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Multigroup kinetic description of X-ray transport in hohlraums

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Abstract

The article presents the multigroup kinetic approximation of radiation transport in hohlraum systems. The view factor-based method of numerical solution is proposed. Its advantages are a detailed account of the problem geometry and accurate handling of the solution angular anisotropy and discontinuities. The method is computationally efficient and easily parallelizable. Coupled to the kinetic + hydro description of the wall dynamics, it represents a powerful tool for integrated investigation of the systems with strongly inhomogeneous optical properties.

Keywords: Angular anisotropy; High-resolution; Hohlraums; Inertial fusion energy; X-ray

1. INTRODUCTION

The problem of inertial fusion energy (IFE) production is addressed by many research groups. Its attractiveness has increased recently with the development of high power laser [e.g., National Ignition Faciltiy (NIF) (USA), MEGAJOULE (France)] and accelerator [e.g., Heavy Ion Synchrotron (SIS) (Germany), Terawatt Accumulator (TWAC) (Russia)] facilities. The estimated parameters of these machines would allow researchers to achieve high plasma parameters, and, therefore, make a good step forward toward the investigation of IFE targets. The laser Phelix constructed at present at Gessellschaft für Schwerionenforschung (GSI, Germany) provides the unique capability to investigate the physics of high energy density in matter using both laser and ion beams.

The investigation of such complex systems requires a combination of experimental studies using high-resolution diagnostics methods and advanced numerical techniques. One of the basic physical processes is soft X-ray transport in hohlraums. The reliable modeling of radiation transport requires taking into account various parameters, such as radiation intensity, spectrum, and angular and spatial flux distribution, as well as optical and thermodynamical properties of the medium. The selection of an adequate approximation for description of radiation transport in these conditions becomes crucial.

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In most of the systems of interest both for IFE research and current and near-future experimental investigations of high energy density in matter, optical properties of the medium and thermal conditions change both from one region to another and in time. Therefore, a self-consistent combination of different approximations of radiation transport is required.

2. THE KINETIC EQUATION OF RADIATION TRANSPORT

Let us consider in more detail the process of radiation transport in a hohlraum, be it an inertial confinement fusion (ICF) or an experimental target. The optical properties of the medium are known to vary strongly within the problem. The radiation is absorbed practically completely in a surface layer of the optically thick casing, heating the material. The material, under the impact of radiation, changes its internal and kinetic energy and starts expanding, and, therefore, its optical properties change as well (the optical depth of the surface layer, heated and expanded, decreases). In the optically thin interior of the casing (vacuum or filled with low-density and/or low-Z material), on the contrary, radiation penetrates through the whole region with only a small fraction of its energy lost (absorbed and scattered) on the way. Therefore, it is necessary to account for the long-range effects of X-ray interaction with the medium. The radiation diffusion approximation, which implies local equilibration of radiation field and material thermal properties, cannot account for remote energy sources. So, to adequately describe the physics, the kinetic equation of radiation transport should be applied in this case:

$$\frac{1}{c}\frac{\partial I_\nu(\mathbf{r},\boldsymbol{\omega},t)}{\partial t}+\boldsymbol{\omega}\cdot\nabla I_\nu(\mathbf{r},\boldsymbol{\omega},t)$$
$$=\varepsilon(\mathbf{r},\boldsymbol{\omega},t)-\kappa_\nu(\mathbf{r},t)I_\nu(\mathbf{r},\boldsymbol{\omega},t), \quad (1)$$

where t is time, c is the light velocity, $I_\nu(\mathbf{r},\boldsymbol{\omega},t)$ is spectral radiation intensity at point $\mathbf{r}$ in the direction $\boldsymbol{\omega}$, $\varepsilon_\nu(\mathbf{r},\boldsymbol{\omega},t)$ is the equilibrium radiation flux at point $\mathbf{r}$ in the direction $\boldsymbol{\omega}$, $\kappa_\nu(\mathbf{r},t)$ is the spectral radiation attenuation coefficient.

Assuming that the surface radiates according to the Lambert law, $I_\nu(\boldsymbol{\omega}) = const$, we get for the radiation flux into half-space

$$J_\nu(\mathbf{r},t)=I_\nu(\mathbf{r},t)\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\cos\theta\cdot\sin\theta\cdot d\theta\,d\varphi=\pi\cdot I_\nu(\mathbf{r},t). \quad (2)$$

The radiation intensity as a function of frequency is, according to the Planck law

$$J_\nu\,d\nu=\pi\cdot I_\nu\,d\nu=\frac{-2\pi h\nu^3}{e^{h\nu/kT}-1}\,d\nu, \quad (3)$$

where h is Planck's constant, k is Boltzmann's constant, and T is the effective temperature.

Neglecting the interaction of radiation with the medium inside the hohlraum, the transport equation can be written in the integral form:

$$J_\nu^-(P,t)=\int_{S(P)}K(P,Q)J_\nu^+\left(Q,t-\frac{r(P,Q)}{c}\right)dS, \quad (4)$$

$$K(P,Q)=\cos(\mathbf{n}_P,\mathbf{PQ})\cdot\cos(\mathbf{n}_Q,\mathbf{QP})/\pi r^2, \quad (5)$$

$$J_\nu^-(P,t)=J_\nu^+(P,t)+q_\nu(P,t). \quad (6)$$

Here t is time, c is the light velocity, P, Q are the points on the boundary surface S of the vacuum region, $r(P,Q)$ is the distance between these points, $\mathbf{n}$ is the inner normal to the surface S, J_ν^+, J_ν^- are one-way fluxes along the inner and the outer normals to S, respectively, q_ν is the normal component of the total radiation flux, and $S(P)$ is the part of the surface S visible from the point P.

One of the specific features of Eq. (4) is that it can have a discontinuous solution integrable over the surface, depicting the geometry of the system or the discontinuities of the boundary conditions (Babaev *et al.*, 1978).

There are clear physical reasons why such solutions occur. The boundary of the system may be "spotted," with sharp interfaces between illuminated and shaded regions due to the system and source geometry. The other reason is the discontinuity of the boundary conditions, that is, a strong local change of absorbing and reflecting properties of the walls, for example, for different wall materials, or in case of slits or holes in the surface.

For a numerical solution, the surface is approximated by a set of patches, discretization over the frequency range is made, and $J^+(\nu)$, $J^-(\nu)$ are substituted by $J^+_{\nu_k}$, $J^-_{\nu_k}$:

$$J^+_{\nu_k}=\int_{\nu_{k-1}}^{\nu_k}J^+(\nu)\,d\nu, J_{\nu_k}=\int_{\nu_{k-1}}^{\nu_k}J^-(\nu)\,d\nu, k=1,\ldots,N_\nu, \quad (7)$$

where N_ν is the number of frequency intervals. So, Eq. (4) is replaced by the following set of equations:

$$J^-_{\nu_k i}(t)=\sum a_{ij}J^+_{\nu_k j}\left(t-\frac{\rho_{ij}}{c}\right),\quad k=1,\ldots,N_\nu, \quad (8)$$

and Eqs. (5) and (6) by

$$a_{ij}=\frac{1}{S_i}\int_{S_i}\int_{S_j}K(P,Q)\,dS_i\,dS_j, \quad (9)$$

$$\rho_{ij}=\frac{1}{a_{ij}}\int_{S_i}\int_{S_j}r(P,Q)K(P,Q)\,dS_i\,dS_j. \quad (10)$$

The relation between $J^+_{\nu_k}$ and $J^-_{\nu_k}$ is

$$J^+_{\nu_k i}(t)=J^+_{\nu_k i}(t)+q_{\nu_{ki}}(t),\quad k=1,\ldots,N_\nu. \quad (11)$$

In the case of the "grey" (single-group) approximation, the equations take the form

$$J_i^-(t)=\sum a_{ij}J_j^+\left(t-\frac{\rho_{ij}}{c}\right), \quad (12)$$

$$J_i^-(t)=J_i^+(t)+q_i(t), \quad (13)$$

where $q_i(t)\sim T^4$. The coefficients a_{ij} are the view factors between the surface elements i and j. The accuracy of the numerical solution of Eqs. (8) is determined by the accuracy of the view factor calculation, which is often more complicated than the solution of the radiation transport problem itself. The view factor for isotropic surface sources is determined by the formula (see, e.g., Siegel & Howell, 1998)

$$a_{ij}=\int_{S_i}\int_{S_j}\frac{\cos\alpha_i\cos\alpha_j}{\pi r_{ij}^2}\,dS_i\,dS_j, \quad (14)$$

where S_i, S_j are the areas of the surface elements i,j; α_i, α_j are the angles between the normals to the surfaces at certain points of these elements and the straight line connecting these points; and r_{ij} is the distance between these surface points. Integration is done over the visible fractions of the surfaces S_i, S_j. Thus the defined view factor is symmetric: $a_{ij}=a_{ji}$. It is evident also, that $\sum_j a_{ij}=S_i$. Source anisotropy may easily be accounted for by introducing the weighting function $w(\theta_i)$ in the view factor integral to determine nonuniform input of different directions into the radiation intensity integrated over the solid angle:

$$a_{ij} = \int_{S_i}\int_{S_j} \frac{\cos\alpha_i \cos\alpha_j}{\pi r_{ij}^2} w(\theta_i)\, dS_i\, dS_j, \tag{15}$$

The efficient method of view factor calculation for axisymmetrical two-dimensional (2D) geometries is described by Vasina (1997) and Vasina and Chekshin (1998). Equations (8) are solved by the "saldo" method using view factors (see, e.g., Abzaev *et al.*, 1999; Vasina *et al.*, 2001).

3. TREATING OF DISCONTINUITIES OF BOUNDARY CONDITIONS AND SOLUTION WITH THE VIEW FACTOR APPROACH

The capability of a numerical model to reproduce such peculiarities of real problems as various kinds of discontinuities is very important in cases when high accuracy of radiation transport simulation is crucial. Two problems having analytical solutions illustrate how they are handled using the view factor approach.

3.1. Radiation transport in a spherical cavity with three types of boundary conditions

There is a time-independent radiation source with the flux q_1 at the fraction of the spherical surface S_1; the surface S_2 is an absolute diffuse reflector, $q_2 = 0$; the surface S_3 is an absolute absorber, $q_3 = J^+$. This stationary problem has an analytical solution. The temperatures T_1, T_2, and T_3 depend only on the value of the source flux and the ratios of the surface areas, rather than on the shape and place of S_1, S_2, and S_3. For $S_1 = S_3$, and $q_1 = Q_1 < 0$ the stationary temperatures are

$$T_1 = \sqrt[4]{-\frac{3Q_1}{2}\frac{4}{\sigma c}}, \qquad T_2 = \sqrt[4]{-Q_1\frac{4}{\sigma c}}, \qquad T_3 = \sqrt[4]{-\frac{Q_1}{2}\frac{4}{\sigma c}}. \tag{16}$$

The numerical solution precisely follows the discontinuities of the boundary conditions (see Table 1).

Table 1. *The deviation of the calculated temperatures from the analytical ones for the different surface grids*

Number of patches	Temperature error on S_1, %	Temperature error on S_2, %	Temperature error on S_3, %	Average error, %
50	0	0.011	0.014	0.0097
100	0	0.0028	0.0036	0.0025
167	0.0008	0.0016	0.0017	0.0015
200	0.0014	0.0019	0.0014	0.0017

3.2. Cooling of a sphere—time-dependent solution and radiation intensity at the boundary as a function of angle

The problem of cooling of a sphere uniformly filled with radiation corresponding to effective temperature T_0 at the initial moment has an analytical solution (V.A. Karepov, pers. comm.). The temperature at the boundary of the sphere is

$$T(t) = T_0 \sqrt[4]{\frac{1}{2}\cdot\left(1-\left(\frac{c\cdot t}{D}\right)^2\right)}, \tag{17}$$

where c is the light velocity, t is the time, and D is the diameter. This problem has an intrinsically three-dimensional (3D) character and cannot be adequately modeled in the 2D approximation. Table 2 shows the accuracy of the calculation of the 3D view factors and temperature at the surface (the errors do not grow in time).

The radiation intensity and flux are, in principle, functions of the solid angle $\boldsymbol{\omega}$ into which a point radiates. The angular dependence of radiation field parameters not only influences redistribution of integral fluxes between different surface patches, but also affects local interaction of radiation illuminating the walls at different angles.

The radiation flux indicatrix at the surface of the sphere is at each moment of time a step function:

$$s = \begin{cases} const, & \vartheta \ge \vartheta(t), \\ 0, & \vartheta \le \vartheta(t), \end{cases} \quad \vartheta(t) = \pi - \arccos\left(2\left(\frac{c\cdot t}{D}\right)^2 - 1\right), \tag{18}$$

where ϑ is the angle of the arc at the maximum cross section of the sphere from the considered point to the point from which radiation reaches the sphere boundary, $0 \le \vartheta \le \pi$. The numerical solution is also a step function, and the accuracy of it is determined by the angular size of the mesh cell at the sphere boundary (no smearing occurs).

4. CONCLUSION

The hohlraum systems are typical in ICF research and experimental investigation of X-ray–matter interaction. Mod-

Table 2. *Numerical "non-sphericity" as a function of the number of surface patches*

Number of patches	Ratio of numerical "nonsphericity"	View factor error, %	Temperature error, %
20×10	1.0038	0.015	1
20×20	1.0012	0.015	0.5
50×20	1.0005	0.003	0.3

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eling of such systems is quite complicated and requires a thorough account of various features characteristic of these problems: the system geometry (shading by occluding bodies, etc.), the discontinuities of initial and boundary conditions, and angular anisotropy of absorbing and reflecting properties of the wall material. The multigroup kinetic approximation of radiation transport allows us to reliably model such problems. The optimal numerical implementation of this model is the integral transport equation solved with the view factor method.

This approach was successfully used to simulate plasma conditions for the proposed experiments at the Phelix + SIS complex (GSI, Germany; Vasina & Vatulin, 2000). The preparatory experimental study in this direction was started recently with the nhelix laser at GSI Plasma Physics group (M. Roth & T. Schlegel, pers. comm.).

The method has been applied to modeling of experiments on X-ray–matter interactions carried out during several years on the ISKRA-5 laser (All-Russian Scientific Research Institute of Experimental Physics (VNIIEF), Russia) and has proven its reliability (see, e.g., Abzaev *et al.*, 1999). Coupled to the kinetic + hydro description of the wall dynamics, it represents a powerful tool for integrated investigation of the systems with strongly inhomogeneous optical properties.

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