

ON THE VALUE OF OPTIONS IN CERTAIN CONTRACTS.

To the Editor.

SIR,—The equation of condition at the end of my letter in the last Number of the *Journal* needs a correction, owing to the accidental omission of a factor. The equation should be

$$Q.P_x + \frac{\lambda_n}{l_{x+n}} \cdot \frac{N_{x+n}}{D_x} = P_x.$$

The annuity portion of the risk is evidently $= \frac{\lambda_n}{l_x} v^n \cdot a_{x+n}$, or $\frac{\lambda_n}{l_{x+n}} \cdot \frac{N_{x+n}}{D_x}$, since $\frac{\lambda_n}{l_x}$ is the probability that any one of the lives will enter upon the annuity. Hence

$$P_x = \frac{\lambda_n}{l_{x+n}} \cdot \frac{N_{x+n}}{(1-Q)D_x}.$$

Taking the example given in my letter, we have, as before, $Q = .42556$ and $\lambda_{10} = 2170.32$; hence $P_x = 5.6094$.

To determine the precise effect of the withdrawals upon the amount of the premium, we must find what the premium would be if it were returnable at the moment of death and no withdrawals were allowed. In this

case it is easily found that $P_x = \frac{N_{x+n}}{D_x - \frac{i}{\log_e(1+i)}(M_x - M_{x+n})} = 6.2828$.

It therefore appears that if the withdrawals which have been assumed were certain to take place, the effect would be to *lessen* the premium, not to increase it.

I did not perceive the omission of the factor referred to until the day the *Journal* was published, otherwise the numerical example before given would have been corrected, and the paradoxical nature of its result would thus have been removed.

At whatever time a withdrawal takes place, it is obvious that the return of the original premium without interest must leave the Office a gainer, because the risk continually increases, while the premium returned merely represents the initial value of the risk. Thus the Office is set free from a liability without being called upon to pay its full equivalent value; hence the reduction which takes place in the premium.

I am, Sir,

Your obedient servant,

316, Regent Street, London,
23th May, 1866.

SAMUEL YOUNGER.