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ON PHASE SHIFTS IN A NEW KEYNESIAN MODEL ECONOMY

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The purpose of this paper is to focus directly on the phase shift. For one thing, we ask whether a New Keynesian sticky-price model economy can account for both countercyclical prices and procyclical inflation. We present findings in which the price level is countercyclical and the inflation rate is procyclical. We proceed to use the model economy as an identification mechanism. What set of individual shocks are necessary to account for the phase shift? That set contains the price markup shock. Next, we ask what set of shocks are sufficient to account for the phase shift. This set contains three elements: the price markup and wage markup shocks along with the government spending shock. The results are important as a building block. We infer that price stickiness is an important model feature; without price stickiness, we are in the real business cycle economies that Cooley and Hansen studied. But, it raises further questions. For instance, is price stickiness of the Calvo form—the one used here—necessary to explain the phase shift?

Keywords: Phase Shift, Countercyclical Price, Procyclical Inflation, Necessary and Sufficient Shocks, Bayesian Estimation

1. INTRODUCTION

Business cycle facts are represented by comovements, typically with real gross domestic product (GDP), and by volatilities. When representing comovements, researchers often characterize the lead–lag relationship, or phase shift, between time series. Since Friedman and Schwartz (1963), researchers have studied the relationship between the price level and output. As researchers applied statistical techniques to extract the cyclical component from the observed time series, there were two postwar correlations that are presented: the price level is countercyclical and the inflation rate is procyclical.[1] There is a simple way to reconcile these two facts; there is a phase shift in the relationship between the price level and output.

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To illustrate the phase shift, consider a case in which the cyclical components of the price level and output had the same periodicity. Note that if the price level and output were in phase in the sense that the peak of the cycle in output occurred simultaneously with the trough in the cycle of the price level, then the rate of change in the price level—that is, the inflation rate—would be negatively correlated with output as the price level is. However, if the price-level waveform was shifted to the left horizontally, then it is possible for the price level to be countercyclical and the inflation rate to be procyclical. It is just such a shift in the price level relative to output that we are looking for in a dynamic stochastic general equilibrium (DSGE) model economy.

In broad terms, business cycle explanations are divided into two camps. The New Keynesian (NK) camp and the Real-Business Cycle (RBC) camp are similar in many ways. The consumption-saving choice is an essential dynamic force through which shocks are propagated. In addition, the aggregate technology is indistinguishable between the two camps. And, expectations are rationally formed. Of course, some forms of market power and price stickiness are important differences between the two camps.[2] Another is the way in which the NK model economies specified monetary policy; Woodford (2003) and others focused on monetary policy as a means of setting the nominal interest rate. In doing so, the so-called "cashless" model economies were being analyzed. Central banks specify nominal interest rate targets and the Taylor rule is justified on that practice. By specifying a Taylor rule, however, the price level can be ignored, or even discarded. Hence, countercyclical prices are essentially excluded from the set of facts considered by researchers specifying NK model economies.

Modern macroeconomics has used the price level and the inflation rate as a dividing line insofar as the cyclical properties of these variables are qualitatively different. Such evidence is consistent with a phase shift in the relationship between the price level and output. King and Watson (1996) provided key background on the existence of a phase shift in the post-World War II data; specifically, they presented evidence that the price level is countercyclical and that movements in the price level temporally precede movements in output. Because there is a negative relationship between the price-level movements and future movements in output, King and Watson are identifying a significant phase shift present between the price level and output.

The purpose of this paper is to focus directly on the phase shift. In doing so, we add this stylized fact to the array of facts that a model economy can account for. There has to be some form of price stickiness to account for the phase shift. Brock and Haslag (2016) explain that in a flexible price rational expectations setting, the price response results in price and output movements that are in phase. With the large literature on sticky-price models, we are not surprised that the model economy with monopolistic competition and Calvo-style price stickiness can account for the phase shift. What matters is that there is some mechanism that keeps the price level from jumping by the full amount that is consistent with rational expectations. Following Sims (2003), we consider price-level adjustments that are sticky in the following sense: the price-level response to shocks is smaller, in absolute value, than response associated with rational expectations. Our main goal, therefore, is to use a representative model economy with sticky prices to identify the source of the underlying shock that accounts for the phase shift. We save comparisons of the different sources for future work; in other words, this paper does not compare rational inattention versus inability to adjust prices versus models with heterogeneous price forecasting technologies.[3]

In this paper, our contribution is twofold. First, we use an NK sticky-price model economy to determine if it can account for both countercyclical prices and procyclical inflation. By examining the phase shift, we use the natural feature that the two facts are part of one fact: the inflation rate is the rate of change in the price level. The working hypothesis is that sticky prices are an important ingredient so that the equilibrium laws of motion will generate the phase shift. Furthermore, note that Brock and Haslag suggest that some kind of stickiness is necessary.

Second, and the key insight compared with the existing literature, is that the model economy allows one to ask whether all, or some subset of, shocks are able to account for the phase shift. In this way, we take a different approach than King and Watson do. Specifically, they focus on models that can match the facts. We start with the King–Watson goal and are able to match the fact, as they do, with some kind of sticky prices. We extend their basic result to consider the set of necessary and sufficient conditions in terms of shocks that could account for the phase shift.

The fact that models with sticky prices can account for the phase shift can be traced to both King and Watson (1996) and Brock and Haslag (2016). In Brock and Haslag, for example, nominal contracting is not the source of sticky prices. They construct a model economy in which prices are sticky because there exists a set of technologies used to forecast future price levels. The most expensive technology is perfect foresight, while cheaper options use the history of price levels to forecast future values. In addition, they specify a model economy that is difference stationary. In first differences, the cyclical components of output and the price level are represented by a smaller phase shift in the sense that the price level is countercyclical and the inflation rate is acyclical. For economies with some positive measure of agents using low-cost forecast technology that relies on past observations of the price level to forecast the future price level, Brock and Haslag are able to account for these two difference-stationary facts.

We use the most common form of DSGE model formulated by Smets and Wouters (2007). Our approach allows us to consider a rich set of shocks, including shocks to production technology, risk premium, government spending, investment-specific technology, monetary policy, the price markup, and the wage markup. We ask whether any of the seven shocks is necessary to account for the phase shift. Necessary conditions are checked by eliminating only one shock and assessing whether the phase shift is evident in the simulated economy. To illustrate, consider an economy in which there is no technology shock; we compute the median cross-correlation coefficients for the modified model economy and if the modified economy cannot account for the phase shift, then the technology shock is necessary for the phase shift. In addition, we check for whether any of the seven shocks are sufficient to account for the observed phase shift. The sufficiency condition is checked by looking at the model economy with only one shock operating. If the phase shift is found, then that shock is sufficient to account for the phase shift.

Our results are easily summarized. We begin by reviewing the data. For the postwar sample through 2017, we verify that the price level is countercyclical and the inflation rate is procyclical. Hence, the evidence of a phase shift between the price level and output is supported with the longer postwar sample. We also include the evidence for the sample period from the Great Moderation, using data from 1985 through 2007. Despite the small sample—only two business cycles—the evidence is mixed with the results indicating some sensitivity to the method used to identify the cyclical component. There is evidence of a phase shift in both the Hodrick-Prescott (HP)-Filter and the first-difference approaches, but both the price level and the inflation rate are acyclical when we apply the Hamilton method. With more lags, the cross-correlation function reveals that the movements in output are significantly related to movements in the future price level. The correlation is slowed by two quarters between output and future values of the inflation rate. With so few observations, the subsample evidence is of lower power than the full sample, but worth monitoring. The evidence does not overturn the presence of a phase shift since the Great Moderation, but is also consistent with a time-varying phase shift or potentially the phase shift is dependent on the monetary policy regime. For the purposes of this paper, our focus is on the entire postwar evidence.[4]

Next, we apply Bayesian methods to estimate the model economy.[5] The numerical results indicate that the model economy can capture both the countercyclical price level and the procyclical inflation rate. Moreover, the results indicate that the cross-correlation function matches the actual pattern insofar as the peak cross-correlation occurs between the price level and two-quarter-ahead output. Thus, the primary numerical result is that the NK model with sticky prices can account for the phase shift and correctly account for the nature, or direction, that the phase shift takes.

In this paper, we use the model economies as an identification mechanism. First, we ask what set of individual shocks are necessary to account for the phase shift? That set contains the price markup shock. Next, we ask what set of shocks are sufficient to account for the phase shift? This set contains three elements: the price markup and wage markup shocks along with the government spending shock. The results are important as a building block. We infer that price stickiness is an important model feature; without price stickiness, we are in the RBC economies that Cooley and Hansen studied. But, it raises further questions. For instance, is price stickiness of the Calvo form—the one used here—necessary to explain the phase shift?

A key motivation for conducting this research is to provide a means of assessing differences between RBC and NK approaches. In the RBC approach, researchers can account for the countercyclical price level by relying on the technology shock as the driving force behind business cycle fluctuations. Alternatively, researchers in the NK camp can account for procyclical inflation by relying on aggregate demand shocks. The point is that each camp ascribes their favorite dominant shock as a key source of business cycle fluctuations. Insofar as an important point of contention lies in two cyclical relationships—one between the price level and output, and the other between the rate of change in the price level and output—our paper seeks to use the phase shift evidence as a means of identification. One previous attempt to account for the phase shift in rational expectations economies failed. Cooley and Hansen (1995) use standard RBC models but find that both the price level and the inflation rate are countercyclical.[6] So, they calibrate a nominal contracting model. The sticky-price results indicate that the price level is acyclical in their model economy. Thus the sticky-price economy can generate a phase shift when the economy is subject to a technology shock, but the evidence suggests that the phase shift is too small.[7]

The paper is organized as follows. We review the empirical evidence supporting the existence of the phase shift between the price level and output in Section 2. We present the numerical analysis of the model economy in Section 3. In Section 4, we determine whether any of the shocks satisfy the necessary or sufficient condition. In Section 5, we offer a brief summary and conclusion.

2. EVIDENCE

Throughout this analysis, the data are quarterly and run from 1958:Q1 through 2017:Q4. The variables are per-capita real GDP (output) and the chain-weighted deflator (price level).[8] Our first aim is to report evidence regarding the nature of the phase shift between the price level and output. Throughout our analysis, we use logs of output and the price level, and the inflation rate is the first-difference values of the log price level.

In work by Haslag and Hsu (2012), the authors explain how a phase shift between the price level and output can account for the two contemporaneous correlation coefficients. More specifically, let the cyclical component of output be the reference line as plotted with respect to time. We refer to the relationship between the cyclical components of the price level and output as a left phase shift if the cyclical component of the price level tends to peak and trough before output. Formally, if $y = \sin(x) + \varepsilon_y$ and $p = -\sin(x + \theta) + \varepsilon_p$, where $\varepsilon_y \sim N(0, \sigma_y^2)$ and $\varepsilon_p \sim N(0, \sigma_p^2)$, then a left phase shift is a value of $\theta > 0$. Haslag and Hsu derive values $\theta \in [\underline{\theta}, \bar{\theta}]$ for which the price level is countercyclical and the inflation rate is procyclical. With $\theta \in [\underline{\theta}, \bar{\theta}]$ and $0 < \underline{\theta} < \bar{\theta}$, a left phase shift is consistent with movements in the cyclical component of the price level leading movements in the cyclical component of output. For $0 < \theta < \underline{\theta}$, the phase shift is too small for the inflation rate to be procyclical. By continuity, there exists $\theta \in [\theta^{'}, \underline{\theta}]$, where $\theta^{'} < \underline{\theta}$, such that prices are countercyclical and the inflation rate is acyclical. A simple way to verify the left phase shift in the data would be to conduct a Granger causality test.

TABLE 1. Business cycle facts

(A) 1958:Q1–2017:Q4		
Variable	Standard Deviation	Correlation with y
	H–P filtered data	
y	1.46	1.00
p	0.75	−0.46***
π	0.26	0.17***
	Log-differenced data	
y	0.86	1.00
p	0.58	−0.21***
π	0.27	−0.05
	Detrended data (Hamilton's method)	
y	0.03	1.00
p	0.02	−0.14**
π	0.02	0.04

(B) 1985:Q1–2007:Q4		
Variable	Std Dev	Corr with y
	H–P filtered data	
y	0.93	1.00
p	0.40	−0.14
π	0.15	0.35***
	Log-differenced data	
y	0.52	1.00
p	0.22	−0.19*
π	0.17	−0.04
	Detrended data (Hamilton's method)	
y	0.02	1.0
p	0.01	−0.10
π	0.01	0.07

Notes: *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Three differents detrending methods are used to identify the cyclical components. We use the Hodrick–Prescott (H–P) filter and in each case, we set $\lambda = 1600$. In addition, we identify the cyclical component as the first difference of each series and from the method developed recently by Hamilton (2017). Table 1 reports the standard deviation for detrended levels of output (y), the price level

(p), and the inflation rate (π), as well as the contemporaneous price–output correlation and inflation–output correlation. In Table 1A, we report data for the sample period 1958:Q1 through 2017:Q4. Based on Table 1A, the evidence is consistent with a phase shift. The H–P filter is qualitatively different in the sense that the price level is countercyclical and the inflation rate is procyclical, while the first-difference and Hamilton methods find that the price level is countercyclical and the inflation rate is acyclical. As Haslag and Hsu demonstrate, the pattern with countercyclical price level and acyclical inflation rate is also consistent with a phase shift in the relationship between the price level and output. Thus, in all three cases, evidence over the postwar sample is consistent with the representation of a phase shift in the relationship between the cyclical components of the price level and the cyclical components of output.

We also consider a subsample of the postwar data. Table 1B reports the standard deviations and contemporaneous cross-correlations for the period 1985:Q1 through 2007:Q4.[9] The Great Moderation and the Volcker disinflation are two factors motivating us to examine this subsample. Table 1 does provide evidence validating the Great Moderation moniker since the standard deviation of the detrended components is lower in each case. For the post-1985 period, the evidence regarding the phase shift is mixed. In the H–P filter case, the price level is acyclical and the inflation rate is procyclical. In the log-difference case, the price level is countercyclical and the inflation rate is acyclical. In the Hamilton method, both price level and inflation rate are acyclical. Thus, we observe evidence that is consistent with the presence of a phase shift in two cases and no phase shift in the case of the third detrending method. For the Great Moderation sample, the evidence for a phase shift between the price level and output is weakened.[10]

The following question becomes critical for this paper. Is the post-1985 sample period evidence consistent with a state-contingent frequency such that output and price level correlations move into phase over time? Alternatively, the post-1985 sample has only two complete business cycles. Another hypothesis is implied, namely, that the small sample is not indicative of the average cross-correlation. In this paper, we are going to take the stand that there is a phase shift present between the detrended measures of the price level and output. The full-sample evidence and the subsample evidence for two of the three detrending methods are consistent with this interpretation. As we proceed with the numerical analysis in a model economy, we will keep in mind how the model economy can account for the observations presented in the post-1985 sample.[11]

Table 2A reports the cross-correlation function for the full sample. The evidence indicates that the covariance between output and the contemporaneous price level is negative. So, we can infer directly that the covariance between output and the lagged price level is negative and smaller (greater in magnitude).[12] In Table 2B, we report the cross-correlation function for the 1985–2007 sample. In the Great Moderation sample, however, the evidence is qualitatively similar for the cyclical component measured by the H–P filter, that is, movements in the price level tend to be negatively (and significantly) correlated with movements in *future*

TABLE 2. Lead–lag cross-correlations with output

(A) 1958:Q1–2017:Q4		
i (Lag/Lead)	$\rho\ (p_{t+i}, y_t)$	$\rho\ (\pi_{t+i}, y_t)$
	H–P filtered data	
−4	−0.50***	−0.24***
−3	−0.55***	−0.14**
−2	−0.55***	−0.03
−1	−0.52***	0.06
0	−0.46***	0.17***
1	−0.36***	0.28***
2	−0.24***	0.34***
3	−0.11*	0.38***
4	0.02	0.38***
	Log-differenced data	
−4	−0.19***	−0.10
−3	−0.22***	−0.03
−2	−0.20***	0.05
−1	−0.21***	−0.03
0	−0.21***	−0.05
1	−0.15**	0.13**
2	−0.13**	0.03
3	−0.09	0.09
4	−0.04	0.11*
	Detrended data (Hamilton's method)	
−4	0.28***	0.34***
−3	0.18***	0.28***
−2	0.08	0.21***
−1	−0.03	0.14**
0	−0.14**	0.04
1	−0.26***	−0.09
2	−0.36***	−0.20***
3	−0.44***	−0.30***
4	−0.47***	−0.40***

(B) 1985:Q1–2007:Q4		
i (Lag/Lead)	$\rho\ (p_{t+i}, y_t)$	$\rho\ (\pi_{t+i}, y_t)$
	H–P filtered data	
−4	−0.55***	0.15
−3	−0.49***	0.15
−2	−0.38***	0.28***
−1	−0.27**	0.31***

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TABLE 2. Continued

0	−0.14	0.35***
1	0.03	0.46***
2	0.19*	0.39***
3	0.32***	0.33***
4	0.42***	0.24**
	Log-differenced data	
−4	−0.16	0.06
−3	−0.28***	−0.16
−2	−0.17	0.13
−1	−0.17	0.00
0	−0.19*	−0.04
1	−0.02	0.23**
2	−0.02	0.01
3	0.01	0.05
4	0.04	0.06
	Detrended data (Hamilton's method)	
−4	0.02	0.03
−3	0.01	0.03
−2	0.01	0.06
−1	−0.01	0.09
0	−0.10	0.07
1	−0.17	0.03
2	−0.25**	−0.06
3	−0.33**	−0.15
4	−0.40*	−0.25**

Notes: *, **, and *** denote 10%, 5%, and 1% significance, respectively.

output. For the full sample, the sample pattern is observed in the first-difference and Hamilton measures. While the results are robust in the post-1985 sample when measuring the cyclical component using the H–P filter, there is no clear lead–lag relationship between the price level and output when the cyclical components are identified by the first-difference method. There is a short-lived pattern between output and future values of the price level in the Hamilton method. Note that in the H–P filter section, the correlation coefficient of the one lagged value of the price level and output is negative and smaller than the contemporaneous correlation coefficient. It follows that $\text{cov}(y_t, p_{t-1})^{H-P} < 0$ but satisfies the condition that yields a positive contemporaneous correlation between output and inflation. With Hamilton's method, the inference is that the $\text{cov}(y_t, p_{t-1})^{Ham}$ results in the contemporaneous correlation between output and inflation being close but the coefficients are algebraically smaller than the coefficient between one lagged value of the price level and contemporaneous output. Overall, Table 2 finds evidence consistent with the correlation between output and one lagged value of the price level being insignificantly different from zero, which can account for the observation that a phase shift is present except when the Hamilton method is used to identify the cyclical component in the post-1985 sample. We will take note of these results as we consider the numerical experiments in the model economy.

For the full postwar sample, the evidence suggests that there is a phase shift between movements in the cyclical components of output and the cyclical component of the price level. Others have presented evidence that is consistent with the phase shift in the postwar data, including Kydland and Prescott (1990) and Cooley and Ohanian (1991). In relation to the findings presented in these papers, our results are not surprising. The evidence presented here confirms, over a longer postwar sample, what previous researchers have found. Perhaps because of the conduct of monetary policy or because of the small sample with only three business cycles, the 1985–2017 sample is more muddled in terms of whether a phase shift is present or not.

Other researchers have scrutinized the contemporaneous relationship between the price level and output. Even with all the scrutiny, perhaps the biggest concern is that the phase shift is not stable. For example, Cooley and Ohanian have data going back to the middle of the 19th century. They divide the sample into subperiods. According to Cooley and Ohanian, the phenomenon that the price level is countercyclical and inflation rate is procyclical does occur in the postwar sample period. In the interwar period, for example, both the price level and the inflation rate are procyclical. We use the model economy to match features in the data, focusing principally on the period after World War II but also examining the Great Moderation period in terms of the model economy's ability to capture the relationship between the price level and output.

3. NUMERICAL ANALYSIS

In this section, we use the canonical DSGE model developed in Smets and Wouters (2007). The Smets–Wouters model is well known and not replicated here.[13] The Calvo-clock dictates how unfettered price-level changes are implemented in the model economy, thus introducing a nominal rigidity that is consistent with the ideas of inertia put forward by Sims (2003).[14]

3.1. Bayesian Approach

With this version of the NK DSGE model economy, there are 41 parameters. We follow Smets and Wouters (2007) in fixing values for five parameters, including δ, g_y, ϕ_w, ε_p, and ε_w. The quarterly depreciation rate is set such that annual depreciation is approximately 10%; hence, $\delta = 0.025$. Let $g_y = 0.18$, which means that the government spending–GDP ratio is 18%. The steady-state labor market markup ϕ_w is set at 1.5. Finally, the curvature parameters of the Kimball aggregators in the goods and labor market (ε_p and ε_w) are both set at 10.

Throughout this analysis, we use the macroeconomic data over 1958:Q1 through 2017:Q4 to estimate the model parameters, especially the Taylor-type monetary policy rule coefficients. This sample period covers the crisis episode from 2009 through 2015 when the short-term nominal interest rates approach their zero lower bound (ZLB). The estimation results, especially the estimated interest rate rule, might be biased because the estimation is conducted using a log-linearization approximation which ignores the ZLB problem. To address this concern and to avoid using computationally intensive nonlinear methods to enforce the ZLB, we replace the effective Federal Funds rate with the "shadow Federal Funds rate" constructed by Wu and Xia (2016).[15] The "shadow Federal Funds rate" essentially coincides with the effective Federal Funds rate over the normal years, but it becomes negative when the Federal Funds rate approaches its ZLB during the crisis time from 2009 through 2015.[16] Wu and Zhang (2017) argue that the "shadow Federal Funds rate" salvages the NK model from the structural break induced by the ZLB.

We use Bayesian methods to obtain posterior distributions of the remaining 36 parameters. We follow Smets and Wouters (2007) in choosing the prior distributions of the 36 estimated parameters. A Monte-Carlo-based optimization routine is applied to compute the mode values for the 36 parameters. With the 5 calibrated parameters and 36 estimated parameters, we solve the first-order Taylor expansion around the steady state. Because the model economy's dynamics are subject to seven exogenous shocks, we follow Smets and Wouters (2007) to use seven observed series to avoid stochastic singularity. The seven observable variables are real GDP, real personal consumption expenditures, real fixed private investment, weekly hours worked, the chain-weighted GDP deflator, real wage, and the effective/shadow federal funds rate.[17]

The results of the Bayesian estimation are based on quarterly data spanning the period 1958:Q1 through 2017:Q4.[18] Two tables are made available in the Supplementary Material. Specifically, the full-sample structural parameters are reported in Table OA1, while parameters characterizing the distribution of shock processes are presented in Table OA2. For the 1985:Q1 through 2007:Q4 sample, structural parameters and distribution parameters are reported in Tables OA3 and OA4, respectively.

3.2. Business Cycle Properties

With the model economy and its parameter values set, we simulate the model and obtain business cycle properties of key endogenous variables in the following way. First, we stochastically simulate series of seven endogenous variables, including output, consumption, investment, wage, hours worked, inflation rate, and the nominal interest rate, for 10,000 times, conditioning on the posterior mean of the structural parameters and of the standard deviations of the shocks. For each of the 10,000 simulations, we then infer the "observable variables" in Smets and Wouters (2007) from the measurement equation in their paper, that is, equation (15).[19] The next step is to compute the log values of real GDP, real consumption, real investment, and the real wage from the inferred log difference of these

TABLE 3. Standard deviations: comparing simulated with actual economy

Variable	y	p	π	c	i	w	l	R
(A) 1958:Q1–2017:Q4								
Simulated Economy	1.90	0.83	0.28	1.52	5.60	1.23	1.36	0.36
Actual	1.46	0.75	0.26	1.14	4.77	0.95	1.33	0.37
(B) 1985:Q1–2007:Q4								
Simulated Economy	1.18	0.63	0.22	1.02	3.25	1.66	0.88	0.18
Actual	0.93	0.40	0.15	0.79	3.28	1.14	0.94	0.27

Notes: y denotes per-capita real output, p denotes the price level, π denotes inflation rate, c denotes per-capita real consumption, i denotes per-capita real investment, w denotes real wage, l denotes hours worked, and r denotes the nominal interest rate. In both the simulated and actual economies, the period is 240 quarters. The median standard deviation in the simulated economy is computed from 10,000 simulations. The cyclical components are obtained by using the H–P filter and are measured in percentage terms.
See Table 3A for notation. In both the simulated and actual economies, the period is 92 quarters. The median standard deviation in the simulated economy is computed from 10,000 simulations. The cyclical components are obtained by using the H–P filter and are measured in percentage terms.

variables and their initial values, that is, their log values at 1957:Q4. Then, we compute log hours worked from the inferred log hours worked and the average hours worked over the full sample period. For the price level, we recover its "simulated" values from the inferred log difference of the GDP deflator and log GDP deflator at 1957:Q4. Then, we obtain the H–P filtered values of all the above variables and compute their business cycle properties for each simulation. We regard the median values of the business cycle properties across the 10,000 simulations as the business cycle properties in the simulated economy.

Table 3 reports the standard deviations for the actual and simulated economies for eight key macroeconomic variables; specifically, per-capita real output (y), the price level (p), the inflation rate (π), per-capita real consumption (c), per-capita real investment (i), the real wage rate (w), hours worked (l), and the nominal interest rate (R). Except for the nominal interest rate, the simulated model economy reports standard deviations that are greater than the standard deviations reported in the actual data. In several cases, the increase in volatility is nearly negligible (see inflation and hours worked). Even in the other five cases, the increase in volatility represented by the model economy is quite reasonable.

Table 4 reports the contemporaneous correlation coefficient with output for the price level, the inflation rate, per-capita real consumption, per-capita real investment, the real wage rate, hours worked, and the nominal interest rate. In each case, the contemporaneous cross-correlation in the model economy matches the sign and is quite close to the value reported for the actual economy. Given our focus, we point to the fact that the model economy accounts for the phase shift. The price level is countercyclical and the inflation rate is procyclical. Moreover, in most cases, one can reject the null hypothesis that the correlation coefficient is equal to zero at the 1% confidence level in both the actual economy and the model

TABLE 4. Median contemporaneous correlation with output: comparing simulated with actual economy

(A) 1958:Q1–2017:Q4							
Variable	P	π	c	i	w	l	R
Simulated Economy	−0.20***	0.12*	0.79***	0.78***	0.33***	0.85***	0.47***
Actual	−0.46***	0.17***	0.89***	0.87***	0.18***	0.87***	0.39***
(B) 1985:Q1–2007:Q4							
Simulated Economy	−0.37***	0.10	0.76***	0.75***	0.34***	0.80***	0.09
Actual	−0.14	0.35***	0.87***	0.89***	0.07	0.83***	0.60***

Notes: See Table 3A for notation. In both the simulated and actual economies, the period is 240 quarters in Table 4A and 92 quarters in Table 4B. The median standard deviation in the simulated economy is computed from 10,000 simulations. The cyclical components are obtained by using the H–P filter and are measured in percentage terms. *, **, and *** denote 10%, 5%, and 1% significance, respectively. See Ashley et al. (1980) for the calculation of the standard errors of the cross-correlation coefficients.

economy. Perhaps the weakest match between the model economy and the actual economy occurs with respect to the price level. The evidence indicates the H–P filter cyclical component is more negatively correlated with output than the model economy indicates.

To provide a more complete picture, we compare the model economy to the actual economy for the lead–lag cross-correlation function. The findings are presented in Table 5. The model economy does a very nice job qualitatively matching the lead–lag cross-correlations with output. The actual economy indicates that the correlations are larger in magnitude for lagged values of the price level and output. The relationship between output and future values of the price level are also slightly greater in magnitude for the actual economy compared with the model economy. A similar pattern is also observed when comparing the lead–lag cross-correlation between inflation and output for the model economy to the cross-correlation function for the actual economy when the H–P filter cyclical component is used. Table 5 shows that the model economy does not match as closely the cross-correlation function reported when the Hamilton method is used to identify the cyclical component. In particular, the signs of the correlation coefficients between lagged prices and output and lagged inflation rates and output are opposite. We do see that with the Hamilton method, the match is closer when measured immediately surrounding the contemporaneous correlation coefficients, that is, at one lead value and the contemporaneous value. However, the model economy reports that cross-correlation coefficients between output and future inflation are positive, while the Hamilton method reports a negative relationship between output and lead values of the inflation rate.

Overall, our results show that the different methods used to measure the cyclical component do give different results. Our findings raise some questions about the goodness-of-fit of the model economy. In terms of the H–P filter, the model

TABLE 5. Lead–lag cross-correlations with output: comparing simulated with actual economy

(A) 1958:Q1–2017:Q4

i (Lag/Lead)	$\rho\ (p_{t+i}, y_t)$		$\rho\ (\pi_{t+i}, y_t)$	
	Simulated Economy	Actual (H–P) (Hamilton in brackets)	Simulated Economy	Actual (H–P) (Hamilton in brackets)
−4	−0.23***	−0.50*** (0.28***)	−0.09	−0.24*** (0.34***)
−3	−0.25***	−0.55*** (0.18***)	−0.06	−0.14** (0.28***)
−2	−0.25***	−0.55*** (0.08)	−0.01	−0.03 (0.21***)
−1	−0.24***	−0.52*** (−0.03)	0.05	0.06 (0.14**)
0	−0.20***	−0.46*** (−0.14**)	0.12*	0.17*** (0.04)
1	−0.13**	−0.36*** (−0.26***)	0.19***	0.28*** (−0.09)
2	−0.07	−0.24*** (−0.36***)	0.18***	0.34*** (−0.20***)
3	−0.02	−0.11* (−0.44***)	0.15**	0.38*** (−0.30***)
4	0.03	0.02 (−0.47***)	0.12*	0.38*** (−0.40***)

(B) 1985:Q1–2007:Q4

i (Lag/Lead)	$\rho\ (p_{t+i}, y_t)$		$\rho\ (\pi_{t+i}, y_t)$	
	Simulated Economy	Actual (H–P) (Hamilton in brackets)	Simulated Economy	Actual (H–P) (Hamilton in brackets)
−4	−0.25**	−0.55*** (0.02)	−0.25**	0.15 (0.03)
−3	−0.34***	−0.49*** (0.01)	−0.21**	0.15 (0.03)
−2	−0.39***	−0.38*** (0.01)	−0.14	0.28*** (0.06)
−1	−0.41***	−0.27** (−0.01)	−0.04	0.31*** (0.09)
0	−0.37***	−0.14 (−0.10)	0.10	0.35*** (0.07)

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TABLE 5. Continued

1	−0.30***	0.03 (−0.17*)	0.20*	0.46*** (0.03)
2	−0.21**	0.19* (−0.25***)	0.24**	0.39*** (−0.06)
3	−0.12	0.32 (−0.33***)	0.24**	0.33*** (−0.15)
4	−0.03	0.42*** (−0.404***)	0.22*	0.24** (−0.25**)

Notes: *, **, and *** denote 10%, 5%, and 1% significance, respectively.

TABLE 6. Median contemporaneous correlation with output: consider model economy with only one shock off at a time

Shock (off)	$\rho\ (p_t, y_t)$	$\rho\ (\pi_t, y_t)$
TFP	−0.20***	0.16**
Risk premium	−0.26***	0.11*
Government spending	−0.20***	0.12*
Investment-specific technology	−0.22***	0.13**
Monetary policy	−0.22***	0.11*
Price markup	−0.01	0.19***
Wage markup	−0.19***	0.12*

Notes: In both the simulated and actual economies, the period is 240 quarters. The median correlation coefficient is computed from 10,000 simulations. The cyclical components are obtained by using the H–P filter and are measured in percentage terms. *, **, and *** denote 10%, 5%, and 1% significance, respectively. See Ashley et al. (1980) for the calculation of the standard errors of the cross-correlation coefficients.

economy does quite well. But the findings are qualitatively different for post-war data when the Hamilton method is adopted. The goodness-of-fit question is outside the scope of this paper, and we leave that to other researchers.

4. NECESSARY AND SUFFICIENT SHOCKS

With seven different shocks affecting the model economy, we consider two alternative approaches that may help shed some light on whether there is a set of either necessary or sufficient shocks that can account for the phase shift between the price level and output.

Table 6 reports the median contemporaneous correlation coefficients for the price level and output and for the inflation rate and output for seven model economies. Each model economy is identified by the first column in Table 6; specifically, the column identifies the particular shock for which standard deviation is set to zero. This method identifies whether the particular shock is necessary to account for the phase shift. If the model economy with one shock eliminated

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cannot match the pattern with a countercyclical price level and either a procyclical or acyclical inflation rate, then we conclude that the shock is necessary. Basically, we use the "not A, then not B" method to find whether a particular shock is necessary or not.

We report the median value of the contemporaneous correlation coefficient from 10,000 simulations of the model economy. Based on the results reported in Table 6, there is one candidate shock that satisfies the necessary condition to account for the phase shift. In particular, the model economy without the price markup shock is a possible candidate because the price level is acyclical while the inflation rate is procyclical. While this feature is suggestive of a phase shift, it is not the same phase shift that appears in the data. In the Supplementary Material, the reader can find Figure OA1, which plots the cross-correlation function for the model economy in which the price markup shock is set to zero along with the cross-correlation for the actual data using the H–P filter. The evidence shows that the cross-correlation function is not as pronounced in the model economy when compared with the actual economy. By looking at the lead–lag cross-correlation between the price level and output, the simulated data show that the lagged values of the price level are negatively correlated with output, matching the observation in the actual economy. Whatever phase shift is indicated by lead–lag pattern is small compared with the actual economy since the cross-correlation function exhibits nearly an acyclical pattern and has no clear temporal precedence. The upshot of this investigation is that the phase shift is much less pronounced in the model economy when the price markup shock is set equitable to zero. Note that in the other six cases, we find that by setting the particular shock equal to zero yields the pattern: $\text{sgn}\left[\rho\left(p_t, y_t\right)\right] < 0$ and $\text{sgn}\left[\rho\left(\pi_t, y_t\right)\right] > 0$. Therefore, the results suggest that the price markup shock is necessary for the model economy to account for a phase shift between the price level and output.

Can the model economy account for the cross-correlation pattern observed in the actual post-1985 data? For the model economy with the price markup shock set equal to zero, we observe the same pattern in the model economy as when we use the Hamilton method to identify the cyclical component in the actual data, that is, the price level is acyclical and the inflation rate is procyclical. Based on this evidence, the price markup shock is not necessary to account for the pattern in the actual data. However, the other set of shocks—total factor productivity (TFP), risk premium, government spending, investment-specific technology, monetary policy, and wage markup—are each necessary since the model economies generate countercyclical prices and procyclical inflation. Overall, the evidence suggests that a combination of the shocks are necessary to account for why the price level and output movements are unrelated in the Great Moderation (1985–2007) sample.

With the price markup shock shutdown, the model economy is subject to the set of six other shocks operating. What the model economy is telling us is that without the additional inefficiency associated with the price markup shock, the price level will be acyclical and the inflation rate will be procyclical. Note this

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TABLE 7. Median contemporaneous correlation with output: consider model economy with only one shock at a time (1958:Q1—2017:Q4)

Shock (on)	$\rho\ (p_t, y_t)$	$\rho\ (\pi_t, y_t)$
TFP	−0.50***	−0.92***
Risk premium	0.20***	0.98***
Government spending	−0.12*	0.92***
Investment-specific technology	0.68***	0.38***
Monetary policy	0.26***	0.97***
Price markup	−0.82***	0.31***
Wage markup	−0.54***	0.73***

Notes: In both the simulated and actual economies, the period is 240 quarters. The median correlation coefficient is computed from 10,000 simulations. The cyclical components are obtained by using the H–P filter and are measured in percentage terms. *, **, and *** denote 10%, 5%, and 1% significance, respectively. See Ashley et al. (1980) for the calculation of the standard errors of the ross-correlation coefficients.

is the pattern that we observe when the H–P filter is applied to the 1985–2007 sample. In terms of the full postwar sample, the effect is to eliminate a source of cost–push inflation by setting the price markup shock equal to zero. The analysis shows that without this source, and its persistence, the remaining shocks result in price level adjustment that are no longer correlated contemporaneously without output. However, the rate of change in the price level is contemporaneously positively related to output. In other words, the Phillips curve tradeoff is observed but there is no systematic relationship between the price level and output.

Next, we examine model economies in which only one shock is operating. The approach yields a kind of sufficiency condition; if, for example, there is only a monetary policy shock, do we observe countercyclical price level and procyclical inflation in the model economy? If yes, then the monetary policy shock is sufficient for the left phase shift observed in the data. Table 7 presents contemporaneous cross-correlations for seven simulations of the model economy. In each simulation, the shock listed in the first column is the only one with a nonzero standard deviation. We report the median value of the contemporaneous correlation coefficient from 10,000 simulations of the model economy. In three of the seven cases, we observe a model economy with only one shock in which the price level is countercyclical and the inflation rate is procyclical: government spending, the price markup, and the wage markup shocks. Thus, each one satisfies the condition as sufficient in terms of accounting for the phase shift we observe in the actual data.

For the government spending shock, the price markup shock, and the wage markup shock, the intuition basically involves the same operating premise. With price markup and wage markup shocks, the model economy is capturing a kind of inefficiency that does not have a strict “market-power-only” interpretation. As such, a positive shock means greater inefficiency and an immediate reduction in output. The price level increases directly (price markup shock) or indirectly through wage markup shocks affecting input prices. So, the negative contemporaneous relationship between the price level and output is manifested by the shocks. With Calvo clocks, the price stickiness means that the inflation rate response is muted initially. In other words, as the price level increases, the rate of change is directly related to output through the propagation process. In short, the price stickiness imparts a phase shift onto the comovements of output and the price level. Price and wage markups are interesting shocks to interpret. They reflect an underlying form of market power which begs the next question; what is the primitive that is driving such changes in market power?

With respect to the government spending shock, the intuition is that there is some crowding out of private spending. From the impulse responses to a government spending shock, output increases at a falling rate because the positive shock to government spending crowds out private consumption and investment. The rise in labor and wage leads to a drop in the price markup, which raises the inflation rate, because inflation depends negatively on the current price markup.[20] The following drop in the inflation rate means slowly falling price level, which leads to a mild negative contemporaneous correlation between the price level and output.

Note that not one of the seven shocks is capable of accounting for the post-1985 observation. Six of the shocks can account for procyclical inflation, namely, the risk premium, government spending, investment-specific technology, monetary policy, price markup, and wage markup. However, the price level is either procyclical—risk premium, investment-specific technology, and monetary policy shocks—or countercyclical as in the cases of the three candidate sufficient shocks discussed above. There is not one individual shock that can account for acyclical prices and acyclical inflation. Thus, the combination of all shocks except the price markup shock are necessary to account for the pattern of acyclical prices and procyclical inflation observed with the Hamilton detrending method and Great Moderation sample.[21]

Even with costly price changes imparting some price stickiness into the model economy, the equilibrium we consider are rational expectations. The rational expectations assumption continues to impart some jumpiness to the price level reactions. Brock and Haslag (2016) derive conditions in which there could be a phase shift in a Woodford-style (2003) model economy.[22] Basically, in a model economy with rational expectations, the price level responds quickly to the new information such that the relationship between the price level and output is in phase. Price stickiness helps, but for the simulated model economy presented here, the numerical results indicate that only one kind of aggregate demand shock—government spending—and two measures of market power are sufficient to account for a phase shift.

5. SUMMARY AND CONCLUSION

In this paper, we examine an NK model economy with Calvo-type sticky prices, attempting to quantitatively assess the model economy's ability to account for a phase shift evident in the relationship between the price level and output. Here, the phase shift is consistent with the observation that the price level is countercyclical and the inflation rate is procyclical. We are able to demonstrate that the NK sticky-price model can account for the phase shift.

This result may not be surprising to many. We investigate further, and our main contribution lies in looking for the source of the model economy's ability to account for the price-level-led phase shift, that is, movements in the cyclical component of the price level temporally precede movements in the cyclical component of output. We examine whether one shock is necessary to be able to account for the countercyclical price level and procyclical inflation. We find that the price markup shock meets the simple criterion to be considered necessary for the phase shift. Specifically, we consider a separate model economy in which the price markup shock is set to zero. In this case, the model economy cannot account for both the countercyclical price level and the procyclical inflation. We systematically consider six other versions, setting one shock equal to zero, and the phase shift is present in these other six model economies. In short, "not price markup shocks" imply "not a phase shift." Therefore, we conclude that there is a set of necessary shocks consisting of the price markup up shock that accounts for the observation.

In addition, we proceed to ask if one type of shock is sufficient to account for the observed pattern in the data. The answer is yes. We consider seven versions of the model economy in which each is distinguished by setting the standard deviations of the other six shocks equal to zero. In three of the one-shock-only model economies, the price–output and inflation–output contemporaneous correlation coefficients qualitatively match the actual correlation coefficients. Hence, we conclude that the government spending shock, the price markup shock, and the wage markup shock are sufficient to account for the observed phase-shift pattern.

We consider the possibility of a structural break in the phase shift, estimating the cross-correlation for the period from 1985 through 2007. In the subsample, we find evidence consistent with a phase shift that is qualitatively different than that observed in the full sample. In the case of the H–P filter measure, the price is acyclical and the inflation rate is procyclical; in the case of the first difference, the price level is countercyclical and the inflation rate is acyclical. Thus, in these two cases, there exists evidence consistent with a phase shift. In the case of the Hamilton method, both the price level and the inflation rate are acyclical. There is not one numerical experiment that matches the pattern observed when one identifies the cyclical component with the Hamilton detrending method and uses data for the 1985–2007 sample. If we use the full-sample model economy, it is possible to match the pattern in the HP and first-difference identification schemes. For example, with the price markup shock missing, the model economy can account for the H–P filter pattern, that is, the price level is acyclical and the inflation rate is procyclical. Thus, the model economy suggests that price markup is not necessary to be able to account for the pattern of acyclical price level and procyclical inflation that is observed when the H–P filter is used to construct the cyclical components and the sample period for 1985–2007 is applied. Thus, a combination of all shocks except the price markup shock is necessary to account for the pattern of acyclical prices and procyclical inflation. Note that we could not identify a necessary condition nor a sufficient condition that would account for the pattern of acyclical price level and acyclical inflation observed with the Hamilton detrending method and the Great Moderation sample.

Our results are first, important step that could possibly explain one of the main modeling differences that is present in analyses of business cycle fluctuations. The two principal camps studying business cycle fluctuations have divided between emphasizing the relationship between inflation and output versus emphasizing the relationship between the price level and output. By focusing on the phase shift in the relationship between the price level and output, some notion of sticky prices is important. As researchers move forward, the phase shift provides a focal point. In addition, it becomes even more important to offer a fundamental interpretation of the price markup shock. Are there any deeper structural forces affecting market power that are being driven through the price markup shock? An important next step would be to look more deeply into the nature of the phase shift movements over time. Is there a time-varying process that is driving the periodicity of the output responses relative to the price-level responses? Or, is phase shift state contingent, depending on some rule change that drives the dynamic responses? We leave these additional questions for future research.

SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit https://doi.org/10.1017/S136510051900097X.

NOTES

1. The evidence is presented in Cooley and Ohanian (1991) for the postwar period and in Cooley and Hansen (1995). We see evidence of the dividing lines in two quotes. In view of the countercyclical price level, Kydland and Prescott (1990) state: "We caution that any theory in which procyclical prices figure crucially in accounting for postwar business cycle fluctuations is doomed to failure." (p. 17). Later den Haan (2000) countered, saying "a theory in which prices do not have some procyclical feature is, at best, missing a part of the explanation of U.S. business cycle fluctuations." (p. 5).

2. To our knowledge, the endogenous business cycle literature has ignored the phase shift properties in the data. See Farmer (2016) for an excellent discussion of the key features of the endogenous business cycle models.

3. See, for example, Brock and Hommes (1997) and economies specified by Branch et al. (2009) and De Grauwe (2011).

4. We do not deny the possibility that the phase shift result is not robust. Rather, our focus is motivated by the full postwar sample, starting with the premise that postwar business cycles share common features. Blanchard and Watson (1986) asked the important question about similarities across business cycles. We leave the test as to whether there is a significant quantitative difference in business cycles after 1985 compared with other postwar cycles to researchers with more observations.

5. See DeJong et al. (1996) for a description.

6. Brock and Haslag (2016) argue that it is difficult to generate countercyclical price level *and* procyclical inflation in a model economy with technology shocks and rational expectations. With rational expectations, the price level responds rapidly to the information that a persistent technology shock has occurred. The upshot is that the price movement is in phase with the output movement.

7. Haslag and Hsu (2012) construct additively stochastic sine functions to demonstrate the parametric values of the phase shift that will yield a negative correlation in levels and a positive correlation in the rate of change of one variable and the level of the other.

8. Per-capita real GDP is obtained by real GDP (chained 2012 dollars) divided by the civilian noninstitutional population, age 16 and over.

9. We have also computed the summary statistics for the detrended values for the 1985 through 2017 period. By including data from the Great Recession, the results are somewhat weaker when we include nine years' worth of data in which the zero lower bound is observed. We do not report these results to save space. The results are available from the authors upon request.

10. The test statistic is computed following Ashley et al. (1980) and indicates that one can reject the null hypothesis that each contemporaneous correlation coefficient is equal to zero.

11. The notion that a time series divides into trend, cyclical and seasonal components is only way to decompose the series. Permanent and transitory shocks also give another way to decompose the series. See Keating and Valcarcel (2015) for a discussion of presence of time-varying factors in the postwar data.

12. Here, we use the fact that $\text{cov}\,(y_t, \pi_t) = \text{cov}\,(y_t, p_t) - \text{cov}\,(y_t, p_{t-1})$. Note that $|\text{cov}\,(y_t, p_{t-1})| > |\text{cov}\,(y_t, p_t)|$ is consistent with the notion that there is a phase shift. See Ross (2014) for Lemma 4.7.1 and the proof on p. 122 for the complete derivation of this statement.

13. See Uhlig (2018) for an excellent description of the Smets–Wouters model. In a separate set of slides, Uhlig lays out the basics of the Smets–Wouters model, even clearing up some of the technical problems present in the published paper. One of the attractive features of the Smets–Wouters model is the goodness-of-fit characteristics compared to other off-the-shelf DSGE model economies.

14. Woodford (2003) asserts that price stickiness is an important determinant that affects equilibrium resource allocation. Sims (2003) approaches the problem as a cognitive technology that limits information processing. See also Maćkowiak and Wiederholt (2009), and Moscarini (2004) for other examples of information-theoretic approaches to sticky prices.

15. We thank an anonymous referee for suggesting this approach.

16. See Wu and Xia (2016) for more details.

17. Following Smets and Wouters (2007), we use per-capita measures of output, consumption, investment, and hours worked to estimate the model. Per-capita measures are obtained by dividing the aggregate value by the civilian noninstitutional population, age 16 and over.

18. Draws from the posterior distribution of the parameters are obtained using the Random-Walk Metropolis-Hastings algorithm. The reported estimation results are based on a sample of 100,000 MCMC draws, for which the first 40,000 are discarded as burn-in draws.

19. The "observable variables" include the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log difference of the GDP deflator, and the federal funds rate.

20. See equations (9) and (10) in Smets and Wouters (2007) for detail.

21. In order to save space, we do not report the results from the model economy estimated with the 1985–2007 data. The results are available for anyone interested by request. We have results from the model economy with all shocks but one operating and from cases in which only one shock is operating at a time. None of the results can account for the contemporaneous cross-correlation pattern reported for the actual 1985–2007 period.

22. Brock and Haslag (2016) provide a detailed exposition on the role that rational expectations play in keeping the price level and output in phase. In short, they find that the price-level response is essentially too quick for the phase shift to be evident in a flexible-price rational expectations model.

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