STABILITY AND CONSTANT BOUNDARY-VALUE PROBLEMS OF HARMONIC MAPS WITH POTENTIAL

QUN CHEN

Abstract

Let M, N be Riemannian manifolds, $f : M \to N$ a harmonic map with potential H, namely, a smooth critical point of the functional $E_H(f) = \int_M [e(f) - H(f)]$, where $e(f)$ is the energy density of f. Some results concerning the stability of these maps between spheres and any Riemannian manifold are given. For a general class of M, this paper also gives a result on the constant boundary-value problem which generalizes the result of Karcher-Wood even in the case of the usual harmonic maps. It can also be applied to the static Landau-Lifshitz equations.

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Let $f : (M, g) \to (N, h)$ be a map between Riemannian manifolds, H a smooth function on N. Consider the following functional

$$(1) \qquad E(f) = \int_M [e(f) - H(f)],$$

where $e(f) = \frac{1}{2} \operatorname{Trace}_g f^* h$ is the energy density of f. The Euler-Lagrange equation of $E_H(f)$ is

$$(2) \qquad \tau(f) + \nabla H(f) = 0,$$

where $\tau(f)$ is the tension field of f, and ∇H is the gradient of H on N. We call a smooth solution f of (2) a harmonic map with potential H. This is a new kind of generalized harmonic maps recently introduced by Fardoun and Ratto in [FR]. Besides the usual harmonic maps, this kind of maps also includes the Landau-Lifshitz equations as a special case. Consider the following static Landau-Lifshitz equation

$$(3) \qquad \Delta f + f |\nabla f|^2 - (H_0 \cdot f)f + H_0 = 0,$$

where $|f(x)|^2 = 1, x \in \Omega \subset \mathbb{R}^m, H_0 \neq 0$ is a constant vector in $\mathbb{R}^3$, and $\cdot$ denotes the inner product in $\mathbb{R}^3$. In fact, the solution f of (3) can be seen as a harmonic map with potential: $\Omega \to S^2$ with the potential $H(y) = H_0 \cdot y, y \in S^2$.

Fardoun and Ratto considered harmonic maps with potential of spheres and tori. In the author's previous papers [C1, C2, C3], some general properties such as maximum principles, uniqueness, existence and Liouville type results were obtained, also some direct applications to Landau-Lifshitz equation were given. In this paper, we will consider the stability and constant boundary-value problems of harmonic maps with potential. Here the stability is defined in the following sense.

Let $f : M \to N$ be a smooth map, then for any $w \in \Gamma(f^{-1}TN)$, there is a family of maps $f_t : M \to N$ such that

$$(4) \qquad \begin{cases} f_0 = f \\ \dfrac{df_t}{dt}\bigg|_{t=0} = w \end{cases}$$

for $t \in [0, \varepsilon], \varepsilon > 0$. Let H be a smooth function on N, f a harmonic map with potential H. Denote

$$(5) \qquad E_H(f_t) = \int_M [e(f_t) - H(f_t)].$$

We say the map f is *stable* if for all $w \in \Gamma(f^{-1}TN)$, $d^2E_H(f_t)/dt^2|_{t=0} \geq 0$. Let us first state our results.

THEOREM 1. *If $n > 2$, then all stable harmonic maps with potential from S^n into any Riemannian manifold N must be constant.*

THEOREM 2. *Let M be a compact Riemannian manifold, $H \in C^\infty(S^n)$.*

(1) *If $n \geq 2$, $\nabla^2 H > 0$, then any nonconstant harmonic map with potential H from M into S^n is unstable.*

(2) *If $n > 2$, $\nabla^2 H \geq 0$, then any nonconstant harmonic map with potential H from M into S^n is unstable.*

Here $\nabla^2 H$ denotes the Hessian of H.

REMARK 1. For the usual harmonic maps, Xin [Xin1] proved a well-known result:

(A)   If $n > 2$, then all stable harmonic maps from S^n into any Riemannian manifold must be constant.

By a similar method, Leung [Lg] obtained:

(B)   If $n > 2$, then all stable harmonic maps from any compact Riemannian manifold into S^n must be constant.

Since in general, the properties of harmonic maps with potential H depend heavily on H, the conclusion of Theorem 1 is surprising. It means that the result (A) is essential.

REMARK 2. In [FR], by α-join method, Fardoun and Ratto constructed a family of harmonic maps with potential: $S^{p+r-1} \to S^{q+s-1}$, $p, r \geq 2$. From Theorem 1, those maps are all unstable.

As for the constant boundary-value problems, in the case of the usual harmonic maps, Karcher and Wood [KW] proved that any harmonic map $f : B^m \to N$ ($m \geq 3$) which is constant on ∂B^m must be constant in B^m. On the other hand, Hong [Ho] asserted that the static Landau-Lifshitz equation (3) with constant boundary-value $f|_{\partial\Omega} = H_0/|H_0|$ has only constant solution, if $\Omega = B^3$. In fact, these two kinds of problems can be treated simultaneously in the more general frame of harmonic maps with potential.

We consider rather general cases and obtain the following theorem.

THEOREM 3. *Let M be an m-dimensional complete, simply connected and nonpositive curved Riemannian manifold, $m > 2$. Suppose its sectional curvature satisfies one of the following conditions*:

(1) $-a^2 \leq K_M \leq -b^2$, $(m-1)b/2 \geq a$;
(2) $-A/(1 + r^2(x)) \leq K_M(x) \leq 0$, $A < m(m-2)/4$,

where a, b, A are positive constants, $r(x)$ denotes the distance from x to a fixed point $p \in M$. Let $B_p(R)$ be the geodesic ball of radius R centered at p, N be any Riemannian manifold, $H \in C^\infty(N)$, $f : M \to N$ be a harmonic map with potential H such that $f|_{\partial B_p(R)} \equiv P \in N$. If $H(P) = \max_{y \in N} H(y)$, then f must be constant in $B_p(R)$.

REMARK 3. Let $H \equiv 0$, then Theorem 3 gives a result for the usual harmonic maps which generalizes the above mentioned result of Karcher and Wood. If we choose $M = \mathbb{R}^m$ ($m > 2$), $N = S^2$, $H(y) = H_0 \cdot y$, $y \in S^2$, then Theorem 3 leads to a conclusion for the static Landau-Lifshitz equation, in particular, when $m = 3$, it is just the result of Hong. Our result also generalizes Theorem 3 in [FR].

REMARK 4. If M satisfies $-a^2 \leq K_M \leq 0$ and $Ric_M \leq -b^2 < 0$ with $b \geq 2a$, then the same conclusion as in Theorem 3 also holds. This kind of M concludes the bounded symmetric domains and complex hyperbolic spaces discussed in [Xin4].

Now let us prove the above theorems. To prove Theorem 1 and Theorem 2, we first establish the second variation formula.

LEMMA 1. *Let $f : M \to N$ be a harmonic map with potential H, f_t, w and $E_H(f_t)$ be as in (4) and (5), then*

$$\frac{d^2 E_H(f_t)}{dt^2}\bigg|_{t=0} = -\int_M [\langle \nabla^2 w + R^N(f_* e_i, w) f_* e_i, w \rangle + \nabla^2 H(w, w)],$$

where $\{e_i, i = 1, 2, \ldots, m\}$ are local orthonormal frames around the considered points x in M such that $\nabla_{e_i} e_j = 0$ for $i, j = 1, 2, \ldots, m$.

PROOF. It is easy to see that

$$\frac{dE(f_t)}{dt} = -\int_M \left\langle \frac{df_t}{dt}, \tau(f_t) \right\rangle.$$

So,

$$\frac{dE_H(f_t)}{dt} = -\int_M \left[\left\langle \frac{df_t}{dt}, \tau(f_t) \right\rangle + \nabla_{\partial_t} H(f_t) \right],$$

but $\nabla_{\partial_t} H(f_t) = \partial H \circ f_t / \partial t = \langle \nabla H, df_t/dt \rangle$, hence,

$$\frac{dE_H(f_t)}{dt} = -\int_M \left\langle \frac{df_t}{dt}, \tau(f_t) + \nabla H \right\rangle.$$

Noting that at $t = 0$, $\tau(f) + \nabla H = 0$, we have

(6) $$\frac{d^2 E_H(f_t)}{dt^2}\bigg|_{t=0} = -\int_M \left\langle \frac{df_t}{dt}, \nabla_{\partial_t} \tau(f_t) + \nabla_{\partial_t}(\nabla H) \right\rangle\bigg|_{t=0}.$$

At any point $x \in M$, we choose local orthonormal frame fields $\{e_i, i = 1, 2, \ldots, m\}$ such that at x,

$$\nabla_{\partial_t} \partial_t = \nabla_{\partial_t} e_i = \nabla_{e_i} e_i = 0.$$

Then,

$$\begin{aligned}
\nabla_{\partial_t} \tau(f_t) &= \nabla_{\partial_t}[(\nabla_{e_i} df_t)(e_i)] = (\nabla_{\partial_t} \nabla_{e_i} df_t)(e_i) \\
&= (R(e_i, \partial_t) df_t)(e_i) + (\nabla_{e_i} \nabla_{\partial_t} df_t)(e_i) \\
&= \nabla_{e_i}[(\nabla_{\partial_t} df_t)(e_i)] + R^N \left(f_{t*} e_i, \frac{df_t}{dt} \right) f_{t*} e_i \\
&= \nabla_{e_i} \nabla_{e_i} \frac{df_t}{dt} + R^N \left(f_{t*} e_i, \frac{df_t}{dt} \right) f_{t*} e_i,
\end{aligned}$$

so,

(7) $$\nabla_{\partial_t} \tau(f_t)|_{t=0} = \nabla^2 w + R^N(f_{t*} e_i, w) f_{t*} e_i.$$

On the other hand,

$$\left\langle \frac{df_t}{dt}, \nabla_{\partial_t}(\nabla H) \right\rangle = \left\langle \frac{df_t}{dt}, \nabla_{\frac{df_t}{dt}}(\nabla H) \right\rangle = \nabla^2 H \left(\frac{df_t}{dt}, \frac{df_t}{dt} \right),$$

hence,

(8) $$\left\langle \frac{df_t}{dt}, \nabla_{\partial_t}(\nabla H) \right\rangle \bigg|_{t=0} = \nabla^2 H(w, w).$$

Putting (7) and (8) into (6), we get the desired conclusion.   □

REMARK 5. [FR] also mentioned a second variation formula without proof (but missed the integral of the Hessian of potential).

PROOF (of Theorem 1). Following [Xin1], we use the conformal vector fields on S^n. Let $\alpha \in \mathbb{R}^{n+1}$, define $h(x) = \langle \alpha, x \rangle, x \in S^n \subset \mathbb{R}^{n+1}$, where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathbb{R}^{n+1}$. Construct $v = \operatorname{grad} h$. At any $x \in S^n$, we choose local orthonormal frame fields $\{e_i\}$ such that $\nabla_{e_i} e_j(x) = 0$. It is known that

(9) $$v = \langle \alpha, e_i \rangle e_i, \quad \nabla_X v = -hX, \quad \nabla^2 v = -v$$

for all $X \in \Gamma(TS^n)$. Suppose $f : M \to N$ is a harmonic map with potential H. Denote

$$I_H(w, w) = \frac{d^2 E_H(f_t)}{dt^2} \bigg|_{t=0}, \quad I(w, w) = \frac{d^2 E(f_t)}{dt^2} \bigg|_{t=0},$$

where w and f_t satisfy (4). Then Lemma 1 implies

(10) $$I_H(w, w) = I(w, w) - \int_{S^n} \nabla^2 H(w, w),$$

and

(11) $$I(w, w) = -\int_{S^n} \langle \nabla^2 w + R^N(f_* e_i, w) f_* e_i, w \rangle.$$

Choose $w = f_* v$ and compute $I_H(f_* v, f_* v)$. Firstly,

(12) $$I(f_* v, f_* v) = -\int_{S^n} \langle \nabla^2 f_* v + R^N(f_* e_i, f_* v) f_* e_i, f_* v \rangle.$$

Since

$$\begin{aligned}
-\nabla^2 f_* v &= -\nabla_{e_i} \nabla_{e_i} f_* v \\
&= -\nabla_{e_i}[(\nabla_{e_i} df)(v) + df(\nabla_{e_i} v)] \\
&= -(\nabla_{e_i} \nabla_{e_i} df)(v) - (\nabla_{e_i} df)(\nabla_{e_i} v) - (\nabla_{e_i} df)(\nabla_{e_i} v) - df(\nabla_{e_i} \nabla_{e_i} v) \\
&= -(\nabla^2 df)(v) - 2(\nabla_{e_i} df)(\nabla_{e_i} v) - df(\nabla^2 v),
\end{aligned}$$

by (9) we have

$$(13) \qquad -\nabla^2 f_* v = -(\nabla^2 df)(v) + 2h\tau(f) + f_* v,$$

and using the Weitzenbök formula [EL, (1.30)] we obtain

$$-(\nabla^2 df)(v) = (\Delta df)(v) + R^N(f_* e_i, f_* v) f_* e_i - f_* \operatorname{Ric}_{S^n} v.$$

Note that $\Delta(df) = -d\tau(f) = d(\nabla H)$ and consequently,

$$(\Delta df)(v) = (d(\nabla H))(v) = \nabla_v(\nabla H),$$

we then have

$$(14) \qquad -(\nabla^2 df)(v) = \nabla_v(\nabla H) + R^N(f_* e_i, f_* v) f_* e_i - (n-1) f_* v.$$

Substituting (14) into (13) yields

$$-\nabla^2 f_* v = \nabla_v(\nabla H) + R^N(f_* e_i, f_* v) f_* e_i - (n-1) f_* v + 2h\tau(f) + f_* v,$$

putting this into (12) we get

$$\begin{aligned} I(f_* v, f_* v) &= \int_{S^n} \langle \nabla_v(\nabla H), f_* v \rangle + 2 \int_{S^n} h \langle \tau(f), f_* v \rangle + (2-n) \int_{S^n} |f_* v|^2 \\ &= \int_{S^n} \nabla^2 H(f_* v, f_* v) + (2-n) \int_{S^n} |f_* v|^2 + 2 \int_{S^n} h \langle \tau(f), f_* v \rangle . \end{aligned}$$

By (10),

$$(15) \qquad I_H(f_* v, f_* v) = (2-n) \int_{S^n} |f_* v|^2 + 2 \int_{S^n} h \langle \tau(f), f_* v \rangle ,$$

but

$$\begin{aligned} 2 \int_{S^n} h \langle \tau(f), f_* v \rangle &= \int_{S^n} \langle \tau(f), df(2hv) \rangle \\ &= \int_{S^n} \langle \tau(f), df \left(\operatorname{grad} h^2 \right) \rangle = \int_{S^n} \langle \tau(f), df \left(\operatorname{grad} \langle \alpha, x \rangle^2 \right) \rangle , \end{aligned}$$

so,

$$(16) \qquad I_H(f_* v, f_* v) = (2-n) \int_{S^n} |f_* v|^2 + \int_{S^n} \langle \tau(f), df \left(\operatorname{grad} \langle \alpha, x \rangle^2 \right) \rangle .$$

Denote $\{\varepsilon_A, A = 1, 2, \ldots, n+1\}$ the standard orthonormal basis in $\mathbb{R}^{n+1}$. In (16) we choose $\alpha = \varepsilon_A$, and $v = v_A = \langle \varepsilon_A, e_i \rangle e_i$, $A = 1, 2, \ldots, n+1$, and compute the sum

$$\sum_{A=1}^{n+1} I_H(f_* v_A, f_* v_A) = \sum_{A=1}^{n+1} (2-n) \int_{S^n} |f_* v_A|^2 + \int_{S^n} \left\langle \tau(f), df \left(\operatorname{grad} \sum_{A=1}^{n+1} \langle \varepsilon_A, x \rangle^2 \right) \right\rangle$$

$$= 2(2-n) \int_{S^n} e(f) + \int_{S^n} \langle \tau(f), df (\operatorname{grad} 1) \rangle$$

$$= 2(2-n) \int_{S^n} e(f) = 2(2-n) E(f).$$

If f is stable, then $2(2-n) E(f) \geq 0$, thus f must be constant.   □

PROOF (of Theorem 2). At any $x \in M$, we choose local orthonormal frame fields $\{e_i, i = 1, 2, \ldots, m\}$ such that $\nabla_{e_i} e_j(x) = 0$. Also choose orthonormal frame fields $\{\bar{e}_s, s = 1, 2, \ldots, n\}$ in S^n. Let $\alpha \in \mathbb{R}^{n+1}$. Similar to the proof of Theorem 1, we set $h(y) = \langle \alpha, y \rangle$, $y \in S^n$, and $v = \operatorname{grad} h$, then $v = \sum_{s=1}^{n} \langle \alpha, \bar{e}_s \rangle \bar{e}_s$.

By Lemma 1,

(17) $\qquad I_H(v, v) = - \int_M \langle \nabla^2 v + R^{S^n}(f_* e_i, v) f_* e_i, f_* v \rangle - \int_M \nabla^2 H(v, v).$

Note that

$$- \int_M \langle \nabla^2 v, v \rangle = \int_M |\nabla v|^2 = \int_M |\nabla_{f_* e_i} v|^2,$$

but from (9), $\nabla_{f_* e_i} v = -h f_* e_i$, so,

(18) $\qquad - \int_M \langle \nabla^2 v, v \rangle = \int_M h^2 |f_* e_i|^2 = 2 \int_M e(f) \langle \alpha, f(x) \rangle^2.$

On the other hand,

$$\langle R^{S^n}(f_* e_i, v) f_* e_i, v \rangle = 2e(f) |v|^2 - \langle f_* e_i, v \rangle^2.$$

Substituting this and (18) into (17), we obtain

$$I_H(v, v) = 2 \int_M e(f) \langle \alpha, f(x) \rangle^2 - 2 \int_M e(f) |v|^2 + \int_M \langle f_* e_i, v \rangle^2 - \int_M \nabla^2 H(v, v)$$

$$= 2 \int_M e(f) \left[\langle \alpha, f(x) \rangle^2 - \sum_{s=1}^{n} \langle \alpha, \bar{e}_s \rangle^2 \right] + \int_M \langle f_* e_i, v \rangle^2 - \int_M \nabla^2 H(v, v).$$

Hence,

$$\sum_{A=1}^{n+1} I_H(v_A, v_A) = 2(2-n) \int_M e(f) - \sum_{A=1}^{n+1} \int_M \nabla^2 H(v_A, v_A).$$

Then the conclusions follow from the above equality.   □

To prove Theorem 3, we introduce the following useful formula.

LEMMA 2 ([Xin2]). *Let $f : (M, g) \to (N, h)$ be a smooth map, $D \subset M$ a compact domain such that ∂D is a smooth hypersurface in M. Let n denotes the outer unit normal vector of ∂D. Let X be any vector field in M. Then*

$$(19) \qquad \int_{\partial D} e(f) \langle X, n \rangle = \int_{\partial D} \langle f_* X, f_* n \rangle + \int_D \operatorname{div} S_f (X) + \int_D \langle S_f, \nabla X \rangle,$$

where $S_f = e(f) g - f^* h$.

PROOF (of Theorem 3). Choose the geodesic polar coordinates (θ, r) in $B_p(R)$. In Lemma 2 we set $D = B_p(R)$, $X = r\partial/\partial r$, and $n = \partial/\partial r$.
It is known that (see [BE])

$$\operatorname{div} S_f = -\langle \tau(f), df \rangle,$$

so,

$$\operatorname{div} S_f (X) = -\left\langle \tau(f), rf_* \frac{\partial}{\partial r} \right\rangle = r \left\langle \nabla H, f_* \frac{\partial}{\partial r} \right\rangle = r \frac{\partial H \circ f}{\partial r}.$$

From Lemma 2,

$$(20) \qquad R \int_{\partial B_p(R)} e(f) = R \int_{\partial B_p(R)} \left| f_* \frac{\partial}{\partial r} \right|^2 + \int_{B_p(R)} r \frac{\partial H \circ f}{\partial r} + \int_{B_p(R)} \langle S_f, \nabla X \rangle.$$

By the computation in [Xin3], there is a number $\delta > 0$ such that

$$(21) \qquad \langle S_f, \nabla X \rangle \geq \delta e(f).$$

Noting that f is constant at $\partial B_p(R)$, and using (20) and (21), we have

$$(22) \qquad \int_{B_p(R)} r \frac{\partial H \circ f}{\partial r} + \delta \int_{B_p(R)} e(f) \leq 0.$$

Now let us estimate the integral

$$I = \int_{B_p(R)} r \frac{\partial H \circ f}{\partial r}.$$

Denote $J(\theta, r) d\theta dr$ the volume element in $B_p(R)$. Recall that ([Li])

$$(23) \qquad \Delta r = \frac{\partial J(\theta, r)}{\partial r} \frac{1}{J(\theta, r)}.$$

Since $K_M \leq 0$, by a new Laplacian comparison theorem [Xin4, Di],

$$\Delta r \geq \frac{1}{r}.$$

From this and (23), we have

$$\frac{1}{J(\theta, r)} \frac{\partial J(\theta, r)}{\partial r} \geq \frac{1}{r}.$$

Hence,

(24) $$\frac{\partial}{\partial r}(rJ(\theta, r)) \geq 2J(\theta, r) > 0.$$

Write

$$I = \int_{S^{m-1}} \left(\int_0^R r \frac{\partial H \circ f}{\partial r} J(\theta, r) dr \right) d\theta,$$

and note that the integrand

$$\tilde{I} \equiv \int_0^R r \frac{\partial H \circ f}{\partial r} J(\theta, r) dr$$

$$= R J(\theta, R) H(P) - \int_0^R H \circ f(\theta, r) \frac{\partial}{\partial r}(rJ(\theta, r)) dr$$

$$\geq R J(\theta, R) H(P) - H(P) \int_0^R \frac{\partial}{\partial r}(rJ(\theta, r)) dr$$

$$= 0.$$

Therefore, $I \geq 0$. From (22), we conclude $e(f) \equiv 0$ in $B_p(R)$, namely, f is constant in $B_p(R)$.   □

Acknowledgement

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Mathematics Department
Central China Normal University
Wuhan 430079
China

School of Mathematical Science
Wuhan University
Wuhan 430072
China
e-mail: qchen@mail.ccnu.edu.cn