at $x_1 \simeq 0$) from the lungs, the arrival of which at $t \simeq 0$ initiates motion of the glottis and the production of sound.

2.3. *Compact Green's function for free space radiation*

Let the observation point $\boldsymbol{x}$ in free space (figure 1*b*) be at a large distance $|\boldsymbol{x}| \gg \sqrt{A_L}$ from the glottis. When the glottis and duct diameter are acoustically compact, the advanced potential converging onto $\boldsymbol{y} = \boldsymbol{x}$ at time $\tau = t$ is dominated by the corresponding free space Green's function (Howe 2002), i.e.

$$G \simeq \frac{1}{4\pi|\boldsymbol{x} - \boldsymbol{y}|} \delta\left(t - \tau - \frac{|\boldsymbol{x} - \boldsymbol{y}|}{c_o}\right). \tag{2.7}$$

Near the glottis, where $|\boldsymbol{y}| \sim O(\sqrt{A_L}) \ll |\boldsymbol{x}|$, this becomes

$$G \simeq \frac{1}{4\pi|\boldsymbol{x}|} \delta([t] - \tau), \tag{2.8}$$

where $[t] = t - |\boldsymbol{x}|/c_o$ is the retarded time for propagation from the glottis.

Within and in the immediate neighbourhood of the glottis, the compact form of Green's function reduces to a solution of Laplace's equation, of the form

$$G = \alpha(\tau) + \beta(\tau)Y(\boldsymbol{y}, \tau), \tag{2.9}$$

where $\alpha(\tau)$ and $\beta(\tau)$ are functions to be determined, $\nabla^2 Y(\boldsymbol{y}, \tau) = 0$, and $Y(\boldsymbol{y}, \tau)$ denotes the velocity potential of flow at unit speed within the duct out through the glottis such that the normal derivative $\partial Y/\partial y_n = 0$ on the instantaneous surface $S(\tau)$ of the glottis and the neighbouring duct, so that

$$Y(\boldsymbol{y}, \tau) \sim \begin{cases} -A_L/4\pi|\boldsymbol{y}|, & |\boldsymbol{y}| \gg \sqrt{A_g} \text{ in free space,} \\ y_1 - \bar{\ell}(\tau), & |y_1| \gg \sqrt{A_L} \text{ within the duct.} \end{cases} \tag{2.10}$$

The length $\bar{\ell}(\tau)$ is a time-dependent 'end correction', defined in terms of the Rayleigh conductivity $K_g \equiv K_g(\tau)$ ($\leqslant 2\sqrt{A_g/\pi}$) of the glottis and the duct cross-sectional area A_L by

$$\bar{\ell}(\tau) = \frac{A_L}{K_g(\tau)} \tag{2.11}$$

(Rayleigh 1945; Howe 1998).

Within the duct at distances $|y_1| \gg \sqrt{A_L}$ from the glottis, the advanced potential G reduces to a plane, *incoming* wave of the form

$$G = f\left(\tau - \frac{y_1}{c_o}\right). \tag{2.12}$$

The functions α, β and f are found by equating the representations (2.8), (2.9) and (2.12) of G correct to $O(\boldsymbol{y})$ in the overlap regions $\sqrt{A_g} \ll |\boldsymbol{y}| \ll c_o/\omega$ and $-c_o/\omega \ll y_1 \ll -\sqrt{A_L}$ respectively outside and inside the duct, where $\omega \sim \partial/\partial\tau$ is a characteristic frequency. This procedure yields the equations (with use of the relations (2.10))

$$\left.\begin{aligned} \alpha(\tau) &= \delta([t] - \tau)/4\pi|\boldsymbol{x}|, \\ f(\tau) &= \alpha(\tau) - \beta(\tau)\bar{\ell}(\tau), \\ f'(\tau) &= -c_o\beta(\tau), \end{aligned}\right\} \tag{2.13}$$

where the prime denotes differentiation with respect to τ.

Therefore,

$$\frac{\partial}{\partial\tau}(\bar{\ell}\beta) - c_o\beta = \frac{\partial\alpha}{\partial\tau}, \tag{2.14}$$

and the function $\beta(\tau)$ (vanishing for $\tau > [t]$) is readily found to be given by

$$\beta(\tau) = \frac{1}{4\pi\bar{\ell}([t])|\boldsymbol{x}|}\left(\delta([t] - \tau) - \frac{c_o}{\bar{\ell}(\tau)}H([t] - \tau)\exp\left[-\int_\tau^{[t]} \frac{c_o\,\mathrm{d}\xi}{\bar{\ell}(\xi)}\right]\right). \tag{2.15}$$

2.4. *Monopole radiation from the glottis*

Consider the representation (2.3) of the sound radiated from the glottis to a point $\boldsymbol{x}$ in the free space far field. To avoid unnecessary complications, attention is confined to the case in which the volume of the vocal fold tissue is constant during vibration and the viscous drag within the glottis is negligible. The first integral in (2.3) is then restricted to the section of the subglottal tract where v_n given by (2.4) is non-zero, and the vortex source integral is confined to the unsteady jet just downstream of the glottis. Then, $B \simeq B_q + B_\omega$, where

$$B_q = \int_{-\infty}^{\infty} \oint_{S(\tau)} G \frac{\partial v_n}{\partial \tau} \,\mathrm{d}S(\boldsymbol{y})\,\mathrm{d}\tau = \int_{-\infty}^{\infty} \oint_{S(\tau)} f\left(\tau - \frac{y_1}{c_o}\right) \frac{\partial v_n}{\partial \tau} \,\mathrm{d}S(\boldsymbol{y})\,\mathrm{d}\tau$$
$$= \frac{2A_L p_{\mathrm{I}}}{\rho_o} \int_{-\infty}^{\infty} \beta(\tau) H(\tau)\,\mathrm{d}\tau, \quad (2.16)$$

where the final result is obtained from (2.4) after integration by parts with respect to τ (noting that v_n, f respectively vanish at $\tau = \mp\infty$) and by use of the third of equations (2.13), and the vortex contribution is

$$B_\omega = -\int_{-\infty}^{\infty} \int_{V(\tau)} \frac{\partial G}{\partial \boldsymbol{y}} \cdot \boldsymbol{\omega} \wedge \boldsymbol{v} \,\mathrm{d}^3\boldsymbol{y}\,\mathrm{d}\tau = -\int_{-\infty}^{\infty} \int_{V(\tau)} \beta(\tau) \frac{\partial Y}{\partial \boldsymbol{y}} \cdot \boldsymbol{\omega} \wedge \boldsymbol{v} \,\mathrm{d}^3\boldsymbol{y}\,\mathrm{d}\tau, \quad (2.17)$$

where the volume integral is over the jet vorticity.

The overall radiation $B = B_q + B_\omega$ at large distances from the glottis accordingly becomes (by use of (2.15))

$$B(\boldsymbol{x}, t) \simeq \int_{-\infty}^{\infty} \beta(\tau) \mathscr{F}(\tau)\,\mathrm{d}\tau \quad (2.18a)$$
$$= \frac{1}{4\pi \bar{\ell}([t])|\boldsymbol{x}|} \left(\mathscr{F}([t]) - \int_{-\infty}^{[t]} \frac{c_o \mathscr{F}(\tau) \mathrm{e}^{-\int_\tau^{[t]} c_o \,\mathrm{d}\xi/\bar{\ell}(\xi)}}{\bar{\ell}(\tau)} \,\mathrm{d}\tau \right), \quad |\boldsymbol{x}| \to \infty, \quad (2.18b)$$

where $[t] = t - |\boldsymbol{x}|/c_o$ and

$$\mathscr{F}(\tau) = \frac{2A_L p_{\mathrm{I}}}{\rho_o} H(\tau) - \int_{V(\tau)} \frac{\partial Y}{\partial \boldsymbol{y}} \cdot \boldsymbol{\omega} \wedge \boldsymbol{v} \,\mathrm{d}^3\boldsymbol{y}. \quad (2.19)$$

The acoustic wave $B(\boldsymbol{x}, t)$ exhibits the expected omnidirectional characteristics of a monopole field, and is fully determined when the source strength Q is known. The magnitude of Q depends on the incident wave p_{I}, on the temporal variations of the glottis cross-sectional area A_g, and on the glottis–jet interaction. This dependence is governed by the Fant equation, which is derived from the condition that the far-field aeroacoustic solution (2.18) should be consistent with the acoustic formula $B(\boldsymbol{x}, t) = -\partial\varphi/\partial t$, where the velocity potential φ is given in terms of Q by (1.1), with r replaced by $|\boldsymbol{x}|$.

Hence,

$$B(\boldsymbol{x}, t) \simeq \frac{1}{4\pi|\boldsymbol{x}|} \frac{\partial Q}{\partial t}([t]) = \int_{-\infty}^{\infty} \beta(\tau) \mathscr{F}(\tau)\,\mathrm{d}\tau, \quad (2.20)$$

which from (2.18b) implies the explicit relation

$$\bar{\ell}(t) \frac{\mathrm{d}Q}{\mathrm{d}t}(t) = \mathscr{F}(t) - \int_{-\infty}^{t} \frac{c_o \mathscr{F}(\tau) \mathrm{e}^{-\int_\tau^{t} c_o \,\mathrm{d}\xi/\bar{\ell}(\xi)}}{\bar{\ell}(\tau)} \,\mathrm{d}\tau. \quad (2.21)$$

The integrated term in this formula is removed by elimination between (2.21) and the equation obtained by differentiation with respect to t, to obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\bar{\ell} \frac{\mathrm{d}Q}{\mathrm{d}t}\right) + c_o \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathrm{d}\mathscr{F}}{\mathrm{d}t}. \quad (2.22)$$

Integrating once, using the initial condition $Q = \mathscr{F} = 0$ for $t < 0$, and introducing the representation (2.19) for $\mathscr{F}$, then yields the required Fant equation

$$\bar{\ell} \frac{\mathrm{d}Q}{\mathrm{d}t} + c_o Q + \int_{V(t)} \frac{\partial Y}{\partial \boldsymbol{y}} \cdot \boldsymbol{\omega} \wedge \boldsymbol{v} \,\mathrm{d}^3\boldsymbol{y} = \frac{2A_L p_{\mathrm{I}}}{\rho_o} H(t), \quad (2.23)$$

where the integrand is evaluated at time t.

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This determines the evolution of Q after the arrival at time $t=0$ of the pressure wave p_I. The linear term $c_o Q$ accounts for damping produced by radiation back into the lungs (radiation losses into free space are negligible, and are omitted from the Green's function approximation of § 2.3); the nonlinear, integrated term represents the back-reaction on the glottis of vorticity produced by 'jetting' of the flow out of the glottis. The equation must be solved simultaneously with the elastic equation of motion of the vocal folds, which determines the changes in the glottal minimum cross-section $A_g(t)$ and also accounts for the coupling of the fold motion with the flow (cf. Ishizaka & Flanagan 1972; Fulcher *et al.* 2006; Zanartu *et al.* 2007; Howe & McGowan 2010).

2.5. *Fant equation derived from the adjoint Green's function equation*

The Fant equation (2.23) was obtained by use of the explicit Green's function formula (2.15) determined by (2.13). The corresponding equations for more complicated systems involving one or more resonant cavities are not usually amenable to a closed-form solution. In those cases, the Fant equation must be derived by suitable adaptation of the following formal argument.

Substitute for α in (2.14) from the first of (2.13):

$$\frac{\partial}{\partial \tau}(\bar{\ell}\beta) - c_o\beta = \frac{-1}{4\pi|\boldsymbol{x}|}\,\delta'([t]-\tau), \tag{2.24}$$

where the prime denotes differentiation with respect to the argument of the δ-function. It may be remarked that the right-hand side of this equation is just equal to $\partial G_o/\partial\tau$, where $G_o \equiv G_o(\boldsymbol{x}, t-\tau)$ is the free space acoustic Green's function for a point source just downstream of the glottis when the latter is *closed*.

Form the Fant equation (2.23) by writing down the *adjoint* of differential equation (2.24),

$$\bar{\ell}\frac{\mathrm{d}Q}{\mathrm{d}t} + c_o Q = \mathscr{F}(t), \qquad Q = 0 \quad \text{for} \quad t < 0, \tag{2.25}$$

driven by the volume-integrated aeroacoustic sources $\mathscr{F}(t)$ defined as in (2.19).

The following alternative proof is required to establish the validity of this procedure. Multiply (2.24) by $Q(\tau)$, replace t by τ in (2.25) and multiply by $\beta(\tau)$. Add the resulting equations and integrate over all source times $-\infty<\tau<\infty$ to obtain

$$\left[\bar{\ell}\beta Q\right]_{-\infty}^{\infty} - \frac{1}{4\pi|\boldsymbol{x}|}\int_{-\infty}^{\infty} Q(\tau)\frac{\partial}{\partial\tau}(\delta([t]-\tau))\,\mathrm{d}\tau \;=\int_{-\infty}^{\infty}\beta(\tau)\mathscr{F}(\tau)\,\mathrm{d}\tau. \tag{2.26}$$

The first term on the left vanishes identically, because $Q(\tau)=\beta(\tau)=0$ respectively at $\tau=\mp\infty$, so that

$$\frac{1}{4\pi|\boldsymbol{x}|}\frac{\partial Q}{\partial t}([t]) = \int_{-\infty}^{\infty}\beta(\tau)\mathscr{F}(\tau)\,\mathrm{d}\tau \tag{2.27}$$

which confirms that Q defined by (2.25) coincides with the acoustic formula (2.20), and is therefore the required volume flux.

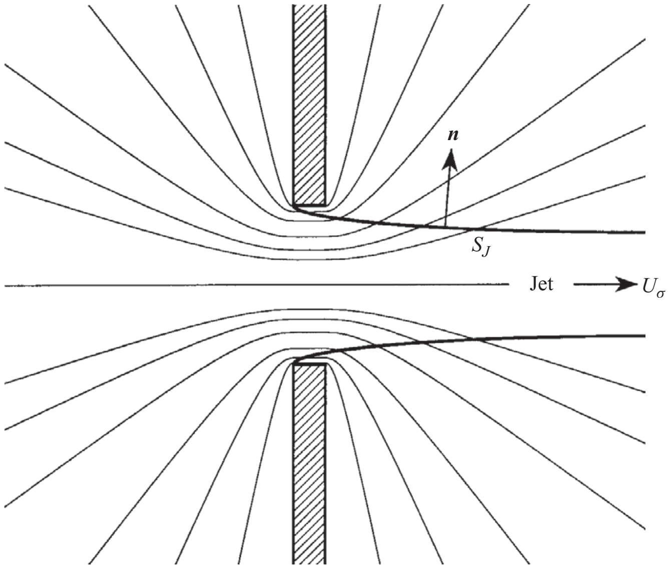

FIGURE 2. Streamsurfaces of the potential function $Y(\boldsymbol{y}, \tau)$ intersecting the vortex sheet boundary of the idealised jet.

2.6. *Interpretation of the Fant equation*

Fant's (1960) original equation (1.2) is similar to the analogous Cummings equation (Cummings 1984, 1986), both of which were derived by heuristic arguments. They can be reconciled with the general Fant equation (2.23) by the following argument of Howe & McGowan (2007) which estimates the value of the vortex integral in (2.23) using a quasi-static, 'free-streamline' model of the jet.

The jet vorticity is assumed to be confined to a free streamsurface at the outer edge of the jet, as indicated in figure 2. The vortex lines are peripheral circles with $\boldsymbol{\omega} = U_\sigma \delta(s_\perp)\hat{\boldsymbol{\theta}}$, where U_σ is the jet velocity just inside the shear layer, $s_\perp$ is distance measured in the direction of the outward unit normal $\boldsymbol{n}$ from the jet, and $\hat{\boldsymbol{\theta}}$ is a unit azimuthal vector in the clockwise sense when viewed along the direction of motion of the jet. Vorticity on the jet boundary is convected at half the local jet velocity, so that $\boldsymbol{\omega} \wedge \boldsymbol{v} = \frac{1}{2} U_\sigma^2 \delta(s_\perp)\boldsymbol{n}$. In a steady, free-streamline approximation, U_σ is equal to the uniform flow speed in the downstream, uniformly contracted section of the jet (Birkhoff & Zarantonello 1957; Gurevich 1965; Batchelor 1967; Milne-Thomson 1968).

Figure 2 also displays a meridional section of the family of streamlines of the instantaneous flow through the glottis defined by the velocity potential $Y(\boldsymbol{y}, \tau)$. Evidently, the main contribution to the vortex integral of (2.23) is from the section of the jet close to the glottis, where these streamlines cut across the edge of the jet. The velocity U_σ does not vary significantly over this short distance ($\sim O(A_g^{1/2})$) along the jet, provided the Strouhal number $\sim f_o\sqrt{A_g}/U_\sigma$ is small (where f_o is the glottal frequency), which is typically the case for normal voiced speech, so that

$$\int_{V(t)} \frac{\partial Y}{\partial \boldsymbol{y}} \boldsymbol{\cdot} \boldsymbol{\omega} \wedge \boldsymbol{v} \, \mathrm{d}^3 \boldsymbol{y} \approx \frac{U_\sigma^2(t)}{2} \oint_{S_J} \frac{\partial Y}{\partial \boldsymbol{y}} \boldsymbol{\cdot} \boldsymbol{n} \, \mathrm{d}S \equiv \frac{A_L U_\sigma^2}{2}, \tag{2.28}$$

where the second integral is over the curved surface S_J of the jet at time t, and the normalisation condition (2.10) of Y has been used. Hence

$$\int_{V(t)} \frac{\partial Y}{\partial \boldsymbol{y}} \cdot \boldsymbol{\omega} \wedge \boldsymbol{v} \, \mathrm{d}^3 \boldsymbol{y} \approx \frac{A_L}{2A_g^2} \frac{Q^2}{\sigma^2}, \tag{2.29}$$

because $Q = \sigma A_g U_\sigma$, where $\sigma \sim \sigma(t)$ is the (area) contraction ratio of the jet relative to its cross-section $A_g(t)$.

Equation (2.23) may now be recast in the form of Fant's original equation (1.2):

$$\rho_o \ell \frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{\rho_o Q^2}{2\sigma^2 A_g} = A_g p_{\mathrm{I}} H(t) + A_g \left[p_{\mathrm{I}} H(t) - \frac{\rho_o c_o Q}{A_L} \right], \tag{2.30}$$

where $\ell = A_g/K_g \equiv (A_g/A_L)\bar{\ell}$ is the effective length of the slug of fluid within and near the glottis that contributes to the inertia of the unsteady glottal flow (Rayleigh 1945). The quadratic term on the left represents the influence on the glottis of vorticity in the jet, and its explicit reduction to a form consistent with Fant's (1960) use of the Bernoulli equation depends on the low-Strouhal-number assumption $f_o\sqrt{A_g}/U_\sigma \ll 1$. The terms on the right are respectively the pressure forcing of the glottal motion over its effective cross-section A_g produced by the incident overpressure p_{I} from the lungs and the pressure wave of amplitude $p_{\mathrm{I}} - \rho_o c_o Q/A_L$ reflected back towards the lungs. When the motion is excited by a small amplitude sound wave incident from $x_1 = -\infty$, so that terms quadratic in Q can be discarded, the inertia term 'end correction' ℓ determines the phase of the wave reflected back towards the lungs. In the more general case, this term is usually small except near glottal closure, when ℓ becomes very large. In the absence of the jet and glottal inertia, the glottis behaves like an 'open end', at which the lung wave is reflected with a reflection coefficient of -1 and the free space monopole source strength $Q = 2p_{\mathrm{I}}/\rho_o c_o$. The linear and quadratic terms $\rho_o c_o (A_g/A_L)Q$ and $\rho_o Q^2/2\sigma^2 A_g$ are likely to be of comparable magnitudes, because, typically,

$$\frac{\rho_o Q^2/2\sigma^2 A_g}{\rho_o c_o (A_g/A_L)Q} = \frac{M_\sigma}{2\sigma(A_g/A_L)} \sim O(1), \tag{2.31}$$

where $M_\sigma = U_\sigma/c_o$ is the jet Mach number, which can attain a maximum of about 0.1 in voiced speech.

3. Throttled flow into the cavity of a Helmholtz resonator

Consider next the problem illustrated schematically in figure 3, where the flow from the glottis enters the cavity of a Helmholtz resonator of volume V which communicates with the atmosphere via a mouth of Rayleigh conductivity $K_m \gg K_g$. This arrangement is a simplified model for determining the influence on sound production of a supraglottal tract with a single resonance frequency.

The problem now is to express the sound radiated from the mouth in terms of the unsteady glottal volume flux $Q(t)$. The characteristic frequencies of the unsteady motions are assumed to be sufficiently small that all relevant acoustic wavelengths are large compared with the scale $\sim V^{1/3}$ of the cavity. This would occur in practice, for example, for a small female vocal tract when the lips are partially closed, where the first formant of the supraglottal tract (identified here with the Helmholtz resonance frequency) $\sim$500 Hz and the characteristic vibration frequency of the vocal folds $\sim$125 Hz. It may be assumed, therefore, that the dominant pressure fluctuations

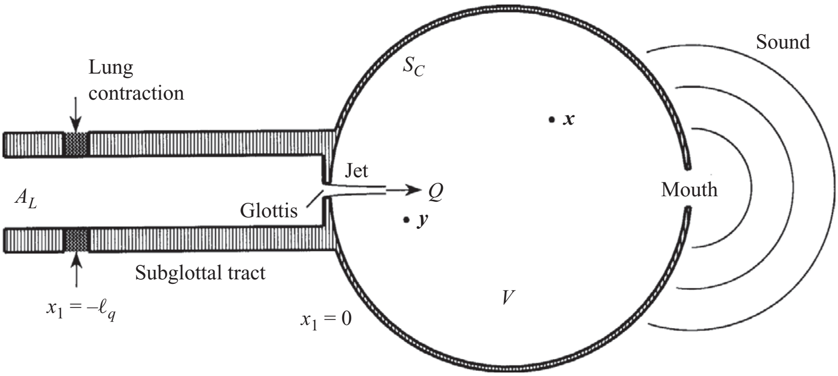

FIGURE 3. Free space radiation produced by throttled flow into the cavity of a Helmholtz resonator. The Fant equation is derived by consideration of the sound produced at an arbitrary point $\boldsymbol{x}$ within the body of the cavity.

forced by the throttled flow into the Helmholtz resonator are uniform across the cavity.

This interior pressure fluctuation can be written $-\rho_o \partial\Phi/\partial t$, where the potential function $\Phi = \Phi(t)$ is uniform across the cavity. When Φ has been obtained in terms of the glottal flux Q (using the appropriate Fant equation), the volume flux from the mouth of the resonator Q_m, say, then is

$$Q_m(t) \approx -K_m \Phi(t), \tag{3.1}$$

and the far-field sound is supplied by (1.1) with Q replaced by Q_m.

3.1. *Green's function*

The Fant equation is derived using the compact Green's function equations for an observer at $\boldsymbol{x}$ *within the body of the cavity*. The formal representations (2.9) and (2.12) of G at the glottis and in the subglottal tract are unchanged in terms of appropriate functions α, β and f. Within the cavity $G = \bar{G}(t, \tau)$, a function of t and τ, except in the immediate vicinity of the source point, glottis and mouth, so that when (2.2) is integrated (with respect to its dependence on $\boldsymbol{y}$) over the interior of the cavity, we find

$$\frac{V}{c_o^2}\frac{\partial^2 \bar{G}}{\partial \tau^2} + \oint_{S_c} \frac{\partial G}{\partial \boldsymbol{y}} \cdot \mathrm{d}\boldsymbol{S} = \delta(t-\tau), \tag{3.2}$$

where S_c is the interior surface of the cavity, extended to include the inner faces of the glottis and mouth, and $\mathrm{d}\boldsymbol{S}$ is directed into the cavity. The surface integral determines the Green's function flux *into* the cavity from the glottis and mouth, equal respectively to $A_L\beta(\tau)$ and $K_m\bar{G}$. The matching of expression (2.9) with Green's function within the cavity requires that $\bar{G} = \alpha(\tau)$, so that (3.2) becomes

$$\frac{V}{c_o^2}\left(\frac{\partial^2}{\partial \tau^2} + \Omega^2\right)\alpha = -A_L\beta + \delta(t-\tau), \tag{3.3}$$

where $\Omega = \sqrt{K_m c_o^2/V}$ may be interpreted as the resonance frequency of the resonator provided $K_m \gg K_g$ (Rayleigh 1945; Howe 1998). This yields the following formula for

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$\alpha(\tau)$ in terms of β:

$$\alpha(\tau)=\frac{c_o^2}{\Omega V}\left(H(t-\tau)\sin[\Omega(t-\tau)]-A_L\int_{-\infty}^{\infty}\beta(\xi)H(\xi-\tau)\sin[\Omega(\xi-\tau)]\,\mathrm{d}\xi\right), \tag{3.4}$$

where $\beta(\xi)=0$ for $\xi>t$.

3.2. *The Fant equation*

The application of the Green's function formulae as in §2 yields the aeroacoustic representation of $B(\boldsymbol{x},t)$ within the cavity in the form

$$B(\boldsymbol{x},t)\equiv-\frac{\partial\Phi}{\partial t}\simeq\int_{-\infty}^{\infty}\beta(\tau)\mathscr{F}(\tau)\,\mathrm{d}\tau, \tag{3.5}$$

where $\mathscr{F}$ is defined as in (2.19), and β is determined from (2.14) and (3.4) by the integro-differential equation:

$$\frac{\partial}{\partial\tau}(\bar{\ell}\beta)-c_o\beta-\frac{c_o^2A_L}{V}\int_{-\infty}^{\infty}\beta(\xi)H(\xi-\tau)\cos[\Omega(\xi-\tau)]\,\mathrm{d}\xi=-\frac{c_o^2}{V}H(t-\tau)\cos[\Omega(t-\tau)]. \tag{3.6}$$

The right-hand side of this equation is equal to $\partial G_o/\partial\tau$, where $G_o=(c_o^2/\Omega V)H(t-\tau)\sin[\Omega(t-\tau)]$ is the compact cavity acoustic Green's function when the glottis is closed.

The Fant equation is derived by consideration of the adjoint equation driven by the aeroacoustic source term $\mathscr{F}(t)$:

$$\bar{\ell}\frac{\partial\bar{Q}}{\partial t}+c_o\bar{Q}+\frac{c_o^2A_L}{V}\int_{-\infty}^{\infty}\bar{Q}(\xi)H(t-\xi)\cos[\Omega(t-\xi)]\,\mathrm{d}\xi=\mathscr{F}(t),\quad \bar{Q}=0 \text{ for } t<0. \tag{3.7}$$

To show that $\bar{Q}\equiv Q$ and that (3.7) with $\bar{Q}$ replaced by Q is the desired Fant equation, replace t by τ in (3.7) and form the sum of (3.6) $\times\,\bar{Q}(\tau)$ and (3.7) $\times\,\beta(\tau)$. Integration of the result over $-\infty<\tau<\infty$ (as in the second method of §2.5) then supplies, using (3.5),

$$\frac{\partial\Phi}{\partial t}=-\frac{c_o^2}{V}\int_{-\infty}^{\infty}\bar{Q}(\tau)H(t-\tau)\cos[\Omega(t-\tau)]\,\mathrm{d}\tau. \tag{3.8}$$

However, when the uniform potential $\Phi(t)$ within the cavity is ascribed to a monopole source of strength Q at the glottal opening, in accordance with elementary acoustic theory, integration of the wave equation over the cavity interior yields

$$Q=-\frac{V}{c_o^2}\left(\frac{\partial^2}{\partial t^2}+\Omega^2\right)\Phi. \tag{3.9}$$

Hence, (3.8) implies that

$$\begin{aligned}\frac{\partial Q}{\partial t}&=\left(\frac{\partial^2}{\partial t^2}+\Omega^2\right)\int_{-\infty}^{\infty}\bar{Q}(\tau)H(t-\tau)\cos[\Omega(t-\tau)]\,\mathrm{d}\tau\\&=-\int_{-\infty}^{\infty}\bar{Q}(\tau)\frac{\partial}{\partial\tau}(\delta(t-\tau))\,\mathrm{d}\tau=\frac{\partial\bar{Q}}{\partial t}.\end{aligned} \tag{3.10}$$

This confirms that $\bar{Q}\equiv Q$ and that (3.7) is the Fant equation.

The substitution from (2.19) of the explicit expression for $\mathscr{F}(t)$ yields

$$\bar{\ell}\frac{\mathrm{d}Q}{\mathrm{d}t} + c_o Q + \int_{V(t)} \frac{\partial Y}{\partial \boldsymbol{y}} \boldsymbol{\cdot} \boldsymbol{\omega} \wedge \boldsymbol{v}\, \mathrm{d}^3\boldsymbol{y} + \frac{c_o^2 A_L}{V} \int_{-\infty}^{t} Q(\tau)\cos[\Omega(t-\tau)]\,\mathrm{d}\tau = \frac{2A_L p_{\mathrm{I}}}{\rho_o} H(t), \tag{3.11}$$

where $Q=0$ for $t<0$. The two integrals in this equation respectively represent the back-reactions of the jet vorticity and the resonator pressure on the motion in the glottis. Equation (3.8) and the approximation (2.29) reduce (3.11) to the following modification of the non-resonator equation (2.30):

$$\rho_o \ell \frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{\rho_o Q^2}{2\sigma^2 A_g} = A_g \left[2p_{\mathrm{I}} H(t) - \frac{\rho_o c_o Q}{A_L} \right] + A_g \rho_o \frac{\partial \Phi}{\partial t}, \tag{3.12}$$

where the two terms on the right-hand side respectively represent the pressure forces over the subglottal and supraglottal endfaces of the glottal slug of length $\sim\ell$.

4. Predictions of the glottal flux and radiated sound

4.1. *The governing equations*

Consider the simplified case where the quasi-static approximation (2.29) is applicable. Then, the substitutions

$$Q_1(t) = \int_{-\infty}^{t} Q(\tau)\cos[\Omega(t-\tau)]\,\mathrm{d}\tau, \qquad Q_2(t) = \frac{\mathrm{d}Q_1}{\mathrm{d}t}(t) - Q(t) \tag{4.1}$$

permit the integro-differential Fant equation (3.11) to be replaced by the differential system

$$\left.\begin{aligned} \frac{\mathrm{d}Q}{\mathrm{d}t} &= \left(\frac{2A_L p_{\mathrm{I}}}{\rho_o} - c_o Q - \frac{A_L Q^2}{2\sigma^2 A_g^2} - \frac{c_o^2 A_L Q_1}{V} \right) \Big/ \bar{\ell} \\ \frac{\mathrm{d}Q_1}{\mathrm{d}t} &= Q + Q_2 \\ \frac{\mathrm{d}Q_2}{\mathrm{d}t} &= -\Omega^2 Q_1 \end{aligned}\right\} \quad t>0, \tag{4.2}$$

where $Q=Q_1=Q_2=0$ at $t=0$.

The solution of this system supplies both the glottal flux $Q(t)$ and (from (1.1), (3.1) and (3.8) with $\bar{Q}=Q$) the free space acoustic pressure

$$p(\boldsymbol{x},t) \simeq \frac{\rho_o \Omega^2}{4\pi r} Q_1\left(t - \frac{r}{c_o}\right), \tag{4.3}$$

where r is distance from the mouth of the resonator to the free space observer position $\boldsymbol{x}$.

During voiced speech, the vocal folds are controlled by unsteady pressure and friction forces and by muscular restoring forces. These are responsible for self-sustaining oscillations of the glottis and the throttling of the nominally steady flow from the lungs. A full discussion of the application of (4.2) to voicing therefore involves the simultaneous consideration of equations describing these fluid–tissue interactions. We shall not attempt to do this, however, because our objective is to examine the dependence of the radiated sound on the oscillation frequency f_o of the glottis, which cannot necessarily be controlled in a nonlinearly coupled system. It will therefore be assumed that the time dependence of the glottal area A_g is prescribed in a manner consistent with observations, in what Titze (2008) calls a ‘Level I’ model. This approach ignores hydroacoustic feedback on the movement of the vocal folds, and is a familiar approximation during an initial application of a new methodology to voicing theory (see e.g. Zhao *et al.* 2002; Howe & McGowan 2007). Thus, it will be assumed that

$$\frac{A_g}{A_L} = a_0 + a_1\{1 - \cos(\omega_o t)\}, \tag{4.4}$$

where the coefficients a_0, a_1 and the glottal radian frequency $\omega_o \equiv 2\pi f_o$ are constant. The corresponding slug length $\bar{\ell}$ will be approximated by Rayleigh's formula (for an aperture in a thin wall, Rayleigh 1945; Howe 1998)

$$\bar{\ell} = \frac{A_L}{2}\sqrt{\frac{\pi}{A_g}}. \tag{4.5}$$

4.2. *Numerical results*

The nominal Helmholtz resonance frequency $\Omega/2\pi$ will be identified with the first formant f_1, say, of the human supraglottal tract. This and other vocal tract and glottis parameters will be assigned the following values:

$$f_1 = 500\,\text{Hz}, \quad K_m = 0.5\,\text{cm}, \quad A_L = \pi\,\text{cm}^2, \quad a_0 = 0.001, \quad a_1 = 0.05, \quad c_o = 340\,\text{m}\,\text{s}^{-1}. \tag{4.6}$$

The value $K_m = 0.5$ cm for the conductivity of the resonator mouth (communicating with free space) is equivalent to that of a circular opening of radius 0.25 cm in a thin wall. These values also imply that the cavity volume $V \sim 58\,\text{cm}^3$, and are typical of those for a small adult female. The values of a_0, a_1 determine sinusoidal variations of A_g between $0.001A_L$ and $0.1A_L$ at radian frequency $\omega_o = 2\pi f_o$. The very small but non-zero value of a_0 implies that there will always be some 'leakage' through the glottis. However, this effect is negligible and our choice for a_0 is actually governed by the need to avoid instabilities associated with the numerical integration of the equations. The lung overpressure is taken to be $p_{\mathrm{I}} = 5$ cm of water, so that the maximum subglottal overpressure $\sim 2p_{\mathrm{I}} = 10$ cm of water ($\sim$1 kPa), which is typical of measured values (Fant 1960; Flanagan 1972; Stevens 1998).

The validity of the assumption that $2\pi f_1 = \Omega \equiv \sqrt{c_o^2 K_m/V}$ is easily checked by consideration of the homogeneous, linear acoustic approximation to the system (4.2), in which each of Q, Q_1, Q_2 varies like $\mathrm{e}^{-\mathrm{i}\omega t}$. To do this, it must be assumed that the glottis cross-section A_g is constant, so that the end-correction $\bar{\ell} = \bar{\ell}_o$, say, is also constant. This yields the characteristic frequencies

$$\omega \simeq \pm\Omega\left(1 + \frac{A_L\bar{\ell}_o}{V}\right)^{1/2} - \frac{\mathrm{i}c_o A_L}{2V}. \tag{4.7}$$

The first term on the right-hand side represents the cavity resonance frequency, which fractionally exceeds Ω by about $A_L\bar{\ell}_o/2V \ll 1$. The imaginary term determines the damping of resonator oscillations produced by radiation into the lungs. A corresponding very much smaller loss from the mouth of the resonator has been neglected by use of the approximation (3.1) for the volume flux Q_m from the mouth (involving a *real*-valued conductivity K_m). The damping experienced by sound in the human supraglottal tract is in practice very much larger, however. In particular, dissipative tissue compliance can greatly increase damping and also modify the resonant frequencies. The effect will be to broaden considerably acoustic resonance peaks relative to those predicted by the present model.

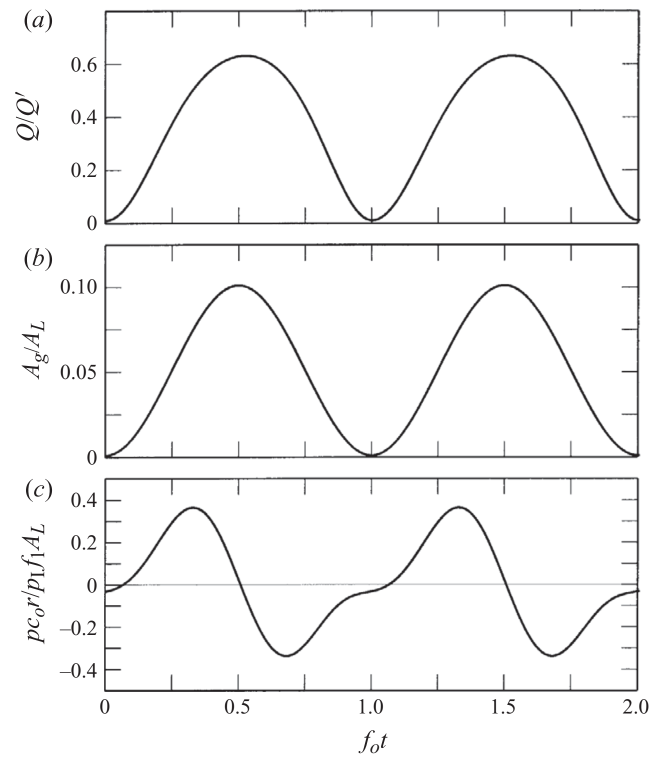

FIGURE 4. Numerical predictions of (4.2)–(4.4) for conditions (4.6) when $p_\mathrm{I} = 0.5\,\mathrm{kPa}$, typified by the non-special case where $f_o/f_1 = 0.6$: (*a*) the glottal volume flux Q normalised by $Q' = 2p_\mathrm{I}A_L/\rho_o c_o$; (*b*) the glottis area ratio A_g/A_L; (*c*) the monopole, far-field acoustic pressure at distance r from the mouth.

It is known that the jet contraction ratio σ actually varies with time during the opening and closing phases of the glottis (Pelorson *et al.* 1994; Park & Mongeau 2007; Zanartu *et al.* 2007; Howe & McGowan 2010). A referee has emphasised that these variations can influence strongly the characteristics of voiced speech, although model calculations (Howe & McGowan 2010) indicate that a time-dependent σ modifies principally only the fine details of the predicted acoustic waveform. However, these waveform changes depend on the precise functional form of $\sigma(t)$ and on the assumed geometry of the glottis, and it will therefore be convenient to ignore them in the present discussion and make use of the classical approximation $\sigma = 0.62$ (Howe 1998) in the quasi-static term $A_L Q^2/2\sigma^2 A_g^2$, which accounts for the vortex back-reaction in the first of (4.2).

The system of (4.2), (4.3) and (4.5) has been solved using a fourth-order Runge–Kutta procedure (Abramowitz & Stegun 1970). Results are first presented for conditions (4.6) in the most general, non-special case typified by an imposed glottal oscillation frequency $f_o = 0.6f_1$. Figure 4(*a*) depicts two cycles of the calculated non-dimensional volume flux Q/Q', where $Q' = 2p_\mathrm{I}A_L/\rho_o c_o$ is the nominal flux associated with the maximal subglottal pressure. The plots in this and subsequent figures represent predictions after the decay of transients triggered by the arrival of the lung overpressure at the glottis, so that the time origin ($f_o t = 0$) is chosen for convenience, and does not correspond to the actual start of the glottal motion. In practice, steady-state voicing is achieved only after many cycles, depending on damping and the mouth

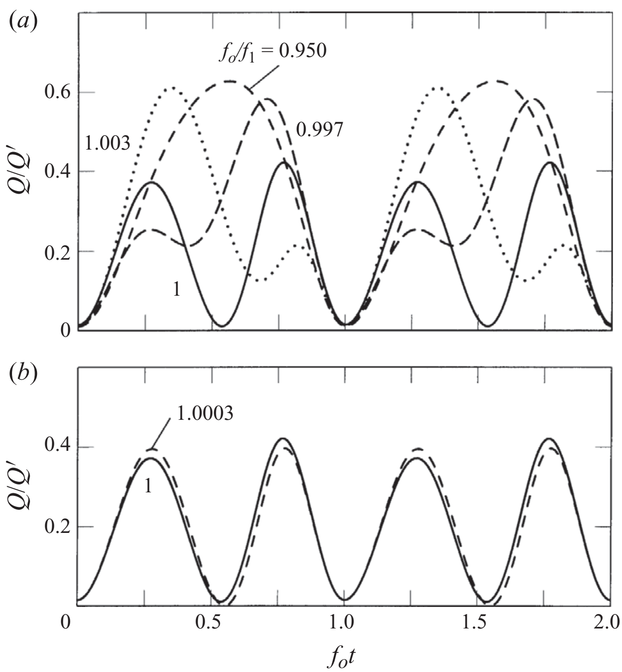

FIGURE 5. Dependence of the volume flux ratio Q/Q' on glottal frequency f_o near the cavity resonance frequency f_1 for the conditions of figure 4: (*a*) $f_o/f_1 = 1$ (———), 0.95 (– – – –), 0.997 (— — — —), 1.003 (· · · · · ·); (*b*) $f_o/f_1 = 1$ (———), 1.0003 (– – – –).

and glottal conductivities. During each cycle, the volume flux profile exhibits a single maximum that is in phase with the corresponding variation of the glottal area ratio A_g/A_L shown in figure 4(*b*); this is typical of all cases in which the glottal frequency f_o is *not* close to a subharmonic of the resonance frequency f_1. The corresponding free space sound pressure (figure 4*c*) is *forced*, periodic at the glottis frequency f_o (in agreement with Titze's (2008) numerical treatment of a duct-like model of the vocal tract).

The shape of the volume flux waveform changes rapidly when the glottal frequency f_o is very close to the resonator frequency f_1. At $f_o/f_1 = 0.95$, the predicted variation of Q/Q' (figure 5*a*) is similar to that in figure 4 for $f_o/f_1 = 0.6$. As f_o increases or decreases towards f_1, however, it is clear by inspection of the curves for $f_o/f_1 = 0.997$ and 1.003 in figure 5(*a*) that the component of Q of frequency f_o is increasingly suppressed, and that ultimately the motion is dominated by volume flux oscillations at *twice* the frequency ($2f_o = 2f_1$). This is analogous to the situation in which the cavity is used to filter sound of frequency f_1 – under ideal circumstances a wave of this frequency incident from $x_1 = -\infty$ on a constant area glottis would be totally reflected (Lighthill 1978). In the present case of a constant subglottal overpressure $2p_{\mathrm{I}}$, throttling of the steady mean flow at $f_o = f_1$ causes pulsatile entry of air into the cavity at frequency $2f_o$. Actually, the solid-line curve in figure 5(*b*) indicates that the volume flux waveform is not precisely periodic at frequency $2f_o$ when $f_o = f_1$. This presumably reflects changes in the effective resonance frequency of the cavity produced by nonlinearity and its weak dependence on the glottis cross-section A_g and slug length $\bar{\ell}$. Also, according to (4.7) the linear acoustic resonance frequency *does* exceed Ω. This appears to be confirmed by the broken-line plot in

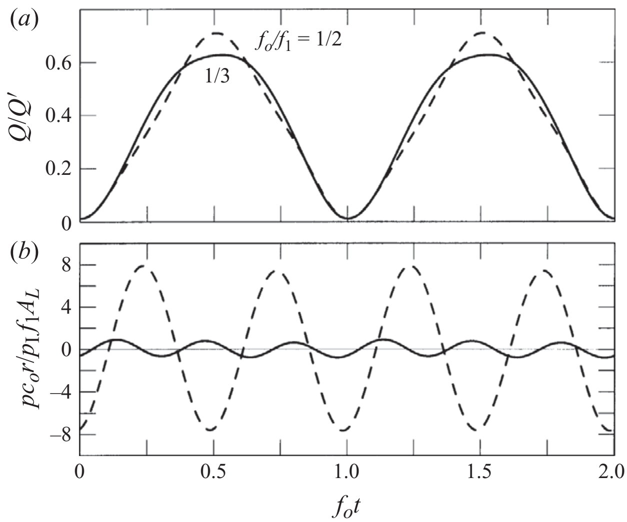

FIGURE 6. Volume flux ratio Q/Q' and the free space acoustic wave profile $pc_or/p_I f_1 A_L$ for the subharmonic frequencies $f_o/f_1 = 1/3, 1/2$ for the conditions of figure 4: (*a*) Q/Q': $f_o/f_1 = 1/3$ (———); $f_o/f_1 = 1/2$ (– – – –). (*b*) $pc_or/p_I f_1 A_L$: $f_o/f_1 = 1/3$ (———); $f_o/f_1 = 1/2$ (– – – –).

figure 5(*b*) for $f_o = 1.0003 f_1 \equiv 1.0003\Omega/2\pi$; the volume flux is now noticeably more periodic at frequency $2f_o$.

In this special case in which $f_o = f_1$, the acoustic radiation is dominated by the contribution from the cavity resonance frequency, the amplitude being very large because of the resonant growth of the pressure fluctuations within the cavity. A similar resonant enhancement of the sound occurs for excitation at subharmonics of the resonance frequency, i.e. for $f_o = f_1/2,\ f_1/3,\ \ f_1/4, \ldots,$ when f_1 is a harmonic of the glottal frequency f_o. Then, the overall sound pressure is periodic at the glottal frequency f_o, but nonlinearity causes a transfer of energy to the resonance frequency of the cavity, and the resulting energy storage within the cavity causes the sound to be dominated by this source. This is illustrated in figure 6 for the principal subharmonics $f_o/f_1 = 1/3, 1/2$. The near-triangular waveform of the variations of Q in figure 6(*a*) for $f_o/f_1 = 1/2$ (as compared, for example, with the more characteristic rounded waveform in figure 4*a*) indicates the presence of strong nonlinear interactions. Figure 6(*b*) confirms that the corresponding acoustic pressure (dashed line) is periodic with frequency f_o, although the sound is dominated by fluctuations at the resonance frequency $f_1 = 2f_o$. Nonlinear effects change rapidly with frequency when f_o is close to $f_1/2$; this causes the peak in the symmetric triangular form of the volume flux profile in figure 6(*a*) to be skewed to smaller/later times accordingly as $f_o \lesssim f_1/2$ (cf. Titze 2008). The same sort of interactions dominate the production of sound at $f_o/f_1 = 1/3$, although nonlinearity is now much less important, and figure 6(*b*) shows that the acoustic amplitude is very much reduced.

This type of skewing has been noted in numerical studies by Titze (2008), but over a much broader range of frequencies in the neighbourhoods of formant subharmonics. The broadening is presumably a consequence of the elevated and more realistic levels of tissue-related damping considered in Titze's analysis, which controls both the heights and widths of resonance peaks. The present results show, however, that skewing of the volume flux wave profile is not necessary to generate voice harmonics, which arise purely as a result of the nonlinearity of the Fant equation.

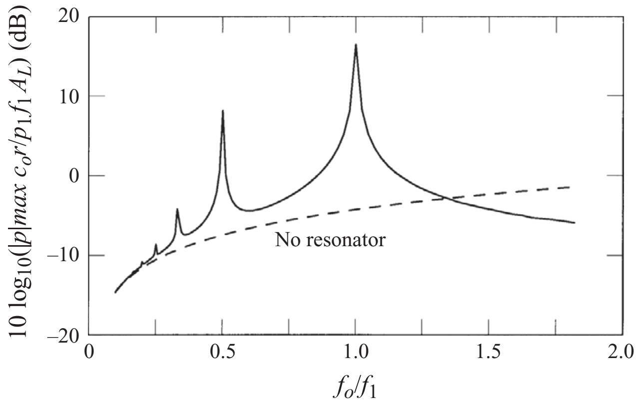

FIGURE 7. The peak free space acoustic pressure level $10 \log_{10}\{|p|_{max} c_o r/p_I f_1 A_L\}$ (dB) with (———) and without (– – – –) the cavity resonator, for the conditions of figure 4. The peaks visible to the left of the main peak for the resonator case occur at the subharmonics $f_o/f_1 = 1/4, 1/3, 1/2$.

The solid-line curve in figure 7 provides an overall picture of the efficiency of sound generation at different frequencies. This is a plot against f_o/f_1 of the maximum sound pressure level $10 \log_{10}(|p|_{max} c_o r/p_I f_1 A_L)$ (dB) in the far field at distance r from the cavity mouth. The curve is peaked at $f_o = f_1$ and at the visible subharmonic frequencies, at which the sound is dominated by the contribution from the cavity resonance. The widths of these peaks are determined by damping of the cavity oscillations; this is a combination of radiation losses into the lung complex and nonlinearity in the glottal flow. The peaks would be broadened in practice by additional tissue-related dissipative mechanisms. At all other frequencies, the dominant sound occurs at the forcing frequency f_o of the glottal oscillations, but the amplitude is still influenced by cavity back-reaction.

The dashed curve is the corresponding maximum sound pressure level when the cavity resonator is removed, i.e. for the configuration of figure 1 where the sound is always forced at the glottal frequency f_o. The calculation for this case is performed by replacing the system (4.2) by (2.30) for the volume flux Q, and the acoustic pressure $p = -\rho_o \partial\varphi/\partial t$ is calculated at distance r from the glottis using (1.1). This comparison illustrates the extent to which the back-reaction of the cavity on the glottis acts to increase acoustic levels over a wide range of non-subharmonic frequencies. The effect disappears at very low frequencies, for $f_o \leqslant 0.2 f_1$, say, when the cavity has little or no influence on the radiation, and the unsteady flux from the resonator mouth is in phase and equal to that from the glottis.

5. Conclusion

Unsteady flow from an orifice into free space is a source of ‘monopole’ sound. The source strength $Q(t)$ is equal to the volume flux from the orifice. When the orifice exhausts into a through-flow cavity, the subsequent radiation from the cavity exit is ‘filtered’ by cavity resonances. Classical acoustics represents the sound pressure (in free space for the case discussed in § 2, or within the cavity for the resonators of §§ 3, 4 and the Appendix) directly as the linear functional

$$\frac{p(\boldsymbol{x},t)}{\rho_o} = -\int_{-\infty}^{\infty} Q(\tau) \frac{\partial G_o}{\partial \tau}(\boldsymbol{x}, t-\tau)\, \mathrm{d}\tau, \tag{5.1}$$

where G_o denotes the acoustic 'source-filter' Green's function for a source just downstream of the throttle when the latter is closed. Voicing theory determines Q from the 'reduced complexity' Fant equation, which is similar to the Cummings equation used to solve throttling problems in engineering acoustics. Our theory of Fant's equation is based on Lighthill's theory of aerodynamic sound, which supplies a representation of the sound pressure (as in (2.3)) in terms of the mean flow, the throttling mechanism and the back-reactions on the orifice flow of cavity resonances and 'jetting'. A self-consistent Fant equation is obtained by equating this alternative representation to the right-hand side of (5.1).

When the throttle is time dependent, the aerodynamic sound problem is non-self-adjoint and the Green's function G required to solve Lighthill's equation must be determined as an advanced potential solution of the wave equation. For complex systems, it is not possible to implement directly the procedure of the previous paragraph because G cannot be obtained in a convenient analytical form. In these circumstances,

$$\frac{p(\boldsymbol{x},t)}{\rho_o} = \int_{-\infty}^{\infty} \beta(\tau)\mathscr{F}(\tau)\,\mathrm{d}\tau, \tag{5.2}$$

where $\mathscr{F}(\tau)$ is given in terms of the aeroacoustic sources, and $\beta(\tau)$ governs the behaviour of G in the neighbourhood of the throttle and is the solution of

$$\mathscr{L}(\tau)\beta(\tau) = \frac{\partial G_o}{\partial \tau}(\boldsymbol{x}, t-\tau), \tag{5.3}$$

here $\mathscr{L}(\tau)$ being a known time-dependent, linear integro-differential operator. Equations (5.1)–(5.3) and causality then yield Fant's equation in the form

$$\hat{\mathscr{L}}(t)Q(t) = -\mathscr{F}(t), \tag{5.4}$$

where $\hat{\mathscr{L}}$ is the adjoint of $\mathscr{L}$.

Two interesting conclusions have emerged from the application of (5.4) to voicing and to the canonical problem of throttled flow into a Helmholtz resonator that have hitherto been largely unremarked in speech science. First, there can be a substantial loss into the subglottal tract when it is assumed to be completely non-reflecting, although it may still be small compared to tissue-related damping. Second, and as pointed out by Titze (2008), nonlinearity of the Fant equation implies that a skewed glottal pulse is not a necessary prerequisite for the generation of voice harmonics.

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Appendix. Throttled flow into a circular cylindrical resonator

A summary overview is given here of the application of the 'adjoint' method of § 3 to the configuration of figure 8, where the resonant cavity is a rigid-walled circular cylinder of length L and cross-sectional area A, with axisymmetric openings at $x = 0$, L corresponding respectively to the glottis and mouth of §§ 3 and 4. Consider an 'open' mouth characterised by an end correction that increases the effective length of the tract to $x_1 = \bar{L} > L$, at which the acoustic pressure may be assumed to vanish.

Fant's equation is derived by calculating the sound at an arbitrary point $\boldsymbol{x}$ in the supraglottal tract by the method of linear acoustics for a given monopole source Q at the glottis, and by application of Lighthill's theory. The compact approximations (2.9)

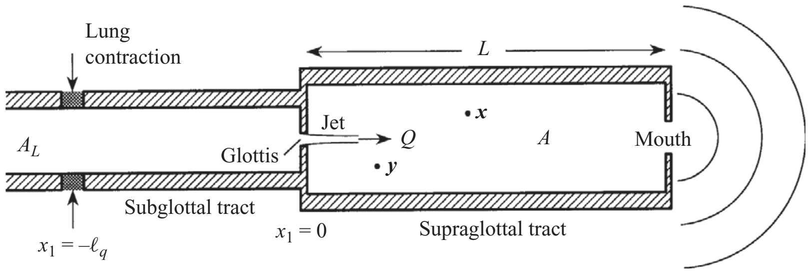

FIGURE 8. Throttled flow into the supraglottal tract modelled by a cylindrical tube of interior length L and cross-section A. The Fant equation is derived by consideration of the sound produced at an arbitrary point $\boldsymbol{x}$ within the body of the tract.

and (2.12) for the advanced potential Green's function for positions $\boldsymbol{y}$ respectively near and in the glottis and within the subglottal tract are still applicable. Within the resonator the compact approximation is

$$\begin{aligned} G = \frac{A_L}{2\pi A} \iint_{-\infty}^{\infty} \frac{\sin[k_o(y_1 - \bar{L})]}{k_o \cos(k_o \bar{L})} \beta(\xi) \mathrm{e}^{-\mathrm{i}\omega(\xi-\tau)} \, \mathrm{d}\omega \, \mathrm{d}\xi \\ - \frac{1}{2\pi A} \int_{-\infty}^{\infty} \Big\{ H(x_1 - y_1) \cos(k_o y_1) \sin[k_o(x_1 - \bar{L})] \\ + H(y_1 - x_1) \cos(k_o x_1) \sin[k_o(y_1 - \bar{L})] \Big\} \frac{\mathrm{e}^{-\mathrm{i}\omega(t-\tau)} \, \mathrm{d}\omega}{k_o \cos(k_o \bar{L})}, \end{aligned} \tag{A 1}$$

where $k_o = \omega/c_o$, and the integrations with respect to ω are taken along paths that pass above singularities in the ω-plane. Matching of the glottal and supraglottal formulae for G requires that G given by (A 1) should equal $\alpha(\tau)$ as $y_1 \to +0$, so that

$$\begin{aligned} \alpha(\tau) = - \frac{A_L}{2\pi A} \iint_{-\infty}^{\infty} \frac{\sin(k_o \bar{L})}{k_o \cos(k_o \bar{L})} \beta(\xi) \mathrm{e}^{-\mathrm{i}\omega(\xi-\tau)} \, \mathrm{d}\omega \, \mathrm{d}\xi \\ - \frac{1}{2\pi A} \int_{-\infty}^{\infty} \frac{\sin[k_o(x_1 - \bar{L})]}{k_o \cos(k_o \bar{L})} \mathrm{e}^{-\mathrm{i}\omega(t-\tau)} \, \mathrm{d}\omega. \end{aligned} \tag{A 2}$$

By evaluating the ω-component of the first integral on the right-hand side, and substituting into (2.14), we find that β is determined by

$$\begin{aligned} \frac{\partial}{\partial \tau}(\bar{\ell}\beta) - c_o\beta - \frac{2A_L c_o^2}{A\bar{L}} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \beta(\xi) H(\xi - \tau) \cos\left[\left(n + \frac{1}{2}\right) \frac{\pi c_o}{\bar{L}} (\xi - \tau)\right] \mathrm{d}\xi \\ = - \frac{\mathrm{i}c_o}{2\pi A} \int_{-\infty}^{\infty} \frac{\sin[k_o(x_1 - \bar{L})]}{\cos(k_o \bar{L})} \mathrm{e}^{-\mathrm{i}\omega(t-\tau)} \, \mathrm{d}\omega. \end{aligned} \tag{A 3}$$

The term on the right-hand side is equal to $\partial G_o/\partial \tau$, where $G_o(x_1, t - \tau)$ is the compact cavity acoustic Green's function for a source just downstream of the glottis when the latter is closed (i.e. the second term on the right-hand side of (A 1) as $y_1 \to +0$).

It may now be asserted, as before, that the required Fant equation is

$$\bar{\ell}\frac{\partial Q}{\partial t} + c_o Q + \frac{2A_L c_o^2}{A\bar{L}} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} Q(\xi) H(t - \xi) \cos\left[\left(n + \frac{1}{2}\right) \frac{\pi c_o}{\bar{L}} (t - \xi)\right] \mathrm{d}\xi = \mathscr{F}(t), \tag{A 4}$$

where $\mathscr{F}(t)$ is defined as in (2.19).

The proof (by the procedure described in § 3.2 for (3.7)) depends on showing that Q determined by

$$\frac{\mathrm{i}c_o}{2\pi A}\iint_{-\infty}^{\infty}\frac{Q(\tau)\sin[k_o(x_1-\bar{L})]}{\cos(k_o\bar{L})}\mathrm{e}^{-\mathrm{i}\omega(t-\tau)}\,\mathrm{d}\omega\,\mathrm{d}\tau = \int_{-\infty}^{\infty}\beta(\tau)\mathscr{F}(\tau)\,\mathrm{d}\tau \tag{A 5}$$

corresponds to the unsteady glottal flux. In other words, that the left-hand side of (A 5) is just the acoustic field of a monopole source at $y_1=0$ predicted according to linear acoustics.

The right-hand side of (A 5) is the acoustic field $B(x_1,t)$ within the supraglottal tract produced by lung contraction and the jet vorticity. When this field is represented by the plane wave velocity potential $\Phi(x_1,t)$ produced by the glottal monopole flow, then (A 5) implies that

$$\frac{\partial\Phi}{\partial t}(x_1,t) = -\frac{\mathrm{i}c_o}{2\pi A}\iint_{-\infty}^{\infty}\frac{Q(\tau)\sin[k_o(x_1-\bar{L})]}{\cos(k_o\bar{L})}\mathrm{e}^{-\mathrm{i}\omega(t-\tau)}\,\mathrm{d}\omega\,\mathrm{d}\tau, \tag{A 6}$$

from which it follows that

$$\lim_{x_1\to+0} A\frac{\partial^2\Phi}{\partial t\,\partial x_1} = \frac{\partial}{\partial t}\left(\frac{1}{2\pi}\iint_{-\infty}^{\infty}Q(\tau)\,\mathrm{e}^{-\mathrm{i}\omega(t-\tau)}\,\mathrm{d}\omega\,\mathrm{d}\tau\right) = \frac{\partial Q}{\partial t}, \tag{A 7}$$

and therefore that $Q(t)$ is indeed the glottal flux.

The explicit form of the Fant equation (A 4) is

$$\bar{\ell}\frac{\partial Q}{\partial t} + c_oQ + \int_{V(t)}\frac{\partial Y}{\partial \boldsymbol{y}}\cdot\boldsymbol{\omega}\wedge\boldsymbol{v}\,\mathrm{d}^3\boldsymbol{y} + \frac{2A_Lc_o^2}{A\bar{L}}\sum_{n=0}^{\infty}\int_0^t Q(\tau)\cos\left[\left(n+\frac{1}{2}\right)\frac{\pi c_o}{\bar{L}}(t-\tau)\right]\mathrm{d}\tau = \frac{2A_Lp_{\mathrm{I}}}{\rho_o}H(t). \tag{A 8}$$

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