J. Plasma Physics (2013), *vol.* 79, *part* 6, *pp.* 995–998. © Cambridge University Press 2013
doi:10.1017/S0022377813000779

An electromagnetic gamma-ray free electron laser*

BENGT ELIASSON[1] and CHUAN SHENG LIU[2]

[1]Institute for Theoretical Physics, Ruhr University Bochum, D-44780 Bochum, Germany
(beliass@yahoo.se)
[2]Department of Physics, University of Maryland, College Park, MD 20742, USA
(csliu@umd.edu)

(Received 11 June 2013; revised 16 June 2013; accepted 17 June 2013; first published online 9 August 2013)

Abstract. We present a theoretical model for the generation of coherent gamma rays by a free electron laser, where a high-energy electron beam interacts with an electromagnetic wiggler. By replacing the static undulator with a 1-μm laser wiggler, the resulting radiation would go from X-rays currently observed in experiments, to gamma rays. Coherent light in the gamma-ray range would have wide-ranging applications in the probing of matter on sub-atomic scales.

1. Introduction

Free electron lasers now produce coherent X-ray laser light in the Ångstrom range (Hand 2009; Ishikawa et al. 2012), which can be used to probe matter on atomic and molecular scales. On the other hand, narrow-band X-rays are used to investigate collective oscillations in solid density matter (Glenzer et al. 2007; Glenzer and Redmer 2009; Neumayer et al. 2010). There also exist theoretical suggestions for producing coherent light in the gamma-ray range, but which yet has to be realized in experiments (Baldwin and Solem 1997; Rivlin 2007; Rivlin and Zadernovsky 2010; Tkalya 2011; Son and Moon 2012). A functioning gamma-ray laser would have wide-ranging applications in experiments where matter could be investigated on sub-atomic scales. In the free electron laser (Madey 1971; Robertson and Sprangle 1985), an energetic electron beam is injected into an undulator consisting of a periodic array of magnets, resulting in micro-bunching of the electrons and the generation of coherent laser light. In the frame of the electron beam, the electrons see an electromagnetic wave propagating in the opposite direction, giving rise to relativistic parametric instabilities (e.g. Stenflo 1976; Shukla et al. 1986). For static wigglers, the wavelength of the laser radiation is decreased by a factor $2\gamma_0^2$ compared with the wiggler wavelength, where γ_0 is the relativistic gamma factor of the electrons. Hence, to convert a wiggler wavelength of a few centimeters to X-rays with wavelengths in the Ångstrom range, the beam electrons need to have a gamma factor of the order of 10^4 and a kinetic energy of several GeV. Also, the wiggler has to be several hundreds of meters long so that the micro-bunching of electrons can have time to take place. To further decrease the radiation wavelength to the gamma-ray range of about 0.024 Å would require a significant increase of the energy of the electron beam and of the size of the wiggler, with a corresponding increase of the costs to build the experiment. An attractive idea to shrink the size of the experiment is to replace the static wiggler by an electromagnetic wave (e.g. Bonifacio et al. 2007; Sprangle et al. 2009). For the electromagnetic wiggler, the radiation wavelength is decreased by a factor $4\gamma_0^2$ compared with the wiggler wavelength. Hence, using a 1-μm laser as a wiggler would require an electron beam with $\gamma_0 \approx 300$ to achieve a radiation wavelength comparable to the Compton wavelength 0.024 Å. The short wavelengths of the laser radiation and beam oscillations require a relativistic quantum mechanical treatment of the electron beam. While the original treatment by Madey (1971) of the free electron laser was quantum mechanical, the final result was classical. Models for quantum free electron lasers have been developed and discussed by Preparata (1988), Bonifacio et al. (2005b), Piovella et al. (2008), Serbeto et al. (2009), Eliasson and Shukla (2012a,b) and others. We shall use a collective Klein–Gordon equation to derive a dynamic model for the electromagnetic gamma-ray free electron laser. In our model, we assume that the wave function ψ represents an ensemble of electrons, so that the resulting charge and current densities act as sources (Takabayasi 1953; Kuzelev 2011; Eliasson and Shukla 2011, 2012a,b) for the self-consistent electromagnetic fields. The governing equations will be used to derive a formula for the growth rate of the backscattering instability for the electron beam and the corresponding gain length of the laser.

2. Mathematical model

For the collective interactions between electromagnetic waves and the electron beam, we will use a collective Klein–Gordon model (Takabayasi 1953), in which a single wave function represents an ensemble of relativistic electrons. The electromagnetic wave is treated classically and the electrons quantum mechanically. This cold plasma model incorporates the effects of special relativity and quantum diffraction (or quantum recoil) on an equal footing, but neglects electron spin, electron degeneracy and kinetic effects in general. Kinetic effects

* This paper is devoted to the memory of Professor Padma Kant Shukla.

https://doi.org/10.1017/S0022377813000779 Published online by Cambridge University Press

can be included using multi-stream (Haas et al. 2012) or Wigner (e.g. Piovella et al. 2008; Mendonça 2011) models. The Klein–Gordon equation in the presence of the electrodynamic fields reads

$$\mathscr{W}^2\psi - c^2\mathscr{P}^2\psi - m_e^2c^4\psi = 0, \tag{2.1}$$

where the energy and momentum operators are $\mathscr{W} = i\hbar\partial/\partial t + e\phi$ and $\mathscr{P} = -i\hbar\nabla + e\mathbf{A}$, respectively. Here, $\hbar$ is Planck's constant divided by 2π, e is the magnitude of the electron charge, m_e is the electron rest mass, and c is the speed of light in vacuum. The electrodynamic potentials are obtained self-consistently from the Maxwell equations

$$\frac{\partial^2\mathbf{A}}{\partial t^2} + c^2\nabla\times(\nabla\times\mathbf{A}) + \nabla\frac{\partial\phi}{\partial t} = \mu_0c^2\mathbf{j}_e \tag{2.2}$$

and

$$\nabla^2\phi + \nabla\cdot\frac{\partial\mathbf{A}}{\partial t} = -\frac{1}{\varepsilon_0}(\rho_e+\rho_i), \tag{2.3}$$

where μ_0 is the magnetic permeability, $\varepsilon_0 = 1/\mu_0c^2$ is the electric permittivity in vacuum, and $\rho_i = en_0$ is a neutralizing positive charge density due to ions, where n_0 is the equilibrium electron number density. The electric charge and current densities of the electrons are $\rho_e = -e[\psi^*\mathscr{W}\psi + \psi(\mathscr{W}\psi)^*]/2m_ec^2$ and $\mathbf{j}_e = -e[\psi^*\mathscr{P}\psi + \psi(\mathscr{P}\psi)^*]/2m_e$, respectively. They obey the continuity equation $\partial\rho_e/\partial t + \nabla\cdot\mathbf{j}_e = 0$.

The interaction between the large-amplitude electromagnetic wave and plasma leads to collective oscillations and parametric instabilities, where the electromagnetic wave is scattered against plasma oscillations. The calculations of the growth rates of the instabilities are significantly simplified if they are carried out in the beam frame, where the plasma is at rest, and the result Lorentz transformed to the laboratory frame. Following Eliasson and Shukla (2012a,b), we assume that the beam is propagating in the negative z direction, in the opposite direction of the laser wiggler beam. For simplicity, we consider a circularly polarized laser wiggler of the form $\mathbf{A}_0 = (1/2)\widehat{\mathbf{A}}_0\exp(-i\omega_0t + i\mathbf{k}_0\cdot\mathbf{r})+$ complex conjugate, with $\widehat{\mathbf{A}}_0 = (\widehat{\mathbf{x}} + i\widehat{\mathbf{y}})\widehat{A}_0$, where ω_0 is the wave frequency and $\mathbf{k}_0 = k_0\widehat{\mathbf{z}}$ is the wave vector, and $\widehat{\mathbf{x}}$, $\widehat{\mathbf{y}}$ and $\widehat{\mathbf{z}}$ are the unit vectors in the x, y and z directions, respectively. Due to circular polarization, the oscillatory parts in the terms proportional to A^2 in the Klein–Gordon equation vanish. The growth rate for four-wave interactions, where the pump electromagnetic wave decays into two electromagnetic sidebands and one electrostatic wave, is given by (Eliasson and Shukla 2011)

$$1 - \frac{(\omega_{pe}')^2}{4\gamma_A^3m_e^2c^2}\frac{D_A'(\Omega',\mathbf{K}')}{D_L'(\Omega',\mathbf{K}')}\sum_{+,-}\frac{e^2|\mathbf{k}_\pm'\times\widehat{\mathbf{A}}_0'|^2}{(k_\pm')^2D_A'(\omega_\pm',\mathbf{k}_\pm')} = 0, \tag{2.4}$$

where the electron plasma oscillations in the presence of the EM field are represented by

$$D_L'(\Omega',\mathbf{K}') = \frac{(\omega_{pe}')^2}{\gamma_A} - (\Omega')^2 + \frac{[c^2(K')^2-(\Omega')^2]}{4\gamma_A^2\omega_C^2}D_A'(\Omega',\mathbf{K}'), \tag{2.5}$$

with $\omega_C = m_ec^2/\hbar$. Here, Ω' and $\mathbf{K}'$ are the frequency and wave vector of the plasma oscillations, respectively, $\gamma_A = (1+a_0^2)^{1/2}$ is the relativistic gamma factor due to the large amplitude EM field, $a_0 = e|\widehat{A}_0'|/m_ec$ is the normalized amplitude of the EM wave (corresponding to the wiggler parameter for a static wiggler), and $\omega_{pe}' = (e^2n_0'/\varepsilon_0m_e)^{1/2}$ is the plasma frequency with n_0' being the electron number density in the beam frame. The dispersion relation for the beam oscillations in the presence of a large amplitude EM wave is given by $D_L'(\Omega',\mathbf{K}') = 0$. The EM sidebands are governed by

$$D_A'(\omega_\pm',\mathbf{k}_\pm') = c^2(k_\pm')^2 - (\omega_\pm')^2 + (\omega_{pe}')^2/\gamma_A, \tag{2.6}$$

where $\omega_\pm' = \omega_0' \pm \Omega'$ and $\mathbf{k}_\pm' = \mathbf{k}_0' \pm \mathbf{K}'$, and ω_0' and k_0' are related through the nonlinear dispersion relation $(\omega_0')^2 = c^2(k_0')^2 + (\omega_{pe}')^2/\gamma_A$. We also denoted $D_A'(\Omega,\mathbf{K}) = c^2(K')^2 - (\Omega')^2 + (\omega_{pe}')^2/\gamma_A$.

To move from the beam frame to the laboratory frame, the time and space variables are Lorentz transformed as $t' = \gamma_0(t - v_0z/c^2)$, $x' = x$, $y' = y$ and $z' = \gamma_0(z - v_0t)$, where $\mathbf{v}_0 = v_0\widehat{\mathbf{z}}$ is the beam velocity and $\gamma_0 = 1/\sqrt{1-v_0^2/c^2}$ is the gamma factor due to the relativistic beam speed. The corresponding frequency and wavenumber transformations, found from the relations $\omega't' = \omega t$ and $\mathbf{k}'\cdot\mathbf{r}' = \mathbf{k}\cdot\mathbf{r}$, are $\omega' = \gamma_0(\omega - v_0k_z)$, $k_x' = k_x$, $k_y' = k_y$, and $k_z' = \gamma_0(k_z - v_0\omega/c^2)$. They apply to the frequency and wave vector pairs $(\Omega,\mathbf{K})$, $(\omega_0,\mathbf{k}_0)$ and $(\omega_\pm, \mathbf{k}_\pm)$. Hence, expressions such as $(\omega')^2 - c^2(k')^2 = \omega^2 - c^2k^2$ are Lorentz invariant. Due to spatial contraction, the number density is transformed as $n_0' = n_0/\gamma_0$ and the plasma frequency as $\omega_{pe}' = \omega_{pe}/\sqrt{\gamma_0}$. The transverse vector potential of the laser wiggler is unaffected by the Lorentz transformation, hence $\widehat{\mathbf{A}}_0' = \widehat{\mathbf{A}}_0$. The total gamma factor is $\gamma = \gamma_A\gamma_0 = (1 + p_0^2/m_e^2c^2 + a_0^2)^{1/2}$, where $\mathbf{p}_0 = \gamma m_ev_0\widehat{\mathbf{z}}$ is the relativistic electron momentum along the z axis. This yields $D_L'(\Omega',\mathbf{K}') = \gamma_0^2D_L(\Omega,\mathbf{K})$, with

$$D_L(\Omega,\mathbf{K}) = \frac{\omega_{pe}^2\gamma_A^2}{\gamma^3} - (\Omega - v_0K_z)^2 + \frac{(c^2K^2-\Omega^2)}{4\gamma^2\omega_C^2}D_A(\Omega,\mathbf{K}), \tag{2.7}$$

$D_A'(\omega_\pm',\mathbf{k}_\pm') = D_A(\omega_\pm,\mathbf{k}_\pm) \equiv c^2k_\pm^2 - \omega_\pm^2 + \omega_{pe}^2/\gamma$, and $D_A'(\Omega',\mathbf{K}') = D_A(\Omega,\mathbf{K})$. In the laboratory frame, (2.4) is of the form

$$1 - \frac{\omega_{pe}^2}{4\gamma^3m_e^2c^2}\frac{D_A(\Omega,\mathbf{K})}{D_L(\Omega,\mathbf{K})}\sum_{+,-}\frac{e^2|\mathbf{k}_\pm'\times\widehat{\mathbf{A}}_0|^2}{(k_\pm')^2D_A(\omega_\pm,\mathbf{k}_\pm)} = 0. \tag{2.8}$$

Using $\mathbf{K} = K_x\widehat{\mathbf{x}} + K_y\widehat{\mathbf{y}} + K_z\widehat{\mathbf{z}}$, we have $K^2 = K_z^2 + K_\perp^2$ with $K_\perp^2 = K_x^2 + K_y^2$, so that $|\mathbf{k}_\pm'\times\widehat{\mathbf{A}}_0|^2 = \{2\gamma_0^2[k_0 \pm K_z + (v_0/c^2)(\omega_0\pm\Omega)]^2 + K_\perp^2\}|\widehat{A}_0|^2$, and $(k_\pm')^2 = \gamma_0^2[k_0 \pm K_z + (v_0/c^2)(\omega_0\pm\Omega)]^2 + K_\perp^2$.

For the resonant backscattering instability, we have $|D_A(\omega_+,\mathbf{k}_+)| \gg |D_A(\omega_-,\mathbf{k}_-)|$. Denoting $\omega_- = \omega$ and $\mathbf{k}_- =$

https://doi.org/10.1017/S0022377813000779 Published online by Cambridge University Press

$\mathbf{k}$, (2.8) can then be written

$$D_L(\Omega,\mathbf{K})D_A(\omega,\mathbf{k}) = \frac{\omega_{pe}^2 D_A(\Omega,\mathbf{K})}{4\gamma^3}\frac{e^2|\mathbf{k}'_- \times \widehat{\mathbf{A}}_0|^2}{m_e^2c^2(k'_-)^2}. \quad (2.9)$$

In order to find approximate solutions of (2.9) for the instability, we set $\Omega = \widetilde{\Omega} + i\Gamma$ and $\mathbf{K} = \widetilde{\mathbf{K}}$, where $\widetilde{\Omega}$ and $\widetilde{\mathbf{K}}$ simultaneously solve $D_L(\widetilde{\Omega},\widetilde{\mathbf{K}}) = 0$ and $D_A(\omega_0 - \widetilde{\Omega},\mathbf{k}_0 - \widetilde{\mathbf{K}}) = 0$, and where Γ is the growth rate. We then have

$$D_A(\omega,\mathbf{k}) = 2i\Gamma(\omega_0 - \widetilde{\Omega}) + \Gamma^2 \approx 2i\Gamma(\omega_0 - \widetilde{\Omega}) \quad (2.10)$$

and

$$\begin{aligned} D_L(\Omega,\mathbf{K}) &= \frac{(2i\Gamma\widetilde{\Omega} - \Gamma^2)}{4\gamma^2\omega_C^2} \\ &\quad \times \left(2\widetilde{\Omega}^2 - 2c^2\widetilde{K}^2 - \frac{\omega_{pe}^2}{\gamma} + 2i\Gamma\widetilde{\Omega} - \Gamma^2\right) \\ &\quad - 2i\Gamma(\Omega - v_0K_z) - \Gamma^2 \\ &\approx \frac{i\Gamma\widetilde{\Omega}}{2\gamma^2\omega_C^2}\left(2\widetilde{\Omega}^2 - 2c^2\widetilde{K}^2 - \frac{\omega_{pe}^2}{\gamma}\right) \\ &\quad - 2i\Gamma(\widetilde{\Omega} - v_0\widetilde{K}_z). \end{aligned} \quad (2.11)$$

We assume here that the terms proportional to Γ^2 are negligible in the expression for D_L due to the quantum recoil effects, and that the laser therefore operates in the Raman regime. In the opposite case, if $D_L \approx -\Gamma^2$, we have a reactive instability and the laser operates in the Compton regime.

To solve approximately the resonance conditions $D_A(\omega_0,\mathbf{k}_0) = 0$, $D_A(\omega,\mathbf{k}) = 0$ and $D_L(\widetilde{\Omega},\widetilde{\mathbf{K}}) = 0$ using $\widetilde{\Omega} = \omega_0 - \omega$ and $\widetilde{\mathbf{K}} = \mathbf{k}_0 - \mathbf{k}$, we assume that all frequencies are much larger than ω_{pe}/γ, i.e. the plasma is strongly underdense. We then have

$$\omega_0 = ck_0, \quad (2.12)$$

$$\omega = ck, \quad (2.13)$$

and

$$\widetilde{\Omega} = -\gamma\omega_C + \sqrt{(\gamma\omega_C + v_0\widetilde{K}_z)^2 + c^2(\widetilde{K}_\perp^2 + \widetilde{K}_z^2/\gamma_0^2)}. \quad (2.14)$$

Eliminating ω_0, ω, $\widetilde{\Omega}$ and $\widetilde{\mathbf{K}}$ in (2.14), we obtain the resonance condition

$$\begin{aligned} c(k_0 - k) &= -\gamma\omega_C \\ &\quad + \sqrt{[\gamma\omega_C + v_0(k_0 - k_z)]^2 + c^2[k_\perp^2 + (k_0 - k_z)^2/\gamma_0^2]} \end{aligned} \quad (2.15)$$

for the scattering of a relativistically strong electromagnetic wave off electrons. To put the resonance condition in explicit form, we choose a coordinate system such that $\mathbf{k}_0 = k_0\widehat{\mathbf{z}}$, $k_z = k\cos\theta$ and $\mathbf{k}_\perp = k_x\widehat{\mathbf{x}} + k_y\widehat{\mathbf{y}}$, $k_\perp = k\sin\theta$, and θ is the angle between $\mathbf{k}$ and $\mathbf{k}_0 = k_0\widehat{\mathbf{z}}$ ($\theta > \pi/2$ corresponds to backscattered light). This gives the resonance condition

$$k = k_0R, \quad (2.16)$$

where we denote

$$R = \frac{1 - v_0/c}{1 - (v_0/c)\cos\theta + (k_0/\gamma k_C)(1 - \cos\theta)} \quad (2.17)$$

and $k_C = \omega_C/c$. For backscattered light with $\cos\theta = -1$ and $v_0 \approx -c$, we have $R \approx 4\gamma_0^2 = 4\gamma^2/\gamma_A^2$ for $k_0/k_C \ll \gamma/4\gamma_0^2$, and $R \approx \gamma k_C/k_0$, giving the wavenumber $k \approx \gamma k_C = \gamma m_e c/\hbar$, for $k_0/k_C > \gamma/4\gamma_0^2$. Expressed in terms of the wavelengths $\lambda_0 = 2\pi/k_0$ and $\lambda = 2\pi/k$, (2.16) is equivalent to

$$\left(1 - \frac{v_0}{c}\right)\lambda - \left(1 - \frac{v_0}{c}\cos\theta\right)\lambda_0 = \frac{2\pi c}{\gamma\omega_C}(1 - \cos\theta), \quad (2.18)$$

which recovers Compton's (1923) result for the scattering of a photon off an electron at rest in the limit $v_0 = 0$ and $\gamma = 1$.

The resonance condition (2.16) can be used to express the other quantities as functions of k_0 and θ via the relations $\omega = ck = ck_0R$, $\widetilde{\Omega} = \omega_0 - ck = ck_0(1 - R)$, $\widetilde{K}_z = k_0 - k\cos\theta = k_0(1 - R\cos\theta)$ and $\widetilde{K}_\perp = -k\sin\theta = -k_0R\sin\theta$, so that $\widetilde{K}^2 = \widetilde{K}_z^2 + \widetilde{K}_\perp^2 = k_0^2(1 + R^2 - 2R\cos\theta)$. This gives $D_A(\omega,\mathbf{k}) \approx 2i\Gamma ck_0R$, $D_L(\Omega,\mathbf{K}) \approx -2ick_0\Gamma\{(1 - R)[(k_0/\gamma k_C)^2R(1 - \cos\theta) + 1 + \omega_{pe}^2/(4\gamma^3\omega_C^2)] - (v_0/c)(1 - R\cos\theta)\}$ and $D_A(\Omega,\mathbf{K}) \approx 2c^2k_0^2R(1 - \cos\theta) + \omega_{pe}^2/\gamma$. Furthermore, we evaluate $|\mathbf{A}_0 \times \mathbf{k}|^2/k^2 = 2|A_0|^2$ for $v_0 \approx -c$ and $|\sin(\theta/2)| \gg 1/\gamma_0$. Inserting these expressions into (2.9) gives, in the limit $\omega_{pe}^2/\gamma \ll c^2k_0^2R(1 - \cos\theta)$ and $v_0 \approx -c$, the growth rate

$$\begin{aligned} \frac{\Gamma}{\omega_{pe}} &= a_0 \Bigg/ 2^{1/2}\gamma^{3/2}R^{1/2}\left\{\left(\frac{v_0}{c} + \frac{k_0}{\gamma k_C}\right)\right. \\ &\quad \left.\times\left[\frac{k_0^2}{\gamma^2k_C^2}R(1 - \cos\theta) + 1\right] - \frac{v_0}{c}\left(1 + \frac{k_0}{\gamma k_C}\right)\right\}^{1/2}. \end{aligned} \quad (2.19)$$

3. Numerical examples and discussion

Figure 1 shows resonant radiation wave vectors and corresponding growth rates for different gamma factors of the electron beam and using a laser wiggler with a wavelength of 1 μm ($k_0 = 6.28 \times 10^6\,\mathrm{m}^{-1}$) and amplitude $a_0 = 1$. As an example, we will use possible electron densities for free electron lasers (Piovella et al. 2008), $n_0 = 2.2\times 10^{22}\,\mathrm{m}^{-1}$, corresponding to a plasma frequency of $\omega_{pe} = 8.4 \times 10^{12}\,\mathrm{s}^{-1}$. In general, the backscattered radiation have wave vectors with much larger parallel than perpendicular components. Using an electron beam with a gamma factor of about 300, the radiation wavelength for backscattered light (i.e. for large negative k_z in Fig. 1a) approaches the Compton wavenumber $k_C \approx 2.6\times 10^{12}\,\mathrm{m}^{-1}$ corresponding to a photon energy of about 0.5 MeV. The corresponding growth rate for backscattered light (cf. Fig. 1b) is $\Gamma \approx 10a_0\omega_{pe}/\gamma^{3/2} \approx 10^{10}\,\mathrm{s}^{-1}$. The gain length is $L_g = c/\Gamma \approx 0.035$ m and an expected interaction length of the order of $10L_g = 0.35$ m. Increasing the gamma factor to about 10^4 (corresponding to an electron energy of about 2.5 GeV), which is used

https://doi.org/10.1017/S0022377813000779 Published online by Cambridge University Press

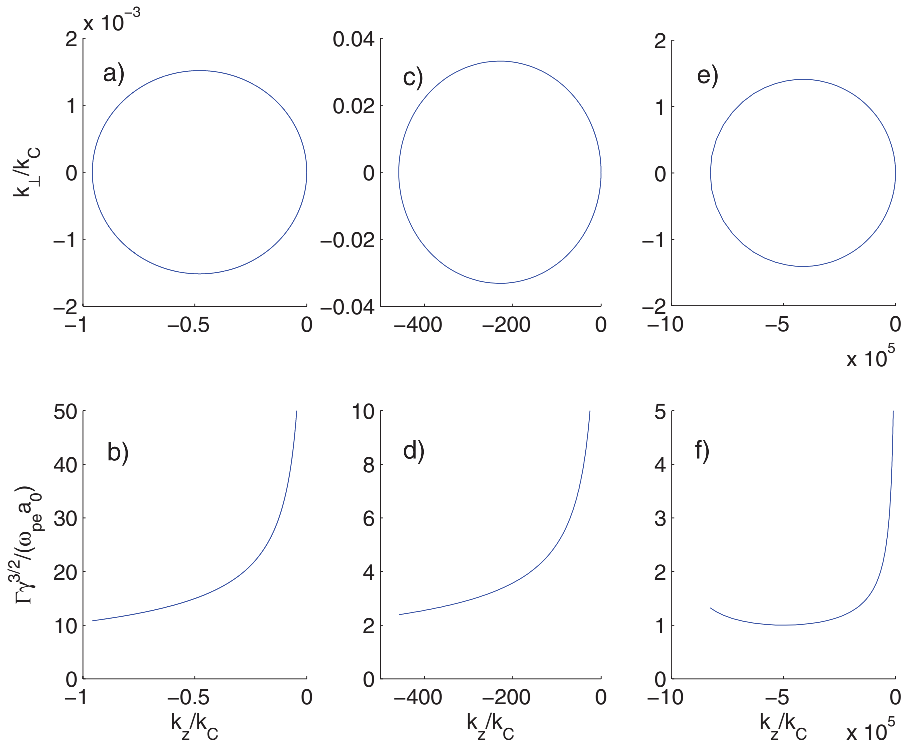

Figure 1. (Colour online) The resonant radiation wave vectors (top row) and the corresponding growth rates (bottom row) for a laser wiggler with 1-μm wavelength ($k_0 = 6.28 \times 10^6\,\mathrm{m}^{-1}$) and amplitude $a_0 = 1$, and for different gamma factors of the electron beam: $\gamma_0 = 320$ (left column), $\gamma_0 = 0.71 \times 10^4$ (middle column) and $\gamma_0 = 0.71 \times 10^6$ (right column).

in today's free electron lasers with static wigglers, the radiation goes into the hard gamma-ray regime with photon energies of the order of 100 MeV (cf. Fig. 1c). The growth rate for backscattered light, seen in Fig. 1(d), is $\Gamma \approx 2a_0\omega_{pe}/\gamma^{3/2} = 1.7 \times 10^7\,\mathrm{s}^{-1}$. In this case, the gain length would be $L_g \approx 20\,\mathrm{m}$ and an expected interaction length of the order of 200 m. This could potentially be realized in experiments by replacing the static wigglers with electromagnetic wigglers. Finally, if the electron beam would have gamma factors of about 10^6 (corresponding to an electron energy of about 250 GeV), the radiation photons would have energies of about 200 GeV (cf. Fig. 1e). In this case, since the photons and relativistic electrons have energies of the same order, one has to take into account Compton shifts in the radiation. The growth rate of the interaction (cf. Fig. 1f) would be $\Gamma \approx 1.5a_0\omega_{pe}/\gamma^{3/2} \approx 10^4\,\mathrm{s}^{-1}$, corresponding to a gain length of $L_g \approx 3 \times 10^4\,\mathrm{m}$. This is probably outside the possibilities of experiments, but similar processes could potentially take place in the extreme environments of astrophysical objects.

References

Baldwin, G. C. and Solem, J. C. 1997 *Rev. Mod. Phys.* **69**, 1085.

Bonifacio, R., Cola, M. M., Piovella, N. and Robb, G. R. M. 2005a *Europhys. Lett.* **69**, 55.

Bonifacio, R., Piovella, N., Cola, M. M. and Volpe, L. 2007 *Nucl. Instrum. Methods Phys. Res.* A **577**, 745.

Compton, A. H. 1923 *Phys. Rev.* **21**, 483.

Eliasson, B. and Shukla, P. K. 2011 *Phys. Rev.* E **83**, 046407.

Eliasson, B. and Shukla, P. K. 2012a *Physical Review* E **85**, 065401(R).

Eliasson, B. and Shukla, P. K. 2012b *Plasma Phys. Control. Fusion* **54**, 124011.

Glenzer, S. H. et al. 2007 *Phys. Rev. Lett.* **98**, 065002.

Glenzer, S. H. and Redmer, R. 2009 *Rev. Mod. Phys.* **81**, 1625.

Haas, F., Eliasson, B. and Shukla, P. K. 2012 *Phys. Rev.* E **85**, 056411.

Hand, E. 2009 *Nature* **461**, 708.

Ishikawa, T., Aoyagi, H. and Asaka, T. et al. 2012 *Nature Photonics*, **6** 540.

Kuzelev, M. V. 2011 *JETP*, **112** 333.

Madey, J. M. J. 1971 *J. Appl. Phys.* **42**, 1906.

Mendonça, J. T. 2011 *Phys. Plasmas* **18**, 062101.

Neumayer, P. et al. 2010 *Phys. Rev. Lett.* **105**, 075003.

Piovella, N., Cola, M. M., Volpe, L., Schiavi, A., and Bonifacio, R. 2008 *Phys. Rev. Lett.* **100**, 044801.

Preparata, G. 1988 *Phys. Rev.* A **38**, 233 (1988).

Rivlin, L. A. 2007 *Quantum Electron.* **37**, 723.

Rivlin, L. A. and Zadernovsky A. A. 2010 *Laser Phys.* **20**(5), 971.

Robertson, C. W. and Sprangle, P. 1989 *Phys. Fluids B* **1**, 3.

Serbeto, A., Monteiro, L. F., Tsui, K. H. and Mendonça, J. T. 2009 *Plasma Phys. Control. Fusion* **51** 124024.

Shukla, P. K., Rao, N. N., Yu, M. Y. and Tsintsadze, N. L. 1986 *Phys. Rep.* **138**, 1.

Son, S. and Moon, S. J. 2012 *Phys. Plasmas* **19**, 063102.

Sprangle, P., Hafizi, B. and Peñano, J. R. 2009 *Phys. Rev. ST Accel. Beams* **12**, 050702.

Stenflo, L. 1976 *Phys. Scr.* **14**, 320.

Takabayasi, T. 1953 *Prog. Theor. Phys.* **9**, 187.

Tkalya E. V. 2011 *Phys. Rev. Lett.*, **106**, 162501.

https://doi.org/10.1017/S0022377813000779 Published online by Cambridge University Press