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Extended dynamic fuzzy logic system for a class of MIMO nonlinear systems and its application to robotic manipulators

M. Hamdy†* and G. EL-Ghazaly‡

†*Department of Industrial Electronics and Control Engineering, Faculty of Electronic Engineering, Menofia University, Menof 32952, Egypt*
‡*Department of Communication, Computer and System Sciences, Faculty of Engineering, University of Genova, Genova 16145, Italy*

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SUMMARY
This paper presents an indirect adaptive fuzzy control scheme for a class of unknown multi-input multi-output (MIMO) nonlinear systems with external disturbances. Within this scheme, the dynamic fuzzy logic system (DFLS) is employed to identify the unknown nonlinear MIMO systems. The control law and parameter adaptation laws of DFLS are derived based on the Lyapunov synthesis approach. The control law is robustified in H^∞ sense to attenuate external disturbance, model uncertainties, and fuzzy approximation errors. It is shown that under appropriate assumptions it guarantees the boundness of all signals in the closed-loop system and the asymptotic convergence to zero of tracking errors. An extensive simulation on the tracking control of a two-link rigid robotic manipulator verifies the effectiveness of the proposed algorithms.

KEYWORDS: MIMO nonlinear systems; DFLS; Lyapunov synthesis approach.

1. Introduction

Identification and control of nonlinear systems has attracted a lot of attention and represents a challenging area in control community during the last two decades. The development of geometric nonlinear control theory and, in particular, feedback linearization methods have led to great success in the development of controllers for nonlinear systems.[1,2] A key assumption in these techniques is that the dynamics of nonlinear systems is exactly known. Some limitations of this theory appear because real systems may have uncertainties. Thus, to deal with uncertain nonlinear systems, many adaptive control approaches have been proposed. Adaptive control approaches are applied to systems with parameter uncertainties. Several results can be found in refs. [3–6], and the references therein.

Since introduced by Zadeh,[7] the fuzzy set theory has received a great deal of attention in both theoretical research and implementation techniques. It has been successfully adopted in many soft-computing applications with special emphasis on control systems. Traditionally, fuzzy logic system has been applied to control a dynamic system without an explicit model,[8] and the design was based on the compositional rule of inference. Fuzzy logic controllers have been synthesized from a collection of fuzzy IF-THEN rules to form rule-based controllers in refs. [9] and [10]. The key features behind the success of fuzzy logic systems that allow it to be one of the most efficient intelligent techniques are that these provide a systematic and efficient framework to incorporate linguistic information from human experts, simulate human thinking procedure, and at the same time universal function approximators. These features allow fuzzy systems to handle the problems of modeling and control of complex and ill-defined nonlinear dynamic systems. Based on the universal approximation theorem,[11] several stable adaptive fuzzy control schemes have been developed for unknown single-input single-output (SISO) nonlinear systems,[11–14] for multi-input multi-output (MIMO) nonlinear systems[15–17] and for large-scale interconnected nonlinear systems[18–20] to achieve stable performance criterion. The stability analysis in such schemes is performed using the Lyapunov synthesis approach. However, these adaptive fuzzy control schemes are static in nature. Motivated by the fact that most of the physical systems are generally dynamic, this suggests that one may introduce some sort of dynamics to these static fuzzy models in order to cope with the dynamic nature of physical systems. This would provide a new tool in the control of dynamic systems. A dynamic structure called the dynamic fuzzy logic system (DFLS) was introduced by Lee and Vukovich,[21,22] who successfully applied this concept to the identification of single-link robotic manipulator.[21] Stable identification and adaptive control based on DFLS was performed in ref. [22]. This work extended to a larger class of SISO nonlinear systems in ref. [23].

However, previous works on DFLS are limited to only SISO nonlinear systems, and still MIMO nonlinear systems have not been addressed. Based on the initial results of DFLS for SISO nonlinear systems,[22,23] an extended adaptive fuzzy control scheme for MIMO nonlinear systems based on the DFLS approach is developed in this paper. Furthermore, the proposed control scheme is designed via an H^∞ tracking performance, which can greatly attenuate disturbances, model uncertainties, and fuzzy approximation errors.

The paper is organized as follows. A class of MIMO nonlinear systems and control objectives are described in Section 2. Section 3 presents a brief description of static fuzzy

* Corresponding author. E-mail: mhamdy72@hotmail.com

systems and DFLS. The DFLS-based adaptive control design is presented in Section 4. In Section 5, the proposed control algorithm is used to control a two-link robot manipulator. Section 6 concludes this paper.

2. System Description

In this paper, we consider a class of MIMO nonlinear system represented by the following set of differential equations:

$$\begin{aligned} y_1^{(n_1)} &= f_1(x) + \sum_{j=1}^{p} g_{1j}(x)u_j + d_1 \\ &\vdots \\ y_p^{(n_p)} &= f_p(x) + \sum_{j=1}^{p} g_{pj}(x)u_j + d_p \end{aligned}, \qquad (1)$$

where $x = [y_1, \dot{y}_1, \ldots, y_1^{(n_1-1)}, \ \ldots, \ y_p, \dot{y}_p, \ldots, y_p^{(n_p-1)}]^T$ is the overall state vector which is assumed to be available for measurements, $u = [u_1, \ldots, u_p]^T$ is the control input vector,$y = [y_1, \ldots, y_p]^T$ is the output vector, $D = [d_1, \ldots, d_p]^T$ denotes the external disturbance, and $f_i(x)$, and $g_{ij}(x)$, $i, j = 1, \ldots p$, are smooth unknown nonlinear functions. Let us denote

$$y^{(n)} = [y_1^{(n_1)} \ldots y_p^{(n_p)}]^T, \ F(x) = [f_1(x) \ldots f_p(x)]^T,$$

$$G(x) = \begin{bmatrix} g_{11}(x) \cdots g_{1p}(x) \\ \vdots \quad \ddots \quad \vdots \\ g_{p1}(x) \cdots g_{pp}(x) \end{bmatrix}, \quad \text{and} \quad D = \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}.$$

Then the dynamic system described by Eq. (1) can be rewritten in the following compact form:

$$y^{(n)} = F(x) + G(x)u + D. \qquad (2)$$

Throughout this paper, the following assumptions are considered for system (1).

Assumption 1. The matrix $G(x)$ is bounded away from singularity over compact set $U_c \subset R^n$, specifically $\|G(x)\|^2 = Trace(G^T(x)G(x)) \geq b_1 \geq 0$, where b_1 represents the smallest singular value of $G(x)$.

Assumption 2. The reference trajectories, y_{mi}, $i = 1, \cdots, p$, are known bounded functions of time with known bounded derivatives, and it is assumed to be r_i- times differentiable.

Control objectives: Develop a feedback control law $u(t)$ (based on DFLS), which ensures the boundness of all variables in closed-loop systems and the parameters of DFLS, and guarantees output tracking of a specified desired trajectory $y_{mi} = [y_{m1}, \cdots, y_{mp}]^T$. In addition, for a given disturbance attenuation level $\rho > 0$, the following H^∞ tracking performance index is achieved:

$$\begin{aligned} \frac{1}{2}\int_0^T \underline{e}^T Q \underline{e}\, dt \leq & \frac{1}{2}\underline{e}_i^T(0)P_i\underline{e}_i(0) + \frac{1}{2}h_i\tilde{z}^T\tilde{z}(0) \\ & + \frac{1}{2}\Delta^T(0)\Delta(0) + \frac{1}{2}\rho^2\int_0^T \delta^T\delta\, dt, \end{aligned} \qquad (3)$$

where e is the error vector, $\delta \in L_2[0, T]$ is the combined disturbance and approximation error for $T \in [0, \infty]$, Q and P are positive matrices of proper dimensions, Δ is a parameter approximation error vector, $\tilde{z}$ is a identification error vector, and h is a design parameter.

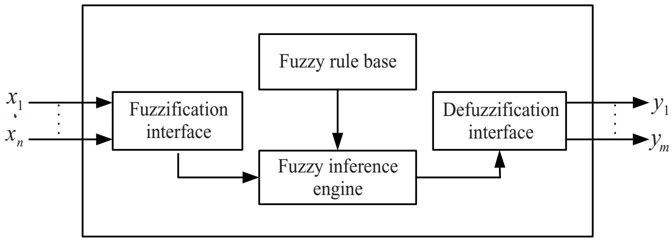

Fig. 1. MIMO fuzzy logic system.

3. Description of DFLS

The DFLS is composed of an ordinary fuzzy logic system (also referred as a static fuzzy logic system) and a dynamic element. The basic structure of a fuzzy logic system considered in this paper, which has been widely used in identification and control of nonlinear systems, is shown in Fig. 1; it is composed of four major components, namely, a fuzzification interface, a fuzzy rule base, a fuzzy inference engine, and a defuzzification interface.

For MIMO fuzzy systems, the fuzzy rule base is made up of the following inference rule:

R^l : IF x_1 is F_1^l AND x_2 is F_2^l AND ... AND x_i is F_i^l AND ... AND x_n is F_n^l
THEN y_1 is G_1^l AND y_2 is G_2^l AND ... AND y_j is G_j^l ' AND ... AND y_p is G_p^l (4)

where F_1^l and G_1^l are fuzzy sets in R, $l = 1, 2, \cdots, N$; $i = 1, 2, \cdots, n$; $j = 1, 2, \cdots, p$. Fuzzy inference Eq. (4) can be decomposed and expressed as:

R^l : IF x_1 is F_1^l AND x_2 is F_2^l AND ... AND x_i is F_i^l AND ... AND x_n is F_n^l
THEN y_j is G_j^l, $(j = 1, 2, \cdots, p)$.

Through center-average defuzzifier, product inference, and singleton fuzzifier,[11] the output of a fuzzy logic system can be expressed as

$$y_j(x) = \frac{\sum_{l=1}^N \bar{y}^l \left(\prod_i^n \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^N \left(\prod_i^n \mu_{F_i^l}(x_i)\right)}, \qquad (5)$$

where $\bar{y}^l$ is the center of the fuzzy set G^l at which μ_G^l achieves its maximum value, and we assume that $\mu_G^l(\bar{y}^l) = 1$.

Equation (5) can be written as

$$y_j(x) = \bar{Y}_j^T\phi(x), \quad j = 1, 2, \cdots, p, \qquad (6)$$

where $\bar{Y}_j^T = [\bar{y}_j^1, \cdots, \bar{y}_j^N]^T$ is a vector of adjustable parameters, and $\phi(x) = [\phi_1, \cdots, \phi_N]^T$ is a regression vector

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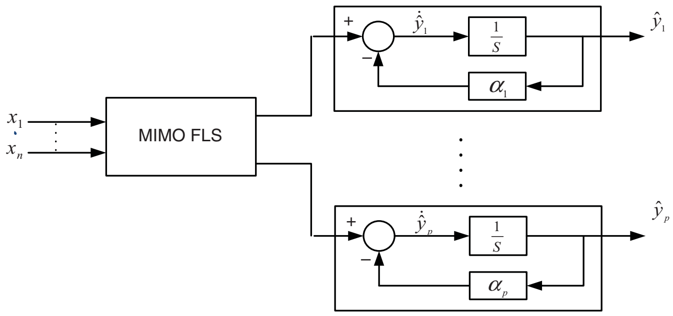

Fig. 2. MIMO dynamic fuzzy logic system.

with each ϕ_l variable defined as a fuzzy basis function (FBF)[11] as

$$\phi_l = \frac{\prod_i^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^N \left(\prod_i^n \mu_{F_i^l}(x_i)\right)}. \tag{7}$$

Define $\bar{Y} = [\bar{Y}_1, \cdots, \bar{Y}_p]_{l\times p}$ as a matrix of adjustable parameters. MIMO fuzzy system can be expressed as

$$y = \bar{Y}^T\phi(x). \tag{8}$$

The MIMO DFLS, shown in Fig. 2, can now be described by the following differential equations:

$$\begin{aligned} \dot{\hat{y}}_1 &= -\alpha_1\hat{y}_1 + \bar{Y}_1^T\phi(x), \\ &\vdots \\ \dot{\hat{y}}_p &= -\alpha_p\hat{y}_p + \bar{Y}_p^T\phi(x). \end{aligned} \tag{9}$$

Using the definition in Eq. (8), Eq. (9) can be written in the following compact form:

$$\dot{\hat{y}} = -\alpha\hat{y} + \bar{Y}^T\phi(x), \tag{10}$$

where $\hat{y} = [\hat{y}_1, \cdots, \hat{y}_p]$ is the output of the MIMO DFLS, $\alpha = diag[\alpha_1, \cdots, \alpha_p]$ is a positive constant matrix.

The DFLS described by Eq. (10) was shown to possess universal approximating capabilities to a large class of nonlinear dynamic systems.[21]

Let $z \in R^p$, a vector defined as

$$z = \begin{bmatrix} z_1 \\ \vdots \\ z_p \end{bmatrix} = \begin{bmatrix} y_1^{(n_1-1)} \\ \vdots \\ y_p^{(n_p-1)} \end{bmatrix}. \tag{11}$$

System (1) can be written as

$$\begin{aligned} \dot{z}_1 &= f_1(x) + \sum_{j=1}^p g_{1j}(x)u_j + d_1 \\ &\vdots \\ \dot{z}_p &= f_p(x) + \sum_{j=1}^p g_{pj}(x)u_j + d_p \end{aligned}. \tag{12}$$

Alternatively, Eq. (12) can be written in the compact form as

$$\dot{z} = F(x) + G(x)u + D. \tag{13}$$

According to the universal approximation theorem,[21] the following DFLS can be used to identify the unknown MIMO nonlinear system (12),

$$\dot{\hat{y}} = -\alpha\hat{y} + \bar{Y}^T\phi(x, u). \tag{14}$$

Our objective now is to develop an appropriate control law for input u in Eq. (1), and an adaptation law for the parameter matrix $\bar{Y}$ of the DFLS (14) such that the closed-loop system is stable in the sense that the tracking errors $e_j = y_{mj} - y_j$, $j = 1, \cdots, p$, as well as the identification errors and identifier parameters are all uniformly bounded.

4. DFLS-Based Adaptive Control

In this section, we develop an adaptive control scheme for system (1) based on DFLS-based identification.

Consider system (1) for the given reference trajectories, $y_m = [y_{m1}, \cdots, y_{mp}]^T$. Let us define the tracking errors as

$$\begin{aligned} e_1 &= y_{m1} - y_1 \\ &\vdots \\ e_p &= y_{mp} - y_p \end{aligned}. \tag{15}$$

Denote $e = [e_1, \cdots, e_p]^T$, then $e = y_m - y$.

If system (1) is known, i.e., $F(x)$ and $G(x)$ are known and $D = 0$, then the feedback law

$$\begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix} = \begin{bmatrix} g_{11}(x)\cdots g_{1p}(x) \\ \vdots \quad \ddots \quad \vdots \\ g_{p1}(x)\cdots g_{pp}(x) \end{bmatrix}^{-1} \left(-\begin{bmatrix} f_1(x) \\ \vdots \\ f_p(x) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix}\right) \tag{16}$$

yields the linearize systems,

$$\begin{bmatrix} y_1^{(n_1)} \\ \vdots \\ y_p^{(n_p)} \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix}. \tag{17}$$

For reference trajectories to be asymptotically tracked, we choose

$$\begin{aligned} v_1 &= y_{m1}^{(n_1)} + k_{1r_1}e_1^{(n_1-1)} + \cdots + k_{11}e_1 \\ &\vdots \\ v_p &= y_{mp}^{(n_p)} + k_{pr_p}e_p^{(n_p-1)} + \cdots + k_{p1}e_p \end{aligned}. \tag{18}$$

Substituting Eq. (16) in Eq. (1) yields

$$\begin{aligned} e_1^{(n_1)} + k_{1r_1}e_1^{(n_1-1)} + \cdots + k_{11}e_1 &= 0 \\ &\vdots \\ e_p^{(n_p)} + k_{pr_p}e_p^{(n_p-1)} + \cdots + k_{p1}e_p &= 0 \end{aligned}. \tag{19}$$

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If the coefficients k_{ij} are chosen such that all polynomials in Eq. (19) are Hurwitz stable, then we can conclude that $\lim_{t\to\infty} e_i(t) = 0$, which is a main objective of control. However, the nonlinear functions $f_i(x)$, $G_{ij}(x)$ are unknown.

Consider the MIMO DFLS in the form of Eq. (9),

$$\begin{aligned} \dot{\hat{z}}_1 &= -\alpha_1 \hat{z}_1 + \bar{Y}_1^T \phi(x,u) \\ &\vdots \\ \dot{\hat{z}}_p &= -\alpha_p \hat{z}_p + \bar{Y}_p^T \phi(x,u) \end{aligned}, \tag{20}$$

which can be used to identify the unknown MIMO nonlinear system (12). Define identification errors as $\tilde{z}_i = \hat{z}_i - z_i$, $i = 1, 2, \cdots p$. Our objective is to develop an appropriate control law for input u_i and an adaptive law for the identifier parameters $\bar{Y}_i$ such that closed-loop system is stable.

The expression for $\dot{z}_i$ in Eq. (12) can be written as

$$\begin{aligned} \dot{z}_1 &= -\alpha_1 z_1 + \bar{Y}_1^T \phi(x,u) - r_1(x,u,\phi,\bar{Y}_1), \\ &\vdots \\ \dot{z}_p &= -\alpha_p z_p + \bar{Y}_p^T \phi(x,u) - r_p(x,u,\phi,\bar{Y}_p), \end{aligned} \tag{21}$$

where $r_i(x,u,\phi,\bar{Y}_i)$ represents the static modeling error of the DFLS identifier and can be expressed as

$$\begin{aligned} r_i(x,u,\phi,\bar{Y}_i) = -\alpha_1 z_1 + \bar{Y}_i^T \phi(x,u) - f_i(x) \\ - \sum_{j=1}^{p} g_{ij}(x) u_j - d_i. \end{aligned} \tag{22}$$

By Lemma 1 in ref. [21], there are existed optimal parameter vectors,

$$\bar{Y}_i^* = \min_{\|\bar{Y}_i\|} \{\bar{Y}_i : \|\bar{Y}_i\| \le M_{\bar{Y}_i}\}, \tag{23}$$

which minimize the static modeling error, r_i, such that

$$\sup_{(x,u)\in\Omega} |r_i(x,u,\phi,\bar{Y}_i^*)| \le M_i^r, \tag{24}$$

where $M_{\bar{Y}_i}$ and M_i^r are positive design constants. In the following, we develop an adaptive law for $\bar{Y}_i$. Replacing $\bar{Y}_i$ by $\bar{Y}_i^*$ in Eq. (21) results in

$$\begin{aligned} \dot{z}_1 &= -\alpha_1 z_1 + \bar{Y}_1^{*T} \phi(x,u) - r_1(x,u,\phi,\bar{Y}_1^*) \\ &\vdots \\ \dot{z}_p &= -\alpha_p z_p + \bar{Y}_p^{*T} \phi(x,u) - r_p(x,u,\phi,\bar{Y}_p^*) \end{aligned}. \tag{25}$$

Subtracting Eq. (25) from Eq. (20) yields

$$\begin{aligned} \dot{\tilde{z}}_1 &= -\alpha_1 \tilde{z}_1 + \Delta_1^T \phi(x,\ u) + r_1(x,\ u,\ \phi,\ \bar{Y}_1^*) \\ &\vdots \\ \dot{\tilde{z}}_p &= -\alpha_p \tilde{z}_p + \Delta_p^T \phi(x,\ u) + r_p(x,\ u,\ \phi,\ \bar{Y}_p^*) \end{aligned}, \tag{26}$$

where $\Delta_i = \bar{Y}_i - \bar{Y}_1^*$ is the parameter estimation error.

In this situation, we propose the following control law, which is based on DFLS:

$$\begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix} = \begin{bmatrix} \hat{g}_{11}(x) \cdots \hat{g}_{1p}(x) \\ \vdots \quad \ddots \quad \vdots \\ \hat{g}_{p1}(x) \cdots \hat{g}_{pp}(x) \end{bmatrix}^{-1} \left(\begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \alpha_p \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_p \end{bmatrix} - \begin{bmatrix} \bar{Y}_1^T \phi(x,0) \\ \vdots \\ \bar{Y}_p^T \phi(x,0) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} + \begin{bmatrix} u_{r1} \\ \vdots \\ u_{r2} \end{bmatrix} \right). \tag{27}$$

Alternatively, Eq. (27) can be written in the compact form as

$$u = \hat{G}(x)^{-1} [\alpha z - \bar{Y}^T \phi(x,0) + v + u_r], \tag{28}$$

where $\hat{G}(x)$ is a static fuzzy logic estimation of $G(x)$. Each element $g_{ij}(x)$ of the control gain matrix $G(x)$ is a nonlinear function of the state vector x and can be approximated by a fuzzy logic system in the form of Eq. (6) as

$$\hat{g}_{ij}(x) = \bar{Y}_{ij}^T \phi(x). \tag{29}$$

The adaptive law for the parameter vectors $\bar{Y}_{ij}$ will be defined later. $\phi(x,0) = \phi(x,u)|_{u=0}$ and u_r is a robust compensator, which is defined as

$$u_{ri} = \frac{1}{\lambda_i} B_i^T P_i \underline{e}_i, \tag{30}$$

where λ_i and P_i are the solutions of the following Riccati-like equation:

$$A_i^T P_i + P_i A_i - Q_i - \left(\frac{2}{\lambda_i} - \frac{1}{\rho^2} \right) P_i B_i B_i^T P_i = 0. \tag{31}$$

It is noticed that the Riccati equation (31) has a solution $P = P^T \ge 0$ if and only if $2\rho^2 \ge \lambda_i$.

Using Eq. (27), we can rewrite Eq. (12) as follows:

$$\begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_p \end{bmatrix} = \begin{bmatrix} f_1(x) \\ \vdots \\ f_p(x) \end{bmatrix} + \left(G(x) + \hat{G}(x) - \hat{G}(x) \right) G^{-1}(x) \times \left(\begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \alpha_p \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_p \end{bmatrix} - \begin{bmatrix} \bar{Y}_1^T \phi(x,0) \\ \vdots \\ \bar{Y}_p^T \phi(x,0) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} + \begin{bmatrix} u_{r1} \\ \vdots \\ u_{r2} \end{bmatrix} \right) + \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}. \tag{32}$$

Using Eqs. (17), (18), and (32), and after straight-forward manipulations, it can be easily obtained:

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$$\begin{bmatrix} e_1^{(n_1)} + k_{1r_1} e_1^{(n_1-1)} + \cdots + k_{11} e_1 \\ \vdots \\ e_p^{(n_p)} + k_{pr_p} e_p^{(n_p-1)} + \cdots + k_{p1} e_p \end{bmatrix} = \begin{bmatrix} f_1(x) + \alpha_1 z_1 - \bar{Y}_1^T \phi(x,0) \\ \vdots \\ f_p(x) + \alpha_p z_p - \bar{Y}_p^T \phi(x,0) \end{bmatrix} + (G(x) - \hat{G}(x)) \begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix} + \begin{bmatrix} u_{r1} \\ \vdots \\ u_{r2} \end{bmatrix} + \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}. \quad (33)$$

It is clear from Eq. (33) that u_{ri} can attenuate external disturbance and fuzzy approximation errors.

Equation (32) can be written as

$$e_i^{(n_i)} + k_{ir_i} e_i^{(n_i-1)} + \cdots + k_{i1} e_i = f_i(x) + \alpha_i z_i - \bar{Y}_i^T \phi(x,0) + \sum_{j=1}^{p} (g_{ij}(x) - \hat{g}_{ij}(x)) u_j - u_{ri} + d_i, \quad i = 1, 2, \cdots p. \quad (34)$$

The state-space form for Eq. (34) can be written as

$$\underline{\dot{e}_i} = A_i e_i + B_i u_{ri} + B_i \left[f_i(x) + \alpha_i z_i - \bar{Y}_i^T \phi(x,\,0) + \sum_{j=1}^{p} \left(g_{ij}(x) - \hat{g}_{ij}(x) \right) u_j \right] + B_i\, d_i, \quad (35)$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_{in_i} & -k_{i(n_i-1)} & -k_{i(n_i-2)} & \cdots & -k_{i1} \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$

Using Eq. (22) with $u = 0$, we can write

$$\underline{\dot{e}_i} = A_i e_i + B_i u_{ri} + B_i \left[-\Delta_i^T \phi(x,0) + \sum_{j=1}^{p} (g_{ij}(x) - \hat{g}_{ij}(x)) u_j - r_i(x,u,0,\bar{Y}_i^*) \right]. \quad (36)$$

As stated before, the control gain functions $g_{ij}(x)$ can be approximated by a static fuzzy logic system (6). Thus, it follows that

$$g_{ij}(x) = \hat{g}_{ij}(x) + \kappa_{ij} = \bar{Y}_{ij}^T \phi(x) + \kappa_{ij}, \quad (37)$$

where $\bar{Y}_{ij}$ is an adjustable parameter vector, $\phi(x)$ is an FBF, and κ_{ij} is a fuzzy approximation error. Define optimal parameter estimates $\bar{Y}_{ij}^*$ such that it minimizes the approximation error. Therefore, we can write

$$g_{ij}(x) = \bar{Y}_{ij}^{*^T} \phi(x) + \kappa_{ij}^*, \quad (38)$$

where κ_{ij}^* is the minimum approximation error. Using Eqs. (37) and (38), Equation (36) can be written as

$$\underline{\dot{e}_i} = A_i e_i + B_i u_{ri} + B_i \left[-\Delta_i^T \phi(x,0) - \sum_{j=1}^{p} \Delta_{ij}^T \phi(x) u_j \right] + B_i w_i, \quad (39)$$

where $\Delta_{ij}^T = \bar{Y}_{ij}^T - \bar{Y}_{ij}^{*^T}$ and $w_i = \kappa_{ij}^* + r_i(x,u,0,\bar{Y}_i^*)$.

The adaptive laws are chosen as

$$\dot{\bar{Y}}_i = -\eta_i h_i \tilde{z}_i \phi(x,u) + \eta_i \phi(x,0) B_i^T P_i \underline{e}_i, \quad (40)$$

and

$$\dot{\bar{Y}}_{ij} = -\eta_{ij} \underline{e}_i^T P_i B_i \phi(x) u_j. \quad (41)$$

Theorem 1. *Consider an unknown MIMO nonlinear dynamic system (1) which is controlled by (27) and to be identified by the DFLS (20) by adjusting the parameter vectors $\bar{Y}_i^T$ and $\bar{Y}_{ij}^T$ with the adaptive laws (40) and (41) respectively, then the closed-loop system possesses the following properties:*

(i) *All signals in the closed-loop system are uniformly bounded.*
(ii) *For a given disturbance attenuation level, the proposed tracking performance index (3) is achieved.*

Proof. Choose a Lyapunov function as

$$V = V_1 + \cdots + V_p, \quad (42)$$

$$V_i = \frac{1}{2} \underline{e}_i^T P_i \underline{e}_i + \frac{1}{2} h_i \tilde{z}_i^2 + \frac{1}{2\eta_i} \Delta_i^T \Delta_i + \sum_{j=1}^{p} \frac{1}{2\eta_{ij}} \Delta_{ij}^T \Delta_{ij}. \quad (43)$$

Differentiating V, V_i and using Eqs. (26) and (39), we obtain

$$\dot{V} = \dot{V}_1 + \cdots + \dot{V}_p, \quad (44)$$

$$\dot{V}_i = \frac{1}{2} \underline{\dot{e}}_i^T P_i \underline{e}_i + \frac{1}{2} \underline{e}_i^T P_i \underline{\dot{e}}_i + h_i \tilde{z}_i \dot{\tilde{z}}_i + \frac{1}{\eta_i} \dot{\Delta}_i^T \Delta_i + \sum_{j=1}^{p} \frac{1}{\eta_{ij}} \dot{\Delta}_{ij}^T \Delta_{ij}$$

$$= \frac{1}{2}\Big[\underline{e}_i^T A_i^T P_i \underline{e}_i - \frac{1}{\lambda_i}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i - \phi(x,\,0)^T \Delta_i B_i^T P_i \underline{e}_i - \sum_{j=1}^{p}\phi(x)\Delta_{ij} B_i^T \underline{e}_i u_j + w_i B_i^T P_i \underline{e}_i + \underline{e}_i^T P_i A_i \underline{e}_i - \frac{1}{\lambda_i}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i - \underline{e}_i^T P_i B_i \Delta_i^T \phi(x,\,0) - \sum_{j=1}^{p}\underline{e}_i^T P_i B_i \Delta_{ij}^T \phi(x) u_j + \underline{e}_i^T P_i B_i w_i\Big] + h_i \tilde{z}_i \big[-\alpha_i \tilde{z}_i + \Delta_i^T \phi(x,\,u) + r_i(x,\,u,\,\phi,\,\bar{Y}_i^*)\big] + \frac{1}{\eta_i}\dot{\tilde{Y}}_i^T \Delta_i + \sum_{j=1}^{p}\frac{1}{\eta_{ij}}\dot{\tilde{Y}}_{ij}^T \Delta_{ij}$$

$$= \underline{e}_i^T\left(A_i^T P_i + P_i A_i - \frac{2}{\lambda_i}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i\right)\underline{e}_i - \alpha_i h_i \tilde{z}_i^2 + \frac{1}{\eta_i}\big(\dot{\tilde{Y}}_i^T + \eta_i h_i \tilde{z}_i \phi^T(x,\,u) - \eta_i \phi(x,\,0) B_i^T P_i \underline{e}_i\big)\Delta_i + \sum_{j=1}^{p}\frac{1}{\eta_{ij}}\big(\dot{\tilde{Y}}_{ij}^T + \eta_{ij}\underline{e}_i^T P_i B_i \phi^T(x) u_j\big)\Delta_{ij} + h_i \tilde{z}_i r_i(x,u,\phi,\bar{Y}_i^*) + \frac{1}{2}\big(w_i B_i^T P_i \underline{e}_i + \underline{e}_i^T P_i B_i w_i\big). \tag{45}$$

Using the adaptive laws (40) and (41), Eq. (45) can be simplified into

$$\dot{V}_i \le \underline{e}_i^T\left(A_i^T P_i + P_i A_i - \frac{2}{\lambda_i}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i\right)\underline{e}_i - \alpha_i h_i \tilde{z}_i^2 - h_i \tilde{z}_i r_i(x,u,\phi,\bar{Y}_i^*) - \frac{1}{2}\big(w_i B_i^T P_i \underline{e}_i + \underline{e}_i^T P_i B_i w_i\big). \tag{46}$$

Using the following triangular inequality for the third term in Eq. (46),

$$h_i \tilde{z}_i r_i(x,u,\phi,\bar{Y}_i^*) \le \frac{h_i^2 \tilde{z}_i^2}{2\rho^2} + \frac{\rho^2}{2} r_i^2(x,u,\phi,\bar{Y}_i^*). \tag{47}$$

Substituting Eq. (47) and using the Riccati equation (31), Eq. (46) becomes

$$\dot{V}_i \le -\underline{e}_i^T Q \underline{e}_i - \frac{1}{2\rho_i^2}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i - \alpha_i h_i \tilde{z}_i^2 + \frac{h_i^2 \tilde{z}_i^2}{2\rho^2} + \frac{\rho^2}{2} r_i^2(x,u,\phi,\bar{Y}_i^*) + \frac{1}{2}\big(w_i B_i^T P_i \underline{e}_i + \underline{e}_i^T P_i B_i w_i\big). \tag{48}$$

Note that the third and fourth terms are negative in Eq. (48),

$$\dot{V}_i \le -\underline{e}_i^T Q \underline{e}_i - \frac{1}{2\rho^2}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i - \left(\alpha_i h_i - \frac{h_i^2}{2\rho^2}\right)\tilde{z}_i^2 + \frac{\rho^2}{2} r_i^2(x,u,\phi,\bar{Y}_i^*) + \frac{1}{2}\big(w_i B_i^T P_i \underline{e}_i + \underline{e}_i^T P_i B_i w_i\big). \tag{49}$$

The third term in Eq. (49) can be made negative by choosing $h_i \le 2\alpha_i \rho^2$,

$$\dot{V}_i \le -\underline{e}_i^T Q \underline{e}_i - \frac{1}{2}\left(\frac{1}{\rho}\underline{e}_i^T P_i B - \rho w_i\right)^2 + \frac{\rho^2}{2}\big(r_i^2(x,u,\phi,\bar{Y}_i^*) + w_i^2\big). \tag{50}$$

Since $\frac{1}{2}(\frac{1}{\rho}\underline{e}_i^T P_i B - \rho w_i)^2 \ge 0$, from Eq. (50) we obtain

$$\dot{V}_i \le -\underline{e}_i^T Q \underline{e}_i + \frac{1}{2}\rho^2 \delta_i^2, \tag{51}$$

where $\delta_i^2 = r_i^2(x,u,\phi,\bar{Y}_i^*) + w_i^2$. After some straightforward manipulations, we can deduce

$$\dot{V}_i \le -c_i V_i + \mu_i, \tag{52}$$

where $c_i = \min\{\lambda, \frac{1}{\eta_i}, \frac{1}{\eta_{ij}}\}$ with $\lambda = \frac{\inf \lambda_{\min}(Q_i)}{sub\lambda_{\max}(Q_i)}$ and $\mu_i = \frac{1}{2\rho^2}\sum_{i=1}^{p}\delta_i^2$.

From Eqs. (52) and (44) we obtain

$$\dot{V} \le -cV + \mu, \tag{53}$$

where $c = \sum_{i=1}^{p} c_i$, $\mu = \sum_{i=1}^{p}\mu_i$.

This implies that all signals in the closed-loop system are bounded. Thus, the control objective (i) is realized.

Integrating Eq. (50) from $t = 0$ to $t = T$, we have

$$\frac{1}{2}\int_0^T \underline{e}_i^T Q \underline{e}_i dt \le V_i(0) - V_i(T) + \frac{1}{2}\rho^2 \int_0^T \delta_i^2 dt. \tag{54}$$

Since $V_i(T) \ge 0$, we can write Eq. (54) as follows:

$$\begin{aligned}\frac{1}{2}\int_0^T \underline{e}_i^T Q \underline{e}_i dt &\le V_i(0) + \frac{1}{2}\rho^2\int_0^T \delta_i^2 dt \\ &= \frac{1}{2}\underline{e}_i^T(0) P_i \underline{e}_i(0) + \frac{1}{2}h_i \tilde{z}_i^2(0) + \frac{1}{2\eta_i}\Delta_i^T(0)\Delta_i(0) \\ &\quad + \sum_{j=1}^{p}\frac{1}{2\eta_{ij}}\Delta_{ij}^T(0)\Delta_{ij}(0) + \frac{1}{2}\rho^2\int_0^T \delta_i^2 dt.\end{aligned} \tag{55}$$

Let $Q = diag\,[Q_1, \cdots, Q_P]$, $P = diag\,[P_1, \cdots, P_P]$, $\underline{e} = \big[\underline{e}_1^T, \cdots, \underline{e}_P^T\big]^T$, $\tilde{z} = \big[\tilde{z}_1, \cdots \tilde{z}_p\big]^T$,

$\Delta = \left[\frac{1}{\eta_1}\Delta_1^T, \cdots, \frac{1}{\eta_p}\Delta_p^T, \frac{1}{\eta_{11}}\Delta_{11}^T, \cdots, \frac{1}{\eta_{pp}}\Delta_{pp}^T,\right]^T$, and $\delta = \big[\delta_1, \cdots, \delta_p\big]^T$.

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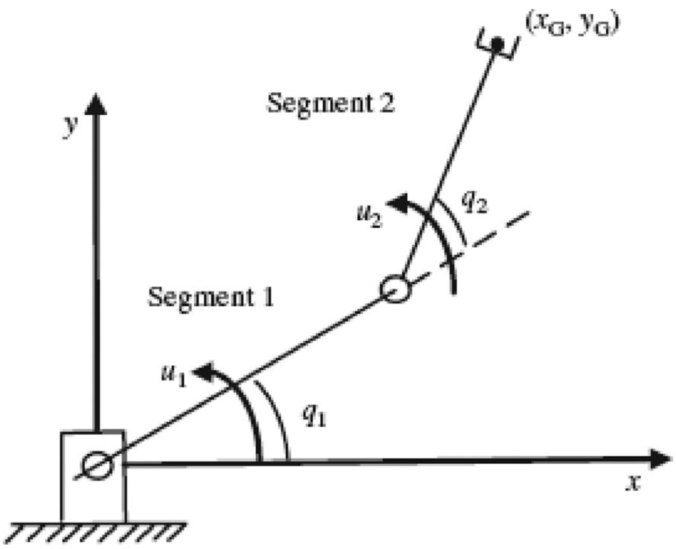

Fig. 3. Two-link robotic manipulator.

Then from Eq. (55) we obtain

$$\frac{1}{2}\int_0^T \underline{e}^T Q\underline{e}dt \leq \frac{1}{2}\underline{e}^T(0)P\underline{e}(0) + \frac{1}{2}h\tilde{z}^T\tilde{z}(0) + \frac{1}{2}\Delta^T(0)\Delta(0) + +\frac{1}{2}\rho^2\int_0^T \delta^T\delta dt. \quad (56)$$

Thus, the control objective (ii) is achieved and the proof of Theorem 1 is completed.

For the sake of clarity of presentation, the overall design procedure of the DFLS scheme is summarized in the following steps:

Step 1. Specify the coefficients k_{ij} such that A_i is Hurwitz stable and meets the required transient response on tracking error dynamics.

Step 2. Specify positive-definite matrices Q_i, desired attenuation level ρ, and the weighting factor λ_i such that $2\rho^2 \geq \lambda_i$.

Step 3. Solve the Riccati equation (31) to obtain positive definite matrices P_i.

Step 4. Select membership functions $\mu_{F_i}(\cdot)$, $i = 1, 2, \ldots, N$ and incorporate expert knowledge as the rule base if available to compute the fuzzy basis vectors $\phi(x)$, $\phi(x, u)$.

Step 5. Apply the control law (27) with adaptive laws (40) and (41).

5. Simulation Results

In order to demonstrate the effectiveness of the proposed scheme, a simulation is performed for the tracking control of a two-link rigid robot manipulator moving in a horizontal plane as shown in Fig. 3.

The dynamics of the robotic manipulator are described by the following differential equation:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_2 - h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad (57)$$

where

$$\begin{aligned} M_{11} &= a_1 + 2a_3\cos(q_2) + 2a_4\sin(q_2), \\ M_{22} &= a_2, \\ M_{21} &= M_{12} = a_2 + a_3\cos(q_2) + a_4\sin(q_2), \\ h &= a_3\sin(q_2) - a_4\cos(q_2) \end{aligned}$$

with

$$\begin{aligned} a_1 &= I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2, \\ a_2 &= I_e + m_e l_{ce}^2, \\ a_3 &= m_e l_1 l_{ce}\cos\delta_e, \\ a_4 &= m_e l_1 l_{ce}\sin\delta_e. \end{aligned}$$

In the simulation, the following parameter values are used:

$$\begin{aligned} &l_1 = 0.1, l_{c1} = 0.5, m_1 = 1.0, I_1 = 0.12, \\ &I_{ce} = 0.6, \delta_e = 0.6, m_e = 2.0, I_e = 0.25. \end{aligned}$$

Since the inertia matrix M is positive definite, the system can be written as

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1}\begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_2 - h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1}\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}. \quad (58)$$

Let $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$, $y_1 = x_1$, $y_2 = x_3$,

$$G(x) = M^{-1}, \quad F(x) = M^{-1}\begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_2 - h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}.$$

Then the dynamics of a two-link robotic manipulator can be expressed as

$$\ddot{y} = F(x) + G(x)u + D. \quad (59)$$

In simulation, we are going to consider the external disturbance as $d_1 = 0.05\sin(20t)$ and $d_2 = 0.05\cos(20t)$. The control objective is to force the angular positions of the robot q_1 and q_2 to track desired reference trajectories. In simulation, the performance of the control scheme is assessed on two different reference signals, the first $y_{m1} = y_{m2} = 0.2\sin(t)$ and the second $y_{m1} = y_{m2} = 0.15\sin(0.5t) + 0.1\sin(2t)$.

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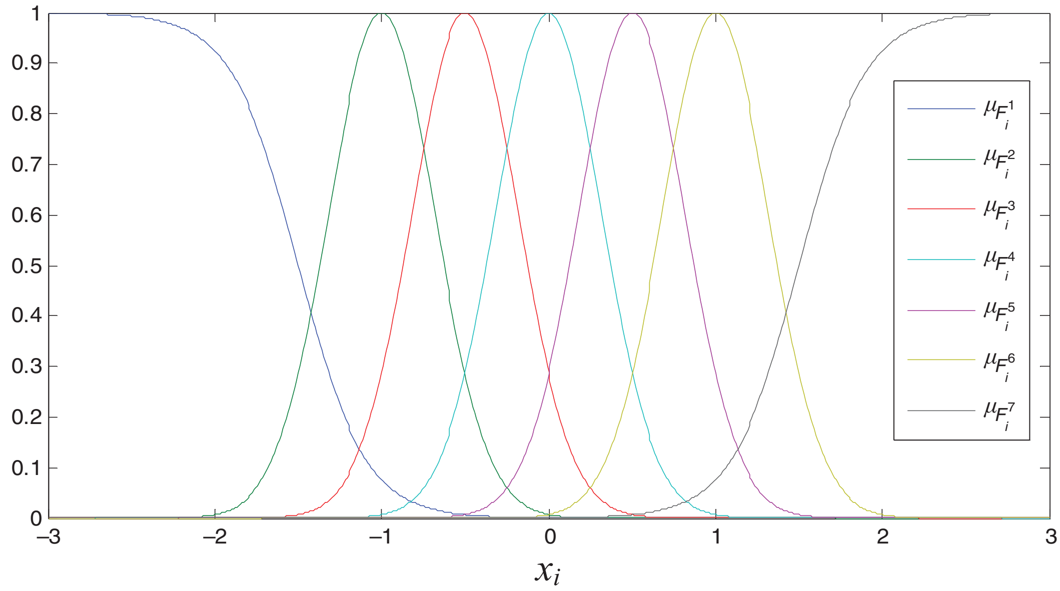

Fig. 4. (Colour online) Fuzzy membership functions.

The DFLS-based control scheme design procedure for robotic manipulator is described in some details in the following steps:

Step 1. $k_{11} = k_{21} = 4, k_{12} = k_{22} = 10$.

Step 2. We are going to perform simulation with two different values of attenuation levels, $\rho = 0.2, 0.1$.

Case 1. $(\rho = 0.2)$:

Select positive definite $Q_1 = Q_2 = diag\,[10, 10]$.

Select $\lambda_1 = \lambda_2 = 0.02$ such that $2\rho^2 \geq \lambda_i$.

Case 2. $(\rho = 0.1)$:

Select positive definite $Q_1 = Q_2 = diag\,[10, 10]$.

Select $\lambda_1 = \lambda_2 = 0.01$ such that $2\rho^2 \geq \lambda_i$.

Step 3. Solving the Riccati equation for both values of attenuation level.

Case 1. $(\rho = 0.2)$:

$$P_1 = P_2 = \begin{bmatrix} 10.044 & 0.041 \\ 0.041 & 0.041 \end{bmatrix}.$$

Case 2. $(\rho = 0.1)$:

$$P_1 = P_2 = \begin{bmatrix} 10.033 & 0.031 \\ 0.031 & 0.031 \end{bmatrix}.$$

Step 4. In this simulation study, seven Gaussian membership functions are employed to construct the DFLS-based control scheme of the following from:

$$\mu_{F_i^1} = 1/(1 + \exp(5(x_i + 1.5))), \quad \mu_{F_i^2} = \exp(-5(x_i + 1)^2),$$
$$\mu_{F_i^3} = \exp(-5(x_i + 0.5)^2), \quad \mu_{F_i^4} = \exp(-5x_i^2),$$
$$\mu_{F_i^5} = \exp(-5(x_i - 0.5)^2), \quad \mu_{F_i^6} = \exp(-5(x_i - 1)^2),$$
$$\mu_{F_i^7} = 1/(1 + \exp(-5(x_i - 1.5))).$$

The shape of these fuzzy membership functions is shown in Fig. 4.

Assuming that there are no linguistic rules, we consider the following fuzzy rule of inference:

$$R^l : \text{ IF } x_1 \text{ is } F_1^l \text{ AND } x_2 \text{ is } F_2^l \text{ AND}$$
$$\ldots \text{ AND } x_n \text{ is } F_n^l \text{ THEN } y \text{ is } G^l.$$

Now we can construct the fuzzy basis vectors $\phi(x)$, $\phi(x, u)$ as follows:

$$\phi_l(x) = \frac{\mu_{F_1^l}(x_1)\mu_{F_2^l}(x_2)\mu_{F_3^l}(x_3)\mu_{F_4^l}(x_4)}{\sum_{l=1}^{7} \mu_{F_1^l}(x_1)\mu_{F_2^l}(x_2)\mu_{F_3^l}(x_3)\mu_{F_4^l}(x_4)}, \tag{60}$$

$$\phi_l(x, u) = \frac{\mu_{F_1^l}(x_1)\mu_{F_2^l}(x_2)\mu_{F_3^l}(x_3)\mu_{F_4^l}(x_4)\mu_{F_5^l}(u_1)\mu_{F_6^l}(u_2)}{\sum_{l=1}^{7} \mu_{F_1^l}(x_1)\mu_{F_2^l}(x_2)\mu_{F_3^l}(x_3)\mu_{F_4^l}(x_4)\mu_{F_5^l}(u_1)\mu_{F_6^l}(u_2)}, \tag{61}$$

$$\phi(x) = [\phi_1(x),\ \phi_2(x), \ldots \phi_7(x)]^T,$$
$$\phi(x, u) = [\phi_1(x, u),\ \phi_2(x, u), \ldots \phi_7(x, u)]^T.$$

Step 5. Using all the data from the previous steps, we can construct the control law (27) with adaptive laws (40) and (41). The parameters of the control and adaptive laws are selected as follows:

$$\alpha_1 = 9, \quad \alpha_2 = 5, \quad h_1 = 100, \quad h_2 = 50, \quad \eta_1 = \eta_1 = 1,$$
$$\eta_{11} = \eta_{22} = 0.1, \quad \eta_{12} = 0.11, \quad \eta_{21} = 0.21.$$

The initial conditions are chosen as $x_1(0) = x_3(0) = 0.1$ and $x_2(0) = x_4(0) = 0$, and the initial conditions for the adaptive parameters are chosen to be zero. Simulation results for the two attenuation levels are shown in Figs. 5–13 for

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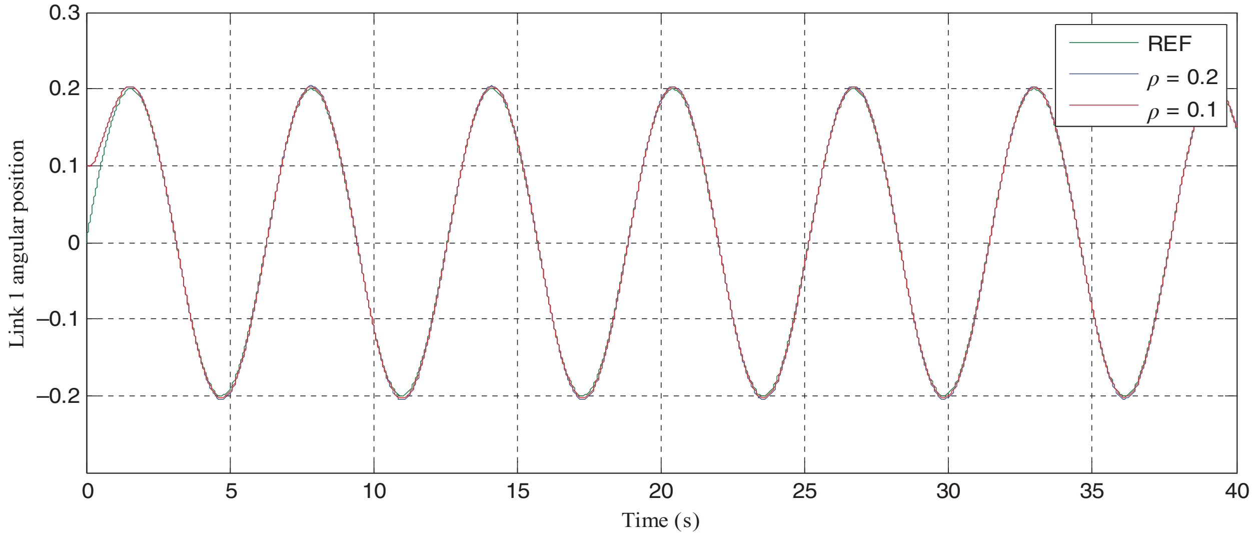

Fig. 5. (Colour online) First link position tracking with $\rho = 0.2$ and $\rho = 0.1$

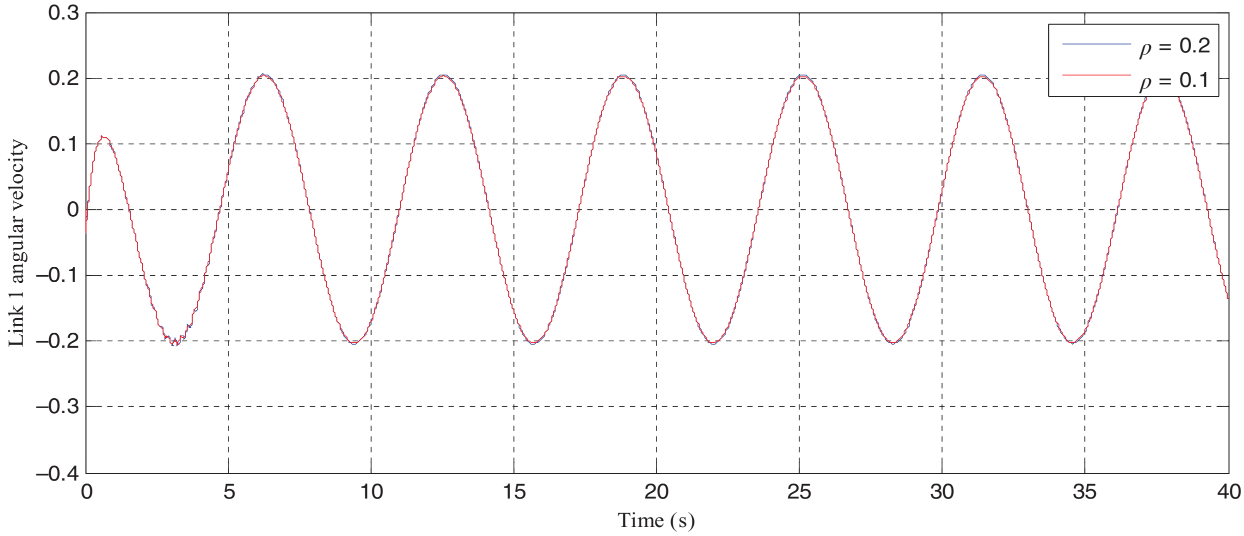

Fig. 6. (Colour online) First link angular velocity with $\rho = 0.2$ and $\rho = 0.1$.

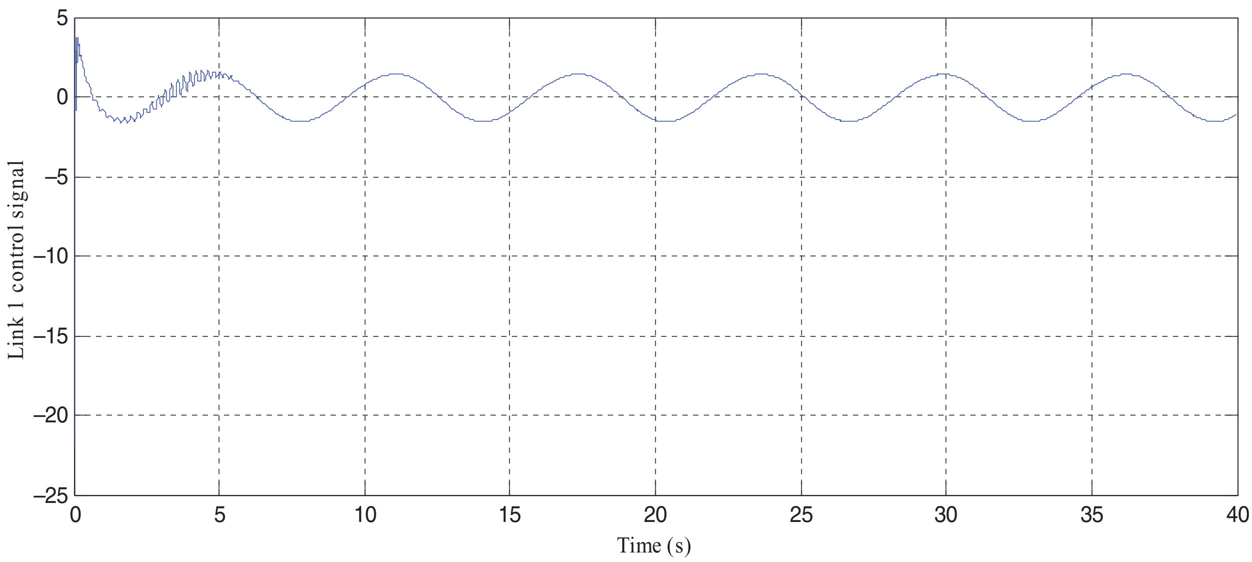

Fig. 7. (Colour online) First link control signal with $\rho = 0.2$.

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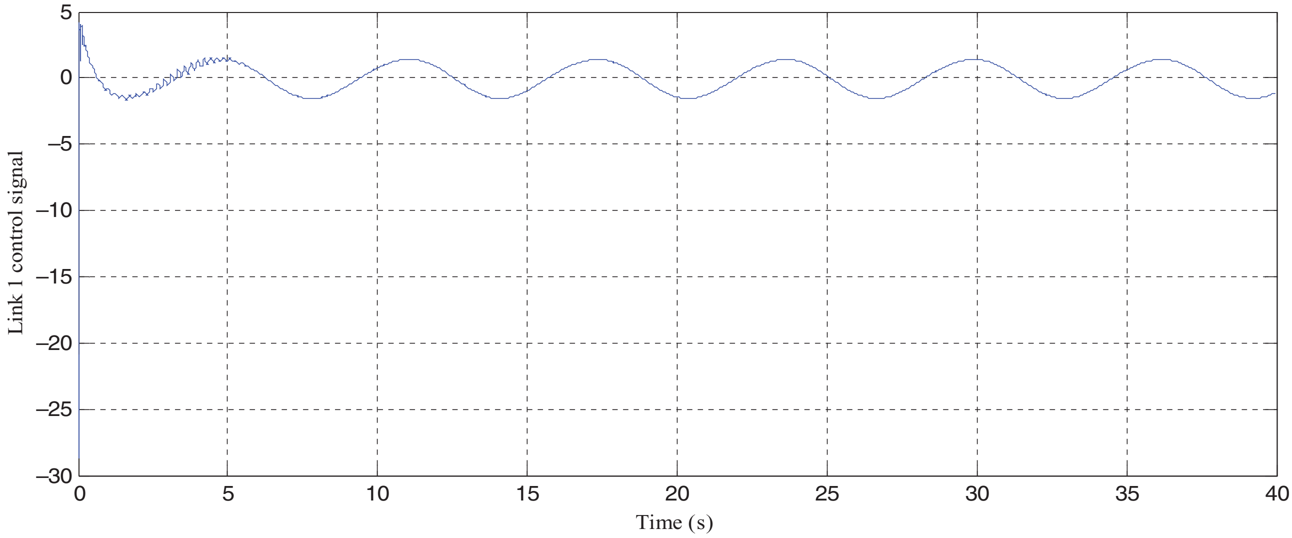

Fig. 8. (Colour online) First link control signal with $\rho = 0.1$.

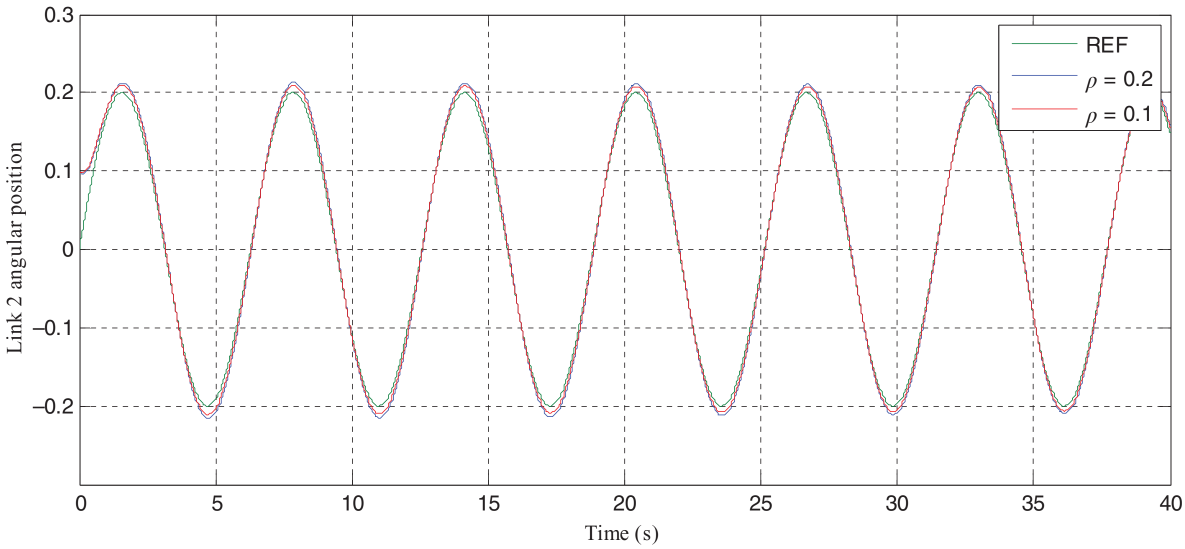

Fig. 9. (Colour online) Second link position tracking with $\rho = 0.2$ and $\rho = 0.1$.

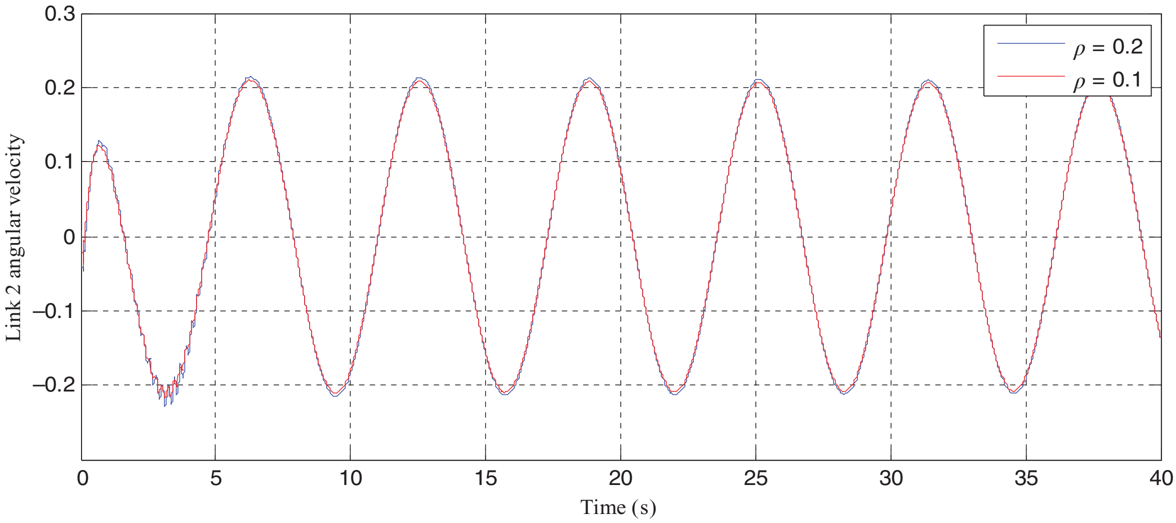

Fig. 10. (Colour online) Second link angular velocity with $\rho = 0.2$ and $\rho = 0.1$.

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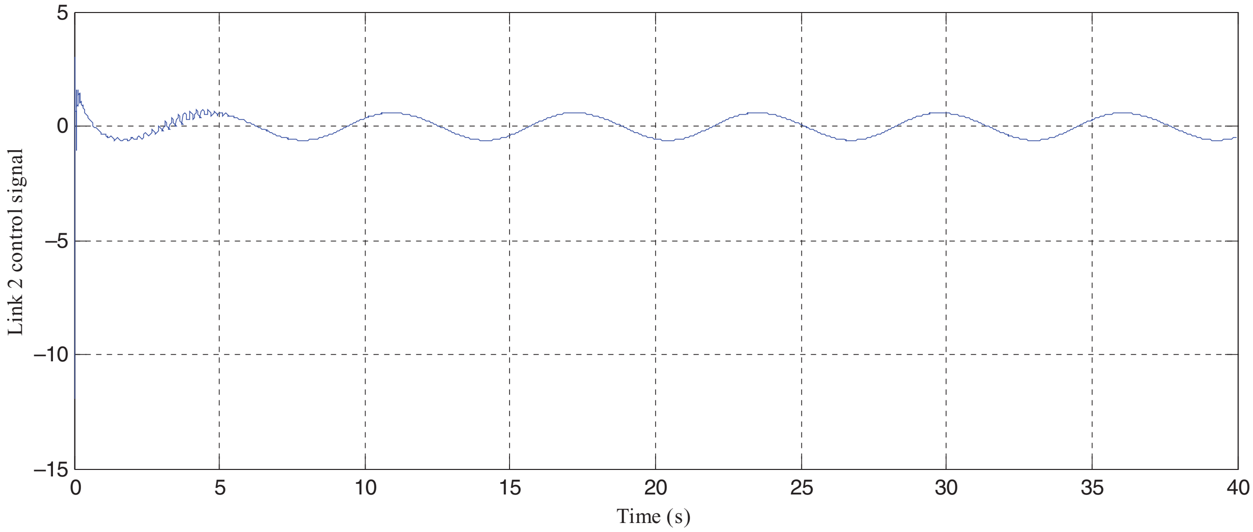

Fig. 11. (Colour online) Second link control signal with $\rho = 0.2$.

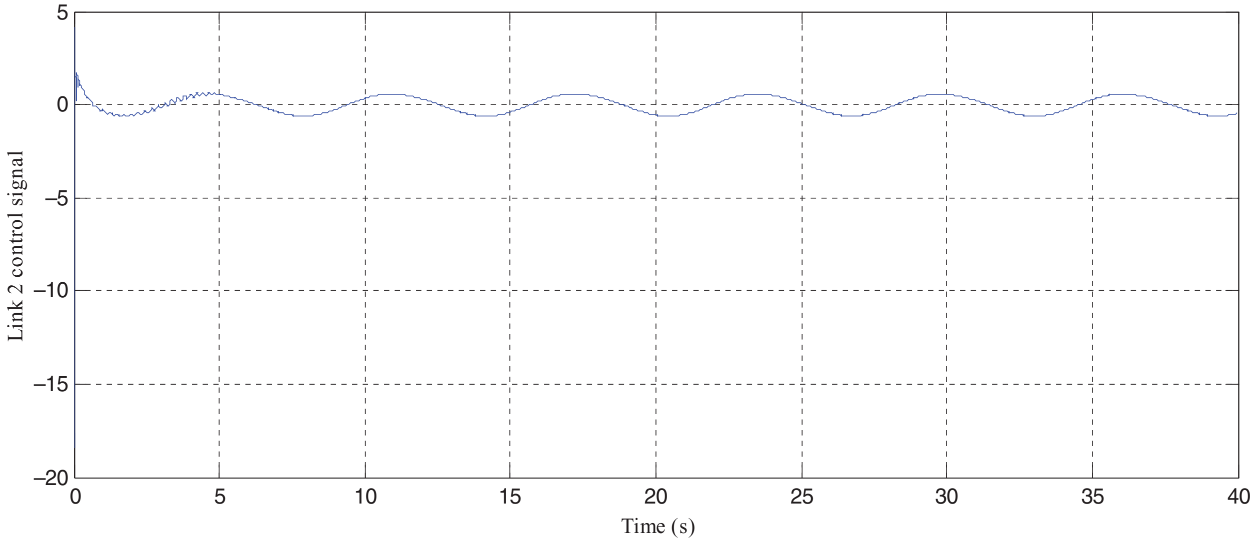

Fig. 12. (Colour online) Second link control signal with $\rho = 0.1$.

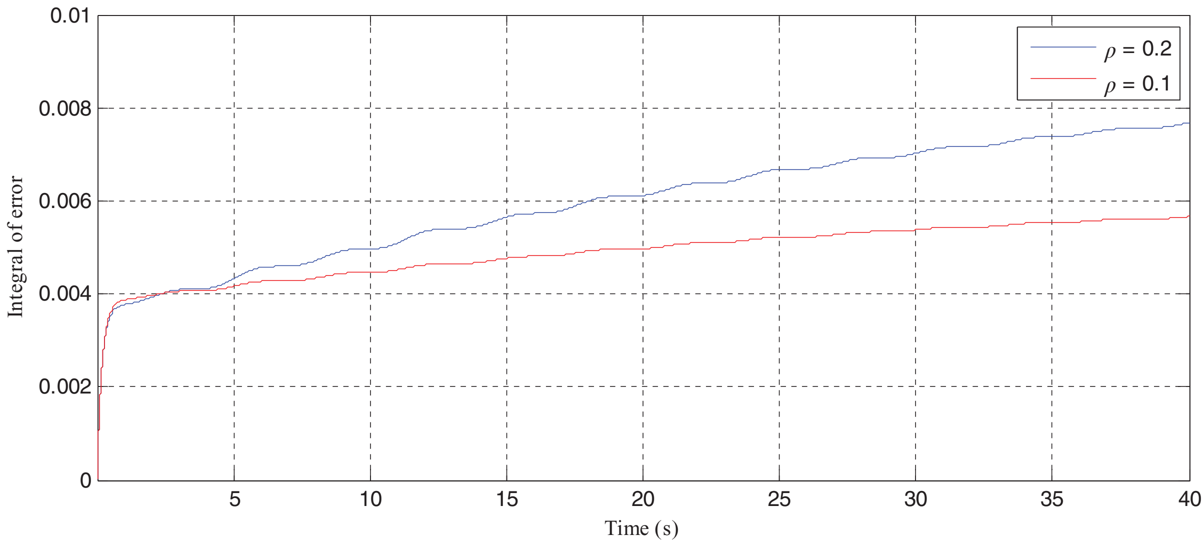

Fig. 13. (Colour online) Integral of error $\int_0^t \|e(t)\|^2\,dt$ with $\rho = 0.2$ and $\rho = 0.1$.

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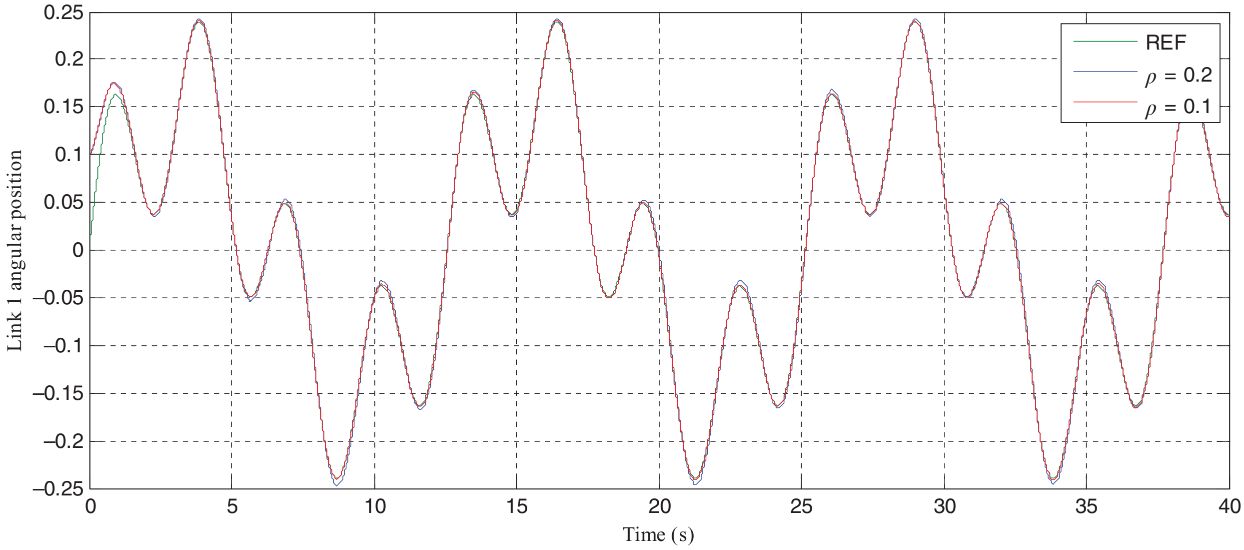

Fig. 14. (Colour online) First link position tracking with $\rho = 0.2$ and $\rho = 0.1$.

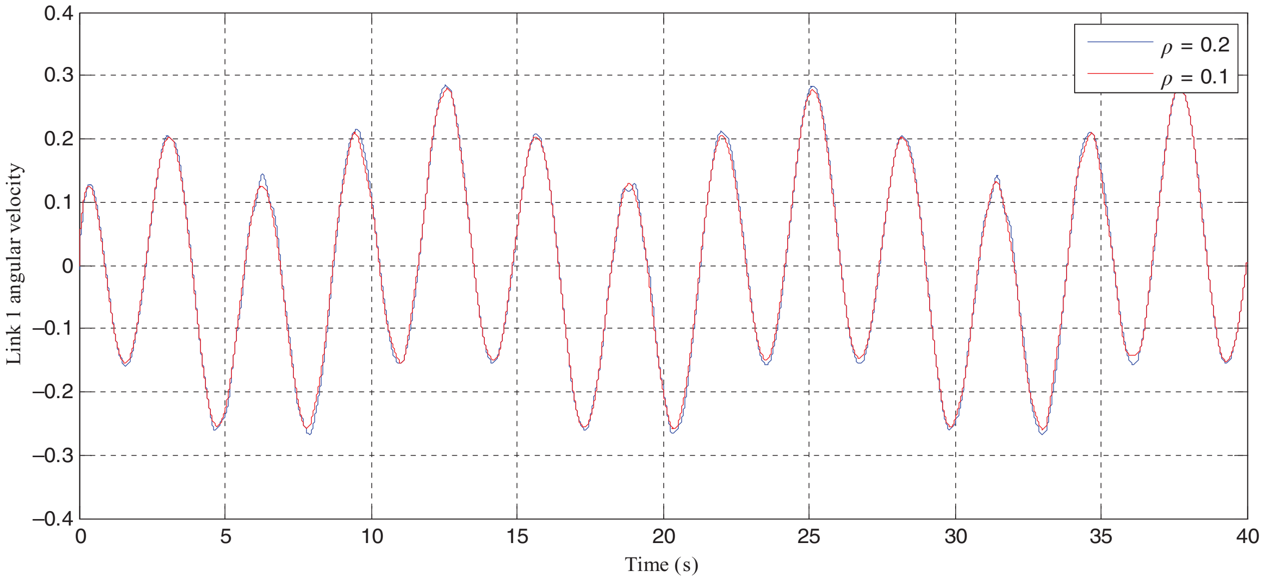

Fig. 15. (Colour online) First link angular velocity with $\rho = 0.2$ and $\rho = 0.1$.

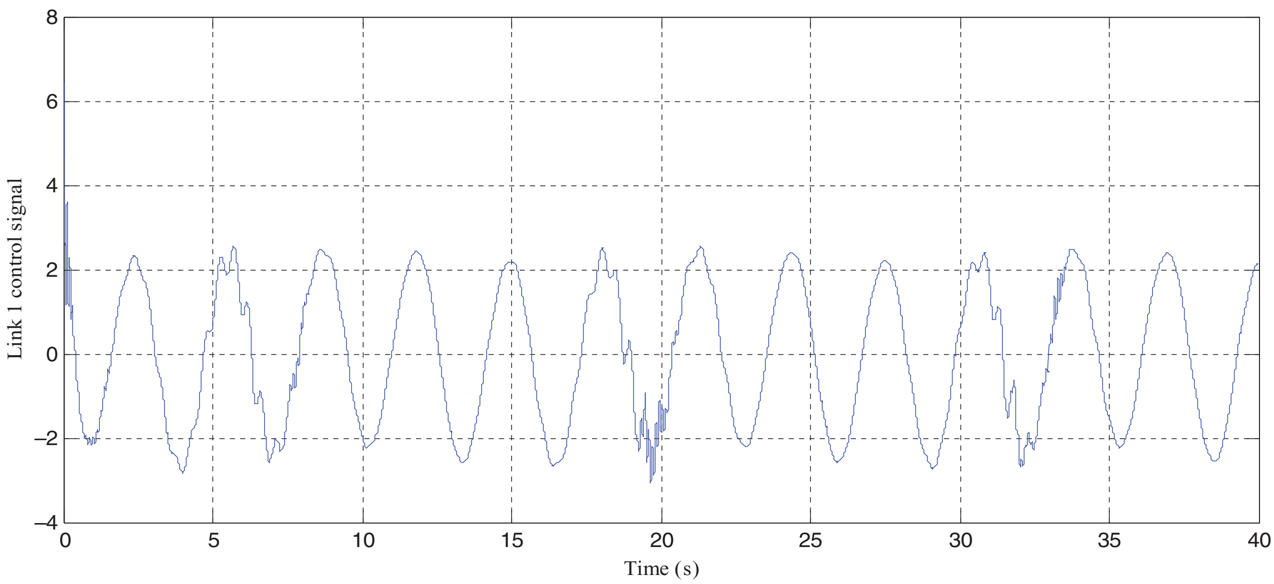

Fig. 16. (Colour online) First link control signal with $\rho = 0.2$.

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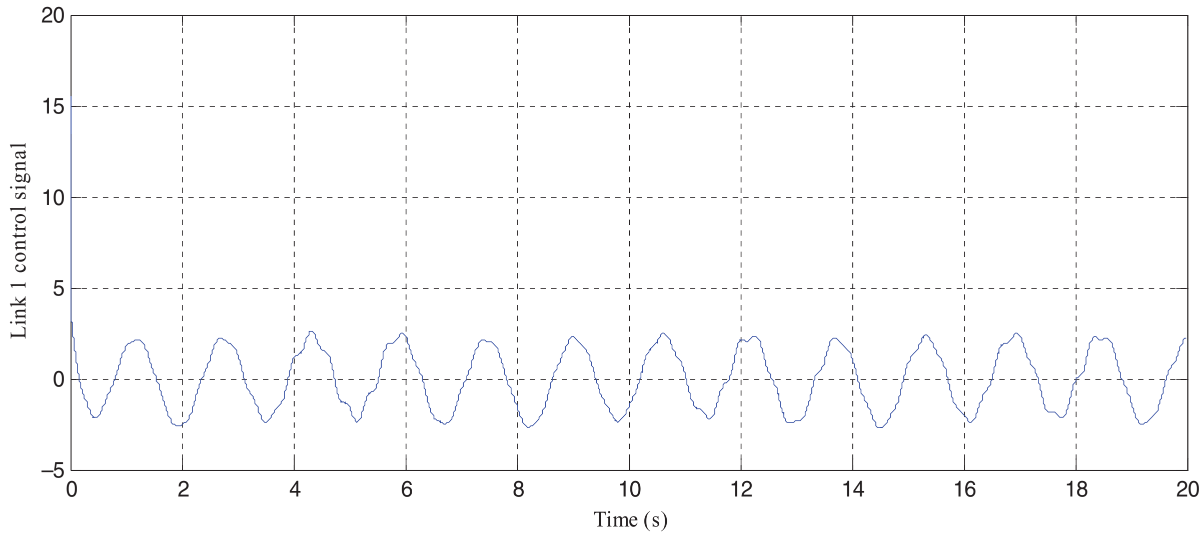

Fig. 17. (Colour online) First link control signal with $\rho = 0.1$.

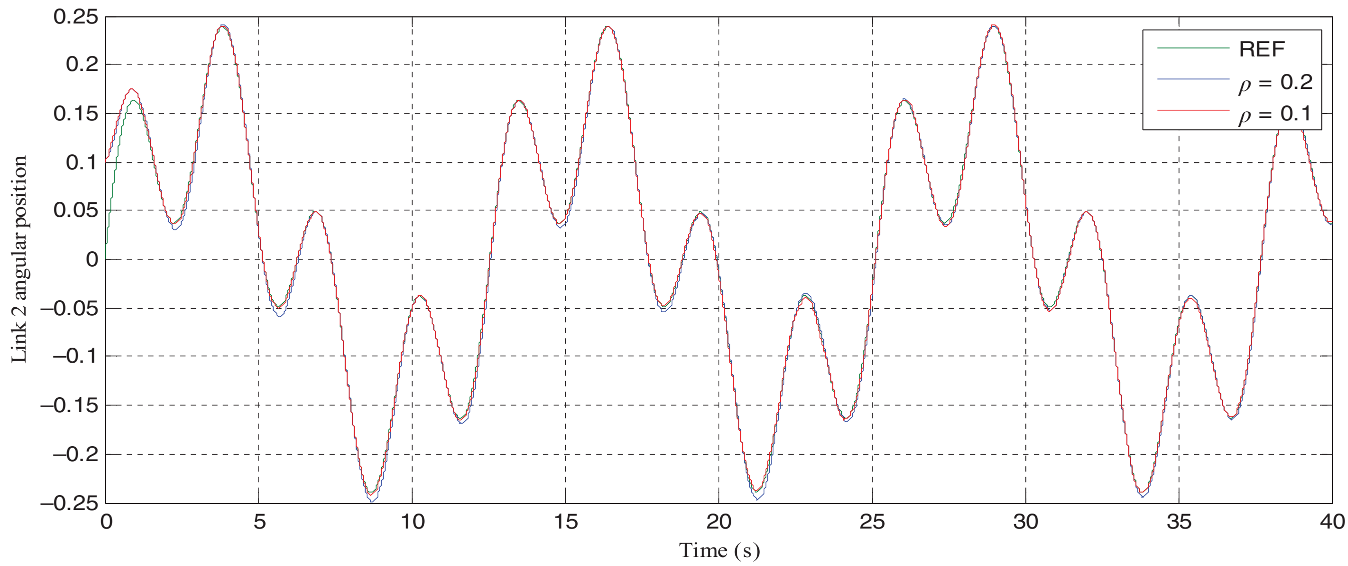

Fig. 18. (Colour online) Second link position tracking with $\rho = 0.2$ and $\rho = 0.1$.

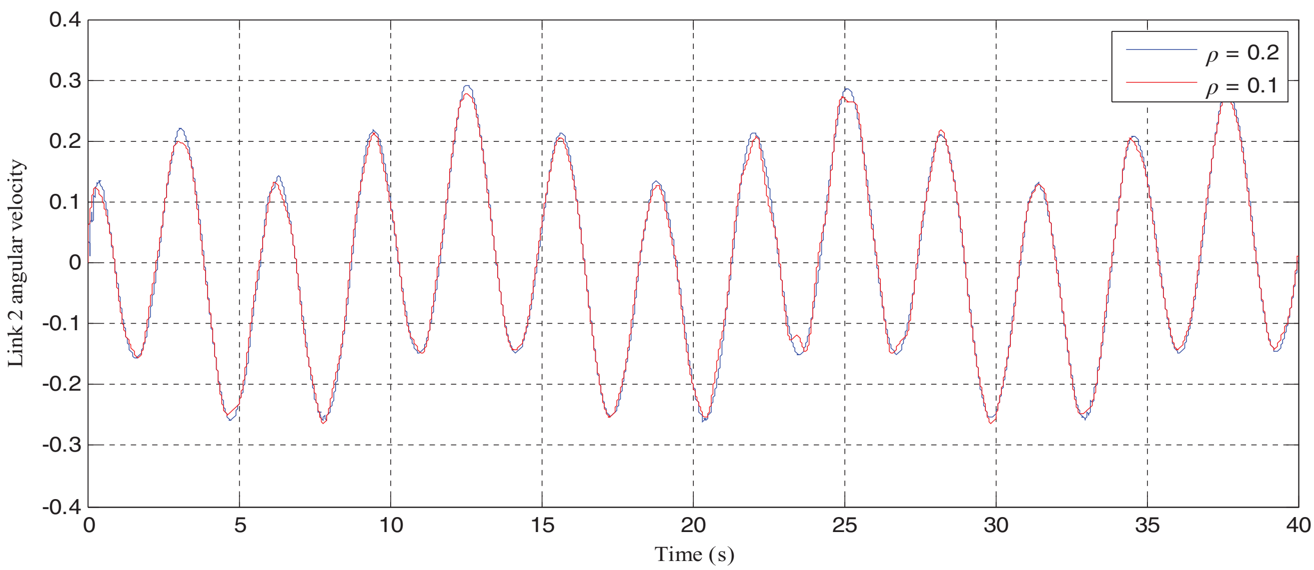

Fig. 19. (Colour online) Second link angular velocity with $\rho = 0.2$ and $\rho = 0.1$.

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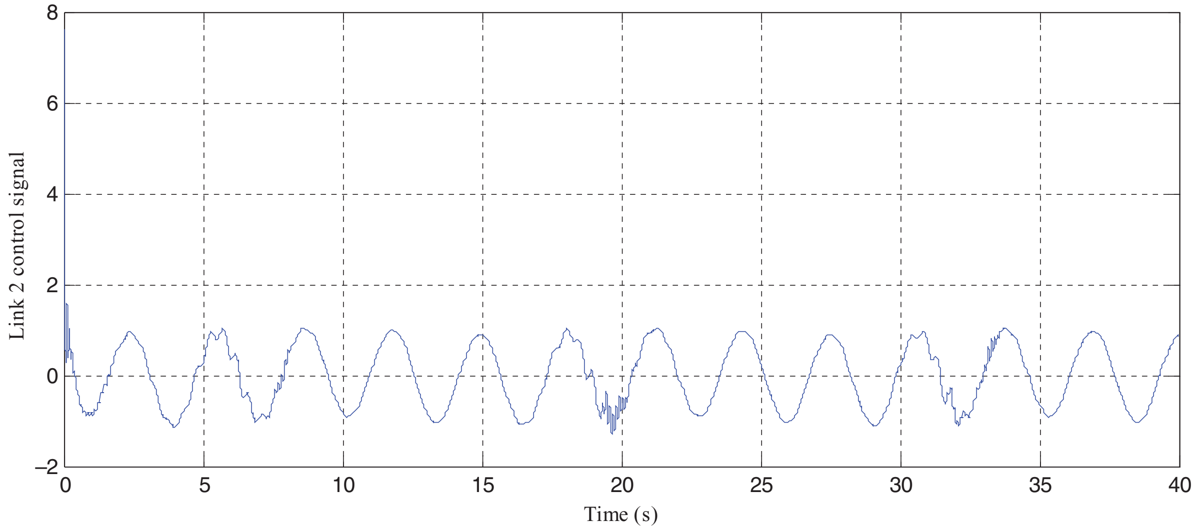

Fig. 20. (Colour online) Second link control signal with $\rho = 0.2$.

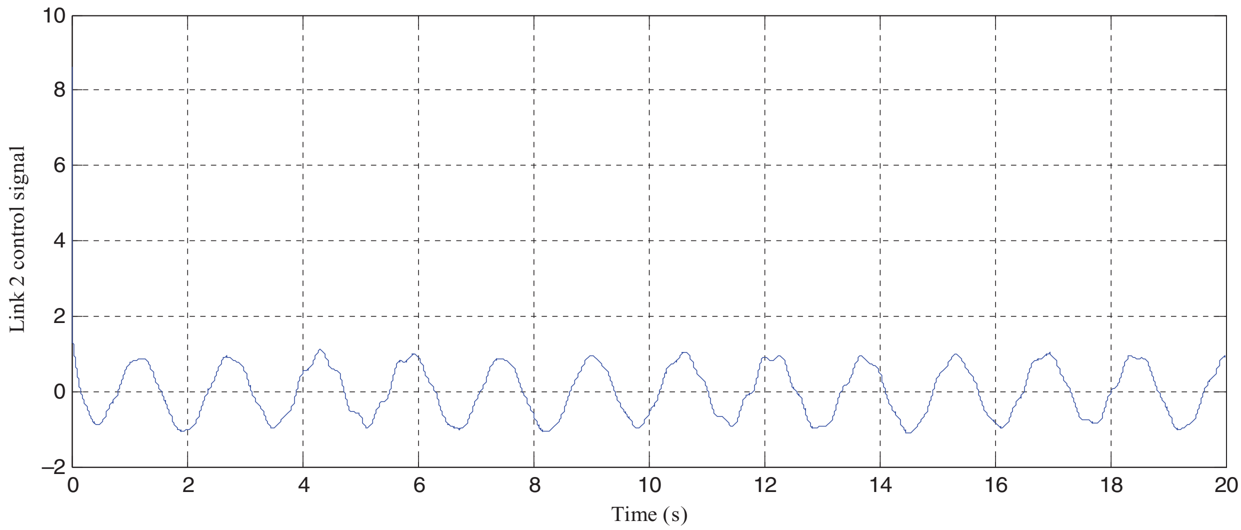

Fig. 21. (Colour online) Second link control signal with $\rho = 0.1$.

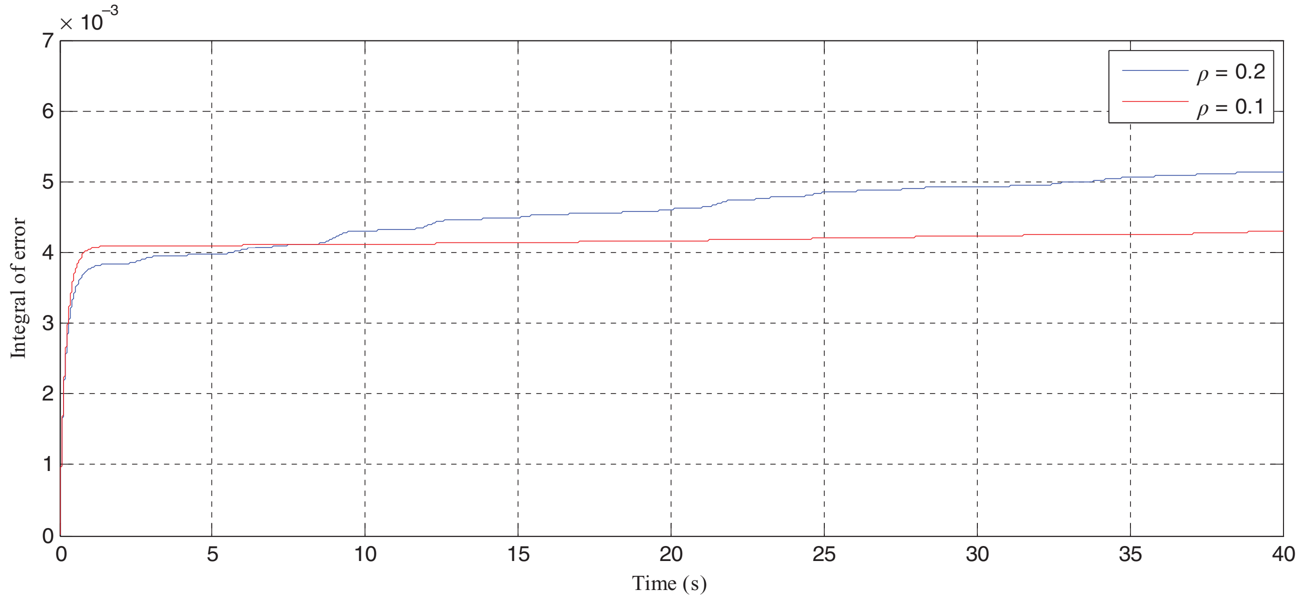

Fig. 22. (Colour online) Integral of error $\int_0^t \|e(t)\|^2\, dt$ with $\rho = 0.2$ and $\rho = 0.1$.

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$y_{m1} = y_{m2} = 0.2\sin(t)$, and Figs. 14–22 for $y_{m1} = y_{m2} = 0.15\sin(0.5t) + 0.1\sin(2t)$. It can be observed from the simulation results that the proposed DFLS-based adaptive control scheme is able to achieve excellent position tracking performances for a two-link robotics manipulator even with unknown model dynamics, presence of external disturbances, and whatever be the reference trajectory. Furthermore, it is clear that smaller the values of the attenuation level ρ, smaller will be the tracking errors, as shown in Figs. 13 and 22. It is also clear that smaller the attenuation level, better the attenuation in the shuttering of control signals. However, these advantages come to the extent of the starting value of the control signal. The starting control signal ($t = 0$) is higher in this case.

6. Conclusion

In this paper, an adaptive fuzzy control scheme is developed based on DFLS for a class of uncertain nonlinear MIMO systems. DFLS is used to identify the unknown nonlinear system as a whole, and an adaptive fuzzy controller is developed from the identified model. The fuzzy control law is robustified by an H^∞ compensator to attenuate the effect of disturbances, model uncertainties, and fuzzy approximation errors. The design of the control scheme is developed by the Lyapunov synthesis approach to guarantee the stability of the overall closed-loop system. It has been shown that the proposed approach guarantees that all signals in the closed-loop system are uniformly bounded, and tracking errors falls to a small neighborhood of the origin. The proposed scheme has been successfully applied to position tracking of a two-link robotics manipulator. Simulation results show the effectiveness of the proposed scheme.

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