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Control Laws synthesis of multi-arm cooperating robots with elastic interconnection at the contacts

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SUMMARY

The problem of the control of the object cooperative manipulation during the work of multiple non-redundant six degrees-of-freedom manipulators is considered in this paper. The problem of the cooperative manipulation control is, like all its problems, solvable only if the system is considered as the elastic one, taking into account all existing constraints. The controlled system is with the output number greater than the available number of inputs, therefore, in the first stage the desired motions are selected from the set of the possible nominal ones, containing the trajectories of the manipulated object mass centre and slave manipulators contacts. Afterward, the classification of control tasks is performed. The procedure for the calculation of the driving torques introduced into the joints of the manipulators, necessary to obtain the nominal trajectory tracking, is proposed. The theoretical analysis of cooperative system closed loop behaviour is exposed, particular attention being paid to the uncontrolled variables. The procedure is illustrated on the example of the simple closed loop cooperative system, consisting of the manipulated object and two one degree-of-freedom manipulators. For this system, the behaviour is determined and the driving torques are calculated.

KEYWORDS: Cooperating robots; Elastic interconnections; Non-redundant manipulators; Control laws.

1. INTRODUCTION – PROBLEM DEFINITION

The determination of the adequate control is the key problem in the object cooperative manipulation realisation. The control laws are based on the cooperative manipulation model, having sense only if that model describes its statics and dynamics with sufficient exactness. The basic property of the cooperative manipulation is that the number of outputs is greater than the number of possible physical inputs, whereas its control task is in essence the desired trajectory tracking one. Therefore, the determination of the cooperative manipulation nominal motion[1] must precede the choice and determination of the control laws. However, when the solution of the cooperative system operation problem is approached form the rigid body mechanics point of view, which is common procedure in the obtainable literature, there appears the force indefiniteness problem, because the cooperative system in the steady state corresponds to the statically undefined space grid. In reference 2 it has been shown for the first time that the problem of the cooperative system force indefiniteness can be solved exclusively by the introduction of the elastic properties assumption of either the whole or the part of the cooperative system, its dynamics being modelled as the elastic structure general motion. It has been shown that the force indefiniteness problem, appearing in the available literature, results from the assumption of cooperative system being non-elastic in this part in which the force acting in the manipulated object mass centre (MC) is decomposed into the contact forces. The expansion in reference 2 yields a mathematical model without force indefiniteness, comprised of the rigid body dynamics model (manipulators and manipulated object) and the set of the equations of the elastic connections. In the choice and determination of the control laws, the basic criterion that must be fulfilled is the realisation of required quality of tracking of this cooperative system coordinated motion, which is defined as the nominal one.

The numerous choices of the cooperative manipulation control laws, given in the available literature, are based on the model with force indefiniteness and, therefore, are not the adequate solution. The number of the proposals of the elastic object cooperative manipulation models and control laws is small.[3–6] The model given in references 3, 4 correctly describing the motion around steady unloaded state is used to make conclusions about the cooperative system general motion. The model given in references 5 and 6 begins with the erroneous implicit assumption that the position and orientation of the elastic system unloaded state are known during the motion. Regardless of the model validity, practically all authors are with the control laws propositions relying on the a *priori* given behaviour of the nominal trajectories or nominal forces deviations. The simulation or experiment has proved the stability of the closed loop cooperative system, not the mathematical analysis.

The cooperative manipulation basic task is the controlled transfer of the workpiece in space and time. From the control theory point of view, the task reduces to the tracking along the selected nominal trajectory. Nominal trajectory defines explicit or implicit requirement of the manipulated object MC motion. It is given as the six dimensional time-variable vector of the manipulated object position and orientation. The model of the cooperative system dynamics[2] is with the greater number of the equations of motion than the number of the physical inputs. Further in the text, the variables upon which the system tracking is performed are defined as the controlled outputs, while the remainders are

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the uncontrolled ones. Similarly, the cooperative system with the feedback is defined as the controlled one, without it as the uncontrolled one. The first problem to be solved is the nominal motion determination. The solution of this problem[1] yields the nominal variables sets ($6m$ inputs and $6m+6$ states) of the uncontrolled cooperative system. As the number of the nominal variables is greater than the number of physical inputs (driving torques), the control definition must begin with the selection of the variable upon which the system is tracked. Therefore, the cooperative manipulation control must be the hierarchical one. For the given class of control tasks, the algorithms performing the selection of the nominal motion character and nominal motion variables defined as the controlled outputs are determined at the higher control level. These algorithms have also to define the transients while changing the master and nominals during the manipulation. The hihger control levels are not the objects of this paper. The control laws for the selected classes of the controlled outputs are defined at the lower control level.

Special analysis is given in determining the answer to the question: What can be required from the cooperative system, *i.e.* which are the classes of the controlled outputs and what are their properties? For example, if from the available driving torques, or inputs, only six are used for the controlled motion along the required trajectory, there remains the problem how to introduce the $6m-6$ remaining ones. Therefore, there arises the problem which, what kind and how many of the $6m+6m$ nominal variables, remaining when the outputs presenting the required controlled trajectory are taken out, can be selected as the controlled outputs?

The control laws in this paper are given as the driving torques calculated on the basis of the cooperative manipulation dynamics model, providing the controlled output errors with the *a priori* determined properties. The advantage of these control laws is that the driving torques are calculated exactly. The disadvantage is their complexity, because all state variables and their derivatives are taking part in them. The theoretical analysis of the cooperative system behaviour is performed relatively easy using the physical relations describing its dynamics and statics. This advantage enables one to perform theoretical analysis of the controlled cooperative system in the whole, for the controlled as well as for the uncontrolled variables, yielding thus a proper conclusion about the whole system stability.

2. MATHEMATICAL MODEL OF COOPERATIVE MANIPULATION

The cooperative system control laws selection is based on the model developed in reference 2, yielding the univocal solution to the force indefiniteness problem. The cooperative action of m six degrees-of-freedom (6DOF) manipulators contacting the body which has no imposed limitations to the motion in the three-dimentional (3D) space is modelled in reference 2. The contact of the manipulators and the manipulated object is elastic, without the possibility of the manipulator's tip relative motion along the object surface. Manipulated object and its contact areas with the manipulators are approximated by the elastic system consisting of $m+1$ bodies rigid in the whole elastically interconnected by the axial and torsion springs. Each body is allowed 6DOF. The gravitational and contact forces are considered as the external ones, acting in the MC of those bodies. This model is briefly presented in this section, in the form suitable for the control laws selection.

The mathematical model of the elastic system dynamics that is performing general motion under the action of the external system contact forces F_c and gravitation G is described by the means of absolute coodinates[2] and given by the relations.

$$\begin{aligned} W_c(Y_c)\ddot{Y}_c + w_c(Y,\dot{Y}) &= F_c \\ W_0(Y_0)\ddot{Y}_0 + w_0(Y,\dot{Y}) &= 0 \end{aligned} \tag{1}$$

where index c denotes values related to the contacts, and index 0 denotes values related to the manipulated object. In these equations, the following notations have been introduced: $Y=col(Y_c,Y_0)\in R^{(6m+6)\times 1}$, $Y_c= col(Y_1,Y_2,\ldots,Y_m)\in R^{6m\times 1}$, $Y_0\in R^{6\times 1}$, $Y_i=col(r_i,\mathcal{A}_i)=col(x_i,y_i,z_i,\psi_i,\theta_i,\varphi_i)\in R^{6\times 1}$ for the absolute coordinates vectors of the elastic system nodes positions and orientations; $F_c= col(F_{c1},F_{c2},\ldots,F_{cm})\in R^{6m\times 1}$ for the contact force vector; $W_c(Y_c)=diag(W_1(Y_1),\ldots, W_m(Y_m))\in R^{6m\times 6m}$, $W_c(Y_c)= W_c^T(Y_c)$, $detW_c(Y_c)\neq 0$, $W_i(Y_i)=W^T_i(Y_i)\in R^{6\times 6}$, $detW_i(Y_i)\neq 0$, $i=0,1,\ldots,m$ for the inertial matrices of all objects; and the arbitrary term $w_c(Y,\dot{Y})=col(w_1(Y,\dot{Y}),\ldots,w_m(Y,\dot{Y}))\in R^{6m\times 1}$. The arbitrary term w_i presents the vector taking into the account all forces depending on elastic system nodes velocities and positions and orientations, is of the form $w_i(Y,\dot{Y})=\dot{W}_i(Y_i)\dot{Y}_i-\partial T_i(Y_i,\dot{Y}_i)/\partial Y_i+D_i(Y)\dot{Y} +0.5\,\partial(Y^T\bar{\pi}_a(Y)Y)/\partial Y_i+\pi_{ia}(Y)Y-G_i(m_ig)=F_{bi} +D_i(Y)\dot{Y}+F_e -G_i\in R^{6\times 1}$, $i=0,1,\ldots,m$; where $T_i=0.5\dot{Y}_i^TW_i\dot{Y}_i$ presents kinetic energy; $Y^T\pi_a(Y)Y\in R^{(6m+6)\times(6m+6)}$ deformation energy; $\bar{\pi}_a(Y)$ denotes that the quadratic form differentiation is performed over π_a matrix only; π_{ia}, D_i are, respectively, submatrices of the stiffness and damping matrix, composed of the $6i+1$-st up to $6i+6$-th row of these matrices; G_i is the weight of the i-th body and $F_{bi}=\dot{W}_i(Y_i)\dot{Y}_i-\partial T_i(Y_i,\dot{Y}_i)/\partial Y_i$. At each elastic system passage through the unloaded state its deformation energy equals zero. Of the $6m+6$ equations (1), only rank $K=6m$ equations are independent whereby matrix $K(Y)$ is determined from the elastic force expression

$$F_e=K(Y)\cdot Y=\frac{1}{2}\,\frac{\partial Y^T\bar{\pi}_aY}{\partial Y}+\pi_a(Y)Y=G+\begin{bmatrix}F_c\\0\end{bmatrix}$$

$$=\begin{bmatrix}A & b\\ c & d\end{bmatrix}\begin{bmatrix}Y_c\\Y_0\end{bmatrix}=\begin{bmatrix}u_v & u_s & u_0\\ A_v & A_s & A_0\\ c_v & c_s & c_0\end{bmatrix}\begin{bmatrix}Y_u\\Y_s\\Y_0\end{bmatrix} \tag{2}$$

$$h(q,0)=\tau+J^Tf_c\in R^{6m\times 1}$$

$$Y_c=\Theta(q)\in R^{6m\times 1}.$$

The partition of the matrix $K(Y)$ has been performed thus that the rank $K(Y)=rangA=6m$, and that the block matrices are consistent to multiplication with vector $Y=col(Y_c,Y_0)=col(Y_v,Y_s,Y_0)=col(Y_v,Y_{so})$, $Y_c=col(Y_v,Y_s)$,

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$Y_{so}=col(Y_s, Y_0)$. In the addition to the indices used in the (1), index 1 (*i.e.* $1 \rightarrow \upsilon$) is used for the master, and indices from 2 to m (*i.e.* $2 \ldots m \rightarrow s_1 \ldots s_{m-1}$ or s) for the slaves. This indexing system holds for the other vectors and matrices. E.g., if vectors Y_υ, Y_s and Y_0 are considered, the stiffness matrix consists of the submatrices given in (2), while the contact and elastic forces are, respectively, $F_{c\upsilon}$, $=F_{c1}$, $F_{cs}=col(F_{c2}, \ldots, F_{cm})$ and $F_e=col(F_{e\upsilon}, F_{es}, F_{e0})$. In the case that the elastic system general motion is performed around its unloaded state only, then the absolute coordinates are $Y=Y_0+y$; $Y_0=col(Y_{c0}, Y_{00})=const$ are the coordinates of the steady unloaded state; $y=col(y_c, y_0)=col(y_\upsilon, y_s, y_0)$ are the coordinates of the loaded state deviations relative to the unloaded one; while $\pi_a(Y)=\pi_a=K=const$ and $D(Y)=D=const$. The model of the elastic system dynamics expressed via the y coordinates possesses the same form as the equation (1), whereby $w_i(y, \dot{y})=\dot{W}_i(y_i)\dot{y}_i - \partial T_i(y_i\dot{y}_i)/\partial \dot{y}_i + D_i\dot{y} + K_i y - G_i(m_i g) \in R^{6\times 1}$, $i=0, 1, 2, \ldots, m$.

The model of the motion of the m6DOF non-elastic manipulators with non-compliant joints and with the gripper force within the internal coordinates space is given by expression[7,8]

$$H(q)\ddot{q}+h(q, \dot{q})=\tau+J^T f_c \tag{3}$$

whereby the notations are introduced: $H(q)=diag(H_1, (q_1), \ldots, H_m(q_m)) \in R^{6m\times 6m}$, $H_i(q_i) \in R^{6\times 6}$ is the positive definite matrix of the manipulators inertia; $h(q, \dot{q})=col(h_1(q_1, \dot{q}_1), \ldots, h_m(q_m, \dot{q}_m)) \in R^{6m\times 1}$, is the vector that takes into account the influences of the gravitation, Coriolis' acceleration and centrifugal forces; $\tau=col(\tau_1, \ldots, \tau_m) \in R^{6m\times 1}$, $\tau_i \in R^{6\times 1}$ is the joint drives vector, $J=diag(J_1, \ldots, J_m) \in R^{6m\times 6m}$, $J_i \in R^{6\times 6}$ is the transformation matrix of the internal coordinates velocity vector into the manipulator tip velocity vector; $f_c=col(f_{c1}, \ldots, f_{cm})=-col(F_{c1}, \ldots, F_{cm}) \in R^{6m\times 1}$, $f_{ci}=-F_{ci} \in R^{6\times 1}$ is the contact force on the manipulator gripper; $q=col(q_1, \ldots, q_m) \in R^{6m\times 1}$, $\dot{q}=col(\dot{q}_1, \ldots, \dot{q}_m) \in R^{6m\times 1}$ are the interior coordinates vector and its derivative.

If the kinematic relation of the internal q and absolute coordinates of the contacts is presented by the expression $Y_i=\Theta_i(q_i) \in R^{6\times 1}$, $i=1, \ldots, m$, then the relation of their velocities and acceleration is determined by the expressions[7] $\dot{Y}_i=\partial\Theta_i(q_i)/\partial q_i \cdot \dot{q}_i= J_i(q_i)\dot{q}_i \in R^{6\times 1}$, $\ddot{Y}_i=\dot{J}_i(q_i)\dot{q}_i+J_i(q_i)\ddot{q}_i \in R^{6\times 1}$, $i=1, \ldots, m$, or, in the united form

$$\begin{aligned} Y_c &= \Theta(q) \in R^{6m\times 1} \\ \dot{Y}_c &= J(q)\dot{q} \in R^{6m\times 1} \\ \ddot{Y}_c &= \dot{J}(q)\dot{q}+J(q)\ddot{q} \in R^{6m\times 1} \end{aligned} \tag{4}$$

Model of cooperative manipulation dynamics is given by (1), (3) and (4). Using the previously described indexing system, unifying the equations (1), (3) and taking into account $F_c=-f_c$ gives the description of the cooperative system dynamics in the form

$$\begin{aligned} N_\upsilon(q_\upsilon)\ddot{q}_\upsilon+n_\upsilon(q, \dot{q}, Y_0, \dot{Y}_0) &= \tau_\upsilon \\ N_s(q_s)\ddot{q}_s+n_s(q, \dot{q}, Y_0, \dot{Y}_0) &= \tau_s \\ W(Y_0)\ddot{Y}_0+w(q, \dot{q}, Y_0, \dot{Y}_0) &= 0 \\ P_\upsilon(q_\upsilon)\ddot{q}_\upsilon+p_\upsilon(q, \dot{q}, Y_0, \dot{Y}_0) &= F_{c\upsilon} \\ P_s(q_s)\ddot{q}_s+p_s(q, \dot{q}, Y_0, \dot{Y}_0) &= F_{cs} \end{aligned} \tag{5}$$

where: $diag(N_\upsilon(q_\upsilon), N_s(q_s))=diag(N_1(q_1), N_2(q_2), \ldots, N_m(q_m))=H(q)+J^T(q)W_c(\Theta(q))J(q) \in R^{6m\times 6m}$, $|N(q)| \neq 0$, $N_i(q_i)=H_i(q_i)+J_i^T(q_i)W_{ci}(\Theta_i(q_i))J_i(q_i) \in R^{6\times 6}$, $|N_i(q_i)| \neq 0$, $i=1, \ldots, m$, $N_\upsilon(q_\upsilon) \in R^{6\times 6}$, $|N_\upsilon(q_\upsilon) \neq 0$, $N_s(q_s) \in R^{6(m-1)\times 6(m-1)}$, $|N_s(q_s)| \neq 0$, $P(q)=W_c(\Theta(q))J(q)=diag(P_1(q_1), P_2(q_2), \ldots, P_m(q_m))=diag(P_\upsilon(q_\upsilon), P_s(q_s)) \in R^{6m\times 6m}$, $|P(q)| \neq 0$, $P_i(q_i)=W_{ci}(\Theta_i(q_i))J_i(q_i) \in R^{6\times 6}$, $|P_i(q_i)| \neq 0$ $i=1, \ldots, m$, $P_\upsilon(q_\upsilon) \in R^{6\times 6}$, $|P_\upsilon(q_\upsilon)| \neq 0$, $P_s(q_s) \in R^{6(m-1)\times 6(m-1)} |P_s(q_s)| \neq 0$, $n(q, \dot{q}, Y_0, \dot{Y}_0)=col(n_\upsilon(q, \dot{q}, Y_0, \dot{Y}_0), n_s(q, \dot{q}, Y_0, \dot{Y}_0))=h(q, \dot{q})+J^T(q)W_c(\Phi(q))\dot{J}(q)\dot{q}+J^T(q)w_c(\Phi(q), J(q)\dot{q}, Y_0, \dot{Y}_0) \in R^{6m\times 6}$, $w(q, \dot{q}, Y_0, \dot{Y}_0)=w_0(\Theta(q), J(q)\dot{q}, Y_0\dot{Y}_0) \in R^{6\times 1}$ and $p(q, \dot{q}, Y_0, \dot{Y}_0)=W_c(\Theta(q))\dot{J}(q)\dot{q}+w_c(\Theta(q), J(q)\dot{q}, Y_0, \dot{Y}_0) \in R^{6m\times 1}$.

The first three equations in (5) describe the cooperative system behaviour. The last two equations express the contact forces as the internal coordinates function. This is the differential function given by the elastic system dynamics model (1) into which the kinematic relations (4) are incorporated.

3. CLASSIFICATION OF CONTROL TASKS

3.1 *Basic postulates*

For the linear system of n_x ordinary first order differential equations with matrices $\bar{A}, \bar{B}, \bar{C}, \bar{D}$, states $x \in R^{n_x\times 1}$, inputs $\upsilon \in R^{n_\upsilon\times 1}$ and outputs $\gamma \in R^{n_\gamma\times 1}$ the state controllability and observability conditions are known[9]. The system output variable $\gamma(0)$ is controllable if and only if there exists control υ which, in the limited time interval, translates this system from the initial state $x(t_0)$, corresponding to the initial output value $\gamma(t_0)$, into the state corresponding to the output value $\gamma(t)=0$. For the linear system with one input ($n_\upsilon=1$) and one output ($n_\gamma=1$) to be controllable with respect to the output, it is necessary and sufficient that $rank(\bar{C}^T\bar{B}, \bar{C}^T\bar{A}\bar{B}, \ldots, \bar{C}^T\bar{A}^{n_x-1}\bar{B}, \bar{D})=1$. Kalman has proved[10] that the linear system with single input and single output is controllable, *i.e.* observable, if and only if its dual system is observable, *i.e.* controllable, respectively. For the linear stationary time continuous dynamic system has been proven that the positive resolving of the controllability problem guarantees the existence of the closed loop system control, providing thus its stability. Intuitively applying the same reasoning it follows that in the selection of the non-linear system control laws, initial stage must be the solution of system controllability problem. The linear system theory criteria cannot be directly applied to non-linear systems. Nevertheless, it can be expected that in the part of the conditions necessary for the non-linear system controllability originate at least the conditions for the number and properties of the requirements that can be imposed upon this system (in this case, the cooperative manipulation one).

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The following considerations lead to the part of the conditions necessary for the non–linear system controllability. Over same input $\mathcal{D}_\upsilon$, state $\mathcal{D}_x$ and output $\mathcal{D}_\gamma$ domains the general form of the solution for the linear and non-linear differential equations system is the same, $x(t)=x(x_0, t_0, t, \upsilon)$, $\gamma(t)=\gamma(x, \upsilon)=\gamma(x(x_0, t_0, t, \upsilon), \upsilon)=\gamma(t, \upsilon)$, where the time t and initial instant t_0 and state x_0 are parameters. Elimination of parameter t yields the functional relations $x=x(x_0, \upsilon)$. $\gamma=\gamma(x, \upsilon)=\gamma(x(\upsilon), \upsilon)$ defining the mapping of the input domain into the state domain and both of these domains into the output domain. The function $x=x(x_0, \upsilon)$ determines the mapping $\mathcal{F}^{\upsilon}_{x}$: $\upsilon \rightarrow x$ of the whole input domain into the whole or the part of the state domain $\mathcal{D}_\upsilon \rightarrow \mathcal{D}^{\upsilon}_{x} \subseteq \mathcal{D}_x$. The controlled outputs function $\gamma(t)$ defines the mapping $\mathcal{F}^{\upsilon_x}_{\gamma}$: $(\upsilon, x(x_0, \upsilon)) \rightarrow \gamma$ of the whole product of the whole input domain and the part of the state domain (obtained by the mapping from the input domain) into the domain of the controlled outputs which is the part of the outputs domain $\mathcal{D}_\upsilon \times \mathcal{D}^{\upsilon}_{x} \rightarrow \mathcal{D}^{\upsilon}_{\gamma} \subseteq \mathcal{D}^{\upsilon_x}_{\gamma}$. The controllability and observability definitions and theorems organise the properties and conditions of the mapping between input, state and output domains. The output controllability definition precisely states the properties of the mapping of the input $\mathcal{D}_\upsilon$ and the part of the state $\mathcal{D}^{\upsilon}_{x}$ domains into the controlled output domain $\mathcal{D}^{\upsilon}_{\gamma}$. From the conditions rank $(\bar{C}^T\bar{B}, \bar{C}^T\bar{A}\bar{B}, \ldots, \bar{C}^T\bar{A}^{n_x-1}\bar{B}, \bar{D})=1$ and the results of Kalman work it follows that the dimensions of the input space $\mathcal{D}_\upsilon$ and controlled output space $\mathcal{D}^{\upsilon}_{\gamma}$ must be the same $dim\{\mathcal{D}_\upsilon\}=dim\{\mathcal{D}^{\upsilon}_{\gamma}\}$ and that there must exist both, the direct and the inverse mapping. In other words, for a system to be output controllable, there must exist the biunivocal correspondence between the whole input space $\mathcal{D}_\upsilon$ and the whole controlled output space $\mathcal{D}^{\upsilon}_{\gamma}$. The following can be summed up. Domain to domain mapping considerations are based on the functional relations between the differential equations system solutions and the controlled outputs. These functional relations have the same form in the case of linear and non-linear systems. In the previous consideration any properties specific to the linear system have not been used. The linear system represents the description of the non-linear one in the sufficiently small vicinity of any point of the state space where the non-linear system is defined. Therefore, under these conditions the output controllability conclusion holds for both, the linear and non-linear systems. For the functions $\gamma_i=\gamma_i\ (\upsilon_1, \ldots, \upsilon_{n_\upsilon})$, $i=1, \ldots, n_\upsilon$ continuously differentiable in some region of the n_υ-dimensional space is known[11] that, if the Jacobian is different from zero $\partial(\gamma_1, \gamma_2, \ldots, \gamma_{n_\upsilon})/\partial(\upsilon_1, \upsilon_2, \ldots, \upsilon_{n_\upsilon}) = det(\partial\gamma_i/\partial\upsilon_j)\neq 0$, the mapping $\gamma=\gamma(\upsilon)$ between sufficiently small vicinity of the selected point υ in the space $\mathcal{D}_\upsilon$ and the vicinity of the point $\gamma(\upsilon)$ in the space $\mathcal{D}^{\upsilon}_{\gamma}$ is biunivocal.

In order that previous Jacobian exists at all, the necessary condition for biunivocal mapping is, obviously, that the dimensions of the input space $\mathcal{D}_\upsilon$ and controlled output space $\mathcal{D}^{\upsilon}_{\gamma}$ are the same. In the mapping, there can be set exactly $dim\{\mathcal{D}_\upsilon\}=dim\{\mathcal{D}^{\upsilon}_{\gamma}\}$ independent variables υ or γ and obtain exactly the same number of dependent variables γ or υ. Each variable υ or γ selected as the independent one can express one independent control requirement to the system. As the independent variables, there can be selected only outputs γ, only inputs υ or their combination. In other words, the system control requirements can be imposed to its output, input or even to their combination. The imposed requirements must be consistant, meaning that to one independent variable or its dependent variable there can be imposed one and only one independent requirement. It follows, that the number of controlled outputs must be equal to the number of control inputs, and that to exactly one input exactly one output must correspond, which is the function of that input and the system state generated by the input action.

In principle, the number of inputs n_υ is not the same as the number of outputs n_γ. *If* $n_\upsilon > n_\gamma$, the problem is easily solvable by cancelling the surplus of inputs. That can be achieved by establishing functional relations between the n_γ inputs and the remainder of $n_\upsilon - n_\gamma$ inputs. That can be also achieved applying the hierarchical control, thus that for defined control task n_γ suitable inputs are selected on the higher control level, while the remainder of the inputs is held constant. If the number of really possible outputs n_γ (mutually independent variables) is greater than the number of inputs n_υ, $n_\gamma > n_\upsilon$ then n_υ outputs can be controlled, while the remainder of $n_\gamma - n_\upsilon$ outputs are uncontrolled, their behaviour determined exclusively by the controlled object dynamics. This relation of outputs and physical inputs exists in the cooperative manipulation. The control task reduces to the selection of the set of $n_\gamma = n_\upsilon$ controlled outputs and the selection of such inputs that the behaviour of the remainder of outputs is acceptable. Therefore, the control system must be the hierarchical one. On the higher hierarchical level, the set of system nominal motions is defined, the controlled outputs are selected and the mode of transfer between nominal motion sets is determined. On the lower hierarchical level, the control of the selected variables is performed with the exactly determined control laws.

The selected control laws must provide such object inputs υ^{ob} that, exclusively in accordance to the controlled object dynamics $\dot{x}=f(x, \upsilon^{ob})$, object states $x=x(x_o, \upsilon^{ob})$ are produced. These states are generating outputs $\gamma=g(x)$ that are satisfying the imposed requirements (Figure 1). Physics of

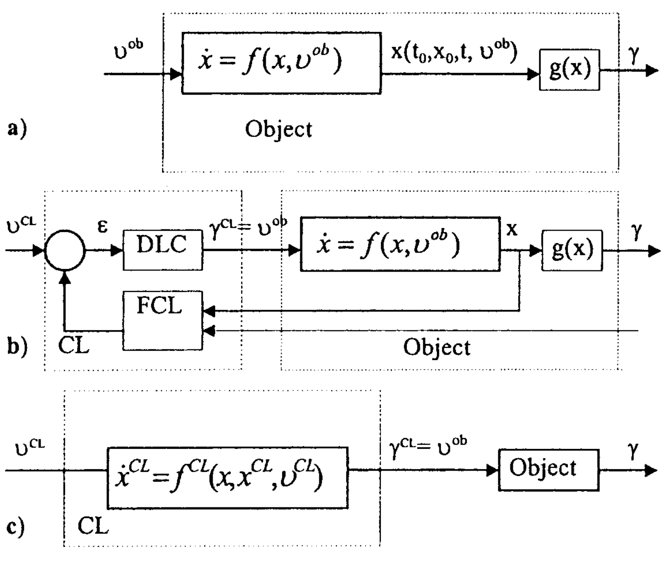

Fig. 1. Control system structure

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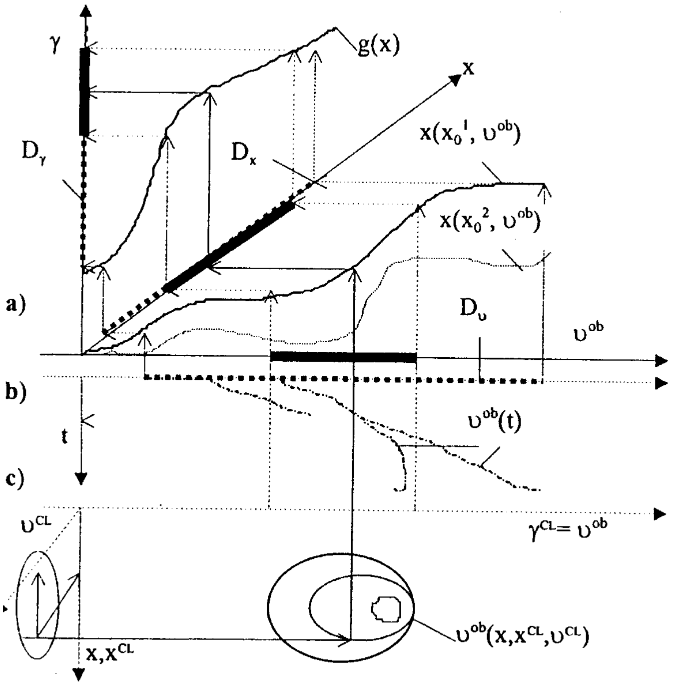

Fig. 2. Mapping of the control object domains

any controlled object determines its input $\mathcal{D}_\upsilon$ state $\mathcal{D}_x$ and output $\mathcal{D}_\gamma$ domains, as well as the mapping functions $x = x(x_o, \upsilon^{pb})$ and $\gamma = g(x)$ between them (Figure 2, longer dotted bold line). The object dynamics is determined by this physics and cannot be changed by any control. The selected control laws are producing only these control system outputs $\gamma^{CL} = \upsilon^{pb}$, *i.e.* controlled object excitations, that the mapping $g(x)$ from the state domain generated by these excitations into the output domain is performed only upon this part of the output function $g(x)$ which is satisfying the imposed requirements (Figures 1b, 1c, and 2a, short bold line).

General form of the control system output is $\gamma^{CL} = \upsilon^{pb}(x, x^{CL}, \upsilon^{CL})$(Figure 2c.). The control system output γ^{CL} is always the function of the imposed required object input υ^{CL}. The control system output can be the function of the object state x. Finally, the control system output is the function of the control system state x^{CL}, for the case of this system being the dynamic one. As the physics of the object determines mapping functions, so it defines the requirements that can be imposed to the object, expressed as the closed loop system requirements υ^{CL}. In addition, the nominal regimes cannot be demanded arbitrarily, but in accordance to the physics defining the controlled object dynamics. **Hereby, the object nominal motion is determined only as this accomplishable object motion which ideally fulfils, in accordance with the object physics, maximally possible number of imposed requirements, equal to the number of object physical inputs.** One solution of this problem is given in reference 1.

The analysis of the object dynamic behaviour presents the considerations about the character of the solutions of the differential equations system describing it, for the excitations that are the functions: for the uncontrolled object only of the time; for the controlled object of the time and state. The analysis of the controlled object dynamic behaviour can be performed on the base of the closed loop model, open loop model and object model only, for the case when its input is completely determined. In this paper, the analysis of the behaviour of the controlled variables is based on the closed loop model. The analysis of the behaviour of the uncontrolled output variables is based on the object model only.

Attention must be paid to the fact that the elastic part of the cooperative system possesses an infinite number of eigenfrequencies and model forms corresponding to them. Choosing the cooperative manipulation model in the form (5) the insight into the $6m$ eigenfrequencies and model forms can be obtained. Then, for the condition that the assumption about the elastic system presentation by springs is the valid one, the lowest eigenfrequency is the closest to the real one.The character of the attack load of the particular elastic system dictates the character of its motion. In other words, in the phase of the nominals selection the character of the elastic, *i.e.* cooperative, system motion is defined. The quasi-static tarnsfer can be selected as the nominal motion, as well as the motion that is far or close to the elastic structure eigen model forms (resonant states). The analysis of the behaviour of the closed loop cooperative system must demonstrate whether the required nominal motion of the system with the selected vector of the controlled outputs and control laws can be asymptotically stable accomplished.

3.2 Task Classification

In the cooperative manipulation control several sets can be observed from or into which mapping is carried out in the input–output sense (Figure 3.). Note some mappings of these sets. The manipulator state set $\mathcal{D}_q$ is obtained by mapping the pair (τ, F_c) into the state q, $(\tau, F_c) \rightarrow q$. The driving torques τ set $\mathcal{D}_\tau$ can be obtained by mapping the pair (q, F_c) into the driving torques, $\tau, (q, F_c) \rightarrow \tau$, and contact forces set $\mathcal{D}_{Fc}$ by mapping the pair (τ, q) into the contact forces F_c, $(\tau, q) \rightarrow F_c$. The mapping law is determined by solving the last two equations (5). The elastic system state set $\mathcal{D}_Y = \mathcal{D}_{Y_c} \cup \mathcal{D}_{Y_o}$ can be obtained by mapping out of the contact forces set $\mathcal{D}_{Fc}$ into the elastic system state set $\mathcal{D}_Y$ by the law defined by solving the first three equations of the differential equations system (5). The elastic forces F_e set $\mathcal{D}_{Fe}$ is obtained by mapping the elastic system state Y into these forces by the law $F_e = K(Y)Y$, *i.e.*$Y \rightarrow F_e$.

As the dimension of the cooperative system input space (driving torques) is $6m$, to this system the $6m$ independent consistent requirements can be maximally imposed, expressed by the controlled output vector properties. As the controlled output vector Y^u some of the following vector types can be selected:

The position and orientation vector of the cooperative system elements. The selection can be part $Y = col\ (Y_v, Y_s, Y_0) \in R^{6m+6}$ of the elastic system state $Y^u = Y_{s0} = col(Y_s, Y_0) \in R^{6m}$, or, expressed by the internal coordinates $Y^u = col(q_s, Y_0) \in R^{6m}$ and $Y^u = q \in R^{6m}$.

The elastic force vector $F_e \in R^{6m+6}$ is the function of the elastic system nodes state vector Y, so for the controlled output can be adopted $Y^u = F_{es0} \in R^{6m}$ or $Y^u = F_{ec} \in R^{6m}$ vector.

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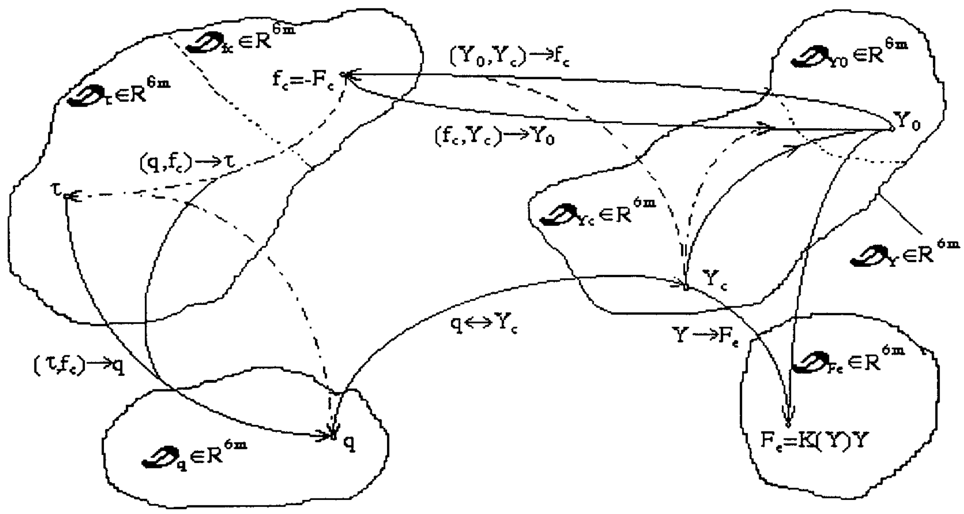

Fig. 3. Cooperative manipulation domain mapping

The contact force vector $Y^u = F_c \in R^{6m}$, where should be noted that, for this controlled output selection, the manipulated object MC spatial position and orientation and, consequently, the whole cooperative system position and orientation can be arbitrary.

The part of the contacts position and orientation vector $\bar{Y}_c$, *or the part of the corresponding internal coordinates* $\bar{q}$ *and the part of the contact forces vector* $\bar{F}_c$. The characteristics selection of the controlled output vector is $Y^u = col(Y_{cv}, F_{cs}) \in R^{6m}$, that is $Y^u = col(q_v, F_{cs}) \in R^{6m}$, in the structure analogue to the vector $Y^u = col(Y_0, F_{cs})$.

From the analysis point of view, in the cooperative manipulation the control laws selection based on the cooperative system elements position and orientation vector is equivalent to this based on elastic force vector. One group of mutually equivalent controlled outputs is $Y^u = Y_c$, $Y^u = q$ and $Y^u = F_{ec}$, while the other is $Y^u = Y_{s0}$, $Y^u = F_{es0}$ and $Y^u = (q_s, Y_0)$, therefore, it is sufficient to make the control laws selection for only one of these outputs.

In accordance to the previously stated, in the cooperative manipulation the typical control tasks are:

- Tracking the nominal trajectory of one elastic system node (manipulated object $MC\ Y_0^0(t) \in R^6$ or master contact $Y_v^o(t) \in R^6$) and tracking of the slave manipulator contacts nominal trajectories $Y_s^0 \in R^{(6m-6)}$, that is, slave manipulators nominal internal coordinates $q_s^0 \in R^{6m-6}$. The cooperative system controlled output is the $6m$-dimensional vector $Y^u = col(Y_s, Y_0)$ or $Y^u = col(Y_v, Y_s) = Y_c$, that is, $Y^u = col(q_s, Y_0)$ or $Y^u = col(q_v, q_s) = q$. Untracked elastic system output variables (uncontrolled outputs) are the contact forces $F_c \in R^{6m}$ and the position and orientation of one contact $\in R^6$).

- Tracking of one elastic system node nominal trajectory (manipulated object $MC\ Y_0^0(t) \in R^6$ or master contact $Y_v^0(t) \in R^6$) and the tracking of the nominal contact forces $F_{cs}^0 \in R^{6m-6}$ in the slave manipulators contact. The cooperative system controlled output is the $6m$-dimensional vector $Y^u = col\ (F_{cs}, Y_0)$ or $Y^u = col(F_{cs}, Y_v)$ that is, $Y^u = col(F_{cs}, q_v)$. Untracked elastic system output variables (uncontrolled outputs) are the position and orientations of m nodes (for the Y_o^o tracking these are the contacts position and orientation $Y_c \in R^{6m}$) and the contact force $F_{cv} \in R^6$ in the master manipulator contact.

4. CONTROL LAWS

In this paper the tracking of the nominal trajectories of the manipulated object MC and slave manipulators contacts is analysed. The controlled output for this case is the vector $Y^u = col(q_s, Y_0)$. It is demanded that, with *a priori* required qualities, controlled cooperative system tracks selected nominal trajectory $Y^0(t) = col(q_s^0(t), Y_0^0(t))$ determined by the particular procedure[1]. The character of the system uncontrolled variable deviations from the nominal values should be tested separately.

The proposition of the procedure for the calculation of the driving torques providing controlled output errors with *a priori* required properties is as follows.

Let the vector of the deviations between real controlled and nominal trajectory as well as the vector of the deviation derivatives be,

$$\overset{(l)}{\eta}_s(t) = \overset{(l)}{q}{}_s^0(t) - \overset{(l)}{q}_s(t) \text{ and } \Delta \overset{(k)}{Y}_0 = \overset{(k)}{Y}{}_0^0(t) - \overset{(k)}{Y}_0(t),\ l, k = 0, 1, 2, \ldots$$

respectively. If $\eta_s(t)$ and $\Delta Y_0(t)$ are the solutions of the homogenous differential equations

$$\chi_s(\overset{(l)}{\eta}_s, \overset{(l-1)}{\eta_s}, \ldots, \overset{(0)}{\eta}_s) = 0,\ \overset{(0)}{\eta}_s = \eta_s$$

and

$$\chi_0(\Delta \overset{(k)}{Y}_0, \Delta \overset{(k-1)}{Y_0}, \ldots, \Delta \overset{(0)}{Y}_0) = 0,\ \Delta \overset{(0)}{Y}_0 = \Delta Y_0,$$

obtained as the response to the initial deviation states $\eta_s(t_0) = q_s^0(t_0) - q_s(t_0)$ and $\Delta Y_0(t_0) = Y_0^0(t_0) - Y_0(t_0)$, then the correspondence between the properties of the preceding differential equations and the variation character of the deviations $\eta_s(t)$ and ΔY_0 can be established. Analogous to the linear regulation loop, it can be required from the closed loop non-linear system that the nominal trajectory deviations satisfy the differential equations with the exactly defined properties regarding stability and quality indices of

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the solution behaviour. Rearranging the previous differential equations yields the deviation highest order as the functional relation of the deviation lower order derivatives as the independent variables. Further calculation renders the highest order derivative values of the controlled variables

$$\overset{(l)}{q}_s = \overset{(l)}{q}{}_s^0(t) - Q_s(\overset{(l-1)}{\eta_s}, \overset{(l-2)}{\eta_s}, \ldots, \eta_s)$$

and

$$\overset{(k)}{Y}_0(t) = \overset{(k)}{Y}{}_0^0(t) - Q_0(\Delta \overset{(k-1)}{Y_0}, \Delta \overset{(k-2)}{Y_0}, \ldots, \Delta Y_0)$$

that the controlled object must posses in order that the deviations between the real controlled and the nominal trajectory satisfy the required differential equations. Based on the requirements that the previous derivatives are realised, after substituting them into (5), the driving torques τ are calculated. The proposed procedure presents the expansion of the procedure based on the requirement that deviations from the nominals are satisfying linear differential equations, applied to the cooperative manipulation. This procedure is regularly met in the available literature. For the case of the manipulator contacting the dynamic environment, this expansion is given in reference 8.

In this tracking case, the calculated value

$$\overset{(k)}{Y}_0(t)$$

should be substituted into the third equation (5). The requirement is imposed through the third derivative ($k=3$) of the deviation between the real and nominal manipulated object MC trajectory. Differentiating the third equation in (5) under the condition that the matrix $\partial w_v | \partial \dot{q}_v$ is non-singular and using the master and slaves indexing convention (2) yields the constrained master accelerations as

$$\ddot{q}_v = -\alpha(\dddot{Y}_0, \ddot{Y}_0, \dot{Y}_0, Y_0, \dot{q}, q) - \beta \cdot \ddot{q}_s, \quad \left| \frac{\partial w_v}{\partial \dot{q}_v} \right| \neq 0, \tag{6}$$

where $\alpha(\dddot{Y}_0, \ddot{Y}_0, \dot{Y}_0, Y_0, \dot{q}, q) = (\partial w_v / \partial \dot{q}_v)^{-1}(\dot{W}(Y_0)\ddot{Y}_0 + W(Y_0)\dddot{Y}_0 + \partial w / \partial q \dot{q} + \partial w / \partial Y_0 \dot{Y}_0 + \partial w / \partial Y_0 \ddot{Y}_0) \in R^{6\times 1}$, $\beta = \beta(\dot{Y}_0, Y_0, \dot{q}, q) = (\partial w_v / \partial \dot{q}_v)^{-1} \partial w / \partial \dot{q}s \in R^{6\times(6m-6)}$. The matrix $\partial w_v / \partial \dot{q}_v$ is singular for the conditions corresponding to the elastic system passage through unloaded state[2]. It is considered that during the transfer the object is clenched and by that loaded. If, in spite of that, the matrix $\partial w_\omega / \partial \dot{q}_v$ is singular, erroneous selection of the master manipulator has been done. That selection must be changed as the manipulated object dynamics has no direct influence on the link accelerations of the manipulator selected as the master.

Let the differential equation

$$\Delta \dddot{Y}_0 = Q_0(\Delta \ddot{Y}_0, \Delta \dot{Y}_0, \Delta Y_0) \Rightarrow \dddot{Y}_0 = \dddot{Y}{}_0^0 - Q_0(\Delta \ddot{Y}_0, \Delta \dot{Y}_0, \Delta Y_0) \tag{7}$$

has the trivial asymptotically stable equilibrium state $\Delta Y_0 = 0$ (*i.e.* $Y_0^0(t) = Y_0(t)$) only.

Let it be required that this differential equation be thus selected that its trivial solution, obtained as the response to the initial deviation $\Delta Y_0(t_0) = Y_0^0(t_0) - Y_0(t_0)$, be asymptotically stable with the desired indices of the dynamic behaviour quality. Substituting the calculated necessary third derivative $\dddot{Y}_0(t)$ from (7) into (6) yields the constrained master accelerations as function of the system state, imposed requirement and slave accelerations $\ddot{q}_s$ in the form $\ddot{q}_v = -\alpha(\dddot{Y}{}_0^0 - Q_0(\Delta \ddot{Y}_0, \Delta \dot{Y}_0, \Delta Y_0), \ddot{Y}_0, \dot{Y}_0, Y_0, \dot{q}, q) - \beta \cdot \ddot{q}_s$. Substituting this acceleration into (5) the driving torques and master contact forces are obtained as the function of the slaves' accelerations, too. Therefore, it results that all driving torques and contact forces are depending on the slave accelerations $\ddot{q}_s$.

Let the differential equation

$$\ddot{\eta}_s(t) = Q_s(\dot{\eta}_s, \eta_s) \Rightarrow \ddot{q}_s(t) = \ddot{q}{}_s^0(t) - Q_s(\dot{\eta}_s, \eta_s) \tag{8}$$

has the trivial asymptotically stable equilibrium state $\eta_s = 0$ (*i.e.* $q_s^0(t) = q_s(t)$). Let it be required that this differential equation be thus selected that its trivial solution, obtained as the response to the initial deviation $\eta_s(t_0) = q_s^0(t_0) - q_s(t_0)$, be asymptotically stable with desired indices of the dynamic behaviour quality.

Substituting the calculated slaves' acccelerations $\ddot{q}_s$ from (8) into (5) the driving torques are calculated as

$$\begin{aligned} \tau_v &= N_v(q_v)[-\alpha(\dddot{Y}{}_0^0 - Q_0(\Delta \ddot{Y}_0, \Delta \dot{Y}_0, \Delta Y_0), \ddot{Y}_0, \dot{Y}_0, Y_0, \dot{q}, q) \\ &\quad - \beta \cdot (\ddot{q}{}_s^0(t) - Q_s(\dot{\eta}_s, \eta_s))] + \eta_v(q, \dot{q}, Y_0, \dot{Y}_0) \\ \tau_s &= N_s(q_s)(\ddot{q}{}_s^0(t) - Q_s(\dot{\eta}_s, \eta_s)) + \eta_s(q, \dot{q}, Y_0, \dot{Y}_0) \end{aligned} \tag{9}$$

These torques should be introduced into the manipulator joints in order to obtain the tracking of the required controlled output $Y^{u0} = col(Y_0^0, q_s^0)$ with the qualities indirectly required *a priori* by (7) and (8). In order to form the driving torques it is necessary to have information about all instantaneous manipulated object kinematic values $\ddot{Y}_0, \dot{Y}_0, Y_0$, information about internal coordinates q and their derivatives $\dot{q}$ and information about nominal outputs derivatives $\dddot{Y}{}_0^0, \ddot{Y}{}_0^0, \dot{Y}{}_0^0, Y_0^0, q_s^0 \dot{q}{}_s^0$ *and* $\ddot{q}{}_s^0$

Let us introduce the calculated driving torques (9) as the input into the cooperative manipulation model (5) and prove that the imposed requirements are realisable. After rearranging is obtained.

$$\begin{aligned} N_v(q_v)[\ddot{q}_v + \alpha(\dddot{Y}{}_0^0 - Q_0(\Delta \ddot{Y}_0, \Delta \dot{Y}_0, \Delta Y_0), \ddot{Y}_0, \dot{Y}_0, Y_0, \dot{q}, q) \\ + \beta \cdot (\ddot{q}{}_s^0(t) - Q_s(\dot{\eta}_s, \eta_s))] = 0 \\ N_s(q_s)[\ddot{q}{}_s^0(t) - \ddot{q}_s - Q_s(\dot{\eta}_s, \eta_s)] = 0 \\ W(Y_0)\dot{Y}_0 + w(q, \dot{q}, Y_0, \dot{Y}_0) = 0 \end{aligned} \tag{10}$$

Due to the matrix $N_s(q_s)$ non-singularity, from the second equation it follows that by introduced driving torques τ_s from (9) the imposed requirement (8) is realised. As the matrix $N_v(q_v)$ is also non-singular, the expression in the square brackets in the first equation must equal zero. The master acceleration $\ddot{q}_v$ has to satisfy (6), so after substituting the value of $\alpha(\ddot{Y}_0, \dot{Y}_0, \dot{Y}_0, \dot{q}, q)$ and rearranging, it is obtained $(\partial w_v / \partial \dot{q}_v)^{-1} W(Y_0) \cdot [\dddot{Y}{}_0^0 - \dddot{Y}_0 - Q_0(\Delta \ddot{Y}_0, \Delta \dot{Y}_0, \Delta Y_0)] = 0$. As the matrices $\partial w_v / \partial \dot{q}_v$ and $W(Y_0)$ are non-singular, it finally follows that by introduced driving torques the required deviation $(\dddot{Y}_0 = \dddot{Y}{}_0^0 - Q_0(\Delta \ddot{Y}_0, \Delta \dot{Y}_0, \Delta Y_0)$ is realised, presenting just the initially imposed requirement (7).

It has been demonstrated that the cooperative system controlled by the driving torques calculated in (9) tracks the

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nominal controlled outputs stably and with required qualities, indirectly imposed by (7) and (8). As the law of the controlled output third $\Delta\dddot{Y}_0$ and second $\ddot{\eta}_s = \ddot{q}_s^0 - \ddot{q}_s = \Delta\ddot{q}_s$ derivitive deviation is realised by the adopted control laws and as $\Delta Y_0(t) = 0$ and $\eta_s(t) = 0$ are the only asymptotically stable solutions of equations (7) and (8), the controlled outputs lower derivatives deviations from the nominal values are with the exponentially decreasing character. Therefore, the controlled outputs are tracked asymptotically stable, *i.e.* after initial deviation, with the increase of time they are tending toward their nominal values. The functional relation $Y_c = \Theta(q)$ causes that the slave manipulator internal coordinates q_s^0 tracking produces the slave manipulator contacts $Y_s^0 = \Theta(q_s^0)$ trajectory tracking. This relation is used to rate the behaviour of the uncontrolled variables q_v, F_{cv} and F_{cs} and calculated driving torques τ.

After the asymptotic tracking of the controlled outputs Y_0 *and* q_s is accomplished, the conclusion about the uncontrolled variables behaviour is made on the basis of the analysis of the elastic system physics. The goal of the analyses is to determine the uncontrolled variable deviations from their nominal values. The analysis results should answer the question if, on the basis of known controlled variable deviations from nominal values, the uncontrolled variable deviations from nominal values can be exactly determined as well as what their properties are.

To illustrate the considerations, motion in vertical plane of the simple elastic structure (elastic system) consisting of two rigid bodies with their MCs in the nodes, interconnected by non-inertial elastic link, is analysed (Figure 4.). Let in one node some external load $F_c = F_c(t)$ be acting. Let the node trajectories $Y_0^0 = Y_0^0(t)$, $Y_c^0(t) = Y_c^0(t)$ and contact force $F_c^0 = F_c^0(t)$ be determined *a priori*. To simplify the presentation, further on the time dependence will not be stated, but it will be considered that all variables refer to the determined observation instant t. During the motion is known that somewhere in the space the elastic structure unloaded state exists, but it is unknown where it exactly is, as the displacement of any node relative to the unloaded state is unknown. Let its position and orientation be determined by the coordinates Y_0^k and Y_c^k. If the nodes are moving along the nominal trajectories, the displacements relative to the unloaded state, equal to y_0^{k0} and $y_*^k = Y_* - Y_*^k$, $\Delta Y_* = Y_*^0 - Y_* = y_*^{k0} - y_*^k$, $* = 0, c$ are valid. The appearance of the elastic forces acting in the elastic system nodes is the consequence of these displacements. During the real motion these forces are $F_e = col(F_{e0}, F_{ec})$, while during the nominal one they are $F_e^0 = col(F_{e0}^0, F_{ec}^0)$. Regardless of the character and origin of the forces acting in the elastic system nodes, the elasticity properties are kept unchanged. Therefore, if in nodes are elastic links with damping properties and some masses and external force actions, the elastic forces F_e *and* F_e^0 are balancing the resultant of the dynamic $F_{d*}^{\#}$, gravitational G_* and contact $F_{c*}^{\#}$, $* = 0, c$, $\# = -, 0$ forces. For easy relation reference, the damping properties are taken out of consideration, resulting that dynamic forces acting in one elastic structure node are only depending on kinematic variables describing the state of that node. For the linear region of stress dilatation dependence the relations for undisturbed (nominal) $F_e^0 = K^k y^{k0}$ and disturbed $F_e = K^k y^k$ motion are valid. The subtraction yields $\Delta F_e = \Delta F_d + col\ (\Delta F_c, 0) = K^k\ (y^{k0} - y^k)$. Taking into account the kinematic relations, developed form of these equations in instant t is

$$\begin{aligned} \Delta F_{dc} + \Delta F_c &= A^k \cdot \Delta_c + b^k \cdot \Delta Y_0 \\ \Delta F_{d0} &= c^k \cdot \Delta Y_c + d^k \cdot \Delta Y_0 \end{aligned} \tag{11}$$

where: $\Delta F_{d*} = F_{d*}^0 - F_{d*}$, $* = c, 0$ and $\Delta F_c = F_c^0 - F_c$ are the dynamic and contact forces deviations from nominal values; $F_{d*}^{\#} = -W_*(Y_*^{\#})\ddot{Y}_*^{\#} - F_{b*}(Y_*^{\#}, \dot{Y}_*^{\#})$, $* = v, s, 0$, $\# = -, 0$ are the realised and nominal dynamic force; $A^k \in R^{3\times3}$, $b^k \in R^{3\times3}$, $c^k \in R^{3\times3}$, $d^k \in R^{3\times3}$ are the submatrices of the constant stiffness matrix K^k. If the position and orientation of the moving unloaded state is known and if the stiffness matrix K of the unloaded state at rest is defined, then the stiffness matrix K^k could be obtained by $K^k = A_r^T(a)KA_r(a)$, where $A_r(a) = diag\ (A(a), I, \ldots A_r(a), I)$ and I is the unit matrix. $A(a)$ is the coodinate transformation matrix in rotation for the orientation $a = \mathcal{A}_i^k - \mathcal{A}_{i0}$ defined by the difference between the moving unloaded state orientation $\mathcal{A}_i^k(t)$ in the instant t and unloaded state at rest orientation $\mathcal{A}_{i0}$. Although the moving unloaded state position and orientation, as well as the orientation $\mathcal{A}_i^k(t)$ of some its node are unknown, for

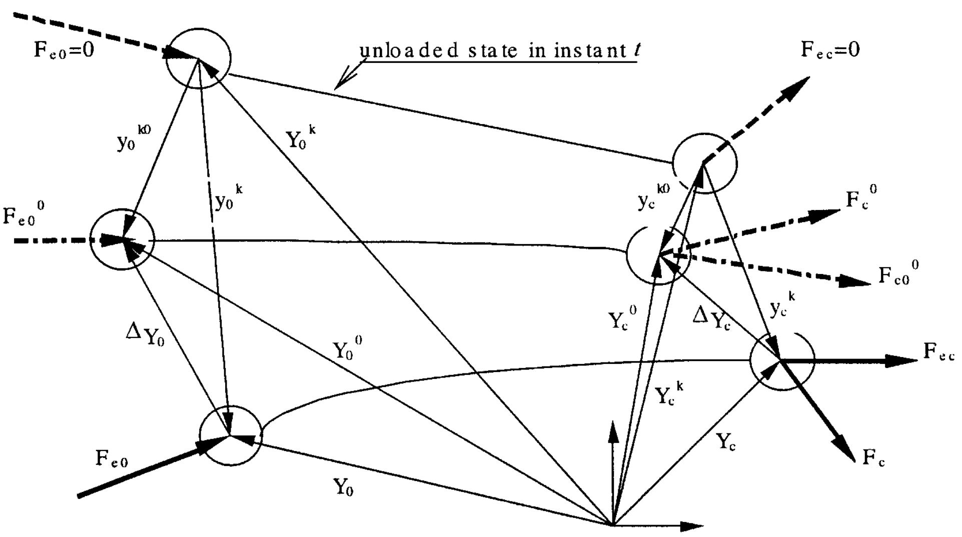

Fig. 4. Clenched elastic object motion in the plane

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conclusions further on, it is essential that in each instant t constant matrix K^k exists.

The elastic structure additional loading is described by (11). These relations can be considered as the equilibrium equations of the fictitious spatial grid, loaded in the nodes by forces $\Delta F_{dc}+\Delta F_c$ and ΔF_{d0} producing the corresponding node displacements ΔY_c and ΔY_0. Let us adopt as the fictitious spatial grid support the point with coordinates Y_0. Then, the support displacement is determined by ΔY_0, whereas the resistance of this support is determined by ΔF_{d0}. If the trajectories Y_0^0 and Y_0 are known (*i.e.* the character of their deviation $\Delta Y_0 = Y_0^0 - Y_0$), the character of F_{d0}^0, F_{d0} and ΔF_{d0} can be explicitly determined. The second node displacement ΔY_c that must be done in order that the support resistance is ΔF_{d0} when displaced by ΔY_0 is determined from the second equation. For known Y_c and ΔY_c Y_c^0 is determined and on their basis F_{dc}^0, F_{dc} and ΔF_{dc}, so that ΔF_c is easily determined from the first equation. Therefore, knowing ΔY_0, ΔY_c can be exactly determined, and on the basis of both of the ΔF_c.

On the basis of previous considerations, for the elastic system (1) guided along the nominal trajectories Y_0^0 and Y_s^0 the equations

$$\begin{aligned} \Delta F_{dv} + \Delta F_{cv} &= u_v^k \Delta Y_v + u_s^k \Delta Y_s + u_0^k \Delta Y_0 \\ \Delta F_{ds} + \Delta F_{cs} &= A_v^k \Delta Y_v + A_s^k \Delta Y_s + A_0^k \Delta Y_0 \\ \Delta F_{d0} &= c_v^k \Delta Y_v + c_s^k \Delta Y_s + c_0^k \Delta Y_0 \end{aligned} \qquad (12)$$

are obtained, describing the equilibrium of the fictitious space grid loaded in the nodes by forces $\Delta F_{dv}+\Delta F_{cv}$, $\Delta F_{ds}+\Delta F_{cs}$ and ΔF_{d0} producing the corresponding node displacements ΔY_v, ΔY_s, ΔF_0.

It is proved in the selection of the control laws (9) for the controlled output $Y^u(t) = col(q_s(t), Y_0(t))$ $6m$-dimensional vector, that the deviation vector $\Delta Y_{s0} = col(\Delta Y_s, \Delta Y_0)$ and its derivatives are with exponentially decreasing character. Considering this and knowing that the object physics imposes relation (12), the conclusion about uncontrolled variable deviations between the real and nominal trajectories must be made.

The vector $\Delta F_{ds0} = col(\Delta F_{ds}, \Delta F_{d0})$ is determined on the basis of ΔY_{s0} and its derivatives. Because of ΔY_{s0} and its derivatives variation character, the vector ΔF_{ds0} is also exponentially decreasing to zero. The variation ΔY_v is determined, for the non-singular matrix c_v^k, from the last equation (12). Because ΔY_v is linear dependant on ΔF_{d0}, ΔY_s and ΔY_0, so is ΔY_v also exponentially decreasing to zero. Therefore, the uncontrolled output variable Y_v is asymptotically tending to the master contact nominal trajectory Y_v^0. Using annalogous procedure to the second equation in (12), the conclusion that the slave contact force increments ΔF_{cs} are also exponentially decreasing to zero is made. Therefore, the slave contact forces are asymptotically tending to their nominal values F_{cs}^0.

For the known realised trajectory Y_v and calculated deviations ΔY_v the nominal trajectory Y_v^0 can be determined, and on their basis also ΔF_{dv}. The bounded value of ΔF_{cv} is finally calculated from the first equation in (12). This demonstrates that, in the case of asymptotic tracking the nominal trajectories of the manipulated object MC and slave contacts, all uncontrolled output variables are also asymptotically tracked. In the case that the trajectories of other m nodes are selected as the nominal ones, e.g. the contact nominal trajectories $Y_c^0 = col(Y_v^0(t), Y_s^0(t))$ only, the same conclusion can be drawn for the selected controlled output vector $Y^u = q$.

Normalising some of the equations of the cooperative system dynamics that is explicitly expressing the driving torques (9), (5) or the manipulator dynamics expression (3), the assessment of the driving torques behaviour can be performed. The simplest way to obtain the norm is to use the manipulator dynamics expression (3), *i.e.* $\|\tau\| \leq \|H(q)\|\,\|\ddot{q}\| + \|h(q, \dot{q})\| + \|J^T(q)\|\|f_c\|$ After initial deviation, with the increase of time q_s^0, q_v and $f_c = -F_c = -col(F_v, F_s)$ are tracked asymptotically stable. For bounded arguments all elements on the right side of the normalised driving torques equation are bounded, therefore, the driving torques are bounded.

Let us conclude, that tracking of the nominal controlled outputs $Y^u = col(Y_0^0, q_s^0)$ in the required manner, indirectly defined by (7) and (8), is provided by introducing the control laws defined by expressions (9) determining the driving torques that should be realised in the manipulator joints. These control laws are providing that, after the transient process generated by initial deviation between the controlled outputs and corresponding nominal values, uncontrolled variables are not unbounded.

5. Example of cooperative system control laws synthesis

In order to illustrate the problem of determining the control laws of the cooperative system coordinated motion, a simple example of the "linear cooperative system" (Figure 5.), elaborated in papers[4, 5], is reviewed. The system in the example consists of one object and two rigid one DOF manipulators. With regard to the examples published in references 4 and 5 this system is modified in reference 1 by the change of the axis along which the motion is performed. The change is done in order to emphazise the properties of the cooperative system at rest. Between the object and manipulators elastic connections are inserted. The masses of the connections are much smaller then the mass of the object, so their influence can be neglected.

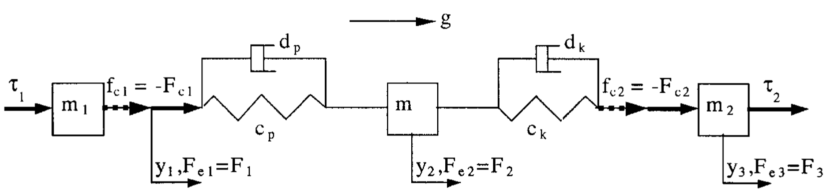

Fig. 5. "Linear cooperative system"

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To describe the general motion, the cooperative system dynamics model is expressed by the absolute coordinates Y in the form[1,12]

$$\begin{aligned} m_1\ddot{Y}_1+d_p\dot{Y}_1-d_p\dot{Y}_2+c_pY_1-c_pY_2+m_1g+c_ps_1&=\tau_1\\ m_2\ddot{Y}_3-d_k\dot{Y}_2+d_k\dot{Y}_3-c_kY_2+c_kY_3+m_2g-c_ks_3&=\tau_2\\ m\ddot{Y}_2-d_p\dot{Y}_1+(d_p+d_k)\dot{Y}_2-d_k\dot{Y}_3&\\ -c_pY_1+(c_p+c_k)Y_2-c_kY_3+mg-c_ps_1+c_ks_3&=0\\ d_p\dot{Y}_1-d_p\dot{Y}_2+c_pY_1-c_pY_2+c_ps_1&=F_{c1}\\ -d_k\dot{Y}_2+d_k\dot{Y}_3-c_kY_2+c_kY_3-c_ks_3&=F_{c2} \end{aligned} \tag{13}$$

The elastic system model (Figure 5) parameters are $s_1=s_2=0.05[m]$ (the distances between nodes in unloaded state); $m=25[kg]$; $c_p=20\cdot10^3[N/m]$; $c_k=10\cdot10^3[N/m]$; $d_p=500[N/(m/s)]$ and $d_k=1000[N/(m/s)]$. The manipulator model parameters are $m_1=12.5[kg]$ and $m_2=12.5$[kg]. The cooperative system initial position before the onset of the clenching process is determined by the nodes coordinates $Y_{10}=0.150[m]$, $Y_{20}=0.200[m]$ and $Y_{30}=0.250[m]$.

Adopting the first manipulator for the master and comparing (13) to (5) it is concluded that is $q_v=q_1=Y_1$, $q_s=q_2=Y_3$, $Y_0=Y_2$, $\tau_v=\tau_1$, $\tau_s=\tau_2$, $N_v(q_v)=m_1$, $N_s(q_s)=m_2$, $W(Y_0)=m$, $P_v(q_v)=0$, $P_s(q_s)=0$, $n_v(q,\dot{q},Y_0,\dot{Y}_0)=d_p\dot{Y}_1-d_p\dot{Y}_2+c_pY_1-c_pY_2+m_1g+c_ps_1$, $n_s(q,\dot{q},Y_0,\dot{Y}_0)=-d_k\dot{Y}_2+d_k\dot{Y}_3-c_kY_2+c_kY_3+m_2g-c_ks_3$, $w(q,\dot{q},Y_0,\dot{Y}_0)=-d_p\dot{Y}_1+(d_p+d_k)\dot{Y}_2-d_k\dot{Y}_3-c_pY_1+(c_p+c_k)Y_2-c_kY_3+mg-c_ps_1+c_ks_3$, $p_v(q,\dot{q},Y_0,\dot{Y}_0)=d_p\dot{Y}_1-d_p\dot{Y}_2+c_pY_1-c_pY_2+c_ps_1$ and $p_s(q,\dot{q},Y_0,\dot{Y}_0)=-d_k\dot{Y}_2+d_k\dot{Y}_3-c_k\dot{Y}_2+c_kY_3-c_ks_3$. In the synthesis of the control laws for tracking the nominal trajectories of the manipulated object MC and slave manipulator contact two functions must be selected. The concreate form of the differential equations describing the realised trajectory deviation relative to nominal trajectory must be selected for both, the manipulated object MC (7) and slave manipulator contact (8). Linear form of these equations is selected

$$\begin{aligned} \Delta\dddot{Y}_2+b_2\Delta\ddot{Y}_2+b_1\Delta\dot{Y}_2+b_0\Delta Y_2=k_{y2}u_{y2}|_{u_{y2}=0}=0\\ \ddot{\eta}_s+a_1\dot{\eta}_s+b_0\eta_s=k_\eta u_\eta|_{u_\eta=0}=0 \end{aligned} \tag{14}$$

where $\Delta Y_2=Y_2^0-Y_2$ and $\eta_s=Y_3^0-Y_3$. Comparing these equations to (7) and (8), it is concluded that $Q_0(\Delta\ddot{Y}_2,\Delta\dot{Y}_2,\Delta Y_2)=-b_2\Delta\ddot{Y}_2-b_1\dot{Y}_2-b_0\Delta Y_2+k_{y2}u_{y2}|_{u_{y2}=0}$ and $Q_s(\dot{\eta}_s,\eta_s)=-a_1\dot{\eta}_s+a_0\eta_s+k_\eta u_\eta|_{u_\eta=0}$. The following coefficient numerical values are selected: $b_0=7106.118[s^{-3}]$; $b_1=8883.0936[s^{-2}]$; $b_2=46.38938[s^{-1}]$; $a_0=355.3059[s^{-2}]$ and $a_1=26.38938[s^{-1}]$. The master acceleration is selected by relation (6), whereby the auxiliary relations $\alpha=-[m\dddot{Y}_2+(d_p+d_k)\ddot{Y}_2-c_p\dot{Y}_1+(c_p+c_k)\dot{Y}_2-c_k\dot{Y}_3]/d_p$ and $\beta=d_k/d_p$, with included $\partial w_v/\partial\dot{q}_v=\partial w/\partial\dot{Y}_1=-d_p$ and $\partial w_s/\partial\dot{q}_s=\partial w/\partial\dot{Y}_3=-d_k$, are selected *a priori*. In accordance to (9), the control laws are determined by

$$\begin{aligned} \tau_1=m_1[-\alpha^0-\beta(\ddot{Y}_3^0-Q_s^0)]+n_v\\ \tau_2=m_2[\ddot{Y}_3^0-Q_s^0]+n_s \end{aligned} \tag{15}$$

where N_v, N_s, n_v and n_s are stated without independent variable notation. The variables with upper index 0 are determined by $\alpha^0=-[m(\dddot{Y}_2^0-Q_0^0)+(d_p+d_k)\ddot{Y}_2-c_p\dot{Y}_1+(c_p+c_k)\dot{Y}_2-c_k\dot{Y}_3]/d_p$, $Q_0^0=-b_2(\ddot{Y}_2^0-\ddot{Y}_2)-b_1(Y_2^0-Y_2)-b_0(Y_2^0-Y_2)$ and $Q_s^0=-a_1(\dot{Y}_3^0-\dot{Y}_3)-b_0(Y_3^0-Y_3)$. The manipulated object MC nominal trajectory and its derivatives are Y_2^0, $\dot{Y}_2^0$, $\ddot{Y}_2^0$ and $\dddot{Y}_2^0$. The slave contact nominal trajectory and its derivatives are Y_3^0, $\dot{Y}_3^0$, $\ddot{Y}_3^0$, in this case being the same as the slave manipulator internal coodinate and its derivatives.

The testing of the synthesized control laws function is performed by observing the nominal trajectories tracking when initially the deviation between the real and nominal trajectories exists. The nominal trajectories along which the system should be guided are, for the same example, determined in reference 1. The nominal trajectories of the manipulated object MC Y_0^0 and slave manipulator contact Y_s^0 are tracked by the synthesized control laws. The simulation results of the controlled cooperative system are presented in the Figures 6a to 6d. The diagrams in the Figures are simultaneously presenting the nominal (the last character for the nominal variables symbol is '0') and the realised trajectories. Observing the results, high quality tracking of the nominal trajectories can be concluded, whereas the remaining uncontrolled variables are tending to corresponding nominal values.

6. CONCLUSION

The analysis of the cooperative system controllability considering it as a non-linear controlled object was performed in this paper on the basis of the cooperative manipulation dynamics model with solved force indefiniteness problem. It was demonstrated on the basis of the mapping between the non-linear system input, state and output domains that the existance of biunivocal correspondence between the input and controlled output domains is necessary condition for non-linear system output controllability. It has been concluded that the number of independent requirements that can be imposed to the cooperative system is exactly the same as the number of independent driving torques. The existence of two control task types based on controlled output vector classification is presented in the paper. The first one is the tracking of the selected point nominal trajectories only, while the second one is the tracking of one point nominal trajectory and nominal contact forces of slave manipulators. The control laws for tracking the nominal trajectories of the manipulated object MC and slave manipulator contacts are determined. They are presented by driving torques calculated from the condition that the controlled output deviations from corresponding nominals satisfy *a priori* the set of differential non-linear equations with exact properties regarding stability and quality indices of the solution behaviour. The closed loop cooperative system tracking the vector of the controlled output nominals in the required way has been proved analytically. For the case of the controlled output asymptotic tracking, conclusion about the uncontrolled variables behaviour has been done on the basis of the elastic system physics analyses. It has been demonstrated that, when the controlled output nominals are tracked asymptotically stable, the uncontrolled variables are tending to their nominal values. The driving torques calculation procedure and the closed loop cooperative system behaviour are illustrated on the

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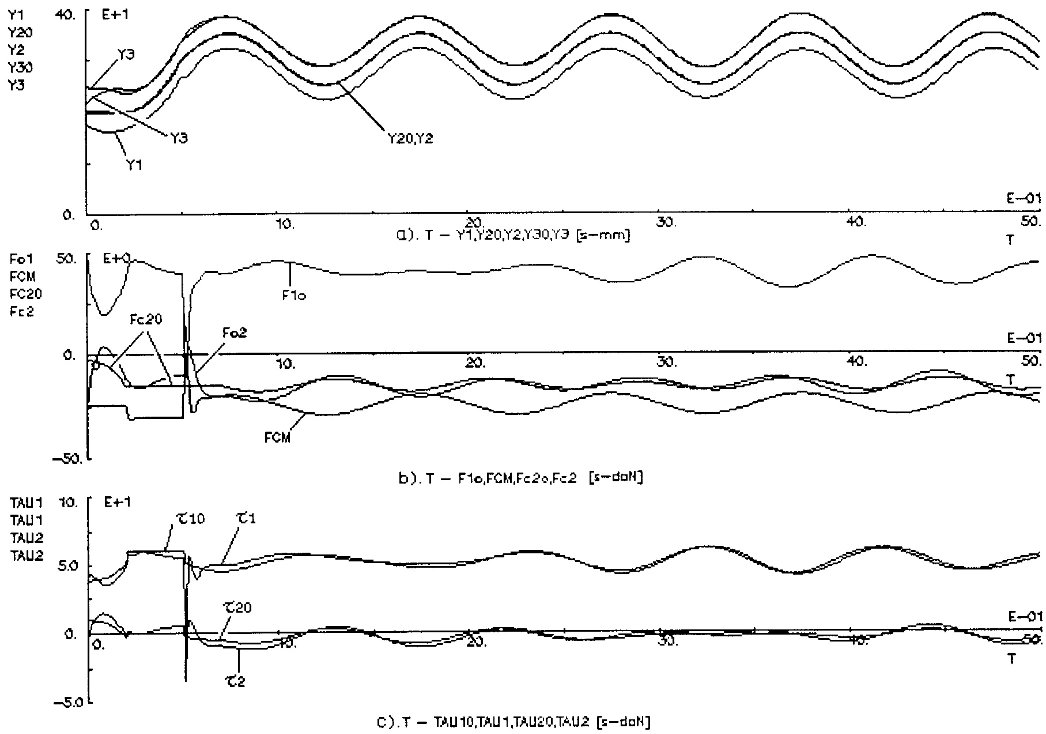

Fig. 6. Controlled "Linear cooperative system"

example of a simple cooperative system consisting of a manipulated object and two one DOF manipulators.

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