

RATIONAL VS. LONG-RUN FORECASTERS: OPTIMAL MONETARY POLICY AND THE ROLE OF INEQUALITY

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This paper builds a stylized simple sticky-price New Keynesian model where agents' beliefs are not homogeneous. We assume that agents choose optimal plans while considering forecasts of macroeconomic conditions over an infinite horizon. A fraction of them (boundedly rational agents) use heuristics to forecast macroeconomic variables over an infinite horizon. In our framework, we study optimal policies consistent with a second-order approximation of the policy objective from the consumers' utility function, assuming that the steady state is not distorted.

Keywords: Monetary Policy, Bounded Rationality, Heterogeneous Expectations

1. INTRODUCTION

Our paper deals with optimal monetary policy when agents are heterogeneous. Central bank policies affect both macroeconomic aggregates and their distribution across individuals. According to the mainstream approach to monetary policy, the effects on the former have always been the main object of analysis, whereas the focus on the latter is relatively new. The study of the effects of monetary policy on distributions clearly needs some departure from the traditional representative agent model to introduce some kind of heterogeneity. In the recent years, a rapid development of heterogeneous agents models related to monetary policy has been observed in the literature. We focus on the heterogeneity of the expectation formation mechanism.¹

Evidence in favor of heterogeneity in expectation formation process is provided by many authors², and different theoretical mechanisms of expectation formation have been recently adopted in the New Keynesian model, which is extensively used by central banks. This strand of the literature derives aggregate demand and supply equations from a microfounded sticky-price model augmented with agents

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who have heterogeneous, possibly boundedly rational expectations. Usually, it is assumed that a proportion of agents use rational expectations and the remaining ones use some bounded rationality schemes. Optimal choices are modeled to be consistent with agents' specific forecasts.

In the above environments, it is possible to study the effects of monetary policy on aggregates and on its distribution across the heterogeneous individuals. Note that heterogeneity could itself matter for the aggregates as long as it affects the transmission of monetary policy to macrovariables. Branch and McGough (2009), Massaro (2013), and others show that the dynamic properties of the model depend crucially on the distribution of expectation operators across agents and differ from those implied by models with purely rational expectations.

Heterogeneity in expectations can be introduced in various ways. Specifically, Branch and McGough (2009) and Massaro (2013) propose two different approaches to derive aggregate demand and supply equations of New Keynesian kind embedding bounded rationality. Branch and McGough (2009) assume that individuals with subjective beliefs choose optimal plans that satisfy their individual Euler equations (short-horizon forecasts). The aggregate dynamics then depend on one-period-ahead subjective heterogeneous forecasts. By contrast, Massaro (2013) focuses on long-horizon forecasts and assumes that agents with subjective expectations choose optimal plans while considering forecasts of macroeconomic conditions over an infinite horizon; therefore, the predicted aggregate dynamics hinge on long-horizon forecasts.³ It is worth noting that assuming short- or long-horizon forecasters leads to the same result if all the agents are assumed to be rational.

In their monetary frameworks, Branch and McGough (2009) and Massaro (2013) investigate the impact of heterogeneity on the existence of sunspot equilibria when the central bank sets the nominal interest rate according to a Taylor-type rule. Both find that heterogeneous expectations can undermine some standard results. They focus on the positive aspects of heterogeneity, i.e., how heterogeneity matters for the transmission of monetary policy to macrovariables. But, they do not consider the normative ones, i.e., optimal policies, distributive effects, and welfare analysis. We aim to fill this gap.

We build a stylized simple sticky-price New Keynesian model where agents' beliefs are not homogeneous, assuming a fraction of long-horizon forecasters. In this modeling framework, we study optimal policies consistent with a second-order approximation of the policy objective from the consumers' utility function, considering that the steady state is not distorted.⁴ Following Benigno and Woodford (2012), Steinsson (2003), Ravenna and Walsh (2011), and Di Bartolomeo and Di Pietro (2016), we define the optimal policy as the first-order conditions that minimize a second-order approximation of the loss function derived by a second-order approximation of consumers' utility function. In this context, we do not discuss the implementation of optimal policy by an interest rate rule.⁵

We find that heterogeneous expectations do not change the nature of the optimal policy. The policy maker is constrained by a trade-off between price dispersion and

output stabilization. However, heterogeneous expectations imply some relevant caveats for the Phillips trade-off. In this setup, inflation cannot be used as a proxy of price dispersion, since it has complex dynamics. Moreover, the trade-off is affected by the existence of output persistence and intrinsic inflation inertia because nonrational agents recognize changes in the economic variables with some period lag. Finally, heterogeneous expectations also imply an additional dimension in the policy trade-off; in implementing its policy, the central banker should also account for inequality (in the form of the cross-sectional variability of consumption).

In the upsurge of scientific contributions on the heterogeneous beliefs in the New Keynesian framework, Gasteiger (2014) and Di Bartolomeo et al. (2016) are closely related to our research. Both study optimal policies in a stylized simple sticky-price model where agents' beliefs are not homogeneous, but they differ from us on the way bounded rationality is introduced. They assume that all the agents' optimal decisions are consistent with their short-horizon forecasts, but a fraction of agents can form their beliefs on the basis of a simple perceived linear law of motion on past observed values [Brock and Hommes (1997), Branch and McGough (2009)]. By contrast, we focus on long-horizon forecasts assuming that agents choose optimal plans while considering forecasts of macroeconomic conditions over an infinite horizon [Preston (2006), Massaro (2013)].

Gasteiger (2014) obtains the optimal path of inflation and output gap by minimizing an ad hoc loss function and studies the implementation problem of the relative optimal path. He shows that fundamental reaction functions lead to indeterminacy, whereas expectation-based reaction functions do not. Di Bartolomeo et al. (2016) focus on an optimal monetary policy by second-order approximation of the policy objective from the consumers' utility function. They find that the introduction of bounded rationality in the New Keynesian framework matters. The trade-off between the price dispersion and aggregate consumption variability is in fact quantitatively affected by heterogeneity. Moreover, they show how commitment marginal gains over discretion are affected by the degree of bounded rationality. Introducing model uncertainty, Di Bartolomeo et al. (2016) also show the additional costs of ignoring expectations heterogeneity under a welfare-maximizing perspective.

The remainder of the paper is organized as follows. Section 2 illustrates the model and the expectations' formation process. Section 3 derives a quadratic Taylor series approximation of the welfare. Section 4 illustrates the path of macroaggregates associated with optimal stabilization policies. It also investigates the relative gains from discretion for different share of rational agents in the economy compared to alternative policy regimes. Section 5 concludes.

2. THE MODEL

The economy is populated by a continuum of infinitely lived households in the interval $[0, 1]$. Households maximize the same utility function, but may have

different expectations about the future. They choose consumption and labor/leisure path to maximize the expected present discounted value of their utility. By using a linear technology, firms hire workers to maximize profit under monopolistic competition. The presence of price stickiness is modeled following Calvo (1983) price setting scheme, each firm resets its price with a certain probability in each period.

2.1. Modeling Heterogeneous Beliefs

Households have the same utility function and firms share the same linear technology; therefore, their choices could only differ as a consequence of different expectations. In solving their optimization problem, all the agents need to forecast the future values of aggregate macroeconomic variables. We describe subjective expectations by $\mathcal{E}_{i,t}(\cdot)$ so that $\mathcal{E}_{i,t}x_{t+1}$ is the time t expectation of individual i on the macroeconomic variable x at time $t + 1$.

The economy is populated by a large number of identical agents of mass one. Furthermore, population is partitioned into two subgroups: rational and boundedly rational agents. Agents belonging to rational group have rational expectations. Boundedly rational individuals use heuristics to forecast macroeconomic variables over an infinite horizon (long-horizon forecasters).

Each rational agent forecasts macroeconomic variables according to the following rule:

$$E_t x_{t+1} = x_{t+1} + \zeta_{i,t}, \quad (1)$$

where E_t is the rational expectation operator and $\zeta_{i,t}$ is the fully rational expectations error. Assuming that a fraction of households, denoted by α , form expectations rationally, while the remaining $(1 - \alpha)$ have boundedly rational beliefs, aggregation of the former yields

$$\int_0^\alpha \mathcal{E}_{i,t} x_{t+1} di = \alpha E_t x_{t+1}. \quad (2)$$

Focusing on the boundedly rational agents, they are modeled as in Preston (2006) and Massaro (2013) to whom we refer for details. In a nutshell, they use heuristics to forecast macroeconomic variables over an infinite horizon. The selection of heuristics takes place at the beginning of period t , when they observe and compare past performances. Each predictor, $\theta_i \in \Theta$, is evaluated according to the past squared forecast error (performance measure). The distribution of beliefs then evolves over time as a function of past performances according to the continuous choice model [Diks and van der Weide (2005)]. The distribution of beliefs is normal and its evolution is characterized by a mean equal to x_{t-1} and a finite variance that is decreasing in the agents' sensitivity to differences in performances.

Aggregating among the boundedly rational agents, we obtain

$$\int_{\alpha}^1 \mathcal{E}_{i,t} x_{t+1} di = (1 - \alpha) \int_{\Theta} \theta_i di = (1 - \alpha) x_{t-1}. \tag{3}$$

The above rules [(2) and (3)] will be used to aggregate individual behavioral decisions and get aggregate demand and supply equations consistent with the presence of heterogeneous expectations.

2.2. A New Keynesian Model with Long-Horizon Forecasters⁶

Household i maximizes the expected value of an intertemporal utility function, depending on consumption [$u(C)$] and labor supplied [$v(H)$], i.e.,

$$\mathcal{E}_{i,t} \sum_{s=t}^{\infty} \beta^{s-t} \left(\underbrace{\frac{C_{i,s}^{1-\sigma}}{1-\sigma}}_{u(C)} - \chi \underbrace{\frac{H_{i,s}^{1+\gamma}}{1+\gamma}}_{v(H)} \right), \tag{4}$$

where $\beta \leq 1$ denotes the discount factor, $C_t^i \equiv [\int_0^1 C_t^i(j)^{(\eta-1)/\eta} dj]^{\eta/(\eta-1)}$ is the composite consumption good, and $C_t^i(j)$ denotes the quantity of good $j \in [0, 1]$ consumed by the household i in period t and⁷ $H_{i,t}$ is the labor supplied. The term η is the elasticity of substitution between goods, restricted to be greater than one.

Assuming a cashless economy, the real budget constraint of household i is defined as follows:

$$C_{i,t} + B_{i,t} \leq W_t H_{i,t} + \frac{\mathcal{I}_{t-1} B_{i,t-1}}{\Pi_t} + D_t, \tag{5}$$

where $B_{i,t}$ indicates bond holdings, W_t is the real wage, \mathcal{I}_t is the gross nominal interest rate, Π_t is the inflation rate, and D_t are dividends agents receive from firms and redistribute within the household, in order to hedge against the Calvo risk. Bonds are zero in net supply and agents can trade among each other.

Household i chooses consumption and labor supplied to maximize (4) s.t. (5). Log-linearizing the first-order conditions of the consumer optimal problem and their budget constraint, we get

$$c_{i,t} = \mathcal{E}_{i,t} c_{i,t+1} - \frac{1}{\sigma} (i_t - \mathcal{E}_{i,t} \pi_{t+1}), \tag{6}$$

$$h_{i,t} = \frac{1}{\gamma} (w_t - \sigma c_{i,t}), \tag{7}$$

$$b_{i,t} - \frac{1}{\beta} b_{i,t-1} - \frac{b}{\beta} (i_{t-1} - \pi_t) - \frac{\eta - 1}{\eta} (w_t + h_{i,t}) - \frac{1}{\eta} d_t + c_{i,t} = 0, \tag{8}$$

where $\sigma \equiv -\bar{C} u_{CC}/u_C > 0$ is the coefficient of relative risk aversion (the inverse of the intertemporal elasticity of substitution of aggregate expenditure)

and lowercase variables are log deviations from the steady-state value [e.g., $c_t \equiv \ln(C_t/C)$], with the exception of $b_t \equiv B_t - B$, which is just the difference.

Iterating the flow budget constraint (8) and imposing the No Ponzi condition, we get the perceived lifetime budget constraint:

$$\mathcal{E}_{i,t} \sum_{s=t}^{\infty} \beta^{s-t} \frac{\sigma + \gamma\mu}{\gamma\mu} c_{i,s} = \mathcal{E}_{i,t} \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{b_{i,t-1}}{\beta} + \frac{d_s}{\eta} + \frac{(1 + \gamma) w_s}{\gamma\mu} \right]. \tag{9}$$

Iterating forward the Euler equation (6) and substituting it in the intertemporal budget constraint (9), we finally derive the individual consumption rule for agent i :

$$c_{i,t} = \zeta_b b_{i,t-1} + \mathcal{E}_{i,t} \sum_{s=t}^{\infty} \beta^{s-t} (\zeta_w w_s + \zeta_d d_s) - \frac{\beta}{\sigma} \mathcal{E}_{i,t} \sum_{s=t}^{\infty} \beta^{s-t} (i_s - \pi_{s+1}), \tag{10}$$

where $\zeta_b \equiv \frac{1}{\beta} \frac{\mu\gamma}{\mu\gamma + \sigma}$, $\zeta_w \equiv \frac{(1-\beta)(1+\gamma)}{\mu\gamma + \sigma}$, $\zeta_d \equiv \frac{(1-\beta)(1+\mu)\gamma}{\mu\gamma + \sigma}$, and $\mu \equiv \eta/(\eta - 1)$ is the steady-state price markup.

The first three terms of the above equation indicate that consumption depends on the expected future discounted wealth, while the last term is due to the time-varying interest rate and reflects deviations from equilibrium as a consequence of variations in the nominal interest rate or inflation.

Regarding the supply side of the economy, firms hire workers to maximize profits constrained by a linear technology. Monopoly firms that reset prices, $P_{i,t}$, at t maximize the following expected discounted profits:

$$\mathcal{E}_{i,t} \sum_{s=t}^{\infty} \omega^{s-t} Q_{s,t} \left(\frac{P_{i,t}}{P_s} - W_s \right) \left(\frac{P_{i,t}}{P_s} \right)^{-\eta} C_s, \tag{11}$$

where $Q_{s,t} = \beta^{s-t} (C_s/C_t)^{-\sigma}$ is the discount factor and W_s is the real marginal cost; $(1 - \omega) \in (0, 1)$ is the probability of resetting price at each time.

After log-linearization of the first-order condition around a zero inflation steady state, the firm i 's maximization problem delivers the following price equation (firm i 's decision rule):

$$p_{i,t} = (1 - \omega\beta) \mathcal{E}_{i,t} \sum_{s=t}^{\infty} (\omega\beta)^{s-t} w_s + \omega\beta \mathcal{E}_{i,t} \sum_{s=t}^{\infty} (\omega\beta)^{s-t} \pi_{s+1}. \tag{12}$$

Our New Keynesian model with long-horizon forecasters is finally obtained by aggregating individual decision rules [(10) and (12)]. After some algebra and

using (2) and (3), we get the aggregate relations:

$$y_t = \alpha E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[(1 - \beta) y_s - \frac{\beta}{\sigma} (i_s - \pi_{s+1}) \right] + (1 - \alpha) \left[y_{t-1} - \frac{\beta}{\sigma} \left(i_t + \frac{\beta}{1 - \beta} i_{t-1} - \frac{1}{1 - \beta} \pi_{t-1} \right) \right], \tag{13}$$

$$\pi_t = \alpha E_t \sum_{s=t}^{\infty} (\omega\beta)^{s-t} [\kappa y_s + (1 - \omega) \beta \pi_{s+1}] + (1 - \alpha) \frac{\kappa y_{t-1} + (1 - \omega) \beta \pi_{t-1}}{1 - \omega\beta} + e_t, \tag{14}$$

where $\kappa \equiv (1 - \omega) (1 - \omega\beta) (\gamma + \sigma) / \omega$.

In deriving (13) and (14), we have also assumed that the current interest rate is observed by boundedly rational agents, while current output and inflation are not. The Phillips curve (14) has been augmented by a supply disturbance, e_t .

3. WELFARE CRITERION

A welfare measure is obtained by aggregating quadratic Taylor series approximations of the utility functions. The utility function (4) of each household depends on two components: consumption and labor supplied.

By approximating the consumption component of (4) to the second order, we obtain

$$\tilde{u}(C_{i,t}) = \bar{C}^{(1-\sigma)} \left(c_{i,t} + \frac{1 - \sigma}{2} c_{i,t}^2 \right) + \text{t.i.p.} + \mathcal{O}(\|\xi^3\|), \tag{15}$$

where we have assumed that in the steady state, all the agents consume the same, \bar{C} , such that all the expectations are correct. The term $\mathcal{O}(\|\xi^3\|)$ indicates the terms of order greater than two, while t.i.p. collects the terms independent of policy. Henceforth, for the sake of brevity, we omit to report $\mathcal{O}(\|\xi^3\|)$ and t.i.p. in our derivation.

Integrating (15) over i , we get

$$\int_0^1 \tilde{u}(C_{i,t}) di = \bar{C}^{(1-\sigma)} \left\{ c_t - \frac{1}{2} \text{var}_i(c_{i,t}) + \frac{1 - \sigma}{2} [c_t^2 + \text{var}_i(c_{i,t})] \right\}. \tag{16}$$

Considering the linear technology, a second-order approximation of the labor component of (4) yields

$$\tilde{v}(H_{i,t}) di = \bar{H}_i^{1+\gamma} \left(y_{i,t} + \frac{1 + \gamma}{2} y_{i,t}^2 \right). \tag{17}$$

After integration, we obtain

$$\int_0^1 \tilde{v}(H_{i,t}) di = \bar{H}^{1+\gamma} \left\{ y_t - \frac{1}{2} \text{var}_i(y_{i,t}) + \frac{1+\gamma}{2} \left[(E_i y_{i,t})^2 + \text{var}_i(y_{i,t}) \right] \right\}, \tag{18}$$

where $\int_0^1 \bar{H}_i di = \bar{H}$, since the price dispersion is zero in the steady state.

Assuming a nondistorted steady state, (18) can be rewritten as⁸

$$\int_0^1 \tilde{v}[H_i(i)] di = \bar{C}^{(1-\sigma)} \left\{ y_t + \frac{\eta^2 \gamma}{2} \text{var}_i[p_t(i)] + \frac{1+\gamma}{2} y_t^2 \right\}. \tag{19}$$

Equation (19) is obtained by considering the Taylor series approximation of the Dixit–Stiglitz index of aggregate demand, $y_t = E_i y_t(i) + \frac{1}{2}(1 - \eta^{-1}) \text{var}_i y_t(i)$, and noting that the cross-sectional variance of $y_t(i)$ can be expressed as $\text{var}_i[y_t(i)] = \eta^2 \text{var}_i[p_t(i)]$.

By combining (16) and (19), we obtain the approximated intertemporal welfare measure⁹

$$\mathcal{W} = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t + \text{t.i.p.} + \mathcal{O}(\|\xi^3\|), \tag{20}$$

where the welfare-based (WB) instantaneous loss is

$$\mathcal{L}_t^{\text{WB}} = (\gamma + \sigma) y_t^2 + \eta^2 \gamma \text{var}_i[p_t(i)] + \sigma \text{var}_i[c_t(i)]. \tag{21}$$

Our welfare measure (20) is made up of three components. Beyond the traditional costs related to aggregate consumption variability (y_t^2) and price dispersion $\{\text{var}_i[p_t(i)]\}$, there is an additional cost which captures the inequality in the consumption of the two different types of agents, i.e., $\text{var}_i[c_t(i)]$.

The cross-sectional variability of consumption can be written as follows¹⁰:

$$\text{var}_i[c_t(i)] = \frac{\alpha}{1 - \alpha} \left[\frac{1}{\sigma} \sum_{t=0}^{\infty} (i_t - E_t \pi_{t+1}) - c_t \right]^2. \tag{22}$$

Price dispersion evolves according to

$$\Delta_t = \omega \Delta_{t-1} + \frac{\omega}{1 - \omega} \left[\pi_t^2 + \frac{(1 - \alpha)\omega}{\alpha} (\pi_t - \beta \pi_{t-1} - \kappa y_t)^2 \right], \tag{23}$$

from which we derive the following equation for the discounted price dispersion:

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{1}{\Phi} \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \frac{\omega(1 - \alpha)}{\alpha} \left[\pi_t - \beta \pi_{t-1} - \frac{\omega^2(1 - \alpha)\kappa}{\alpha(1 - \omega)} y_t \right]^2 \right\}, \tag{24}$$

where $\Phi = \kappa(\gamma + \sigma)$.

4. HETEROGENOUS BELIEFS, INEQUALITY, AND OPTIMAL POLICIES

4.1. Calibration

The model is calibrated to the US economy. The time unit is one quarter. The subjective discount rate β is fixed at 0.99 such that the annual real interest rate is 4%. Following Rotemberg and Woodford (1997) and other studies, the coefficient of relative risk aversion, σ , is set equal to 0.16; the intratemporal elasticity of substitution between the differentiated goods (price elasticity of demand) is fixed at $\eta = 7.88$ —implying a markup of 15%; the elasticity of the marginal disutility of producing output with respect to an increase in labor is $\gamma = 0.47$; and the frequency of price adjustment, ω , is set at 0.63, which implies that prices are reset, on average, every three quarters.

We calibrate the parameter characterizing agents’ heterogeneity, α , to get a value consistent with the observed forecast error in the data on inflation forecasts from the US Survey of Professional Forecasters from 1971 to 2014. Specifically, we follow the methodology applied by Coibion and Gorodnichenko (2015).

In our model, the aggregate time t forecast, F_t , of x at time $t + j$ is given by the weighted average of rational agents j -step ahead perfect foresight on economic variable x , i.e., $\mathcal{E}_t^R x_{t+j} = E_t x_{t+j} = x_{t+j} + \varepsilon_{t+j}$, and of nonrational individuals beliefs based on the simple perceived linear law of motion, $\mathcal{E}_t^B x_{t+j} = x_{t-1}$. Then, the forecast error, $x_{t+j} - F_t x_{t+j}$, can be written as

$$x_{t+j} - F_t x_{t+j} = \frac{1 - \alpha}{\alpha} (F_t x_{t+j} - x_{t-1}) + v_{t+j}, \tag{25}$$

where $v_{t+j} = -\alpha \varepsilon_{t+j}$ and ε_{t+j} is the fully rational expectations error.

By using (25), we test the model consistent with the boundedly rational hypothesis based on the following empirical specification:

$$\pi_{t+3} - F_t \pi_{t+3} = c + \phi (F_t \pi_{t+3} - \pi_{t-4}) + \text{error}_t. \tag{26}$$

Based on our empirical results, we reject the fully rational hypothesis in favor of a heterogeneity expectation model at the 5% level of statistical significance. In particular, our estimates suggest that $\alpha = 0.82$ [i.e., $\phi = 0.21$ in (26)]. Considering the confidence interval of our estimation of α , calibrated values for the share of boundedly rational agents $(1 - \alpha)$ range between 5% and 30%.¹¹

Results are presented in Table 1 [see column (1)]. They are qualitatively and quantitatively confirmed also in the case of augmented empirical estimation to allow for additional controls such as interest rates, oil prices, and unemployment rate [see columns (2)–(4)].

Our calibration is summarized in Table 2.

4.2. Expectation Heterogeneity and Optimal Policies

We are interested in investigating how heterogeneous expectations affect optimal policies. Therefore, we consider a central bank who minimizes a WB loss function

TABLE 1. Forecast error estimation

$\pi_{t+3,t} - F_t\pi_{t+3,t}$	(1)	(2)	(3)	(4)
Constant	-0.051 (0.181)	0.064 (0.192)	-0.081 (0.165)	1.149 (0.742)
$F_t\pi_{t+3,t} - \pi_{t-4}$	0.214** (0.096)	0.208** (0.098)	0.200** (0.094)	0.179** (0.085)
Control variable: z_{t-1}		-0.022 (0.045)	1.547* (0.839)	-0.187** (0.099)
Observations	168	168	168	168

Note: * and ** denote statistical significance at the 90% and 95% confidence levels, respectively.

TABLE 2. Baseline calibration

$\beta = 0.99$	Subjective discount rate.
$\gamma = 0.47$	Elasticity of the marginal disutility of producing output.
$\sigma = 0.16$	Coefficient of relative risk aversion.
$\omega = 0.63$	Frequency of price adjustment.
$\eta = 7.88$	Price elasticity of demand.
$\alpha = 0.82$	Fraction of rational agents.

and compare the case of homogeneous expectations to the case of heterogeneous ones. We assume that the central bank operates under discretion and the economy is perturbed by a stochastic disturbance. Formally, it minimizes (21) s.t. (13), (14), and (24). Results are illustrated in Figure 1 that depicts the impulse response functions (IRFs) of the key macrovariables.¹²

In the standard New Keynesian framework, after the shock, inflation (and price dispersion) tends to increase. Then the policy maker finds optimal to increase the nominal (and real) interest rate to reduce the inflationary pressure at the cost of an output gap fall. In other words, the policy maker aims to stabilize the economy constrained by a Phillips’ kind trade-off.

The nature of policy maker’s problem is unchanged when heterogeneous expectations are considered. The central bank faces a policy trade-off between price stabilization and output variability. However, looking at the policy-relevant variables, compared to the standard case ($\alpha = 1$), it is worth noting that heterogeneous expectations ($\alpha = 0.82$) imply two new facts.

1. The economy exhibits some degree of output persistence and inflation inertia as nonrational agents recognize changes in both inflation and output gap with one-period lag.¹³
2. The path of inflation does not map one-to-one the path of price dispersion. Therefore, differently from the traditional New Keynesian framework, the central bank cannot look at the former as a proxy for the latter. In this case, price dispersion has, in fact, more complex dynamics [see equation (24)].

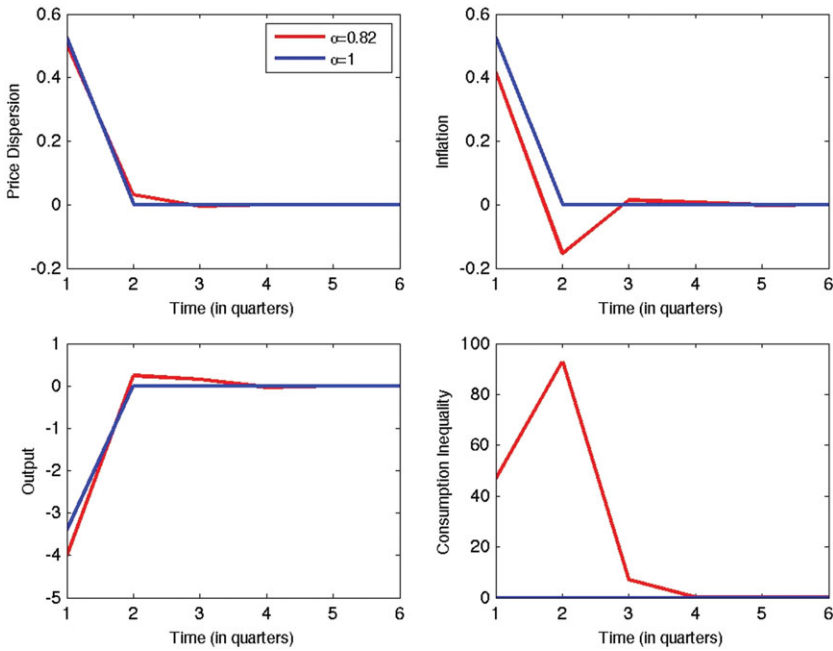


FIGURE 1. Optimal welfare-based policies: IRFs to a cost-push shock.

The paths of both output gap and price dispersion are quite similar when $\alpha = 1$ and $\alpha = 0.82$. However, under heterogeneous beliefs ($\alpha = 0.82$), the policy maker lowers the initial impact of the cost-push shock on price dispersion and inflation (relative to the standard case, $\alpha = 1$), while incurring larger output gap losses in the same period. Resulting inflation increases much less in the first period and deflation is later obtained. In other words, the observed path is similar to that implied by timeless commitment, where such a policy reduces the expectations and improves the current trade-off between price dispersion and output gap. Indeed, as long as nonrational agents recognize changes with one-period lag, current (discretionary) policies act like a commitment. At time t , by choosing a value for inflation, the central bank fixes the nonrational agents' expectations at $t + 1$, and therefore, it affects the trade-off between price dispersion and output gap at $t + 1$.

Finally, the introduction of heterogeneous expectations implies an additional dimension in the policy trade-off. In the homogeneous expectation case, the policy maker's problem is reduced to the traditional trade-off between the variability of price dispersion (inflation) and output. Instead, in our case, the central banker is also constrained to take into account the cross-sectional variability of consumption, implied by consumption's inequality among agents, when implementing its policy. Note that inequality has a hump-shaped IRF due to the backward-looking behavior of boundedly rational agents.

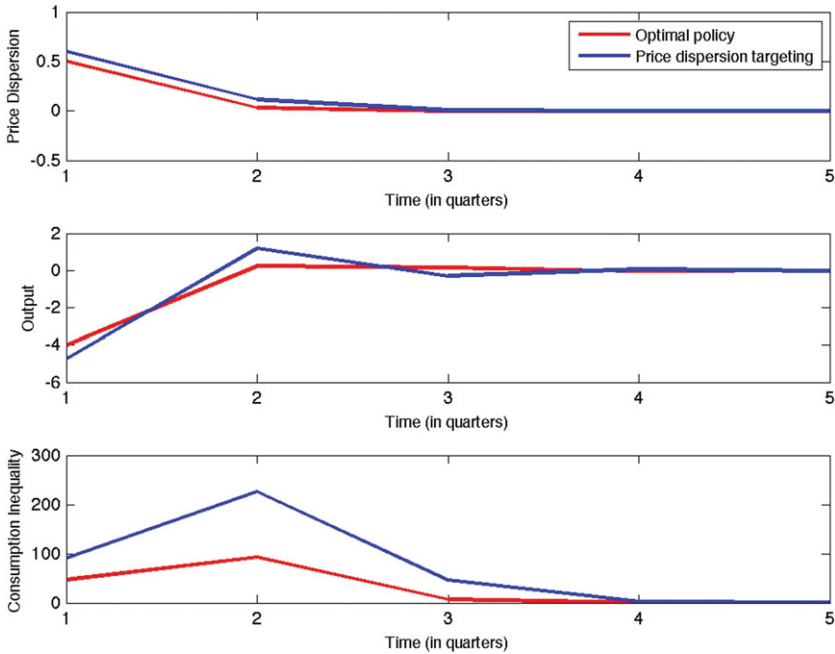


FIGURE 2. Welfare-based policies vs. PT regime: IRFs to a cost-push shock.

Now, we concentrate to our heterogeneous expectation setting ($\alpha = 0.82$) and consider two alternative policy regimes to the WB optimal policy. We first consider the case where the policy maker ignores the minimization of the cross-sectional variability of consumption when implementing optimal monetary policies. Formally, the policy maker minimizes $\mathcal{L}_t^{PT} = (\gamma + \sigma) y_t^2 + \eta^2 \gamma \text{var}_i [p_t(i)]$ s.t. (13) and (14). We refer to this case as the price-dispersion-targeting (PT) regime. Then, we consider the case where the policy maker also considers inflation instead of price dispersion as a policy target. We label this regime as (flexible) inflation-targeting (IT) scenario. The policy maker minimizes $\mathcal{L}_t^{IT} = (\gamma + \sigma) y_t^2 + \eta^2 \gamma \pi_t^2$ s.t. (13) and (14). In both regimes, the policy maker operates under discretion. Policies obtained under these regimes are compared to the WB case in Figures 2 and 3.

Let us consider the comparison between the WB policy and the PT regime (Figure 2). As expected, the latter implies a higher volatility of consumption inequality. Moreover, in the PT regime, the initial response of the policy maker requires to stabilize more price dispersion relatively to output at the cost of larger future fluctuations in both price dispersion and output. The greater importance of price dispersion relative to output when the consumption inequality is neglected is due to the fact that the latter has a stronger impact on inequality compared to the former.

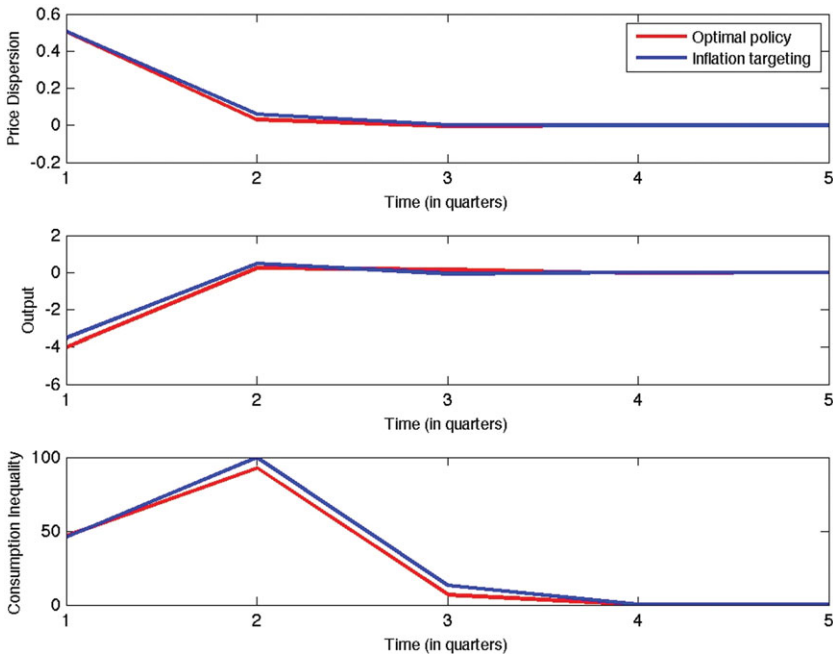


FIGURE 3. Welfare-based policies vs. IT regime: IRFs to a cost-push shock.

The comparison between the WB policy and the IT regime is described in Figure 3. The path of the macrovariables is very similar. The IT regime seems to mimic the WB optimal policies. It implies a slightly increase in consumption inequality. However, the welfare impact of the difference can be relevant. We refer to Table 3 for a more detailed analysis of the differences in welfare.

Table 3 describes the welfare loss obtained under WB optimal policy and in the PT and IT regimes. It reports losses for different fractions of boundedly rational agents. Welfare losses are normalized for the optimal loss obtained in the case of homogeneous expectations ($\alpha = 1$).¹⁴

The table shows that bounded rationality raises welfare costs of fluctuations several times with respect to the case of homogeneous rational expectations. These costs are increasing in the degree of bounded rationality. The relative importance of consumption inequality can be evaluated by comparing columns (1) and (2), since PT optimal policies only differ from the outcomes obtained under WB because of the inequality component. For a realistic range of bounded rationality (i.e., 0.5%–30%), the costs of ignoring inequality are relatively high. By contrast, for higher degrees, they become negligible. Ignoring consumption inequality has almost no effect on welfare when the share of boundedly rational agents is greater than 30%.

Comparing the outcomes of the different policy regimes, it also emerges that IT performs better than PT in the realistic range of bounded rationality. More

TABLE 3. Welfare comparisons

	WB	PT	IT
Bounded rationality ($1 - \alpha$)	(1)	(2)	(3)
90%	88.0	88.0	134.9
80%	51.4	51.4	64.8
70%	35.0	35.0	39.4
60%	24.6	24.6	25.7
50%	17.2	17.2	17.2
40%	11.8	11.9	11.6
30%	7.9	8.4	8.1
20%	5.7	7.6	6.5
10%	5.1	11.8	9.5

generally, optimal policies under IT exhibit a nonmonotonic path for the welfare relative to changes in the bounded rationality degree. Higher losses are obtained when the fraction of boundedly rational agents are either more than 60% or less than 30%. In between, the IT regime performs even better than the policies stemming from the WB optimization. By contrast, as said before, welfare in the PT regime tends to converge to that obtained in WB.

5. CONCLUSIONS

We have examined the effects of monetary policy on welfare under bounded rationality and agents' heterogeneity. In an otherwise standard macrosticky price model, a fraction of agents (boundedly rational individuals) use heuristics to forecast macroeconomic variables over an infinite horizon and choose optimal plans accordingly to these forecasts. Welfare has been evaluated by a second-order approximation from the consumers' utility function.

The presence of heterogeneity introduces a new dimension in the policy maker's policy trade-off; the central banker should now account for the cross-sectional variability of consumption (inequality, in short). The policy maker is constrained by a trade-off between price dispersion and output stabilization. Heterogeneous expectations also imply that inflation can no longer be used as a proxy of price dispersion and that the policy trade-offs are affected by the existence of output persistence and intrinsic inflation inertia.

We consider three policy regimes. First, we study optimal policies consistent with a second-order approximation of the policy objective function from the consumers' utility function (i.e., WB). Second, we still assume that the central bank minimizes the WB loss, but ignoring inequality. So we refer to this regime as the price dispersion (flexible) targeting as the policy makers' targets are the output gap and price dispersion. Finally, we assume that the central bank simply minimizes the WB loss obtained under the rational expectation case, i.e., targeting output gap and inflation. We refer to this regime as (flexible) IT.

In all the policy regimes, bounded rationality significantly raises the welfare cost of economic fluctuations compared to the case of rational expectations. The cost is increasing in the share of agents who use heuristics for their forecasts.

The main normative contributions of our paper are as follows. For a realistic range of bounded rationality, the costs of ignoring inequality are relatively high. The cost of ignoring inequality is lower when the central bank targets inflation instead of price dispersion. By targeting inflation, the central bank is able to somehow control future expectations and to improve the trade-off between price dispersion and output—likewise under timeless perspective. The costs of targeting inflation, however, become very high when large shares of boundedly rational agents are considered. By contrast, for large shares, welfare differences obtained by ignoring and considering inequality become negligible when the central bank's target is price dispersion.

NOTES

1. This development is surveyed by Brzoza-Brzezina et al. (2013), who review studies that analyze the heterogeneity of households' income and preferences, consumers' age, expectations, and firms' productivity and financial position. See also Anufriev et al. (2013) and references therein.

2. See, e.g., Carroll (2003), Branch (2004), Capistrán and Timmermann (2009), Pfajfar and Santoro (2010).

3. See also Preston (2006).

4. See Benigno and Woodford (2012).

5. See Evans and Honkapohja (2003) and Gasteiger (2014).

6. The model is borrowed from Massaro (2013) to whom we refer for a full derivation.

7. Consistently, the aggregate price index for consumption is defined as $P_t \equiv \left[\int_0^1 P_t(j)^{1-\eta} dj \right]^{1/(1-\eta)}$.

8. We used $\bar{C}^{1-\sigma} / \bar{H}^\gamma = \bar{H}$.

9. Welfare is normalized by $1/\bar{C}^{(1-\sigma)}$.

10. The derivation of cross-sectional variability of consumption and price dispersion are available upon request.

11. This range is consistent with other estimations. See Coibion and Gorodnichenko (2015) and reference therein.

12. We consider a white noise shock in the Phillips curve.

13. Simulations show that persistence increases in the fraction of boundedly rational types.

14. In the standard case ($\alpha = 1$), the three regimes considered here do not lead to any differences. In such a case, inequality is always zero and price dispersion is approximated by inflation.

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