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FINITE CAPACITY ENERGY PACKET NETWORKS

YASIN MURAT KADIOGLU
Intelligent Systems and Networks Group,
Department of Electrical and Electronic Engineering,
Imperial College, London SW7 2BT, UK
E-mail: y.kadioglu14@imperial.ac.uk

This paper surveys research on mathematical models that predict the performance of digital devices that operate with intermittent energy sources. The approach taken in this work is based on the "Energy Packet Network" paradigm where the arrival of data to be processed or transmitted, and the energy to operate the system, are modeled as discrete random processes. Our assumption is that these devices will capture energy from intermittent ambient sources such as vibrations, heat or light, and capture it onto electrical energy that may be stored in batteries or capacitors. The devices consume this energy intermittently for processing and for wired or wireless transmission. Thus, both the arrival of energy to the device, and the devices workload, are modeled as random processes. Based on these assumptions, we discuss probability models based on Markov chains that can be used to predict the effective rates at which such devices operate. We also survey related work that models networks of such systems.

Keywords: data packets, energy harvesting, energy packet networks, markov chains, random walk

1. INTRODUCTION

Energy is a primary driver for the manufacturing and operation of information processing and transmission devices such as computers, network nodes, wireless receivers and transmitters, and so on. Thus, there has been increasing concern about the massive amount of electrical energy that is being used in this context (see, e.g., Gelenbe and Caseau [17]), which is approaching 10% of the total amount of electricity consumed worldwide.

In addition, there has been much work done on the design of systems that can exploit "free" energy captured from the ambient environment via energy harvesting (EH) (see, e.g., Rodopln and Meng [35], Meshkati et al. [33], Alippi and Galperti [2], Seah, Eu, and Tan [36]) from thermal, light, chemical, vibrational, or electromagnetic sources and converted into electrical energy. Of course, when both the workload in a computer system and the network are intermittent, and the process of acquiring energy to operate them and service the workload is intermittent, it is important to investigate such systems in terms of stochastic processes.

Significant work by Berl et al. [3] has addressed techniques to reduce or optimize the energy consumption of Cloud servers, which are major consumers of electricity, and energy

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consumption in Data Centers was investigated by Newcombe [34]. Recent work that uses queueing theory and experimentation has considered the option of an optimum workload that will result in a compromise between the cost of energy consumption and the quality of service of a Cloud server (see, e.g., Gelenbe, Lent, and Douratsos [26], Gelenbe and Lent [24], and Gelenbe and Lent [25]) have discussed further how tasks can be dispatched among several Cloud servers so as optimize energy-aware cost functions. In other work, the optimum routing of data packets (DPs) in a packet network has been considered from a theoretical perspective by Gelenbe and Morfopoulou [29], while Gelenbe and Mahmoodi [27] have conducted experiments on a network test-bed so as to minimize the average energy that is consumed per packet that is forwarded. Francois, Abdelrahman, and Gelenbe [5,6] have shown that attacks on networks can also significantly increase their energy consumption. In order to reduce by orders of magnitude the amount of energy that is consumed by computing and communications, systems as in Gelenbe [12,14,16], where he energy and data are carried by the same "packet", that is, the spin of particles represents the data, while the charge of the particles carries the energy.

1.1. The Energy Packet Network Paradigm

The energy packet paradigm (EPN) developed by Gelenbe [9,10] is a queueing network approach based on G-networks theory (see, e.g., Gelenbe [7,8]), which has been used to model a data processing or transmission network that can store and consume harvested energy as discrete units. The analogy that drives this approach is based on the fact that computer jobs and DPs are discrete entities that are queued and then processed in the system. However, Gelenbe proposed [11] that it is also possible to discretize the amount of energy being used, and electrical batteries or capacitors can also be viewed as storing discrete units of energy. Recent work on EPNs has considered the design of optimal energy distribution architectures (see, e.g., Ceran and Gelenbe [4]) and of optimal flows that maximize certain utility functions (see, e.g., Gelenbe and Ceran [18,19]). However, in this paper we principally review work regarding a somewhat simpler paradigm.

1.2. E-Networks

While EPNs consider task service times, so that both the consumption of energy and the processing or transmission of jobs or packets happens over some continuous time, in this paper we will consider another paradigm, which was also introduced by Gelenbe [13,15], motivated by energy harvesting (ES) wireless devices (see, e.g., Gelenbe et al. [20], Gelenbe and Gunduz [21]) where processing or transmission times are very fast (and hence negligible), in comparison to the time it takes to acquire data (as in a sensor) and to acquire energy (as through harvesting). For convenience, we will call this type of model the "E-Network".

Just as in the EPN model, in E-Networks energy packets (EPs) are stored in a battery or a capacitor, and are consumed both for the node's electronics (packet sensing-storing-processing) and for DP transmission. DPs are stored in a data buffer until they are transmitted. While a "perfect" transmitter is assumed for some models where there is no energy waste for node electronics, more practical models are studied by including energy consumption for node electronics, and different transmission power levels can also be considered. Thus, in general we may consider that a finite number of DPs will be transmitted with some (possibly other) finite number of EPs.

While in this paper we consider single hop nodes, in other work two-hop nodes have been analyzed by Gelenbe and Marin [28]. Mixed discrete continuous models for similar

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problems can be found in Abdelrahman and Gelenbe [1], and very low-power models where single particles are used for carrying each bit are discussed in Gelenbe [14].

2. MATHEMATICAL MODEL

We consider a system that can store DPs in a buffer whose maximum capacity is D, and which stores EPs in an energy buffer (battery or capacitor) whose maximum capacity is E. At any given time $t \geq 0$, the number of DPs waiting for the transmission and EPs waiting in the storage are $D(t)$ and $E(t)$, respectively. Thus, the state of the system at any given time t is expressed by the pair $(D(t), E(t))$ so that we will be interested in $p(d, e, t) = Pr[D(t) = d, E(t) = e]$. While the data sensing and formation in the data buffer is assumed to be a Poisson process of rate λ, the ES and placement in the energy buffer is also assumed to be a Poisson process of rate Λ.

In such nodes, the process of harvesting energy from the environment and its conversion to electrical energy may take many milliseconds, while the packet transmission may take shorter time period, such as microseconds, or more likely nanoseconds. Since, the data formation and replacement in the buffer also may take a certain time period in milliseconds, the time needed for packet transmission compared with the time needed for DP and EP arrivals into the sensor node is very small, or negligible. Thus, we assume that the packet transmission is instantaneous, that is, we assume an E-Network model.

Also, we assume that total K EPs are needed to sense, process, store, and transmit one DP in a sensor node where $K = K_e + K_t$, K_e is the number of EPs consuming for sensor node electronics (sensing–processing–storing), and K_t is the number of EPs consuming for packet transmission. In the rest of the paper, we will analyze different system models according to different K_e and K_t values.

3. PACKET TRANSMISSION BY SINGLE EP VIA PERFECT TRANSMITTER

Such systems where one DP can be transmitted by a single EP ($K_t = 1$) with a perfect transmitter ($K_e = 0$) has been studied by Gelenbe [13,15]. To prevent interruption of DP transmission in such sensor nodes, the energy flow into the system should somehow balance the flow of DPs. However, Gelenbe [15] showed an unexpected result that such systems where energy and data flows are exactly balanced exhibit unstable behavior.

Since we assume instantaneous transmission, whenever we have a state $(D(t) > 0, E(t) > 0)$, it will immediately transit to either $(0, E(t) - D(t))$ if $E(t) > D(t)$ or $(D(t) - E(t), 0)$ if $D(t) > E(t)$. Thus, the state space S is composed of integer pairs (d,e) such that:

$$S = \{(0,0), (0,e), (d,0) : d, e > 0\}. \quad \mathbf{(1)}$$

If we assume that energy storage and data buffer both have finite capacity, the system can be modeled as finite Markov chain whose state transitions can be seen in Figure 1. Moreover, $[D(t), E(t) : t \geq 0]$ is an irreducible and aperiodic process, so that the stationary probabilities $p(d, e) = \lim_{t\to\infty} \Pr[D(t) = d, E(t = e)]$ uniquely exist and can be calculated

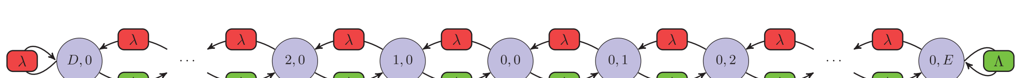

FIGURE 1. Random walk model for the system states.

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by using the following balance equations:

$$p(0,0)[\lambda+\Lambda] = \lambda p(0,1) + \Lambda p(1,0), \tag{2}$$

$$p(d,0)[\lambda+\Lambda] = \lambda p(d-1,0) + \Lambda p(d+1,0), \quad B > n > 0 \tag{3}$$

$$p(0,e)[\lambda+\Lambda] = \lambda p(0,e+1) + \Lambda p(0,e-1), \quad E > m > 0 \tag{4}$$

$$p(0,E)[\lambda] = \Lambda p(0,E-1), \tag{5}$$

$$p(B,0)[\Lambda] = \lambda p(B-1,0). \tag{6}$$

Thus, the stationary probability distributions are calculated as:

$$p(d,0) = p(0,0)\alpha^d, \quad B \geq d > 0, \tag{7}$$

$$p(0,e) = p(0,0)\beta^e, \quad E \geq e > 0, \tag{8}$$

$$p(0,0) = \frac{2-\alpha-\beta}{\alpha^D(1-\alpha)+\beta^E(1-\beta)} = \frac{\alpha^E(1-\alpha)}{1-\alpha^{D+E+1}}, \tag{9}$$

where $\alpha = \frac{\lambda}{\Lambda}$ and $\beta = \frac{\Lambda}{\lambda}$. Also, we can express the marginal probabilities for the queue length of DPs and EPs as:

$$p_d(d) = \sum_{e=0}^{\infty} p(d,e) = p(d,0) = \alpha^d p(0,0), \quad d > 0, \tag{10}$$

$$p_e(e) = \sum_{d=0}^{\infty} p(d,e) = p(0,e) = \beta^e p(0,0), \quad e > 0, \tag{11}$$

and

$$p_d(0) = \sum_{e=0}^{\infty} p(0,e) = \frac{1-\beta^{E+1}}{1-\beta} p(0,0), \tag{12}$$

$$p_e(0) = \sum_{d=0}^{\infty} p(d,0) = \frac{1-\alpha^{D+1}}{1-\alpha} p(0,0). \tag{13}$$

3.1. DP and EP Losses Due to Finite Storage Capacities

Since the energy storage and data buffer are bounded by some finite capacity, the system will have some amount of EP and DP losses. The loss rates L_d and L_e in DP and EP per second can be computed as:

$$L_d = \lambda \sum_{e=0}^{\infty} p(D,e) = \lambda p(D,0) = \frac{\lambda-\Lambda}{1-\beta^{D+E+1}}, \tag{14}$$

$$L_e = \Lambda \sum_{d=0}^{\infty} p(d,E) = \Lambda p(0,E) = \frac{\Lambda-\lambda}{1-\alpha^{D+E+1}}. \tag{15}$$

When buffer sizes are very large, that is, D or/and E tend to infinity, we have the following results:

Case 1: If $\alpha < 1$ or $\beta > 1$, the energy is more than required for the system, so that $L_d \to 0$ and $L_e \to \Lambda - \lambda$.

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Case 2: If $\alpha > 1$ or $\beta < 1$, the energy is less than required for the system, so that $L_d \to \lambda - \Lambda$ and $L_e \to 0$.

Case 3: If $\alpha = \beta = 1$, the energy balances the data, so that $L_d = L_e = \frac{\lambda}{D+E+1} \to 0$.

3.2. System Stability

Stability is a question for such systems where both energy storage and data buffer have unlimited capacity. When the number of DPs (EPs) is finite with probability one as $t, D, E \to \infty$, the system is then called stable with respect to DPs (EPs). A system is called stable if it is stable with respect to both DPs and EPs.

In the steady state, the backlog probabilities of DPs and EPs not exceeding some finite values $D' < D$ and $E' < E$ are as follows:

$$P_d(D') = \lim_{t\to\infty} \Pr[0 \le D(t) \le D' \le D] \tag{16}$$

$$= p_d(0) + \sum_{d=1}^{D'} p_d(d) \tag{17}$$

$$= \frac{\alpha^{D'}(1-\alpha) + \beta^E(1-\beta)}{\alpha^D(1-\alpha) + \beta^E(1-\beta)} \tag{18}$$

and

$$P_e(E') = \lim_{t\to\infty} \Pr[0 \le E(t) \le E' \le E] \tag{19}$$

$$= p_e(0) + \sum_{e=1}^{E'} p_e(e) \tag{20}$$

$$= \frac{\alpha^D(1-\alpha) + \beta^{E'}(1-\beta)}{\alpha^D(1-\alpha) + \beta^E(1-\beta)}. \tag{21}$$

These lead to the following results:

Case 1: If $\alpha < 1$ or $\beta > 1$ as $D, E \to \infty$, $P_d(D') \to 1$ and $P_e(E') \to 0$, so that the system is stable with respect to DPs and is unstable with respect to EPs.

Case 2: If $\alpha > 1$ or $\beta < 1$ as $D, E \to \infty$, $P_d(D') \to 0$ and $P_e(E') \to 1$, so that the system is unstable with respect to DPs and is stable with respect to EPs.

These two results tell us that as long as $\alpha > 1$ or $\beta > 1$, equivalently $\lambda \neq \Lambda$, the system shows an unstable behavior.

Case 3: If $\alpha = \beta = 1$ as $D, E \to \infty$, one can expect it is the best operating point for the system, since equal flowing rates somehow indicate that the energy amount into system matches the energy needed for transmitting all DPs.

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However, when $\alpha = \beta = 1$, the expression for $p(0,0)$ is an indeterminate form. The following is obtained by using L'Hopital's rule:

$$\lim_{\alpha\to 1} p(0,0) = \lim_{\alpha\to 1} \frac{\alpha^E(1-\alpha)}{1-\alpha^{D+E+1}} = \frac{1}{D+E+1}. \quad \textbf{(22)}$$

Accordingly, we can calculate the following:

$$\lim_{\alpha\to 1} p_d(d) = \lim_{\alpha\to 1} p(0,0), \quad \textbf{(23)}$$

$$\lim_{\alpha\to 1} p_e(e) = \lim_{\alpha\to 1} p(0,0), \quad \textbf{(24)}$$

and

$$\lim_{\alpha\to 1} p_d(0) = (E+1) \lim_{\alpha\to 1} p(0,0), \quad \textbf{(25)}$$

$$\lim_{\alpha\to 1} p_e(0) = (D+1) \lim_{\alpha\to 1} p(0,0). \quad \textbf{(26)}$$

Therefore, the backlog probabilities when $\alpha = 1$:

$$\lim_{\alpha\to 1} P_d(D') = \frac{D'+E+1}{D+E+1}, \quad \textbf{(27)}$$

$$\lim_{\alpha\to 1} P_e(E') = \frac{D+E'+1}{D+E+1}. \quad \textbf{(28)}$$

Thus, as $D, E \to \infty$ both $P_d(D'), P_e(E') \to 0$, so that the system is unstable.

An interesting result shows that any relation between λ and Λ cannot provide a stable behavior for such systems with unlimited capacities.

3.3. DP Re-transmission

After an EP is used for transmitting a DP, the re-transmission of the same DP might be needed because of errors caused by noise and interference. Similarly, re-transmission is also used for reducing transmission errors by sending several duplicates of same DP. To model these effects, we define the following probabilities:

- π indicates the probability that an EP arrival is not sufficient to transmit a DP waiting in the queue. Thus, the DP remains in the node and waiting for another EP to independently repeat the transmission process.
- When a DP arrives in the node, the transmission takes place immediately by consuming a stored EP. However, transmission may fail and another EP might be used for re-transmission with a probability of p. This process may continue until providing a successful transmission.

According to these probabilities, we have the following transition rates:

- $\Lambda\pi : (d,0) \to (d,0),\ d > 0$. An EP arrives in the node, but it is not sufficient to transmit DP with probability π, so that the DP remains at the node.
- $\Lambda(1-\pi) : (d,0) \to (d-1,0),\ d > 0$. DP transmission occurs with probability $1-\pi$ when an EP arrives in the node.
- $\lambda p : (0,1) \to (1,0)$. A stored EP is consumed for the transmission, but it fails with probability p when a DP arrives in the node.

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- $\lambda(1-p) : (0,e) \to (0,e-1),\ e>0$. After a DP arrival, transmission will successfully occur with probability $1-p$ by consuming an EP in the storage.
- $\lambda p^{k-1}(1-p) : (0,e) \to (0,e-k),\ e \geq k$ After $k-1$ unsuccessful attempts at transmission and a successful one at the kth attempt with probability $p^{k-1}(1-p)$. Total k EPs are consumed for transmission.
- $\lambda p^{k} : (0,e) \to (1,0),\ \ e=k$. All EPs are depleted after a DP arrival, so that all transmission attempts will fail and one DP will take place in the buffer to be transmitted.

These transitions lead to the balance equations below:

$$p(0,0)[\lambda+\Lambda] = \lambda \sum_{l=1}^{\infty} p^{l-1}(1-p)p(0,l) + \Lambda(1-\pi)p(1,0), \tag{29}$$

$$p(1,0)[\lambda+\Lambda(1-\pi)] = \lambda \sum_{l=0}^{\infty} p^{l}p(0,l) + \Lambda(1-\pi)p(2,0), \tag{30}$$

$$p(d,0)[\lambda+\Lambda(1-\pi)] = \lambda p(d-1,0) + \Lambda(1-\pi)p(d+1,0), \quad d>1, \tag{31}$$

$$p(0,e)[\lambda+\Lambda] = \lambda \sum_{l=1}^{\infty} p^{l-1}(1-p)p(0,e+l) + \Lambda p(0,e-1), \quad e>0. \tag{32}$$

We can reach the following results by using the above equations:

THEOREM 1: *If $p>\pi\geq 0$ and $\Lambda(1-p)<\lambda<\Lambda(1-\pi)$, a stationary distribution exists, and is of the form:*

$$p(0,e) = p(0,0)Q^{e}, \quad e>0, \tag{33}$$

$$p(d,0) = p(1,0)q^{d-1}, \quad d>0, \tag{34}$$

$$Q = \frac{\Lambda}{\lambda+p\Lambda}, \tag{35}$$

$$q = \frac{\lambda}{\Lambda(1-\pi)}, \tag{36}$$

$$p(1,0) = \frac{\lambda+p\Lambda}{\Lambda(1-\pi)}p(0,0), \tag{37}$$

$$p(0,0) = \frac{[\Lambda(1-\pi)-\lambda][\lambda-\Lambda(1-p)]}{\Lambda(p-\pi)[\lambda+p\Lambda]}. \tag{38}$$

PROOF OF THEOREM 1: To proceed with the proof, we substitute Eq. (33) in balance equations stated above:

$$p(0,0)[\lambda+\Lambda] = \frac{\lambda Q(1-p)}{1-pQ}p(0,0) + \Lambda(1-\pi)p(1,0), \tag{39}$$

$$p(1,0)[\lambda+\Lambda(1-\pi)] = \frac{\lambda}{1-pQ}p(0,0) + \Lambda(1-\pi)qp(1,0), \tag{40}$$

$$\lambda+\Lambda(1-\pi) = \frac{\lambda}{q} + \Lambda(1-\pi)q, \tag{41}$$

$$\lambda+\Lambda = \frac{\lambda Q(1-p)}{1-pQ} + \frac{\Lambda}{Q}. \tag{42}$$

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Substituting the values of Q and p:

$$p(0,0)[\lambda+\Lambda] = \Lambda(1-p)p(0,0) + \Lambda(1-\pi)p(1,0), \tag{43}$$

$$p(1,0)[\lambda+\Lambda(1-\pi)] = [\lambda+p\Lambda]p(0,0) + \lambda p(1,0), \tag{44}$$

$$\lambda+\Lambda(1-\pi) = \Lambda(1-\pi)+\lambda, \tag{45}$$

$$\lambda+\Lambda = \Lambda(1-p)+\lambda+p\Lambda. \tag{46}$$

The first two equations reduce to:

$$p(1,0) = \frac{\lambda+p\Lambda}{\Lambda(1-\pi)}p(0,0). \tag{47}$$

Using the fact that the probabilities must sum to one we now have:

$$p(0,0) = \frac{[\Lambda(1-\pi)-\lambda][\lambda-\Lambda(1-p)]}{\Lambda(p-\pi)[\lambda+p\Lambda]}, \tag{48}$$

completing the proof. ■

4. ENERGY AND DATA LOSSES THROUGH STANDBY AND LEAKAGE

Such systems as found in Section 3 store EPs in energy buffers. In fact, every storage loses some amount of energy due to the self-discharging nature of capacitors and batteries. We can model this effect with random EP leakage at a rate of μ when there is no DP waiting to be transmitted. Such systems with energy leakage imperfections have been studied by Gelenbe and Kadioglu [22,23]. In addition to energy losses, we also assume that DPs waiting in the buffer discard themselves at a rate of γ when there is no energy in the node. After introducing the data and energy leakages into the system, the state transitions can be seen in Figure 2. Thus, we may write equilibrium equations as follows:

$$p(0,0)[\lambda+\Lambda] = (\Lambda+\gamma)p(1,0) + (\lambda+\mu)p(0,1), \tag{49}$$

$$p(d,0)[\lambda+\Lambda+\gamma] = (\Lambda+\gamma)p(d+1,0) + \lambda p(d-1,0), \tag{50}$$

$$p(D,0)[\Lambda+\gamma] = \lambda p(D-1,0), \tag{51}$$

$$p(0,e)[\lambda+\Lambda+\mu] = \Lambda p(0,e-1) + (\lambda+\mu)p(0,e+1), \tag{52}$$

$$p(0,E)[\lambda+\mu] = \Lambda p(0,E-1). \tag{53}$$

Similar to the previous analysis, the stationary probability distributions can be calculated as:

$$p(d,0) = p(0,0)\,\alpha^d, \quad B \geq d > 0, \tag{54}$$

$$p(0,e) = p(0,0)\,\beta^e, \quad E \geq e > 0, \tag{55}$$

$$p(0,0) = \frac{1-\alpha-\beta+\alpha\beta}{\alpha^{D+1}(\beta-1)+\beta^{E+1}(\alpha-1)+1-\alpha\beta}, \tag{56}$$

where $\alpha = \frac{\lambda}{\Lambda+\gamma}$ and $\beta = \frac{\Lambda}{\lambda+\mu}$.

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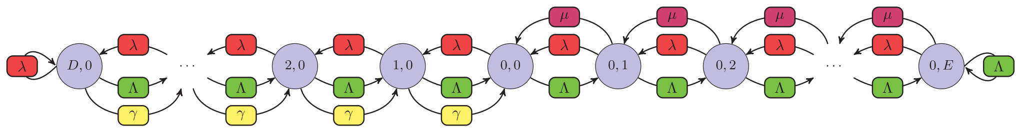

FIGURE 2. Random walk model with energy and data losses

4.1. DP and EP Losses Due to Finite Storage Capacities

After similar analysis in Section 3.1, the following results can easily be obtained:

$$L_d = \lambda p(D,0) = \frac{\Lambda\beta^E(1-\alpha-\beta+\alpha\beta)}{\alpha^{B+1}(\beta-1)+\beta^{E+1}(\alpha-1)+1-\alpha\beta}, \tag{57}$$

$$L_e = \Lambda p(0,E) = \frac{\lambda\alpha^D(1-\alpha-\beta+\alpha\beta)}{\alpha^{D+1}(\beta-1)+\beta^{E+1}(\alpha-1)+1-\alpha\beta}. \tag{58}$$

For the assumption of very large buffer sizes, that is, both D and E tend to infinity, the following cases can be considered:

Case 1: If $\alpha > 1$ and hence $\beta < 1$ or equivalently $\Lambda < \lambda$, so that the energy flow is not sufficient for the data flow and $L_e \to 0$ and $L_d \to \lambda - (\Lambda + \gamma)$.

Case 2: If $\beta > 1$ and hence $\alpha < 1$ or equivalently $\Lambda > \lambda$, so that the energy flow is more than required, and $L_e \to \Lambda - (\lambda + \mu)$ and $L_d \to 0$.

Case 3: If $\alpha < 1$ and $\beta < 1$, in this case there is no leakage for both buffers, and $L_e \to 0$ and $L_d \to 0$.

Case 4: If $\alpha = 1$ and $\beta < 1$, the expressions for L_e and L_d are in indeterminate form. However, after applying some algebra we get $L_e \to 0$ and $L_d \to 0$.

Case 5: If $\alpha < 1$ and $\beta = 1$, the expressions for L_e and L_d are again in indeterminate form. However, after applying some algebra we get $L_e \to 0$ and $L_d \to 0$.

4.2. Optimum Energy Efficiency of the Transmission

The sensor we consider receives Λ EPs/sec (in power units, e.g. milliwatts) from ES; however, it cannot use all this energy due to finite capacity energy loss and the energy leakage of the system. Similarly, the sensor cannot transmit all the DPs gathered from the environment due to finite capacity data loss and data leakage. Thus, its energy consumption per effectively transmitted packet is:

$$\sigma = \frac{\Lambda - f(\Lambda)}{\lambda - g(\Lambda)}. \tag{59}$$

where $f(\Lambda) = \mu\sum_{e=0}^{\infty} p(0,e) + L_e$ and $g(\Lambda) = \gamma\sum_{d=0}^{\infty} p(d,0) + L_d$ Thus, it is of interest to see what the best operating point may be for this system, in terms of the energy it is consuming. We therefore take the derivative of various terms in the expression with respect

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to Λ and see that:

$$\sigma' = \frac{(1 - f'(\Lambda))(\lambda - g(\Lambda)) + g'(\Lambda)(\Lambda - f(\Lambda))}{(\lambda - g(\Lambda))^2}, \quad (60)$$

so that the extremum for σ is reached for the value of Λ which gives:

$$(f'(\Lambda) - 1)(\lambda - g(\Lambda)) = g'(\Lambda)(\Lambda - f(\Lambda)). \quad (61)$$

In addition, we have

$$\sigma'' = \frac{[\lambda - g(\Lambda)][(g(\Lambda) - \lambda)f''(\Lambda) + (\Lambda - f(\Lambda))g''(\Lambda)]}{(\lambda - g(\Lambda))^3} \quad (62)$$

$$+ \frac{-2[\lambda - g(\Lambda)][(f'(\Lambda) - 1)g'(\Lambda)] + 2[(\Lambda - f(\Lambda))g'(\Lambda)^2]}{(\lambda - g(\Lambda))^3}, \quad (63)$$

so that at the value of Λ which satisfies Eq. (61) we have:

$$\sigma'' = \frac{[(g(\Lambda) - \lambda)f''(\Lambda) + (\Lambda - f(\Lambda))g''(\Lambda)]}{(\lambda - g(\Lambda))^2}. \quad (64)$$

When we take derivative of Eq. (61) we have:

$$(g(\Lambda) - \lambda)f''(\Lambda) + (\Lambda - f(\Lambda))g''(\Lambda) = 0 \quad (65)$$

so that Eq. (61) is a point of inflection of the efficiency function σ.

Figure 3 shows σ versus Λ for the case of $\lambda = 10,\ B = 100$ and $E = 100$, with μ and $\gamma = 0.1$. We see that to keep energy efficiency high, that is, to have σ as low as possible, the power Λ that is supplied from harvesting should remain *above* the nominal need to satisfy all the flow λ of DPs that are being harvested.

4.3. System Stability

After similar analysis in Section 3.2, the following results can easily be obtained:

$$P_d(D') = \frac{\alpha^{D'+1}(\beta - 1) + \beta^{E+1}(\alpha - 1) + 1 - \alpha\beta}{\alpha^{D+1}(\beta - 1) + \beta^{E+1}(\alpha - 1) + 1 - \alpha\beta}, \quad (66)$$

$$P_e(E') = \frac{\alpha^{D+1}(\beta - 1) + \beta^{E'+1}(\alpha - 1) + 1 - \alpha\beta}{\alpha^{D+1}(\beta - 1) + \beta^{E+1}(\alpha - 1) + 1 - \alpha\beta}. \quad (67)$$

This leads to the following as $E \to \infty$ and $D \to \infty$:

Case 1: If $\alpha > 1$ and hence $\beta < 1$, $P_d(D') \to 0$ for all finite D' and $P_e(E') \to 1$ for all finite E'; the system is stable with respect to EPs and unstable with respect to DPs.

Case 2: If $\beta > 1$ and hence $\alpha < 1$, $P_d(D') \to 1$ for all finite D' and $P_e(E') \to 0$ for all finite E'; the system is stable with respect to DPs and unstable with respect to EPs.

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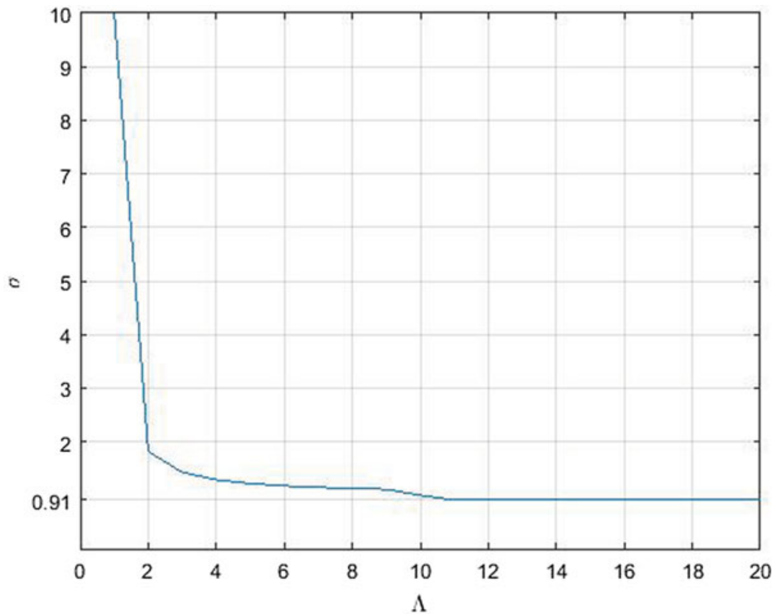

FIGURE **3.** σ as a function of Λ.

Case 3: If $\alpha < 1$ and $\beta < 1$,

$$P_d(E') \to \frac{\alpha^{E'+1}(\beta - 1) + 1 - \alpha\beta}{1 - \alpha\beta}$$

and it is obviously in the interval (0,1), since $-1 < \alpha^{D'+1}(\beta - 1) < 0$ for all finite D' and similarly

$$P_e(E') \to \frac{\beta^{E'+1}(\alpha - 1) + 1 - \alpha\beta}{1 - \alpha\beta}$$

and similarly it is in the interval (0, 1), for all finite E'. Therefore, the system is unstable with respect to both DPs and EPs.

Case 4: If $\alpha = 1$ and hence $\beta < 1$, the expression p(0, 0) is an indeterminate form, so that we apply L'Hospital's rule and obtain:

$$\lim_{(D,E)\to\infty} \left[\lim_{\alpha\to 1} p(0,0) = \frac{\beta - 1}{(D+1)(\beta - 1) + \beta^{E+1} - \beta} \right] \to 0, \tag{68}$$

$$\lim_{\substack{(D,E)\to\infty \\ \alpha\to 1}} \left[P_d(D') = p(0,0) \left(\sum_{m=0}^{E} \beta^m + \sum_{n=1}^{D'} \alpha^n \right) \right] \to 0. \tag{69}$$

A similar analysis can be made for the $P_e(E')$, which leads $P_d(D') \to 0$ for all finite G and $P_e(E') \to 0$ for all finite E'. Therefore, the system is unstable with respect to both DPs and EPs.

Case 5: If $\beta = 1$ and hence $\alpha < 1$, the expression p(0,0) is again an indeterminate form, so that similar from the previous analysis in case 4 we may have $P_d(D') \to 0$ for all finite D' and $P_e(E') \to 0$ for all finite E'; the system is unstable with respect to both DPs and EPs.

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4.4. DP Re-transmission

After considering the same error probabilities and state transitions explained in Section 3.3, we may write the following equilibrium equations:

$$p(0,0)[\lambda+\Lambda] = \lambda\sum_{l=1}^{\infty} p^{l-1}(1-p)p(0,l) + (\Lambda(1-\pi)+\gamma)p(1,0) + \mu p(0,1), \tag{70}$$

$$p(1,0)[\lambda+\Lambda(1-\pi)+\gamma] = \lambda\sum_{l=0}^{\infty} p^{l}p(0,l) + (\Lambda(1-\pi)+\gamma)p(2,0), \tag{71}$$

$$p(d,0)[\lambda+\Lambda(1-\pi)+\gamma] = \lambda p(d-1,0) + (\Lambda(1-\pi)+\gamma)p(d+1,0)), \quad d>1, \tag{72}$$

$$\begin{aligned} p(0,e)[\lambda+\Lambda+\mu] &= \lambda\sum_{l=1}^{\infty} p^{l-1}(1-p)p(0,e+l) \\ &\quad + \Lambda p(0,e-1) + \mu p(0,e+1), \quad e>0. \end{aligned} \tag{73}$$

THEOREM 2: *If* $(\Lambda-\mu)(1-p) < \lambda < \Lambda(1-\pi)+\gamma$, *the stationary distribution exists and is given by:*

$$p(0,e) = p(0,0)Q^{e}, \quad e>0, \tag{74}$$

$$p(d,0) = p(1,0)q^{d-1}, \quad d>0, \tag{75}$$

$$Q = \frac{\lambda+\mu+\Lambda p - \sqrt{(\lambda+\mu+\Lambda p)^2 - 4\mu\Lambda p}}{2\mu p}, \tag{76}$$

$$q = \frac{\lambda}{\Lambda(1-\pi)+\gamma}, \tag{77}$$

$$p(1,0) = \frac{q}{(1-pQ)}p(0,0), \tag{78}$$

$$p(0,0) = \frac{(1-q)(1-Q)(1-pQ)}{q(1-Q)+(1-q)(1-pQ)}. \tag{79}$$

PROOF OF THEOREM 1: To proceed with the proof, we substitute Eq. (74) into Eq. (73), which after applying some algebra becomes:

$$Q^{e}[\lambda+\Lambda+\mu] = (\lambda(1-p)+\mu)Q^{e+1}\frac{1}{1-pQ} + \Lambda Q^{e-1}, \tag{80}$$

$$0 = (Q-1)[Q^2(\mu p) + Q(-\Lambda p - \lambda - \mu) + \Lambda], \tag{81}$$

so that we have

$$Q_{1,2} = \frac{\lambda+\mu+\Lambda p \pm \sqrt{(\lambda+\mu+\Lambda p)^2 - 4\mu\Lambda p}}{2\mu p}, \tag{82}$$

note that Q has to be smaller than 1, while:

$$Q_1 = \frac{\lambda+\mu+\Lambda p + \sqrt{(\lambda+\mu+\Lambda p)^2 - 4\mu\Lambda p}}{2\mu p} \geq \frac{\lambda+\mu+\Lambda p}{2\mu p} > \frac{1}{2}\left(\frac{1}{p}+\frac{\Lambda}{\mu}\right) > 1, \tag{83}$$

where $p<1$ and $\mu<\Lambda$.

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Since $Q_2 1$ has to be smaller than 1, we have:

$$\frac{\lambda+\mu+\Lambda p-\sqrt{(\lambda+\mu+\Lambda p)^2-4\mu\Lambda p}}{2\mu p}<1, \tag{84}$$

$$(\lambda+\mu+\Lambda p-2\mu p)^2<(\lambda+\mu+\Lambda p)^2-4\mu\Lambda p, \tag{85}$$

$$\Lambda+\mu p<(\lambda+\mu+\Lambda p), \tag{86}$$

$$(\Lambda-\mu)(1-p)<\lambda. \tag{87}$$

Equation (72) has a solution of the form $q^{d-1}p(1,0)$ where

$$q=\frac{\lambda}{\Lambda(1-\pi)+\gamma}.$$

Since we also have $q<1$:

$$\lambda<\Lambda(1-\pi)+\gamma. \tag{88}$$

Also, using the fact that the probabilities must sum to one, we have:

$$p(0,0)+p(1,0)+\sum_{d=2}^{\infty}p(d,0)+\sum_{e=1}^{\infty}p(0,e)=1, \tag{89}$$

$$p(0,0)\left[1+R+R\frac{q}{1-q}+\frac{Q}{1-Q}\right]=1, \tag{90}$$

$$p(0,0)\left[R\frac{1}{1-q}+\frac{1}{1-Q}\right]=1. \tag{91}$$

where $R=\frac{p(1,0)}{p(0,0)}$. Thus, we have:

$$p(1,0)=\frac{q}{(1-pQ)}p(0,0), \tag{92}$$

$$p(0,0)=\frac{(1-q)(1-Q)(1-pQ)}{q(1-Q)+(1-q)(1-pQ)}. \tag{93}$$

■

Based on the above results, we notice that:

- The *probabilities that the data queue is empty and non-empty are*:

$$P[d=0]=\frac{1}{1+\frac{q}{1-q}\frac{1-Q}{1-pQ}}, \tag{94}$$

$$P[d>0]=\frac{1}{1+\frac{1-q}{q}\frac{1-pQ}{1-Q}}. \tag{95}$$

- The *probabilities that the energy queue is empty and non-empty are*:

$$P[e=0]=\frac{q(1-Q)+(1-q)(1-pQ)(1-Q)}{q(1-Q)+(1-q)(1-pQ)}, \tag{96}$$

$$P[e>0]=\frac{Q(1-q)(1-pQ)}{q(1-Q)+(1-q)(1-pQ)}. \tag{97}$$

- Obviously, $P[d\geq 0]=P[e\geq 0]=1$.

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5. PACKET TRANSMISSION VIA IMPERFECT TRANSMITTER

In Sections 3 and 4, it is assumed that the node has a perfect transmitter so that the harvested energy is only consumed for data transmission, not for the node electronics. However, more practical models also consume energy for sensing information from the ambient environment, processing and generating DPs, and storing packets in the node. A more realistic scenario with an imperfect transmitter has been studied by Kadioglu [31].

It is assumed that DP transmission occurs by consuming one EP ($K_t = 1$). However, to sense, process, and store data, a node also needs to consume another EP ($K_e = 1$). Thus, without storing single an EP the sensor node cannot sense the data arrival and the information will be lost. If there is only one EP in the storage, it is consumed for node electronics and the DP is queued in the buffer. Whenever two or more EPs stored in the node, DP transmission occurs immediately.

When we consider such a system model, an unbounded rise of DPs or EPs is not allowed since the system has a finite state space. In fact, when one DP arrives to the node whose state is $(D(t) = 0, E(t) = 1)$, the state will change as $(D(t) = 1, E(t) = 0)$ and it is the only state where the data buffer is not empty. Because of such an interesting situation, instead of considering (d, e) integer pairs, we may consider $(d - e)$ as system states. Thus, the state space is:

$$S = \{-1, 0, 1, \ldots, E\} \tag{98}$$

where E is the maximum number of EPs that can be stored in the node. Figure 4 shows the state transitions of the system, including energy leakage at a rate of μ due to the imperfection of energy storage. Thus, the equilibrium equations:

$$p(-1)[\Lambda] = \lambda p(1), \tag{99}$$
$$p(0)[\Lambda] = \Lambda p(-1) + \lambda\, p(2) + \mu p(1), \tag{100}$$
$$p(N)[\Lambda + \lambda + \mu] = \Lambda p(N-1) + \lambda p(N+2) + \mu p(N+1), \tag{101}$$
$$p(E-1)[\Lambda + \lambda + \mu] = \Lambda p(E-2) + \mu p(E), \tag{102}$$
$$p(E)[\lambda + \mu] = \Lambda p(E-1). \tag{103}$$

Equation (101) has a solution of the form:

$$p(N) = c\varphi^N \tag{104}$$

where c is an arbitrary constant, $0 < N < E - 1$, and φ can be computed as:

$$\varphi = \frac{-(\lambda + \mu) + \sqrt{(\lambda + \mu)^2 + 4\Lambda\lambda}}{2\lambda}. \tag{105}$$

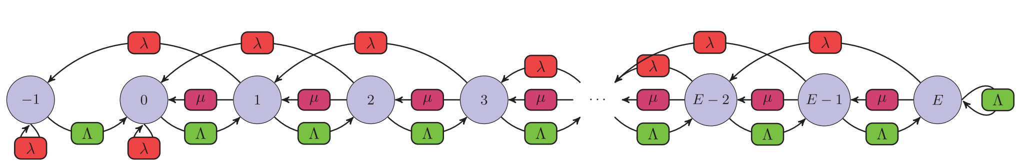

FIGURE 4. Random walk model for $K_e = 1$.

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Thus, we may reach following conclusion:

$$p(-1) = c\frac{\lambda}{\Lambda}\varphi, \tag{106}$$

$$p(0) = c\left(\frac{\lambda}{\Lambda}\varphi^2 + \frac{\lambda+\mu}{\Lambda}\varphi\right), \tag{107}$$

$$p(E-1) = c\left[1 + \frac{\lambda+\mu}{\Lambda} - \frac{\mu}{\lambda+\mu}\right]^{-1}\varphi^{E-2}, \tag{108}$$

$$p(E) = c\left[\left(\frac{\lambda+\mu}{\Lambda}\right)\left(\frac{\Lambda+\lambda+\mu}{\Lambda}\right) - \frac{\mu}{\Lambda}\right]^{-1}\varphi^{E-2}. \tag{109}$$

Using the fact that summation of the probabilities is one:

$$\sum_{N=-1}^{E} p(N) = c\left(\frac{2\lambda+\mu}{\Lambda}\varphi + \frac{\lambda}{\Lambda}\varphi^2\right) + c\sum_{N=1}^{E-2}\varphi^N + c\left[\frac{\lambda+\mu}{\Lambda} - \frac{\mu}{\Lambda+\lambda+\mu}\right]^{-1}\varphi^{E-2} = 1. \tag{110}$$

After further calculations, we may reach the value for c:

$$c = \left[\frac{2\lambda+\mu}{\Lambda}\varphi + \frac{\lambda}{\Lambda}\varphi^2 + \frac{\varphi - \varphi^{E-1}}{1-\varphi} + \frac{\Lambda(\Lambda+\lambda+\mu)\varphi^{E-2}}{(\lambda+\mu)(\Lambda+\lambda+\mu) - \mu\Lambda}\right]^{-1}. \tag{111}$$

5.1. DP and EP Losses

Due to finite capacity, loss rates can be calculated as follows:

$$\Gamma_d = \lambda\sum_{N=0}^{-D} p(N) = \lambda(p(0) + p(-1)) = c\lambda\left(\frac{2\lambda+\mu}{\Lambda}\varphi + \frac{\lambda}{\Lambda}\varphi^2\right), \tag{112}$$

$$\Gamma_e = \Lambda p(E) = c\left[\frac{1}{\Lambda}\left[\left(\frac{\lambda+\mu}{\Lambda}\right)\left(\frac{\Lambda+\lambda+\mu}{\Lambda}\right) - \frac{\mu}{\Lambda}\right]\right]^{-1}\varphi^{E-2}. \tag{113}$$

Since $(d = 1, e = 0)$ is the only state in which the data buffer is not empty, a greater number of excessive DPs is expected for such systems. However, we can observe that the DP loss rate remains low for the $\lambda < \Lambda$ in Figure 5 where we assume, $\Lambda = 10, \mu = 1, E = 100$. After reaching the $\lambda = \Lambda$ threshold, the DP loss rate increases exponentially so that operating as $\lambda < \Lambda$ prevents a massive amount of DP losses.

5.2. System Stability

We need to reconsider the model with unlimited capacity buffers to study the stability of the system. Thus, we may have:

$$p(-1) = c'\frac{\lambda}{\Lambda}\varphi, \tag{114}$$

$$p(0) = c'\left(\frac{\lambda}{\Lambda}\varphi^2 + \frac{\lambda+\mu}{\Lambda}\varphi\right), \tag{115}$$

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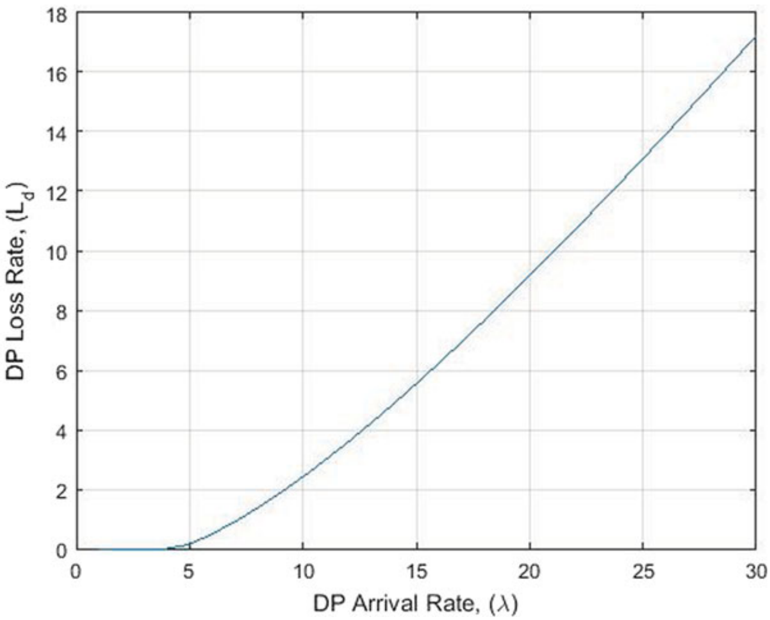

FIGURE 5. DP loss rate L_d versus DP arrival rate λ.

$$p(N) = c'\varphi^N, \tag{116}$$

$$\varphi = \frac{-(\lambda+\mu)+\sqrt{(\lambda+\mu)^2+4\Lambda\lambda}}{2\lambda}. \tag{117}$$

$$c' = \frac{(\lambda+\mu-2\Lambda)+\sqrt{(\lambda+\mu)^2+4\Lambda\lambda}}{2(2\lambda+\mu)}. \tag{118}$$

We can also express the backlog probabilities as:

$$P_d(D') = \sum_{d=0}^{D'}\sum_{e=0}^{\infty} p(e-d) = p_d(1)+p_d(0) = p(-1)+p(0)+\sum_{N=1}^{\infty} c'\varphi^N = 1. \tag{119}$$

and

$$\begin{aligned} P_e(E') &= \sum_{e=0}^{E'}\sum_{d=0}^{\infty} p(e-d) = p_e(0)+p_e(e)1[e>0] \\ &= p(-1)+p(0)+\sum_{N=1}^{E'} c'\varphi^N = 1-c'\frac{\varphi^{E'+1}}{1-\varphi}. \end{aligned} \tag{120}$$

Thus, we can conclude that the system with unlimited storage capacities is always stable with respect to DPs and unstable with respect to EPs.

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5.3. Transmission Errors

Since the energy rate is calculated in units of power, the harvested energy rate Λ is the total power entering the sensor node. Thus, the total power consumed by the node is:

$$\xi = \Lambda - L_e - \mu \sum_{N=1}^{E} p(N), \tag{121}$$

where all power cannot be used due to EP leakages and excessive EPs. The total radiating power from a sensor on average is simply:

$$\phi = \frac{\xi}{2}, \tag{122}$$

since half of the power is used for node electronics. The probability of correctly receiving a DP transmitted with power level K_{t} is:

$$1 - e = f\left(\frac{\eta K_t}{I + B}\right), \tag{123}$$

where f is some increasing function of its argument, I is the interference level, B is the noise level, and $0 \leq \eta \leq 1$ represents the propagation factor of the transmission power.

The total interference level for a single node among M identical sensor nodes is:

$$I = I_1 + I_2 = \eta\frac{\xi}{2}\kappa_0(M-1) + \eta\frac{\xi}{2}\left(\frac{M-m}{M}\right)1[M > m]. \tag{124}$$

where κ_0 is a small valued factor representing the side-band effect, $I_1 = \eta(\xi/2)\kappa_0(M-1)$ is the interference level if the total number of sensor nodes is less than the number of separate frequency channels m, and

$$I_2 = \eta\frac{\xi}{2}\left(\frac{M-\alpha}{M}\right)1[M > m]$$

is the additional interference level if the number of nodes exceeds the number of frequency channels. Thus, we may rewrite Eq. (123) as:

$$1 - e = f\left(\frac{\eta K_t}{\eta\frac{\xi}{2}\kappa_0(M-1) + \eta\frac{\xi}{2}(\frac{M-m}{M})1[M > m] + B}\right). \tag{125}$$

Obviously, the transmission error will rise with an increase in the number of sensor nodes in the network due to greater effect of the interference over the transmission. On the other hand, after a certain number of sensor nodes α is reached, the system will face an additional level of interference I_2, so that the error probabilities will increase. We observe these effects in Figure 6, where we consider single bit transmissions with $\Lambda = 10, \lambda = 10, \mu = 1, E = 100, B = 0.1, \eta = 0.5, \kappa_0 = 0.05, \alpha = 20$ and different values of M. Also, we assume BPSK transmission (see, e.g., Goldsmith [30]), so that:

$$1 - e = Q\left(\sqrt{\frac{\eta K_t}{\eta\frac{\xi}{2}\kappa_0(M-1) + \eta\frac{\xi}{2}(\frac{M-m}{M})1[M > m] + B}}\right), \tag{126}$$

where

$$Q(x) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right].$$

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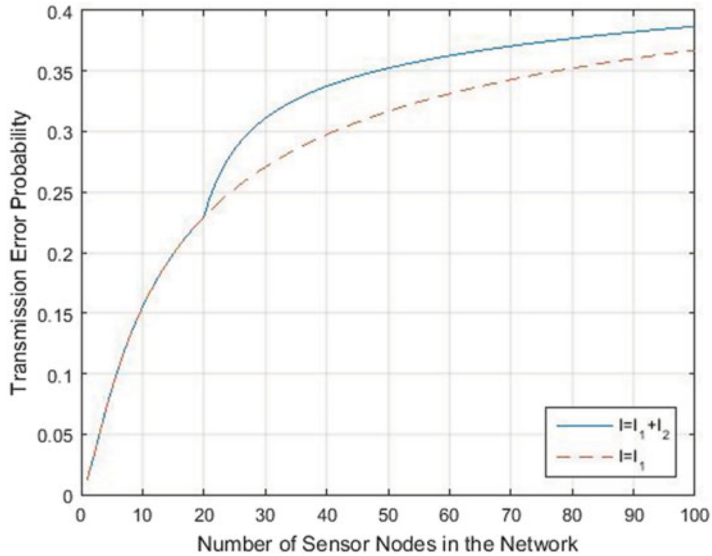

FIGURE 6. Transmission error probability versus number of sensor nodes.

6. DP TRANSMISSION WITH K EPS

In previous sections, a DP can be sent by consuming a single EP, $K_t = 1$, in a sensor node. However, Kadioglu and Gelenbe [32] studied the case where exactly $K > 1$ EPs are needed to successfully transmit a single DP without considering power consumption for node electronics, $K_e = 0$. The reason that a transmitter tends to increase the transmission power level may be that it prevents transmission errors caused by noise and interference. Also, a transmitter may reduce the power level to save energy. Such systems featuring DP transmission by means of several EPs may be modeled as a two-dimensional random walk, which makes analysis harder. In this case, the state space is:

$$S = \{(0,0), (d,0), (0,e), (l,k)\}, \tag{127}$$

where $1 \leq d \leq D, 1 \leq e \leq E, 1 \leq l < D, 1 \leq k < K$. The state transitions can be seen in Figure 7, by which we may write the following equilibrium equations:

$$p(0,0)[\lambda + \Lambda] = \Lambda p(1, K-1) + \lambda p(0, K), \tag{128}$$

$$p(d,0)[\lambda + \Lambda] = \Lambda p(d+1, K-1) + \lambda p(d-1, 0), \quad \mathbf{1 \leq d < D} \tag{129}$$

$$p(D,0)[\Lambda] = \lambda p(D-1, 0), \tag{130}$$

$$p(0,e)[\lambda + \Lambda] = \Lambda p(0, e-1) + \lambda p(0, e+K) 1[E \geq e+K], \quad \mathbf{1 \leq e < E} \tag{131}$$

$$p(0,E)[\lambda] = \Lambda p(0, E-1), \tag{132}$$

$$p(l,k)[\lambda + \Lambda] = \Lambda p(l, k-1) + \lambda p(l-1, k), \quad \mathbf{1 \leq l \leq D-1}, \quad \mathbf{1 \leq k \leq K-1} \tag{133}$$

$$p(D,k)[\Lambda] = \Lambda p(D, k-1) + \lambda p(D-1, k), \quad \mathbf{1 \leq k \leq K-1}. \tag{134}$$

It is not easy to express close-form formulas for the stationary probability distributions by considering the above expressions. Thus, Kadioglu and Gelenbe [32] defined an *one-to-one* and *onto* function to reduce random walk dimension by using cylindrical symmetry

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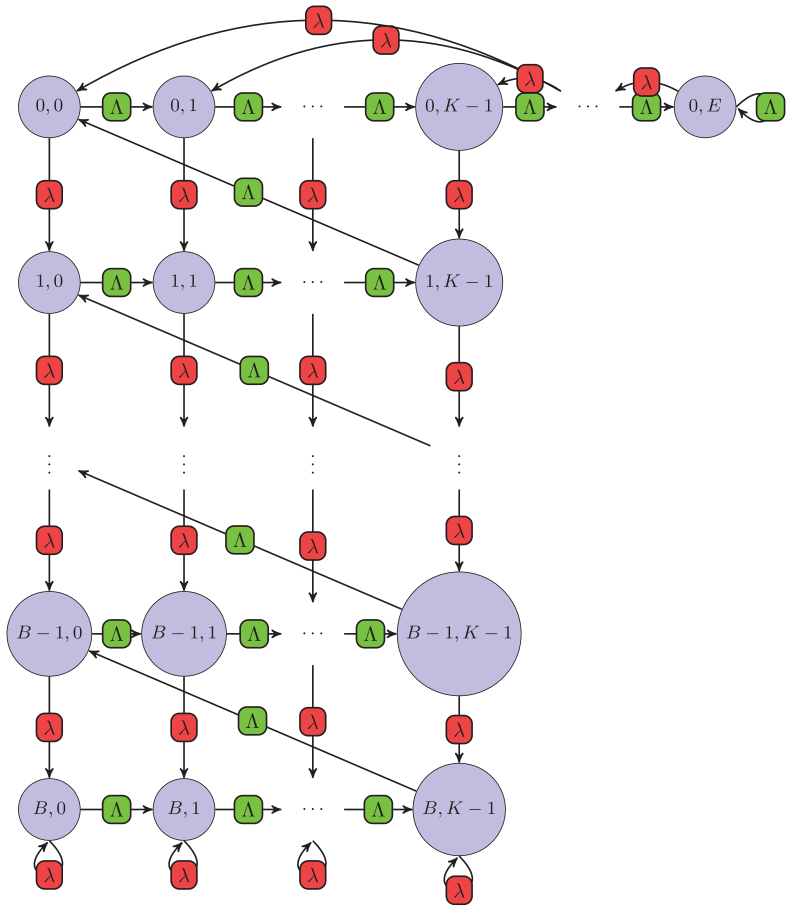

FIGURE 7. Two-dimensional random walk for the state transitions.

among the states, which is:

$$p(d, e) = \tilde{p}(dK - e + E). \quad (135)$$

Figure 8 shows the reduced diagram of the two-dimensional Markov model. The state space is:

$$S = \{0, 1, 2, \ldots, DK + E\}. \quad (136)$$

In order to reduce complication of the analysis, states can be divided into three different regions according to similarities of transition behaviors. Thus, we may write the following equilibrium equations for each region:

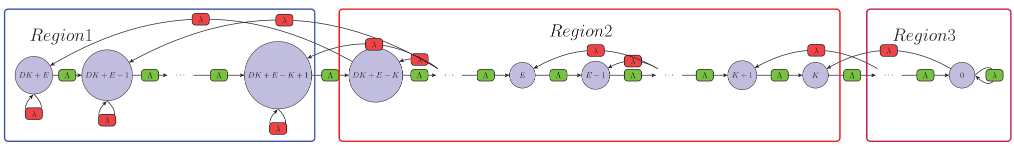

FIGURE 8. One-dimensional random walk model with single index.

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- *Region 1,* $DK + E - K < N \leq DK + E$*:*

$$\tilde{p}(DK+E)\Lambda = \lambda\tilde{p}(DK+E-K), \quad (137)$$

$$\tilde{p}(N)\Lambda = \lambda\tilde{p}(N-K) + \Lambda\tilde{p}(N+1). \quad (138)$$

- *Region 2,* $K \leq N \leq DK + E - K$*:*

$$\tilde{p}(N)[\Lambda+\lambda] = \lambda\tilde{p}(N-K) + \Lambda\tilde{p}(N+1). \quad (139)$$

- *Region 3,* $0 < N < K$*:*

$$\tilde{p}(N)[\Lambda+\lambda] = \Lambda\tilde{p}(N+1), \quad (140)$$

$$\tilde{p}(0)\lambda = \Lambda\tilde{p}(1). \quad (141)$$

After considering equilibrium equations for all regions and assuming a very large data buffer capacity, that is, $D \to \infty$, the closed-form solution is:

$$\tilde{p}(N) = \begin{cases} (1-\Theta)\Theta^N, & K \leq N < \infty, \\ (1-\Theta)\Theta^K \left(\frac{\Lambda}{\Lambda+\lambda}\right)^i, & N = K-i,\ 0 < i < K, \\ (1-\Theta)\Theta^K \frac{\Lambda}{\lambda}\left(\frac{\Lambda}{\lambda+\Lambda}\right)^{K-1}, & N = 0, \end{cases} \quad (142)$$

where Θ is the summation of linearly combined roots of the following equation:

$$\Theta^{K+1} - \left(1+\frac{\lambda}{\Lambda}\right)\Theta^K + \frac{\lambda}{\Lambda} = 0. \quad (143)$$

In fact, Eq. (143) is the characteristic equation of the recurrence relation in Eq. (139).

Note that Eq. (143) cannot be solved in radicals for $K \geq 4$ by the Abel & Ruffini theorem (see, e.g., Zoladek [37]), which means that an expression for the roots of such equations as a function of the coefficients by means of algebraic operations or roots of natural degrees does not exist.

6.1. Transmission Errors

Similar to Section 5.3, the correctly received probability of a single bit among the M identical sensor is:

$$1 - e = Q\left(\sqrt{\frac{\eta K}{\eta\phi\kappa_0(M-1) + \eta\phi(\frac{M-m}{M})1[M > m] + B}}\right), \quad (144)$$

where K is the transmission power level, $\phi = \Lambda - L_e = \Lambda(1 - \tilde{p}(0))$ is the average radiating power from a sensor.

We can observe the effect of a number of sensors on the correctly received probability for different K values in Figure 9, where we assume a single bit transmission with the parameters $\eta = 0.5, \kappa_0 = 0.1, B = 0.1, m = 30, \Lambda = 2\lambda = 1$. Correctly received probability decreases with an increasing number of sensor nodes, while it increases slightly with higher K values.

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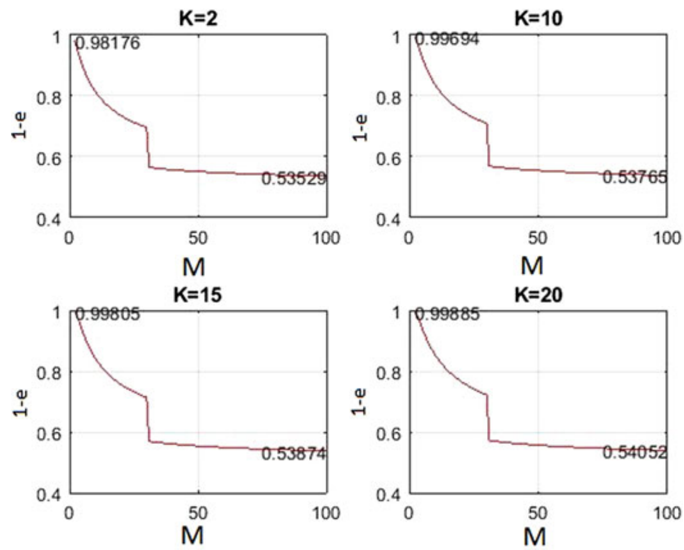

FIGURE 9. Correctly received probability versus number of sensor nodes.

7. DP TRANSMISSION WITH GENERALIZED K_e AND K_t VALUES

In this section, we extend the system model by considering arbitrary $K_e > 1$ and $K_t > 1$ values. The motivation behind this idea lies in the fact that a sensor node may vary the power levels according to speed of processing or transmission errors. This generalization leads us to a two-dimensional random walk model similar to the one studied in Section 6 and state space:

$$S = \{(0,0), (d,0), (0,e), (l,k) : 1 \leq d \leq D, 1 \leq e \leq E, 1 \leq l < D, \\ 1 \leq k < K, K = K_e + K_t\}. \quad (145)$$

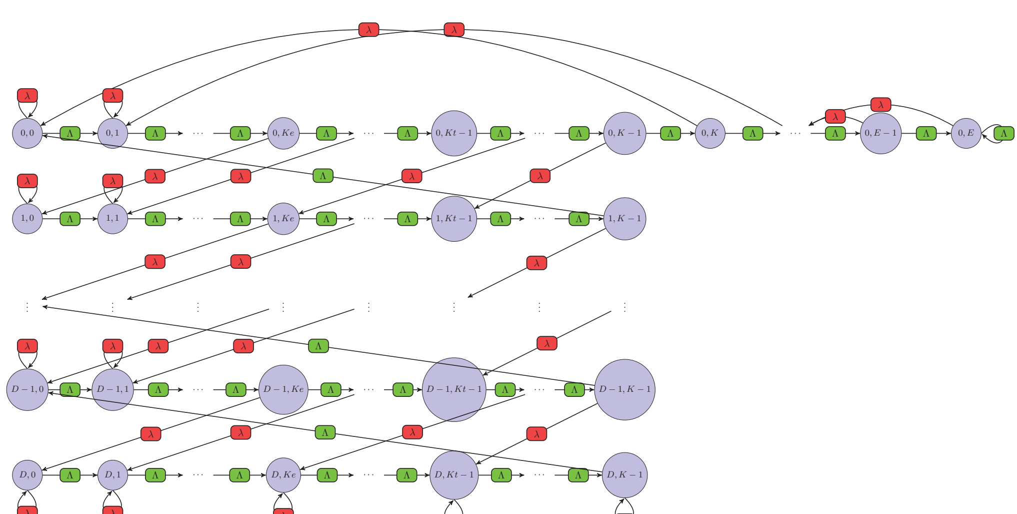

FIGURE 10. Two-dimensional random walk model for generalized K_e and K_t values.

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Due to the fact that energy consumption for such systems is mostly dominated by DP transmission, we assume $K_{\mathrm{t}} > K_{\mathrm{e}}$. Figure 10 shows the state transitions of the system by which we can write the following equilibrium equations:

$$p(0,0)[\Lambda] = \Lambda p(1, K-1) + \lambda p(0, K), \tag{146}$$

$$p(0,e)[\Lambda] = \Lambda p(0, e-1) + \lambda p(0, e+K)1[E \geq e+K], \quad \mathbf{1 \leq e < K_{\mathrm{e}}} \tag{147}$$

$$p(0,e)[\Lambda + \lambda] = \Lambda p(0, e-1) + \lambda p(0, e+K)1[E \geq e+K], \quad \mathbf{K_{\mathrm{e}} \leq e < E} \tag{148}$$

$$p(0,E)[\lambda] = \Lambda p(0, E-1), \tag{149}$$

$$p(d,0)[\Lambda] = \Lambda p(d+1, K-1) + \lambda p(d-1, K_{\mathrm{e}}), \quad \mathbf{1 \leq d < D} \tag{150}$$

$$p(d,e)[\Lambda] = \Lambda p(d, e-1) + \lambda p(d-1, e+K_{\mathrm{e}}), \quad \mathbf{1 \leq d < D}, \quad \mathbf{1 \leq e < K_{\mathrm{e}}} \tag{151}$$

$$p(d,e)[\Lambda + \lambda] = \Lambda p(d, e-1) + \lambda p(d-1, e+K_{\mathrm{e}}), \quad \mathbf{1 \leq d < D}, \quad \mathbf{K_{\mathrm{e}} \leq e < K - K_{\mathrm{e}}} \tag{152}$$

$$p(d,e)[\Lambda + \lambda] = \Lambda p(d, e-1), \quad \mathbf{1 \leq d < D}, \quad \mathbf{K - K_{\mathrm{e}} \leq e < K} \tag{153}$$

$$p(D,e)[\Lambda] = \Lambda p(D, e-1) + \lambda p(D-1, e+K_{\mathrm{e}}), \quad \mathbf{K_{\mathrm{e}} \leq e < K - K_{\mathrm{e}}} \tag{154}$$

$$p(D,e)[\Lambda] = \Lambda p(D, e-1), \quad \mathbf{K - K_{\mathrm{e}} \leq e < K} \tag{155}$$

$$p(D,0)[\Lambda] = \lambda p(D-1, K_{\mathrm{e}}). \tag{156}$$

Finding a closed-form solution by considering the above equations is elusive. To find stationary distributions, we obviously can use a traditional approach where we use an n-dimensional square generator matrix. The solution complexity dramatically increases with the increasing buffer sizes since $n = E + DK + 1$ for such systems. Companion matrices and matrix algebra techniques are introduced to reduce solution complexity and find stationary probability distributions.

7.1. Solution with Companion Matrices

For solution simplicity, we define new state representation S_j such that:

$$S_j = p(d,e) : j = dK - m + E, \quad j \in \{0, 1, \ldots, E + DK\}. \tag{157}$$

Also, each row of Figure 10 is defined as a row such that:

$$V_0 = [S_E, S_{E-1}, \ldots, S_1, S_0], \tag{158}$$

$$V_1 = [S_{E+K}, S_{E+K-1}, \ldots, S_{E+2}, S_{E+1}], \tag{159}$$

$$V_2 = [S_{E+2K}, S_{E+2K-1}, \ldots, S_{E+2+K}, S_{E+1+K}], \tag{160}$$

$$\vdots \tag{161}$$

$$V_D = [S_{E+DK}, \ldots, S_{E+2+DK-K}, S_{E+1+DK-K}]. \tag{162}$$

Besides the fact that complicated state transition behaviors exist among the states, once we carefully observe the diagram in Figure 10, it can be seen that every row, except for the first and the last one, has the exact same state transition behaviors. Therefore, we might have some recurrence relations that reduce the number of total equations and system complexity.

Figure 11 shows the state representation of the ith row of the two-dimensional state diagram or vector V_i where $0 < i < D$. We observe in Figure 11 that for vector V_i, there are

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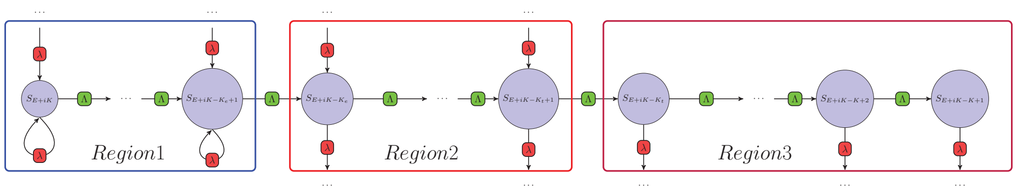

FIGURE **11.** One-dimensional random walk model with single index.

three different transition behaviors among the states so that we can subdivide the vector into three separate regions by which we can write following equations:

- *For Region 1,* $0 \leq e < K_{\mathrm{e}}$*:*

$$S_{N+1} = S_N - \left(\frac{\lambda}{\Lambda}\right) S_{N-K-K_{\mathrm{e}}}, \tag{163}$$

- *For Region 2,* $K_{\mathrm{e}} \leq e < K_{\mathrm{t}}$*:*

$$S_{N+1} = S_N + \left(\frac{\lambda}{\Lambda}\right) (S_N - S_{N-K-K_{\mathrm{e}}}), \tag{164}$$

- *For Region 3,* $K_{\mathrm{t}} \leq e < K$*:*

$$S_{N+1} = \left(1 + \frac{\lambda}{\Lambda}\right) S_N. \tag{165}$$

The linear recurrence relations in Eqs. (163) and (164) are in the order of $K + K_{\mathrm{e}} + 1$, whose minimum value is 8 since $K_{\mathrm{t}} > K_{\mathrm{e}} > 1$. We know that there are no solutions in radicals for polynomial equations having an order greater than 5 in Zoladek [37]. Instead of trying to solve these equations, we may use companion matrices to express stationary probability distributions. We assume that each companion matrix is a square matrix with a dimension of $K + K_{\mathrm{e}} + 1$. Thus, we may write state transitions for V_1 as:

$$\begin{bmatrix} S_{E+2} \\ S_{E+1} \\ \vdots \\ S_{E+2-K-K_{\mathrm{e}}} \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{\lambda}{\Lambda}\right) & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} S_{E+1} \\ S_E \\ \vdots \\ S_{E+1-K-K_{\mathrm{e}}} \end{bmatrix} \tag{166}$$

or equivalently:

$$\overrightarrow{S_{E+2}} = C_3 \overrightarrow{S_{E+1}}. \tag{167}$$

Other state vectors in *Region 3* can also be expressed iteratively as follows:

$$\overrightarrow{S_{E+3}} = C_3 \overrightarrow{S_{E+2}} = C_3^2 \overrightarrow{S_{E+1}}, \tag{168}$$

$$\vdots \tag{169}$$

$$\overrightarrow{S_{E+K_{\mathrm{e}}+1}} = C_3 \overrightarrow{S_{E+K_{\mathrm{e}}}} = C_3^{K_{\mathrm{e}}} \overrightarrow{S_{E+1}}. \tag{170}$$

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Similarly, for *Region 2*:

$$\overrightarrow{S_{E+K_e+2}} = C_2\,\overrightarrow{S_{E+K_e+1}} = C_2\,C_3^{K_e}\,\overrightarrow{S_{E+1}}, \tag{171}$$

$$\vdots \tag{172}$$

$$\overrightarrow{S_{E+K_t+1}} = C_2\,\overrightarrow{S_{E+K_t-1}} = C_2^{K_t-K_e}\,C_3^{K_e}\,\overrightarrow{S_{E+1}}, \tag{173}$$

and for *Region 1*:

$$\overrightarrow{S_{E+K_t+2}} = C_1\,\overrightarrow{S_{E+K_t+1}} = C_1\,C_2^{K_t-K_e}C_3^{K_e}\,\overrightarrow{S_{E+1}}, \tag{174}$$

$$\vdots \tag{175}$$

$$\overrightarrow{S_{E+K+1}} = C_1\,\overrightarrow{S_{E+K}} = C_1^{K_e}\,C_2^{K_t-K_e}\,C_3^{K_e}\,\overrightarrow{S_{E+1}}, \tag{176}$$

where

$$C_1 = \begin{bmatrix} 1 & 0 & \dots & 0 & -\frac{\lambda}{\Lambda} \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} \left(1+\frac{\lambda}{\Lambda}\right) & 0 & \dots & 0 & -\frac{\lambda}{\Lambda} \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}. \tag{177}$$

After further analysis, we may write the following results:

- For the states $N < K$:

$$\overrightarrow{S_K} = \begin{bmatrix} S_K \\ S_{K-1} \\ \vdots \\ S_2 \\ S_1 \\ S_0 \\ S_{-1} \\ \vdots \\ S_{-K_e} \end{bmatrix} = \frac{\lambda}{\Lambda} S_0 \begin{bmatrix} \left(1+\frac{\lambda}{\Lambda}\right)^{K-1} \\ \left(1+\frac{\lambda}{\Lambda}\right)^{K-2} \\ \vdots \\ \left(1+\frac{\lambda}{\Lambda}\right) \\ 1 \\ \frac{\Lambda}{\lambda} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \left(\frac{\lambda}{\Lambda}\right) S_0 \overrightarrow{\gamma}. \tag{178}$$

- For the states $K \le N < E+2$:

$$\overrightarrow{S_N} = \begin{cases} {C_2}^{N-K}\left(\frac{\lambda}{\Lambda}\right) S_0 \overrightarrow{\gamma} & K \le N \le \varsigma_2, \\ {C_1}^{N-\varsigma_1}\,{C_2}^{\varsigma_2}\left(\frac{\lambda}{\Lambda}\right)\, S_0 \overrightarrow{\gamma} & \varsigma_2 < N \le E+1, \end{cases} \tag{179}$$

where $\varsigma_1 = E - K_e + 1$ and $\varsigma_2 = \varsigma_1 - K$.

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- For the states $E+2 \leq N < DK+E+1-K$:

$$\overrightarrow{S_N} = \begin{cases} {C_3}^{\alpha} C^{\lfloor \frac{N-E-1}{K} \rfloor} C' & 0 \leq \alpha \leq K_e, \\ {C_2}^{\alpha-K_e} {C_3}^{K_e} C^{\lfloor \frac{N-E-1}{K} \rfloor} C' & K_e < \alpha \leq K_t, \\ {C_1}^{\alpha-K_t} {C_2}^{K_t-K_e} {C_3}^{K_e} C^{\lfloor \frac{N-E-1}{K} \rfloor} C' & K_t < \alpha < K, \end{cases} \quad (180)$$

where $C' = C_1^{K_e} C_2^{E+1-(K+K_e)} (\frac{\lambda}{\Lambda}) S_0 \overrightarrow{\gamma}$.

- For the states $DK+E+1-K \leq N \leq BK+E$:

$$\overrightarrow{S_N} = \begin{cases} C^{D-1} C' & DK+E+1-K \leq N \leq \varsigma_3, \\ {C_1}^{N-\varsigma_3} C^{D-1} C' & \varsigma_3 < N \leq DK+E, \end{cases} \quad (181)$$

where $\varsigma_3 = DK+E-K_t+1$.

Since the normalization condition holds $\sum_{i=0}^{BK+E} S_i = 1$, we can find stationary probability distributions for S_0 and all the other states in the system.

7.2. Transmission Errors

Since the rate of energy is in power units, the average total power consumed by the sensor node is:

$$\xi = (1 - S_0)\Lambda, \quad (182)$$

where the reduction $S_0\Lambda$ is due to the lost energy packets when the battery or capacitor is full. On the other hand, the average radiated power is:

$$\phi = \kappa\xi, \quad (183)$$

where $\kappa = \frac{K_t}{K}$. Similar to Section 5.3, the correctly received probability of a single bit among the M identical sensor is:

$$1 - e = Q\left(\sqrt{\frac{\eta K_t}{\eta\phi\kappa_0(M-1) + \eta\phi(\frac{M-m}{M})1[M > m] + B}}\right), \quad (184)$$

Figure 12 where we assume, $\eta = 0.5, \kappa_0 = 0.02, B = 1, \Lambda = 10, \lambda = 2, E = 10, D = 10$ shows the effect of the number of sensor nodes for different $K_t = 7, 6, 5$ and $K_e = 2, 3, 4$ values, while the summation of the two remains constant for transmission error probability. Overall, the error characteristics are almost same, while the K is kept constant.

Also, Figure 13, where we assume that similar values for parameters shows the effect of the number of sensor nodes for increasing K values, such that $K_e = 2$ and $K_t = 3, 4, 7$, on transmission error probability. The higher level of K results in smaller errors.

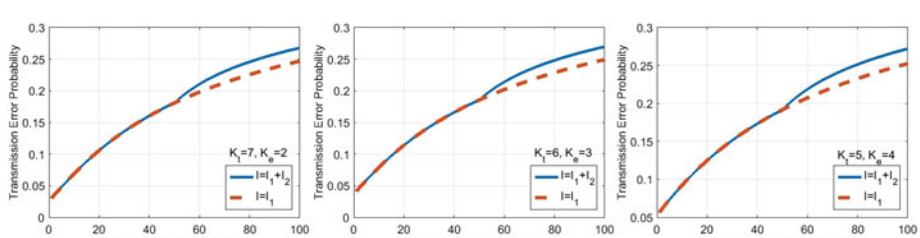

FIGURE 12. Receiving error probabilities with same K and different K_e & K_t values.

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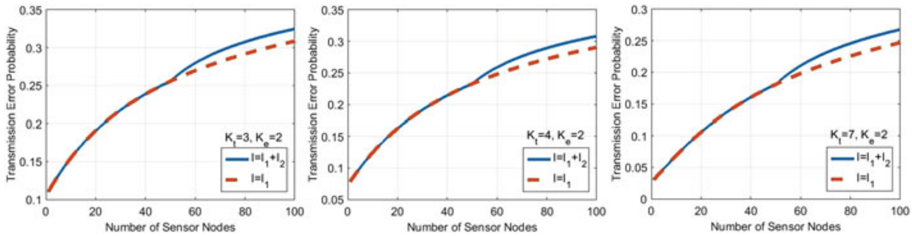

FIGURE **13.** Receiving error probabilities with increasing K_{t} and K values.

8. CONCLUSIONS

This paper surveys recent work on mathematical models of digital devices where energy and data are in discrete packet units. This abstraction is based on the EPN paradigm. While the EPNs examine the task service times for both energy consumption and job processing, the E-Network paradigm considers devices with zero service time. Especially in wireless sensors, the DP transmission usually takes time on a scale of microseconds or nanoseconds, while the packet sensing and processing in the node may take many milliseconds so that the E-Network paradigm is applicable. Wireless sensors studied in this paper receive data from other devices or through sensing, and gather energy through harvesting from photovoltaic or other ambient energy sources. Buffers for DPs and EPs are assumed to have limited capacities of D and E for data and energy buffers, respectively.

A sensor node needs energy not only for packet transmission but also for node electronics, including packet sensing, processing and storing. Therefore, the harvested energy consumption in a sensor node is basically divided into two parts: the number of K_{e} and K_{t} EPs for node electronics and for DP transmission, respectively. Whenever a sensor node stores fewer than K_{e} EPs, it cannot sense and store the data arrival, and the information will be lost. However, if a node stores more than K_{e} EPs, the data arrival can be sensed, processed, and stored immediately by consuming K_{e} EPs. On the other hand, if the remaining EPs are greater than K_{t} EPs, they can also be transmitted in zero time again when all of K_{t} EPs are consumed. Thus, the successful sensing and transmission of a DP requires exactly $K = K_{\mathrm{e}} + K_{\mathrm{t}}$ EPs.

In Section 3, it is assumed that a successful DP transmission can occur by consuming a single EP through a perfect transmitter, that is, $K_{\mathrm{t}} = 1$ and $K_{\mathrm{e}} = 0$. Such systems led us to a one-dimensional random walk diagram to model the behaviour of the state transitions. After modeling the system, we obtained closed-form formulas for the solution of stationary probability distributions. We also studied on excessive packets due to finite buffer capacities and obtained formulas for EP and DP loss rates. The question of system stability is also analyzed by considering infinite buffer sizes. Analysis show that such systems with unlimited capacity buffer can never exhibit a stable behavior. Furthermore, DP re-transmission is studied by introducing the error probabilities π and p into the system. It is shown that closed-form formulas for stationary probability distributions can be found under certain conditions according to new transition rates. In the next section, energy and data leakage caused by the standby operation of the node are introduced into the system. Such leakages are also modeled as Poisson processes with rates μ and γ so that similar analysis in Section 3 can be studied. In Section 5, the more practical model, a packet transmission through an imperfect transmitter is considered. Such imperfect transmitter consumes a single EP for node electronics and another EP for transmission. As a consequence of this energy consumption model, the DP buffer could be either empty or only store a single DP. Thus, one may expect that there would be a great number of excessive DPs; however, this can

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be prevented for certain ratios of data and energy flows. The next section studied data transmission with K EPs. The motivation lies in the fact that a sensor node may vary the transmission power level to decrease the probability of unsuccessful transmissions or prevent energy waste in some cases. This system leads to a two-dimensional random walk to model its state behaviors. Closed-form formulas were obtained by modifying the system where a state can be represented with a single index, not with integer pairs. In Section 7, the generalized model with arbitrary K_{e} and K_{t} values is studied. A solution method is proposed to reduce the computational complexity of the system by introducing companion matrices and using linear algebra.

Future work will address multi-hop systems and different network topologies where DPs and EPs can travel over nodes and hops.

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