THE BULLETIN OF SYMBOLIC LOGIC
Volume 21, Number 1, March 2015

2014 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM '14

Vienna, AUSTRIA
July 14–19, 2014

Logic Colloquium '14, the 2014 European Summer Meeting of the Association for Symbolic Logic, was hosted by the Vienna University of Technology (TU Wien) from July 14 to July 19, 2014. The meeting was part of the Vienna Summer of Logic, which ran from July 9 to July 24, with almost twenty logic-related conferences and numerous workshops.

Funding for the conference and the Vienna Summer of Logic was provided by: the Association for Symbolic Logic (ASL); the Kurt Gödel Society; Technische Universität Wien; Universität Wien; Institute of Science and Technology, Austria; Akademie der Bildenden Künste Wien; Stadt Wien; Austrian Airlines; Bundesminesterium für Wissenschaft, Forschung, und Wirtschaft; the University of Manchester; Catering Kultur; Vienna Convention Bureau; Artificial Intelligence (Elsevier); European Association for Computer Science Logic; and Blacklane Limousines.

The success of the meeting was due largely to the hard work of the Local Organizing Committee under the leadership of its Chair, Matthias Baaz (TU Wien), and Co-chair, Stefan Hetzl (TU Wien). The other members of the committee were Agata Ciabattoni (TU Wien), Sebastian Eberhard (Bern), and Martin Goldstern (TU Wien). In addition, the following students and post-doctoral researchers assisted in the organization: Bahareh Afshari, Gabriel Ebner, Graham Leigh, Bernhard Mallinger, Janos Tapolczai, and Sebastian Zivota (all TU Wien).

The Program Committee consisted of Zofia Adamowicz (Polish Academy of Sciences, Warsaw), Jeremy Avigad (Carnegie Mellon, Chair), Marc Bezem (Bergen), Sy Friedman (Vienna), Jochen Koenigsman (Oxford), Kamal Lodaya (Institute of Mathematical Sciences, Chennai), Paulo Oliva (Queen Mary), Ted Slaman (Berkeley), and Richard Zach (Calgary).

The program included two tutorial courses, eleven invited plenary lectures, and twenty-nine invited lectures in seven special sessions. There were 133 contributed talks and 271 registered participants. Twenty one students and recent Ph.D.'s received ASL travel grants and nineteen students received NSF travel awards under a grant to the ASL.

The following tutorial courses were given:

Krzysztof Apt: *A tutorial on strategic and extensive games*.
Alexandre Miquel: *A tutorial on classical realizability and forcing*.

The following invited plenary lectures were presented:

Andrej Bauer: *Reductions in computability theory from a constructive point of view*.
Paddy Blanchette: *The birth of semantic entailment*.
Kirsten Eisenträger: *Generalizations of Hilbert's Tenth Problem*.
Andrés Cordón Franco: *On local induction schemes*.
Vera Fischer: *Cardinal invariants and template iterations*.
Noam Greenberg: *Applications of admissible computability*.
Leszek Kołodziejczyk: *The problem of a model without collection and without exponentiation*.

1079-8986/15/2101-0004
DOI:10.1017/bsl.2015.3

Ben Miller: *Definable cardinals just beyond* $\mathbb{R}/\mathbb{Q}$.
Mark Reynolds: *Synthesis for monadic logic over the reals*.
Mariya Soskova: *Definability, automorphisms and enumeration degrees*.
Albert Visser: *On a theorem of McAloon*.

The twenty-fifth annual Gödel Lecture was presented by Julia Knight: *Computable structure theory and formulas of special forms*.

The following lectures were presented in honor of the Karp Prize recipients:

Matt Foreman: *The Singular Cardinals Problem after 130 years or so*, on the work of Moti Gitik.

Matthias Aschenbrenner: *Logic meets number theory in o-minimality*, on the work of Ya'acov Peterzil, Jonathan Pila, Sergei Starchenko, and Alex Wilkie.

More information about the meeting can be found at the conference web page, `http://www.logic.at/lc2014/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
JEREMY AVIGAD

Abstracts of Invited Tutorials

▸ KRZYSZTOF R. APT, *A tutorial on strategic and extensive games*.
CWI and University of Amsterdam, Amsterdam, The Netherlands.
E-mail: `apt@cwi.nl`.

The aim of this tutorial is to introduce the most fundamental concepts and results concerning strategic and extensive games. No prior knowledge of the subject is assumed.

Strategic games deal with the analysis of interaction between rational players, where rationality is understood as payoff maximization. In strategic games the players take their actions simultaneously and the payoff for each player depends on the resulting joint action.

We shall begin by introducing the fundamental notions of a Nash equilibrium and of mixed strategies. Then we shall discuss the fundamental result of Nash stating that every finite game has a Nash equilibrium in mixed strategies and compare it with an earlier result of Von Neumann concerning equilibria in zero-sum games.

Subsequently we shall discuss various ways of elimination of strategies, in particular iterated elimination of strictly and of weakly strategies, and the concept of rationalizability due to Bernheim and Pearce.

The final part of the tutorial will deal with extensive games. These are games in which the players take their actions in turn. We shall discuss the so-called Zermelo result about the game of chess. Finally, we shall introduce the notion of a subgame perfect equilibrium due to Selten and relate it to the procedure of backward induction.

A short guide to the literature. The first book on game theory was [15] that profoundly influenced the subsequent developments. There are by now several excellent books on strategic and extensive games. Most of them are written from the perspective of applications to Economics and cover also other topics. [9] is a broad in its scope, undergraduate level textbook, while [10] is probably the best book on the market for the graduate level. Undeservedly less known is the short and lucid [14]. An elementary, short introduction, focusing on the concepts, is [12]. In turn, [11] is a comprehensive book on strategic games and extensive games. Finally, [4] is an insightful and occasionally entertaining introduction to game theory. Several textbooks on microeconomics include introductory chapters on game theory, notably strategic and extensive games. Two good examples are [6] and [5]. In turn, [8] is a collection of surveys and introductions to the computational aspects of game theory, with a number of

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

articles concerned with strategic games. Finally, [7] is a most recent, very comprehensive account of the main areas of game theory, while [13] is an elegant introduction to the subject. We conclude by mentioning three references to our work that we shall rely upon: [3], [1], and [2].

[1] K. R. Apt, *Direct proofs of order independence*. ***Economics Bulletin***, vol. 1 (2011), no. 31, pp. 106–115, available from `http://www.economicsbulletin.com/`.

[2] ———, ***A primer on strategic games***, (K. R. Apt and E. Grädel, editors), Lectures in Game Theory for Computer Scientists, pp. 1–37, Cambridge University Press, available from `http://www.cwi.nl/~apt`, 2011.

[3] K. R. Apt and J. A. Zvesper, *The role of monotonicity in the epistemic analysis of strategic games*. ***Games***, vol. 1 (2010), no. 4, pp. 381–394.

[4] K. Binmore, ***Playing for real: A text on game theory***, Oxford University Press, Oxford, 2007.

[5] G. Jehle and P. Reny, ***Advanced microeconomic theory***, Addison Wesley, Reading, Massachusetts, second edition, 2000.

[6] A. Mas-Collel, M. D. Whinston, and J. R. Green, ***Microeconomic theory***, Oxford University Press, Oxford, 1995.

[7] M. Maschler, E. Solan, and S. Zamir, ***Game theory***, Cambridge University Press, Cambridge, 2013.

[8] N. Nisan, T. Roughgarden, É. Tardos, and V. J. Vazirani, editors, ***Algorithmic game theory***, Cambridge University Press, Cambridge, 2007.

[9] M. J. Osborne, ***An introduction to game theory***, Oxford University Press, Oxford, 2005.

[10] H. Peters, ***Game theory: A multi-leveled approach***, Springer, Berlin, 2008.

[11] K. Ritzberger, ***Foundations of non-cooperative game theory***, Oxford University Press, Oxford, 2002.

[12] Y. Shoham and K. Leyton-Brown, ***Essentials of game theory: A concise, multidisciplinary introduction***, Morgan and Claypool Publishers, Princeton, 2008.

[13] S. Tadelis, ***Game theory: An introduction***, Princeton University Press, Princeton, 2013.

[14] S. Tijs, ***Introduction to game theory***, Hindustan Book Agency, Gurgaon, India, 2003.

[15] J. von Neumann and O. Morgenstern, ***Theory of games and economic behavior***, Princeton University Press, Princeton, 1944.

▸ ALEXANDRE MIQUEL, *A tutorial on classical realizability and forcing*.

IMERL, Facultad de Ingeniería, Universidad de la República (UdelaR), Julio Herrera y Reissig 565, Montevideo C.P. 11300, Uruguay.
E-mail: `amiquel@fing.edu.uy`.

The theory of classical realizability was introduced by Krivine [4] in the middle of the 90's to analyze the computational contents of classical proofs, following the connection between classical reasoning and control operators discovered by Griffin [2]. More than an extension of Kleene's intuitionistic realizability [3], classical realizability is a complete reformulation of the principles of realizability, with strong connections with Cohen forcing [1, 5, 7, 6].

The aim of this tutorial is to present the basics of classical realizability as well as some of its connections with Cohen forcing. For that, I will first present the theory in the framework of second-order arithmetic (PA2), focusing on its computational aspects and on classical program extraction. Then I will show how to combine classical realizability with Cohen forcing (in PAω) and give a computational interpretation of this combination. Finally, I will present some research directions, explaining why classical realizability can be seen as a noncommutative form of forcing.

[1] P. J. Cohen, *The independence of the continuum hypothesis*. ***Proceedings of the National Academy of Sciences of the United States of America***, vol. 50 (1963), no. 6, pp. 1143–1148.

[2] T. Griffin, *A formulae-as-types notion of control*, ***Principles Of Programming Languages*** (***POPL'90***), 1990, pp. 47–58.

[3] S. C. Kleene, *On the interpretation of intuitionistic number theory*. ***The Journal of Symbolic Logic***, vol. 10, 1945, pp. 109–124.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

[4] J.-L. KRIVINE, *Realizability in classical logic*, ***Interactive models of computation and program behaviour***, Panoramas et synthèses, vol. 27, Société Mathématique de France, 2009, pp. 197–229.

[5] ———, *Realizability algebras: A program to well order* $\mathbb{R}$. ***Logical Methods in Computer Science***, vol. 7 (2011), no. 3:02, pp. 1–47.

[6] ———, *Realizability algebras II: New models of* $ZF + DC$. ***Logical Methods in Computer Science***, vol. 8 (2012), no. 1:10, pp. 1–28.

[7] A. MIQUEL, *Forcing as a program transformation*. ***Logic in Computer Science*** (***LICS'11***), 2011, pp. 197–206.

Karp Prize Lectures

▸ MATTHIAS ASCHENBRENNER, *Logic meets number theory in o-minimality*.
Department of Mathematics, University of California, Los Angeles, Box 951555, Los Angeles, CA 90095-1555, USA.
E-mail: `matthias@math.ucla.edu`.

In the past, applications of logic to number theory have mostly come through the model theory of certain *algebraic* structures (such as the field of p-adic numbers, or fields equipped with a derivation). The work of the Karp Prize winners Peterzil, Pila, Starchenko, and Wilkie harnesses the power of model-theoretic structures which have a more *analytic* flavor but are seemingly far removed from arithmetical considerations: o-minimal expansions of the field of real numbers. This leads to novel applications to number theory. A high point of these developments to date is the proof of certain special cases of the André–Oort Conjecture by Pila. Indispensable ingredients in this proof are a counting theorem by Pila–Wilkie as well as definability results due to Peterzil–Starchenko. I plan to survey this circle of ideas, with as few extra-logical prerequisites as possible.

▸ MATTHEW FOREMAN, *The Singular Cardinals Problem after 130 years or so*.
Mathematics Department, UC Irvine, Irvine, CA 92697, USA.
E-mail: `mforeman@math.uci.edu`.

We trace the history of singular cardinals problem from its inception †to the remarkable work of Shelah and Gitik, culminating in the PCF theory and the PCF conjecture.

25th Annual Gödel Lecture

▸ JULIA F. KNIGHT, *Computable structure theory and formulas of special forms*.
University of Notre Dame, Mathematics Department, 255 Hurley Hall, Notre Dame, IN 46556, USA.
E-mail: `knight.1@nd.edu`.

In computable structure theory, we ask questions about complexity of structures and classes of structures. For a particular countable structure $\mathcal{M}$, how hard is it to build a copy? Can we do it effectively? How hard is it to describe $\mathcal{M}$, up to isomorphism, distinguishing it from other countable structures? For a class K, how hard is it to characterize the class, distinguishing members from nonmembers? How hard is it to classify the elements of K, up to isomorphism. In the lecture, I will describe some results on these questions, obtained by combining ideas from computability, model theory, and descriptive set theory. Of special importance are formulas of special forms.

Abstracts of Invited Plenary talks

▸ ANDREJ BAUER, *Reductions in computability theory from a constructive point of view*.
Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia.
E-mail: `Andrej.Bauer@andrej.com`.

In constructive mathematics we often consider implications between nonconstructive reasoning principles. For instance, it is well known that the Limited principle of omniscience implies that equality of real numbers is decidable. Most such reductions proceed by reducing an instance of the consequent to an instance of the antecedent. We may therefore define a notion of *instance reducibility*, which turns out to have a very rich structure. Even better, under Kleene's function realizability interpretation instance reducibility corresponds to Weihrauch reducibility, while Kleene's number realizability relates it to truth-table reducibility. We may also ask about a constructive treatment of other reducibilities in computability theory. I shall discuss how one can tackle Turing reducibility constructively via Kleene's number realizability. One can then ask whether the constructive formulation of Turing degrees relates them to standard mathematical concepts.

► PATRICIA BLANCHETTE, *The birth of semantic entailment*.
Department of Philosophy, University of Notre Dame, Notre Dame, IN 46556, USA.
E-mail: blanchette.1@nd.edu.

The relation of semantic entailment, i.e., of a conclusion's being true on every model of its premises, currently plays a central role in logic, and is arguably the canonical entailment-relation in most contexts. But it wasn't always this way; the relation doesn't come into its own until shortly before its starring role in the completeness theorem for first-order logic. This talk investigates the development of the notion of *model* from the mid-19th century to the early 20th century, and the parallel emergence of logic's concern with the relation of semantic entailment. We will be especially interested in clarifying some of the ways in which the emergence of the modern conceptions of model and of entailment are tied to a changing view of the nature of axiomatic foundations.

► ANDRÉS CORDÓN-FRANCO, *On local induction schemes*.
Department of Computer Science and Artificial Intelligence, Faculty of Mathematics, University of Seville, Avd. Reina Mercedes s/n, 41012, Seville, Spain.
E-mail: acordon@us.es.

First-order Peano arithmetic PA is axiomatized over a finite algebraic base theory by the full induction scheme

$$\varphi(0, v) \wedge \forall x\,(\varphi(x, v) \rightarrow \varphi(x+1, v)) \rightarrow \forall x\, \varphi(x, v),$$

where $\varphi(x, v)$ ranges over all formulas in the language of arithmetic $\{0, 1, +, \cdot, <\}$. Fragments of arithmetic are obtained by restricting, in one way or another, the induction scheme axiomatizing PA. Classical examples include the theories of Σ_n and Π_n induction and their parameter free counterparts.

In this talk we present a new kind of restriction on the induction scheme, giving rise to new subsystems of arithmetic that we collectively call *local induction* theories. Roughly speaking, local indiction axioms have the form

$$\varphi(0, v) \wedge \forall x\,(\varphi(x, v) \rightarrow \varphi(x+1, v)) \rightarrow \forall x \in \mathcal{D}\, \varphi(x, v).$$

That is to say, we restrict the x's for which the axiom claims $\varphi(x, v)$ to hold to the elements of a prescribed subclass $\mathcal{D}$ of the universe. Natural choices for $\mathcal{D}$ are the sets of the Σ_n-definable elements of the universe as well as other related substructures of definable elements.

We will study the basic properties of the local induction theories obtained in this way and derive a number of applications to the study of 'classical' fragments of PA. Remarkably, we show that the hierarchy of local reflection principles can be reexpressed in terms of our local induction theories, thus filling a gap in our understanding of the equivalence between reflection and induction in arithmetic.

($\star$) This is joint work with F. Félix Lara-Martín (University of Seville).
($\star\star$) Partially supported by grant MTM2011-26840, Ministerio de Ciencia e Innovación, Spanish Government.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

▸ KIRSTEN EISENTRÄGER, *Generalizations of Hilbert's Tenth Problem*.
Department of Mathematics, The Pennsylvania State University, University Park, PA 16802, USA.
E-mail: eisentra@math.psu.edu.

Hilbert's Tenth Problem in its original form was to find an algorithm to decide, given a multivariate polynomial equation with integer coefficients, whether it has a solution over the integers. In 1970 Matiyasevich, building on work by Davis, Putnam and Robinson, proved that no such algorithm exists, i.e., Hilbert's Tenth Problem is undecidable. Since then, analogues of this problem have been studied by asking the same question for polynomial equations with coefficients and solutions in other commutative rings. The biggest open problem in the area is Hilbert's Tenth Problem over the rational numbers. In this talk we will construct some subrings R of the rationals that have the property that Hilbert's Tenth Problem for R is Turing equivalent to Hilbert's Tenth Problem over the rationals. We will also discuss some recent undecidability results for function fields of positive characteristic.

▸ VERA FISCHER, *Cardinal invariants and template iterations*.
Kurt Gödel Research Center, University of Vienna, Währingerstrasse 25, 1090 Vienna, Austria.
E-mail: vera.fischer@univie.ac.at.

The cardinal invariants of the continuum arise from combinatorial, topological, and measure theoretic properties of the reals, and are often defined to be the minimum size of a family of reals satisfying a certain property.

An example of such an invariant is the minimum size of a subgroup of S_∞, all of whose nonidentity elements have only finitely many fixed points and which is maximal (with respect to this property) under inclusion. This cardinal invariant is denoted $\mathfrak{a}_g$. Another well-known invariant, denoted non$(\mathcal{M})$, is the minimum size of a set of reals which is not meager. It is a ZFC theorem that non$(\mathcal{M}) \leq \mathfrak{a}_g$. A third invariant, denoted $\mathfrak{d}$, is the minimum size of a family $\mathcal{F}$ of functions in ${}^\omega\omega$ which has the property that every function in ${}^\omega\omega$ is eventually dominated by an element of $\mathcal{F}$. In contrast to the situation between $\mathfrak{a}_g$ and non$(\mathcal{M})$, ZFC cannot prove either of the inequalities $\mathfrak{a}_g \leq \mathfrak{d}$ or $\mathfrak{d} \leq \mathfrak{a}_g$. The classical forcing techniques seem, however, to be inadequate in addressing the consistency of $\mathfrak{d} < \mathfrak{a}_g$ which was obtained only after a ground-breaking work by Shelah and the appearance of his "template iteration" forcing techniques.

We further develop these techniques to show that $\mathfrak{a}_g$, as well as some of its relatives, can be of countable cofinality. In addition we will discuss other recent developments of the technique and conclude with open questions and directions for further research.

▸ NOAM GREENBERG, *Applications of admissible computability*.
School of Mathematics, Statistics and Operations Research, Victoria University of Wellington, PO Box 600, Wellington 6140, New Zealand.
E-mail: greenberg@msor.vuw.ac.nz.

Admissible computability is an extension of traditional computability theory to ordinals beyond the finite ones. I will discuss two manifestations of admissible computability in the study of effective randomness and in the study of effective properties of uncountable structures.

▸ LESZEK KOŁODZIEJCZYK, *The problem of a model without collection and without exponentiation*.
Institute of Mathematics, University of Warsaw, Banacha 2, 02-097 Warszawa, Poland.
E-mail: lak@mimuw.edu.pl.

$\mathrm{I}\Delta_0$ is the fragment of first-order arithmetic obtained by restricting the induction scheme to bounded formulas. $\mathrm{B}\Sigma_1$ extends $\mathrm{I}\Delta_0$ by the collection scheme for bounded formulas, that is by the axioms

$$\forall x < v\, \exists y\, \psi(x, y) \Rightarrow \exists w\, \forall x < v\, \exists y < w\, \psi(x, y),$$

where ψ is bounded (and may contain additional parameters).

It has been known since the seminal work of Parsons and of Paris and Kirby in the 1970s that $B\Sigma_1$ does not follow from $I\Delta_0$, even though it is Π^0_2-conservative over $I\Delta_0$. However, all constructions of a model of $I\Delta_0$ not satisfying $B\Sigma_1$ make use of the axiom Exp, which asserts that 2^x is a total function. From the perspective of $I\Delta_0$, which does not prove the totality of any function of superpolynomial growth, the totality of exponentiation is a very strong unprovable statement. This led Wilkie and Paris [2] to ask whether $I\Delta_0 + \neg\mathrm{Exp}$ proves $B\Sigma_1$.

It is generally believed that the answer to Wilkie and Paris's question is negative, and there are various statements from computational complexity theory, in some cases mutually contradictory, known to imply a negative answer. However, an unconditional proof of a negative answer remains elusive.

I plan to survey some facts related to Wilkie and Paris's question, focusing on two recent groups of theorems:

(i) the results of the paper [1], which seem to suggest that we are a "small step" away from building a model of $I\Delta_0 + \neg\mathrm{Exp}$ without collection,
(ii) some new results suggesting that the "small step" will be very hard to take, because there is a complexity-theoretic statement, almost certainly false but possibly not disprovable using present-day methods, which implies that $B\Sigma_1$ does follow from $\neg\mathrm{Exp}$.

[1] Z. Adamowicz, L. A. Kołodziejczyk, and J. Paris, *Truth definitions without exponentiation and the Σ_1 collection scheme*. ***The Journal of Symbolic Logic***, vol. 77 (2012), no. 2, pp. 649–655.

[2] A. Wilkie and J. Paris, *On the existence of end extensions of models of bounded induction*, ***Logic, Methodology, and Philosophy of Science VIII*** (Moscow 1987), (J. E. Fenstad, I. T. Frolov, and R. Hilpinen, editors), North-Holland, 1989, pp. 143–162.

► BENJAMIN D. MILLER, *Definable cardinals just beyond $\mathbb{R}/\mathbb{Q}$*.
Institut für Mathematische Logik und Grundlagenforschung, Fachbereich Mathematik und Informatik, Universität Münster, Einsteinstraße 62, 48149 Münster, Germany.
E-mail: `glimmeffros@gmail.com`.
URL Address: `http://wwwmath.uni-muenster.de/u/ben.miller`.

Over the last few decades, a definable refinement of the usual notion of cardinality has been employed to great effect in shedding new light on many classification problems throughout mathematics. In order to best understand such applications, one must investigate the abstract nature of the definable cardinal hierarchy.

It is well known that the initial segment of the hierarchy below $\mathbb{R}/\mathbb{Q}$ looks quite similar to the usual cardinal hierarchy. On the other hand, if one moves sufficiently far beyond $\mathbb{R}/\mathbb{Q}$, the two notions diverge wildly.

After reviewing these results, we will discuss recent joint work with Clinton Conley, seeking to explain the difficulty in understanding definable cardinality beyond $\mathbb{R}/\mathbb{Q}$ by showing that the aforementioned wild behavior occurs immediately thereafter.

► MARK REYNOLDS, *Synthesis for monadic logic over the reals*.
CSSE, The University of Western Australia, 35 Stirling Highway, Nedlands 6009, W.A., Australia.
E-mail: `mark.reynolds@uwa.edu.au`.

We say that a first-order monadic logic of order (FOMLO) sentence is satisfiable over the reals if there is some valuation for the monadic predicates which makes the formula true. Burgess and Gurevich showed that satisfiability for this logic is decidable. They built on pioneering work by Läuchli and Leonard who, in showing a similar result for linear orders in general, had presented some basic operations for the compositional building of monadic linear structures.

We look at some recent work in using these basic operations to give a synthesis result. That is, we present an algorithm which given a FOMLO sentence which is satisfiable over the reals, outputs a specific finite description of a model.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

► MARIYA I. SOSKOVA, *Definability, automorphisms and enumeration degrees*.
Faculty of Mathematics and Informatics, Sofia University, 5 James Bourchier Blvd., 1164 Sofia, Bulgaria.
E-mail: msoskova@fmi.uni-sofia.bg.

The enumeration degrees are an upper semi-lattice with a least element and jump operation. They are based on a positive reducibility between sets of natural numbers, enumeration reducibility, introduced by Friedberg and Rogers in 1959. The Turing degrees have a natural isomorphic copy in the structure of the enumeration degrees, namely the substructure of the total enumeration degrees. A long-standing question of Rogers [5] is whether the substructure of the total enumeration degrees has a natural first order definition. The first advancement towards an answer to this question was made by Kalimullin [4]. He discovered the existence of a special class of pairs of enumeration degrees, $\mathcal{K}$-pairs, and showed that this class has a natural first order definition in $\mathcal{D}_e$. Building on this result, he proved the first order definability of the enumeration jump operator and consequently obtained a first order definition of the total enumeration degrees above $\mathbf{0}_e{}'$. Ganchev and Soskova [3] showed that when we restrict ourselves to the local structure of the enumeration degrees bounded by $\mathbf{0}_e{}'$, the class of $\mathcal{K}$-pairs is still first order definable. In [2] they investigated maximal $\mathcal{K}$-pairs and showed that within the local structure the total enumeration degrees are first order definable as the least upper bounds of maximal $\mathcal{K}$-pairs.

The question of the global definability of the total enumeration degrees is finally solved by Cai, Ganchev, Lempp, Miller and Soskova [1]. They show that Ganchev and Soskova's local definition of total enumeration degrees is valid globally. Then using this fact, they show that the relation "c.e. in", restricted to total enumeration degrees is also first order definable. We will discuss these results and certain consequences, regarding the automorphism problem in both degree structures.

This research was supported by a BNSF grant No. DMU 03/07/12.12.2011, by a Sofia University SF grant and by a Marie Curie international outgoing fellowship STRIDE (298471) within the 7th European Community Framework Programme.

[1] M. Cai, H. A. Ganchev, S. Lempp, J. S. Miller and M. I. Soskova, *Defining totality in the enumeration degrees*, submitted.

[2] H. A. Ganchev and M. I. Soskova, *Definability via Kalimullin pairs in the structure of the enumeration degrees*. ***Transactions of the American Mathematical Society***, to appear.

[3] ———, *Cupping and definability in the local structure of the enumeration degrees*. ***The Journal of Symbolic Logic***, vol. 77 (2012), no. 1, pp. 133–158.

[4] I. Sh. Kalimullin, *Definability of the jump operator in the enumeration degrees*. ***Journal of Mathematical Logic***, vol. 3 (2003), pp. 257–267.

[5] H. Rogers, Jr., ***Theory of recursive functions and effective computability***, McGraw-Hill Book Company, New York, 1967.

► ALBERT VISSER, *On a theorem of McAloon*.
Philosophy, Faculty of Humanities, Utrecht University, Janskerkhof 13, 3512 BL Utrecht, The Netherlands.
E-mail: a.visser@uu.nl.

A theory is *restricted* if there is a fixed bound on the complexity of its axioms. In his classical paper [1], Kenneth McAloon proves that every restricted arithmetical theory that is consistent with Peano Arithmetic has a model in which the standard natural numbers are definable. In slogan, one could say that McAloon shows that one needs the full language to exclude the standard numbers in principle.

In this talk we discuss the idea of generalizing McAloon's result to the class of consistent restricted sequential theories. We only obtain a weaker statement for the more general case. Whether the stronger statement holds remains open.

Sequential theories are, as a first approximation, theories with sufficient coding machinery for the construction of partial satisfaction predicates of a certain sort. Specifically, we have satisfaction for classes of formulas with complexity below n for a complexity measure like *depth of quantifier alternations*. Sequential theories were introduced by Pavel Pudlák in [2].

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

There are several salient general results concerning sequential theories. For example the degrees of interpretability of sequential theories have many good properties. Examples of sequential theories are PA^-, S^1_2, $\mathrm{I}\Sigma_1$, PA, ACA_0, ZF, GB.

To any sequential model $\mathcal{M}$ we can uniquely assign an arithmetical model $\mathcal{J}_{\mathcal{M}}$. This is, roughly, the intersection of all definable cuts of an internal model $\mathcal{N}$ of a weak arithmetic like S^1_2. We can show that $\mathcal{J}_{\mathcal{M}}$ is independent of the specific choice of $\mathcal{N}$. Our theorem says that any consistent restricted sequential theory U has a model $\mathcal{M}$ such that $\mathcal{J}_{\mathcal{M}}$ is isomorphic to the standard model.

In the talk, we will briefly indicate how McAloon's proof works and discuss some immediate generalizations. Then, we will outline the basic ideas behind the proof of the result concerning consistent restricted sequential theories.

[1] K. McAloon, *Completeness theorems, incompleteness theorems and models of arithmetic*. ***Transactions of the American Mathematical Society***, vol. 239 (1978), pp. 253–277.

[2] P. Pudlák, *Some prime elements in the lattice of interpretability types*. ***Transactions of the American Mathematical Society***, vol. 280 (1983), pp. 255–275.

Abstracts of invited talks in the Special Session on The Place of Logic in Computer Science Education

▸ BYRON COOK, ALEXANDER LEITSCH, PRAKASH PANANGADEN, NICOLE SCHWEIKARDT, HELMUT VEITH, RICHARD ZACH, *The place of logic in computer science education*.
Microsoft Research, 21 Station Road, Cambridge, CB1 2FB, United Kingdom.
E-mail: `bycook@microsoft.com`.
URL Address: `http://research.microsoft.com/en-us/people/bycook/`.
Institute for Computer Languages, Theory and Logic Group, Vienna University of Technology, Favoritenstrasse 9, A–1040 Vienna, Austria.
E-mail: `leitsch@logic.at`.
URL Address: `http://www.logic.at/staff/leitsch`.
School of Computer Science, McGill University, 3480 rue University, Montreal, QC H3A 0E9, Canada.
E-mail: `prakash@cs.mcgill.ca`.
URL Address: `http://www.cs.mcgill.ca/~prakash/`.
Institute for Computer Science, Goethe-University Frankfurt am Main, Robert-Mayer-Strasse 11–15, D–60054 Frankfurt/Main, Germany.
E-mail: `schweika@informatik.uni-frankfurt.de`.
URL Address: `http://www.tks.informatik.uni-frankfurt.de/schweika`.
Institute for Information Systems, Formal Methods in Systems Engineering Group, Vienna University of Technology, Favoritenstrasse 9, A–1040 Vienna, Austria.
E-mail: `veith@forsyte.at`.
URL Address: `http://forsyte.at/people/veith/`.
Department of Philosophy, University of Calgary, 2500 University Dr. NW, Calgary, AB T2N 0A9, Canada.
E-mail: `rzach@ucalgary.ca`.
URL Address: `http://richardzach.org/`.

Logic has been called the "calculus of computer science"—and yet, while any physics student is required to take several semesters of calculus, the same cannot be said about logic and students of computer science. Despite the great and burgeoning activity in logic-related topics in computer science, there has been very little interest, in North America at least, in developing a strong logic component in the undergraduate curriculum. Meanwhile, in other parts of the world, departments have set up specialized degree programs on logical methods and CS. This special session, organized under the auspices of the ASL's Committee on Logic Education, aims to explore the role of logic in the computer science curriculum. How are computer scientists trained in logic, if at all? What regional differences are there, and why?

Is a greater emphasis on logic in the computer science undergraduate curriculum warranted, both from the point of view of training for research in CS and from the point of view of training for industry jobs? What should an ideal "Logic for Computer Science" course look like?

Byron Cook believes that, in the rush to create engineers and scientists, we have lost sight of the fact that an education should be broad and place emphasis on principles rather than specific skills such as Javascript. Logic is the perfect topic in this setting, as it has application in both humanities and science, and fosters a discussion about mechanics while not requiring a significant amount of technical overhead.

The Association for Computing Machinery has just chartered a new Special Interest Group on Logic and Computation (SIGLOG). Education is one of the prime concerns of this new SIG and one of the activities on the SIG's education committee will be to advocate for a greater presence of logic in the curriculum. Prakash Panangaden discusses the aims of the new SIG with particular emphasis on its educational mission.

Nicole Schweikardt will report on experiences with designing an undergraduate introductory course on logic in computer science at Goethe-University Frankfurt.

The University of Technology Vienna participates in a European Masters program in computational logic and has just started a doctoral program in Logical Methods in Computer Science. Alexander Leitsch describes these initiatives and considers lessons other departments can draw from the Vienna experience.

Presentations will be followed by a panel discussion. Materials will be available on the Committee on Logic Education website at `http://ucalgary.ca/aslcle/`.

Abstracts of invited talks in the Special Session on Logic of Games and Rational Choice

► ROHIT PARIKH, ***Elections and knowledge***.
Brooklyn College and CUNY Graduate Center, 365 Fifth Avenue New York, NY 10016-4309, USA.
E-mail: `rparikh@cuny.gc.edu`.

There are (at least) two ways in which knowledge can enter into elections.

1. When a voter strategizes, i.e., votes for someone who is not her first preference then she needs to know something about how the others are voting. Perhaps they want to know how she is voting. There are various possible scenarios here.
2. When a politician campaigns, he wants to influence the voters' beliefs. What should he say in order to appeal to them in the best way?

We will make use of previous work by ourselves, Samir Chopra, Hans van Ditmarsch, Walter Dean, and Eric Pacuit, as well as suggest some new ideas.

► GABRIEL SANDU, *Nash equilibrium semantics for Independence-Friendly logic*.
Department of Philosophy, University of Helsinki, Finland, Helsinki, FI 00014.
E-mail: `sandu@mappi.helsinki.fi`.

Henkin (1961) enriched first-order logic with so-called branching or Henkin quantifiers such as $\begin{pmatrix}\forall x\\ \exists y\end{pmatrix}$ and $\begin{pmatrix}\forall x & \exists y\\ \forall z & \exists w\end{pmatrix}$. The former is intended to express the fact that the existential quantifier $\exists y$ is independent of the universal quantifier $\forall x$. The latter is more easily introduced in terms of the idea of dependence: the existential quantifier $\exists y$ depends only on the universal quantifier $\forall x$, and the existential quantifier $\exists w$ depends only on the universal quantifier $\forall z$. The notions of independence and dependence are codified in terms of the existence of certain (Skolem) functions. It turns out that prefixing first-order formulas with branching quantifiers results in a logic which is strictly stronger than ordinary first-order logic.

In the first part of my presentation I will quickly review various formalisms which develop Henkin's ideas. One of them is Independence-Friendly logic introduced by Hintikka and Sandu (1989). The first branching quantifier is expressed in IF logic by $\forall x(\exists y/\{x\})$ ("for all x there is a y which is independent of x"). Similarly, the second branching quantifier is expressed by $\forall x \exists y \forall z (\exists w / \{x, y\})$ ("for all x there is a y and for all z there is a w which is independent from both x and y"). The notion of independence is spelled out in game-theoretical terms. With each IF formula φ, model $\mathbb{M}$, and partial assignment s whose domain is restricted to the free variables of φ, we associate an extensive win-lose game of imperfect information $G(\mathbb{M}, \varphi, s)$. When φ is the sentence $\forall x(\exists y/\{x\})x = y$, and s is the empty assignment, in a play of the game $G(\mathbb{M}, \varphi, s)$ $\forall$ chooses an individual $a \in \mathbb{M}$ to be the value of x after which $\exists$ chooses an individual $b \in \mathbb{M}$ to be the value of y without knowing the choice made earlier by $\forall$. Player $\exists$ wins the play if a is the same individual as b. Otherwise player $\forall$ wins. We stipulate that φ is true (false) in $\mathbb{M}$ if there is a winning strategy for player $\exists(\forall)$. The notion of strategy is the standard notion of choice function in classical game theory. In games of imperfect information such a function is required to be uniform. A comprehensive presentation of the model-theoretical properties of IF logic may be found in Mann, Sandu, and Sevenster (2011). Hintikka (1996) explores the significance of IF logic for the foundations of mathematics.

As expected, games of imperfect information may be indeterminate. For instance, on models with at least two elements, the IF sentence $\forall x(\exists y/\{x\})x = y$ is neither true nor false. Blass and Gurevich (1986) follow a suggestion by Aitaj and resolve the indeterminacy of this sentence by applying von Neumann's Minimax theorem: $\forall x(\exists y/\{x\})x = y$ gets the probabilistic value $\frac{1}{n}$ on any finite model with n elements. This value is the expected utility returned to the existential player by any mixed strategy equilibrium in the underlying game. This idea has been explored systematically for the first time in Sevenster (2006), and then further developed in Sevenster and Sandu (2010), and in Mann, Sandu, and Sevenster (2011). In the second part of my talk I will review some of the recent results on probabilistic IF logic.

Finally I will address the question: What kind of probabilistic logic is probabilistic IF logic? Here I shall draw some comparisons to other probabilistic semantics (Leblanc's probabilistic semantics, Bacchus' and Halpern's probabilistic interpretations of first-order logic.)

[1] A. Blass and Y. Gurevich, *Henkin quantifiers and complete problems*. ***Annals of pure and Applied Logic***, vol. 32 (1986), no. 1, pp. 1–16.

[2] L. Henkin, *Some remarks on infinitely long formulas*, ***Intuitionistic methods: Proceedings of the Symposium on Foundations of Mathematics***, (P. Bernays, editor), Pergamon Press, Oxford, 1959, pp. 169–183.

[3] J. Hintikka and G. Sandu, *Informational Independence as a Semantic Phenomenon*, ***Logic, Methodology and Philosophy of Science***, (J. E. Fenstead et al., editors), Elsevier, Amsterdam, 1989, pp. 571–589.

[4] A. I. Mann, G. Sandu and M. Sevenster, ***Independence-friendly logic: A game-theoretic approach***, Cambridge University Press, Cambridge 2011.

[5] M. Sevenster, *Branches of imperfect information: Logic, games, and computation*, PhD Thesis, University of Amsterdam, 2006.

[6] M. Sevenster and G. Sandu, *Equilibrium semantics of languages of imperfect information*. ***Annals of Pure and Applied Logic***, vol. 161 (2010), pp. 618–631.

► JOUKO VÄÄNÄNEN, *Dependence and independence—a logical approach*.
Department of Mathematics and Statistics, University of Helsinki, Finland, Gustaf Hällströmin, Katu 2b, PL 68, FIN-0014 and Institute for Logic, Language and Computation, University of Amsterdam, The Netherlands.
E-mail: jouko.vaananen@helsinki.fi.
URL Address: http://www.math.helsinki.fi/logic/people/jouko.vaananen/.

I will give an overview of dependence logic [1], the goal of which is to establish a basic logical theory of dependence and independence underlying seemingly unrelated subjects such as game theory, causality, random variables, database theory, experimental science, the theory of social choice, Mendelian genetics, etc. There is an abundance of new results in this field demonstrating a remarkable convergence. The concepts of (in)dependence in the different fields of humanities and sciences have surprisingly much in common and a common logic is starting to emerge.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

[1] JOUKO VÄÄNÄNEN, ***Dependence logic***, London Mathematical Society Student Texts, vol. 70, Cambridge University Press, Cambridge, 2007.

▶ JOHAN VAN BENTHEM, *The DNA of logic and games*.
Institute for Logic, Language and Computation, University of Amsterdam, 1090 GE Amsterdam, The Netherlands.
E-mail: johan.vanbenthem@uva.nl.
URL Address: http://staff.science.uva.nl/~johan.

Logic and games are entangled in delicate ways. Logics of games are used to analyze how players reason and act in a game. I will discuss dynamic-epistemic logics that analyze various phases of play in this mode. But one can also study logic as games, casting major logical notions as game-theoretic concepts. The two perspectives create a circle, or double helix if you will, of contacts all around. I will address this entanglement, and the issues to which it gives rise ([1]).

[1] VAN BENTHEM, ***Logic in games***, MIT Press, Cambridge MA, 2014.

Abstracts of invited talks in the Special Session on Model Theory

▶ PANTELIS E. ELEFTHERIOU, *Pregeometries and definable groups*.
Zukunftskolleg Box 216, University of Konstanz, 78457 Konstanz, Germany.
E-mail: Panteleimon.Eleftheriou@uni-konstanz.de.

We will describe a program for analyzing groups and sets definable in certain pairs (M, P). Examples include:

(1) M is an o-minimal ordered group and P is a real closed field with bounded domain (joint work with Peterzil)
(2) M is an o-minimal structure and P is a dense elementary substructure of M (work in progress with Hieronymi)

In each of these cases, a relevant notion of a pregeometry and genericity is used.

▶ MEERI KESÄLÄ, *Quasiminimal structures and excellence*.
Department of Mathematics and Statistics, University of Helsinki, P.O. Box 68, FIN-00014 Helsinki.
E-mail: meeri.kesala@helsinki.fi.

A structure M is quasiminimal if every definable subset of M is either countable or co-countable. The field of complex numbers is a strongly minimal structure and hence quasiminimal, but if we add the natural exponential function, the quasiminimality of the structure becomes an open problem. Boris Zilber defined the nonelementary framework of *quasiminimal excellent classes* in 2005 in order to show that his class of *pseudoexponential fields* is uncountably categorical. He conjectured that the unique pseudoexponential field of cardinality $2^{\aleph_0}$ fitting into this framework is isomorphic to the complex numbers with exponentiation. A key property for the categoricity of quasiminimal excellent classes was the technical axiom of excellence, which was adopted from Shelahs work for excellent sentences in $L_{\omega_1\omega}$. However, the original proof of the categoricity of pseudoexponential fields turned out to have a gap and the problem lay in showing that the excellence axiom holds.

In the paper *Quasiminimal structures and excellence* [1] we fill the gap in the proof with a surprising result: the excellence axiom is actually redundant in the framework of quasiminimal excellent classes. This result elegantly combines methods from classification theory that were generalized to different nonelementary frameworks by a group of people. These methods have a combinatorial core idea that is independent of the compactness of first order logic. We also study whether other quasiminimal structures fit into this uncountably categorical framework.

The paper strengthens the belief that nonelementary methods can provide effective tools to analyse structures that are out of reach for traditional model-theoretic methods.

Different frameworks have been suggested and the methods refined and there are many interesting paths in the ongoing research.

The paper is joint work of Martin Bays, Bradd Hart, Tapani Hyttinen, MK and Jonathan Kirby.

[1] Martin Bays, Bradd Hart, Tapani Hyttinen, Meeri Kesälä and Jonathan Kirby, *Quasiminimal structures and excellence*. ***Bulletin of the London Mathematical Society***, vol. 46 (2014), no. 1, pp. 155–163.

▸ JOCHEN KOENIGSMANN, *Definable valuation rings*.
Mathematical Institute, University of Oxford, Andrew Wiles Bldg., Woodstock Road, Oxford OX2 6GG, UK.
E-mail: koenigsmann@maths.ox.ac.uk.

The question which valuation rings on a field are first-order definable in the language of rings and if so by what kind of formula and in what kind of uniformity in families naturally arises in model theory of valued fields, but also, for example, in the context of Hilbert's 10th Problem or of motivic integration. It has gained momentum in recent years. We shall report on the latest developments and discuss some open problems.

▸ DANIEL PALACÍN, *The Fitting subgroup of a supersimple group*.
Department of Mathematics and Computer Science, Universität Münster, Eisteinstrasse 62, 48149 Münster, Germany.
E-mail: daniel.palacin@uni-muenster.de.

The Fitting subgroup of a given group G is the subgroup generated by all nilpotent normal subgroups of G. While it is always normal, it may not be nilpotent. Wagner proved that the Fitting subgroup of a stable is always nilpotent. However, this is not known for the wider class of groups with a simple theory.

A certain amount of model-theoretic ideas for groups in the stable context can be adapted to the more general framework of simple theories. For instance, stabilizers and generic types exist. In this talk we present some of the main tools and notions of groups in simple theories, and focus on those which have ordinal Lascar rank. Our aim is to prove that the Fitting subgroup of a hyperdefinable supersimple group is nilpotent-by-bounded. This generalizes a proof of Milliet in the finite rank case.

Abstracts of invited talks in the Special Session on Perspectives on Induction

▸ ALAN BUNDY, *Automating inductive proof*.
School of Informatics, University of Edinburgh, 10 Crichton St, Edinburgh, UK.
E-mail: A.Bundy@ed.ac.uk.
URL Address: http://homepages.inf.ed.ac.uk/bundy/.

The automation of inductive proof plays a pivotal role in the formal development of ICT systems: both software and hardware. It is required to reason about all forms of *repetition*, which arises in: recursive and iterative programs; parameterised hardware; traces of program runs; program invariants; etc. Since formal proof is a highly skilled and time-consuming activity, industry requires as much automation as possible to enable formal methods to be used cost effectively.

Unfortunately, inductive reasoning is much harder to automate than, for instance, first-order reasoning. Negative results from mathematical logic underpin these difficulties. These results include incompleteness, the undecidability of termination and the absence of cut elimination. Of these, the absence of cut elimination creates the most practical problems. The proofs of even some very simple and obviously true conjectures require the injection of cut formulae. These formulae typically take the form of intermediate lemmas, generalisations of the conjecture or nonstandard induction rules. Cut rule steps are generally assumed to require human intervention with an interactive prover to provide an appropriate cut formula.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

We have developed a proof technique called *rippling* [1] that guides the manipulation of the induction conclusion until the induction hypothesis can be used in its proof. In fact, rippling can be used in any situation where a given embeds in a goal. It rewrites the goal while preserving and re-grouping the embedding until an instance of the given appears as a subexpression of the goal.

The main contribution of rippling, however, is not its guidance of the step case, but the way it informs the application of the cut rule. It provides a strong expectation of the direction of the proof, but is not always successful. When it fails, an analysis of the failure suggests an appropriate application of cut: the form of a missing lemma, a generalisation or a nonstandard induction rule [2]. This increases the scope of inductive-proof automation, which has economic implications for the use of formal methods in the ICT industry.

[1] A. Bundy, D. Basin, D. Hutter, and A. Ireland, ***Rippling: Meta-level guidance for mathematical reasoning***, Cambridge Tracts in Theoretical Computer Science, vol. 56, Cambridge University Press, Cambridge, 2005.

[2] A. Ireland and A. Bundy, *Productive use of failure in inductive proof*. ***Journal of Automated Reasoning***, vol. 16 (1996), no. 1–2, pp. 79–111.

► MICHAEL DETLEFSEN, *Inductive proofs & the knowledge they give*.
Department of Philosophy, University of Notre Dame, 100 Malloy Hall, University of Notre Dame, Notre Dame, IN 46556, USA.
E-mail: mdetlef1@nd.edu.

Proofs by mathematical induction require or induce certain interdependencies between the instances of the generalizations they prove. The character of these interdependencies and the conditions under which they obtain will be the principal concerns of this talk.

► GEORG GOTTLOB, *Decidable languages for knowledge representation and inductive definitions: From Datalog to Datalog+/-*.
Computer Science Department, University of Oxford, Wolfson Building, Parks Road,break Oxford, OX1 3QD, England.
E-mail: ggottlob@cs.ox.ac.uk.

Datalog is a language based on function-free Horn clauses used to inductively define new relations from finite relational structures. Datalog has many nice computational and logical properties. For example, Datalog captures PTIME on ordered structures, which means that evaluating fixed Datalog programs (i.e., rule sets) over finite structures is in PTIME and, moreover, every PTIME-property on ordered structures can be expressed as a datalog program (see [4] for a survey). After giving a short overview of Datalog we argue that this formalism has certain shortcomings and is not ideal for knowledge representation, in particular, for inductive ontological knowledge representation and reasoning. We consequently introduce Datalog+/- which is a new framework for tractable ontology querying, and for a variety of other applications. Datalog+/- extends plain Datalog by features such as existentially quantified rule heads, negative constraints, and equalities in rule heads, and, at the same time, restricts the rule syntax so as to achieve decidability and tractability. In particular, we discuss three paradigms ensuring decidability: chase termination, guardedness, and stickiness, which were introduced and studied in [1, 2, 3, 5].

[1] A. Calì, G. Gottlob, and M. Kifer, *Taming the infinite chase: Query answering under expressive relational constraints*. ***Journal of Artificial Intelligence Research***, vol. 48 (2013), pp. 115–174.

[2] A. Calì, G. Gottlob, and T. Lukasiewicz, *A general Datalog-based framework for tractable query answering over ontologies*. ***Journal of Web Semantics***, vol. 14 (2012) pp. 57–83.

[3] A. Calì, G. Gottlob, and A. Pieris, *Towards more expressive ontology languages: The query answering problem*. ***Artificial Intelligence***, vol. 193 (2012), pp. 87–128.

[4] E. Dantsin, T. Eiter, G. Georg, and A. Voronkov, *Complexity and expressive power of logic programming*. ***ACM Computing Surveys***, vol. 33 (2001), no. 3, pp. 374–425.

[5] G. Gottlob, M. Manna, and A. Pieris, *Combining decidability paradigms for existential rules*. ***Theory and Practice of Logic Programming***, vol. 13 (2013), no. 4–5, pp. 877–892.

► GERHARD JÄGER, *Weak well orders and related inductions*.
Institut für Informatik und angewandte Mathematik, Universität Bern, Neubrückstrasse 10, 3012 Bern, Switzerland.
E-mail: `jaeger@iam.unibe.ch`.

It is an interesting program to investigate the relationship between the proof theory of second order arithmetic and more general second order systems (e.g., theories of sets and classes such as von Neumann–Bernays–Gödel set theory and Morse–Kelley set theory). Which proof-theoretic results can be lifted from second order arithmetic to theories of sets and classes, for which is this not the case, and what are the reasons? What is specific of second order number theory and what additional insights can we gain?

One of the crucial questions is how to distinguish between "small" and "large" in the various contexts. In second order arithmetic, the small objects are the natural numbers whereas the large objects are the infinite sets of natural numbers. Hence it seems natural to identify the small objects in sets and classes with sets and the large objects with proper classes.

As long as only comparatively weak systems are concerned, the moving up from second order arithmetic to sets and classes seems to be a matter of routine. However, as soon as well orderings enter the picture, the situation becomes interesting. In second order arithmetic, every Π^1_1 statement is equivalent to the question whether a specific arithmetic relation is well ordered; on the other hand, in set theory a simple elementary formulas expresses the well foundedness of a given relation.

We propose studying the (new) notion of *weak well order* in sets and classes as the proof-theoretically adequate analogue of well order in second order arithmetic. To support this claim several results about inductions and recursions in connection with weak well orders will be presented.

This is joint work with D. Flumini.

[1] D. Flumini, ***Weak Well Orders***, PhD Thesis, University of Bern, Bern, 2013.

[2] G. Jäger, *Operations, sets and classes*, ***Logic, Methodology and Philosophy of Science, Proceedings of the Thirteenth International Congress*** (Beijing), (C. Glymour, W. Wei, D. Westerståhl, editors), College Publications, London, 2009, pp. 74–96.

[3] G. Jäger and J. Krähenbühl, Σ^1_1 *choice in a theory of sets and classes*, ***Ways of Proof Theory*** (R. Schindler, editor), Ontos Verlag, Frankfurt, 2010, pp. 283–313.

► THEODORE A. SLAMAN, *Subsystems of first-order arithmetic delineated by second-order principles*.
Department of Mathematics, The University of California Berkeley, Berkeley, CA 94720-3840, USA.
E-mail: `slaman@math.berkeley.edu`.

Principles of first-order induction provide a metric by which we may measure the number theoretic consequences of most familiar infinitary principles. However, some second-order principles, especially combinatorial ones, present fragments of first-order arithmetic other than those from the hierarchy of definable induction. We will survey the area, with an emphasis on recent results.

Abstracts of invited talks in the Special Session on Philosophy of Mathematics

► PATRICIA BLANCHETTE, *Frege on mathematical progress*.
Department of Philosophy, University of Notre Dame, Notre Dame, IN 46556, USA.
E-mail: `blanchette.1@nd.edu`.

Progress in mathematics has often involved a good deal of conceptual clarification, including increasingly precise characterizations of concepts (e.g., those of infinity, of continuity, perhaps of set, etc.) that were less clearly understood by earlier theorists. But the sometimes-vast difference between the earlier and later concepts that go by the same name raises the

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

possibility that such conceptual refinement really brings with it a whole new subject-matter for the branch of mathematics in question, rather than a clarified understanding of the concepts used by earlier generations. This talk investigates Gottlob Frege's approach to understanding this kind of conceptual progress, and assesses the plausibility of his view that a given subject-matter can survive essentially unscathed despite fairly radical changes in the surrounding theory.

▸ LEON HORSTEN, *Reflection, trust, entitlement*.
Philosophy, University of Bristol, 43 Woodland Road, Bristol, BS81UU, UK.
E-mail: `Leon.Horsten@bristol.ac.uk`.

It has been argued by Feferman and others that when we accept a mathematical theory, we implicitly commit ourselves to reflection principles for this theory. When we reflect on this implicit commitment, we come to explicitly believe certain reflection principles. In my talk I will discuss our epistemic warrant for this resulting explicit belief in reflection principles.

▸ LUCA INCURVATI AND BENEDIKT LÖWE, *Restrictiveness relative to notions of interpretation*.
Department of Philosophy and Institute for Logic Language and Computation, University of Amsterdam, Oude Turfmarkt 141–147, 1012 GC Amsterdam, The Netherlands.
E-mail: `L.Incurvati@uva.nl`.
URL Address: `https://sites.google.com/site/lucaincurvati/`.
Faculty of Mathematics, University of Hamburg, Bundesstraße 55, 20146 Hamburg, Germany.
Institute for Logic, Language and Computation, University of Amsterdam, Postbus 94242, 1090 GE Amsterdam, The Netherlands.
E-mail: `B.Loewe@uva.nl`.
URL Address: `http://www.math.uni-hamburg.de/home/loewe/`.

In [4], Maddy gives a semi-formal account of *restrictiveness* by defining a corresponding formal notion based on a class of interpretations. In [2] and [3], Maddy's notion of restrictiveness was discussed and the theory ZF + 'Every uncountable cardinal is singular' was presented as a potential witness to the restrictiveness of ZFC. More recently, Hamkins has given more examples and pointed out some structural issues with Maddy's definition [1]. We look at Maddy's definitions from the point of view of an abstract *interpretation* relation. We consider various candidates for this interpretation relation, including one that is close to Maddy's original notion, but fixes the issues raised in [1]. Our work brings to light additional structural issues that we also discuss.

[1] Joel David Hamkins, *A multiverse perspective on the axiom of constructibility*, ***Infinity and Truth, Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore*** (Singapore), (Chitat Chong, Qi Feng, Theodore A Slaman, and W Hugh Woodin, editors), vol. 25, World Scientific, Singapore 2013, pp. 25–45.

[2] Benedikt Löwe, *A first glance at non-restrictiveness*. ***Philosophia Mathematica***, vol. 9 (2001), no. 3, pp. 347–354.

[3] ———, *A second glance at non-restrictiveness*. ***Philosophia Mathematica***, vol. 11 (2003), no. 3, pp. 323–331.

[4] Penelope Maddy, ***Naturalism in Mathematics***, Clarendon Press, Oxford, 1997.

▸ GABRIEL UZQUIANO, *On Bernays' generalization of Cantor's theorem*.
University of Southern California, School of Philosophy, 3709 Trousdale Parkway, Los Angeles, CA 90089, USA and Arché Research Centre, University of St. Andrews, Edgecliffe, 5 The Scores, St. Andrews, KY16 9AL, UK.
E-mail: `uzquiano@usc.edu`.

Cantor's theorem states that there is no one-to-one correspondence between the members of a set a and the subsets of a. In [1], Paul Bernays showed how to encode the claim that there is no one-to-one correspondence between the members of a class A and the subclasses of A by means of a sentence of the language of class theory. Moreover, he proved his

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generalization of Cantor's theorem by means of a diagonal argument: given a one-to-one assignment of subclasses of A to members of A, he defined a subclass of A, which, on pain of contradiction, is not assigned to any member of A. It follows from Bernays' observation that if one assigns a member of A to every subclass of A, then the assignment is not one-one. Unfortunately, familiar arguments for this claim fail to provide an explicit characterization of two different subclasses of A to which one and the same member of A is assigned by the assignment. George Boolos tackled a related problem in [2], where he showed how to specify explicit counterexamples to the claim that a function from the power set of a set a into the set a is one-one. Similar constructions turn out to be available in the case of classes, but they are sensitive to the presence of global choice and impredicative class comprehension. We explore some ramifications of this observation for traditional philosophical puzzles raised by the likes of Russell's paradox of propositions in Appendix B of [4] and Kaplan's paradox in [3].

[1] Paul Bernays, *A system of axiomatic set theory: Part IV*. ***The Journal of Symbolic Logic***, vol. 7 (1942), no. 4, pp. 133–145.

[2] George Boolos, *Constructing Cantorian counterexamples*. ***Journal of Philosophical Logic***, vol. 26 (1997), no. 3, pp. 237–239.

[3] David Kaplan, ***A problem in possible-world semantics, modality, morality and belief: Essays in honor of Ruth Barcan Marcus***, (Walter Sinnott-Armstrong and Diana Raffman and Nicholas Asher, editors), Cambridge University Press, Cambridge, 1995, pp. 41–52.

[4] Bertrand Russell, ***The Principles of Mathematics***, W. W. Norton & Company, New York, 1903.

Abstracts of invited talks in the Special Session on Recursion Theory

▶ JOHANNA N. Y. FRANKLIN, *UD-randomness and the Turing degrees*.
Department of Mathematics, University of Connecticut, 196 Auditorium Road, Unit 3009, Storrs, CT 06269-3009, USA.
E-mail: johanna.franklin@uconn.edu.

The roots of UD-randomness are firmly analytic: Avigad defined it in 2013 using concepts from a 1916 theorem of Weyl concerning uniform distribution. Avigad showed in his original paper that UD-randomness is very weak. While every Schnorr random real is UD-random, the class of UD-random reals is incomparable with the class of the Kurtz random reals. In this talk, I will present some subsequent work on the Turing degrees of the UD-random reals and the relationships between UD-randomness and other randomness notions.

This work is joint with Wesley Calvert.

▶ RUPERT HÖLZL, *Randomness in the Weihrauch degrees*.
Department of Mathematics, Faculty of Science, National University of Singapore, Block S17, 10 Lower Kent Ridge Road, Singapore 119076, Singapore.
E-mail: r@hoelzl.fr.

It is a recurring theme of theoretical computer science how access to sources of random information can enable the computation of certain mathematical objects. While this is particularly evident in the context of complexity theory, the question can also be studied in more general settings. Many different versions have been studied in the field of algorithmic randomness. It can be argued that this approach better represents the original question of what can be computed with access to randomness than, for example, the complexity theoretic approach, as in this setting space or time bounds are not considered, meaning we are getting a better idea of the computational *content* of random objects—as opposed to a gauge of their ability to *speed up* a computation until it can be performed within polynomial time. For this reason, the results from algorithmic randomness and computability theory are of high importance.

In this talk we will look at the question from yet another angle, and give arguments why we think that this is the correct way to formalize the question of what random information can be used for. In fact, the field of algorithmic randomness does already provide two answers to the question: First there is the Kučera–Gács-Theorem, which, informally stated, says that everything can be computed from some random object. We will argue that despite the high importance of this theorem it does not provide an answer to the initial informal question, when we formalize it in a way that actually captures the intention behind it. Secondly, there is Sacks' theorem, which states that no nontrivial information can be generated from a set of oracles of positive measure. Again we will argue that this is not the answer we are looking for: Sacks' theorem only applies if we want to compute a single set A, as the proof relies essentially on a majority vote argument.

But there are many very valid settings where we do not want to compute a single set: Often we are given a mathematical problem and want to find a solution to it, and we want to know whether randomness can help us to find such a solution. For a given instance of such a task there may be many legal solutions; each of these solutions may have low probability of being produced by a Turing machine, so that a majority vote mechanic would fail.

To overcome this limitation we therefore need a different framework, without abolishing completely the ideas of computability theory. This new framework is provided by the Weihrauch degrees, the lattice induced by Weihrauch reducibility. In the talk we will introduce the framework and give arguments for why we think it is the correct way to approach the initial question.

We will study computation from sets of oracles of positive measure in this framework. Among other results, we will in particular identify two natural models of randomized computation: One is computation with access to Martin-Löf random oracles. The other is computation with what we call a Las Vegas algorithm, a Weihrauch degree version of Babai's similarly named notion from complexity theory. This second model of randomized computation can be naturally identified with Weak Kőnig's Lemma.

We will then see that these two models of randomized computation can be separated in the Weihrauch degrees. This contrasts with results in the related field of reverse maths, where they are known to coincide. We will discuss what the origin of this different behavior is.

To conclude, we will briefly discuss some other ways in which algorithmic randomness and related notions show up in the Weihrauch lattice, to illustrate that the study of algorithmic randomness with Weihrauch tools is a fruitful topic with many open questions to explore.

(Based on joint work with Vasco Brattka and Guido Gherardi, and on joint work with Paul Shafer.)

▸ ISKANDER KALIMULLIN, *Uniform and non-uniform reducibilities of algebraic structures*.
Kazan Federal University, Kremlevskaya st. 18, Kazan, Russia.
E-mail: ikalimul@gmail.com.

The talk will be devoted to various versions of algorithmic reducibility notion between algebraic structures. In particular, the reducibilities under Turing operators, enumeration operators, and under Σ-formulas will be considered. Several constructions of jump inversion where these reducibilities do not coincide. Furthermore, the Σ-reducibility between the direct sums of cyclic p-groups will be studied in detail.

▸ BAKHADYR KHOUSSAINOV, *On finitely presented expansions of semigroups, groups, and algebras*.
Department of Computer Science, The University of Auckland, Auckland, New Zealand.
E-mail: bmk@cs.auckland.ac.nz.

Finitely presented algebraic systems, such as groups and semigroups, are of foundational interest in algebra and computation. Finitely presented algebraic systems necessarily have computably enumerable (c.e. for short) word equality problem and these systems are finitely generated. Call finitely generated algebraic systems with c.e. word equality problem computably enumerable. Computable enumerable finitely generated algebraic systems are not necessarily finitely presented. This paper is concerned with finding finitely presented expansions of finitely generated c.e. algebraic systems. The method of expansions of algebraic systems, such as turning groups into rings or distinguishing elements in the underlying algebraic systems, is an important method used in algebra, model theory, and in various areas

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

of theoretical computer science. Bergstra and Tucker proved that all c.e. algebraic systems with decidable word problem possess finitely presented expansions. Then they and, independently, Goncharov asked if every finitely generated c.e. algebraic system has a finitely presented expansion. In this paper we build examples of finitely generated c.e. semigroups, groups, and algebras that fail to possess finitely presented expansions thus answering the question of Bergstra–Tucker and Goncharov for the classes of semigroups, groups and algebras. We also construct an example of a residually finite, infinite, and algorithmically finite group thus answering the question of Miasnikov and Osin. Our constructions are based on the interplay between key concepts and known results from computability theory (such as simple and immune sets), and algebra (such as residually finiteness and the theorem of Golod–Shafaverevich).

The work is joint with A. Miasnikov, and the authors were partially supported by Marsden Fund, Royal New Zealand Society.

Abstracts of invited talks in the Special Session on Set Theory

► DAISUKE IKEGAMI, *Large cardinals, forcing axioms, and the theory of subsets of ω_1*.
Graduate School of System Informatics, Kobe University, Rokko-dai 1-1, Nada, Kobe 657-8501, Japan.
E-mail: daiske.ikegami@gmail.com.

The goal of this research is to rule out "natural" independence phenomena in Set Theory by maximizing your theory in terms of large cardinals and forcing axioms. Using large cardinals in ZFC, by the results of Woodin [1], we have a clear understanding of the 1st order theory of sets of natural numbers and what it should be.

In this talk, we try to extend this understanding to the 1st order theory of subsets of ω_1 by using large cardinals, forcing axioms, and some hypothesis from inner model theory in ZFC.

This is joint work with Matteo Viale.

[1] W. Hugh Woodin, ***The axiom of determinacy, forcing axioms, and the nonstationary ideal***, de Gruyter Series in Logic and its Applications, volume 1, Walter de Gruyter GmbH & Co. KG, Berlin, 2010.

► DIEGO A. MEJÍA, *Matrix iterations and Cichoń's diagram*.
Kurt Gödel Research Center, University of Vienna, Währinger Strasse 25, 1090 Wien, Austria.
E-mail: damejiag@gmail.com.

Using matrix iterations of ccc posets we prove the consistency, with ZFC, of some constellations of Cichoń's diagram where the cardinals on the right hand side assume three different values. We also discuss the influence of the constructed models on other classical cardinal invariants of the continuum.

[1] D. A. Mejía, *Matrix iterations and Cichoń's diagram*. ***Archive for Mathematical Logic***, vol. 52 (2013), no. 3–4, pp. 261–278.

[2] ———, *Models of some cardinal invariants with large continuum*, ***Kyōto daigaku sūrikaiseki kenkyūsho kōkyūroku***, (2013), pp. 36–48.

► KONSTANTIN SLUTSKY, *Regular cross-sections of Borel flows*.
Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagens Ø, Denmark.
E-mail: kslutsky@gmail.com.

When working with measurable flows, it is sometimes convenient to choose a countable cross-section and to reduce a problem of interest to a similar question for the action induced by the flow on this cross-section. In some cases, one wants to impose additional restrictions on the cross-section, usually by restricting possible distances between points within each orbit.

Historically, cross-sections of flows were studied mainly in the context of ergodic theory. One of the most important results here is a theorem of D. Rudolph [3], which states that any free measure preserving flow, when restricted to an invariant subset of full measure, admits a cross-section with only two possible distances between adjacent points.

Borel dynamics deals with actions of groups on standard Borel spaces, when the latter is not equipped with any measure. In this more abstract context, one needs to construct cross-sections that are regular on all orbits without exceptions, and methods of ergodic theory, which tend to produce cross-sections only almost everywhere, are therefore frequently insufficient. In this regard, M. G. Nadkarni [2] posed a question whether the analog of Rudolph's Theorem holds true in the Borel setting: Does every free Borel flow admit a cross-section with only two different distances between adjacent points?

The talk will provide an overview of these and other results concerning the existence of regular cross-sections, and a positive answer to Nadkarni's question will be given. As an application of our methods, we give a classification of free Borel flows up to Lebesgue Orbit Equivalence, by which we understand orbit equivalence preserving Lebesgue measure on each orbit. This classification is an analog of the classification of hyperfinite equivalence relations obtained by R. Daugherty, S. Jackson, and A. S. Kechris [1].

[1] Randall Dougherty, Steve Jackson, Alexander S. Kechris, *The structure of hyperfinite Borel equivalence relations*. ***Transactions of the American Mathematical Society***, vol. 341 (1994), no. 1, pp. 193–225.

[2] Mahendra G. Nadkarni, ***Basic ergodic theory***, Birkhäuser Advanced Texts: Basler Lehrbücher, Birkhäuser Verlag, 1998.

[3] Daniel Rudolph, *A two-valued step coding for ergodic flows*. ***Mathematische Zeitschrift***, vol. 150 (1976), no. 3, pp. 201–220.

Abstracts of Contributed talks

▸ ANTONIS ACHILLEOS, *Complexity bounds for Multiagent Justification Logic*.
The Graduate Center of CUNY, 365 Fifth Avenue New York, NY 11209, USA.
E-mail: aachilleos@gc.cuny.edu.

We investigate the complexity of systems of Multi-agent Justification Logic with interacting justifications (see [1]). The system we study has n agents, each based on some (single-agent) justification logic (we consider J, $\mathsf{J4}$, JD, $\mathsf{JD4}$, JT, LP) and a transitive, irreflexive binary relation, C. Each agent i has its own set of axioms, depending on the logic it is based on. If iCj, then we include axiom $t :_i \phi \rightarrow t :_j \phi$ (we do not include V-Verification as in [1]). Finally, it has a sufficient amount of propositional axioms and an axiomatically appropriate constant specification, which is in P. Traditionally, to establish upper complexity bounds for satisfiability for Justification Logic, we use a set of tableau rules to generate a branch and then we run the $*$-calculus on it.

A similar system for (diamond-free) modal logic was studied in [2]. We adjust appropriately the tableau for the corresponding system in [2] and the $*$-calculus can be run locally for every prefix, so we can use the same methods as in [2] to establish upper bounds. On the other hand, we can see that if we replace $\Box_i$ by $x :_i$ in a diamond-free modal formula (for all i), then the new formula is satisfiable iff the old one was. Thus, we can prove the same complexity bounds as in [2]—with the exception that where satisfiability for a modal logic is in NP, the corresponding justification logic has its satisfiability in Σ_2^p.

[1] Antonis Achilleos, *Complexity jumps in Multi-agent Justification Logic with interacting justifications*, submitted.

[2] ———, *Modal logics with hard diamond-free fragments*, submitted.

▸ RYOTA AKIYOSHI, *Proof-theoretic analysis of Brouwer's argument of the bar induction*.
Faculty of Letters, Kyoto University, Yoshidahonmachi, Sakyo Ward, Kyoto Prefecture 606-8501, Japan.
E-mail: georg.logic@gmail.com.

In a series of papers, Brouwer had developed intuitionistic analysis, in particular the theory of choice sequences. An important theorem called the "fan theorem" plays an essential role in the development of it. The fan theorem was derived from another stronger theorem called the "bar induction", which is an induction principle on a well-founded tree. We refer to [4, 5] as standard references of Brouwer's intuitionistic analysis.

Brouwer's argument in [1] contains a controversial assumption on canonical proofs of some formula. In many cases, constructive mathematicians have assumed the bar induction as axiom, hence the assumption has not been examined by them.

In this talk, we sketch an approach of Brouwer's argument via infinitary proof theory. We point out that there is a close similarity between Brouwer's argument and Buchholz' method of the Ω-rule ([2, 3]). In particular, Brouwer's argument in [1] seems very close to Buchholz' embedding theorem of the (transfinite) induction axiom of ID_1 in [2], which is a theory of noniterated inductive definition. By comparing these two arguments, we give a natural explanation of why Brouwer needed the assumption. Our conclusion is that Brouwer supposed the assumption in order to avoid the impredicativity or a vicious circle which is essentially the same as one in the Ω-rule for ID_1. In other words, the impredicativity can be explained in a very clear way from the view point of the Ω-rule. Moreover, Brouwer's argument can be formulated in a mathematically precise way by the Ω-rule. Therefore, we conclude that his introduction of the assumption is mathematically well-motivated. If time is permitting, we suggest how to carry out this idea in a mathematical way.

[1] Luitzen Egbertus Jan Brouwer, *Über Definitionsbereiche von Funktionen*. ***Mathematische Annalen***, vol. 97, 1927, pp. 60–75.

[2] Wilfried Buchholz, *The $\Omega_{\mu+1}$-rule*, ***Iterated inductive definitions and subsystems of analysis: Recent proof-theoretical studies***, Lecture Notes in Mathematics vol. 897, 1981, pp. 188–233.

[3] ———, *Explaining the Gentzen-Takeuti reduction steps*. ***Archive for Mathematical Logic***, vol. 40, pp. 255–272.

[4] Michael A. E. Dummett, ***Elements of intuitionism***, 2nd edition, Oxford University Press, Oxford, 2000.

[5] A. S. Troelstra and D. van Dalen, ***Constructivism in mathematics***, vol. 1, North-Holland, Amsterdam, 1988.

► PAVEL ALAEV, *The Δ^0_α-dimension of computable structures*.
Sobolev Institute of Mathematics, pr. Koptuga 4, Novosibirsk, 630090, Russia.
E-mail: alaev@math.nsc.ru.

Let $\alpha \geqslant 1$ be a computable ordinal and $\mathfrak{A}$ be a computable structure. The Δ^0_α-dimension of $\mathfrak{A}$ is maximal $n \leqslant \omega$ such that there exist n computable presentations of $\mathfrak{A}$ without any Δ^0_α isomorphism between them. $\mathfrak{A}$ is Δ^0_α-categorical if this dimension is 1.

In [1], it was noted that if $\mathfrak{A}$ has a Σ^0_α Scott family then it is Δ^0_α-categorical. Moreover, a set of conditions $\Phi(\mathfrak{A})$ was found, under which this sufficient condition becomes necessary: if $\Phi(\mathfrak{A})$ holds then $\mathfrak{A}$ has a Σ^0_α Scott family iff it is Δ^0_α-categorical.

We prove that under a similar set of conditions $\Phi'(\mathfrak{A})$, this equivalence also holds, and, in addition, the Δ^0_α-dimension of $\mathfrak{A}$ is 1 or ω. The main part of this result is the theorem below. In addition, we fix a small error in the original formulation of $\Phi(\mathfrak{A})$.

If $\bar{a}, \bar{b}$ are tuples in $\mathfrak{A}$ of the same length, then $\bar{a} \leqslant_\alpha \bar{b}$ means that every infinite Π_α formula true on $\bar{a}$ is true on $\bar{b}$. $\mathfrak{A}$ is α-friendly if the relations $\leqslant_\beta$ are c.e. uniformly in $\beta < \alpha$. Let $\Rightarrow$ be a binary relation on finite tuples in $\mathfrak{A}$. We define a relation $\mathrm{Free}^{\Rightarrow}_\alpha(\bar{a}, \bar{c})$ on tuples in $\mathfrak{A}$ as follows:

$$\forall \beta < \alpha\, \forall \bar{a}_1\, \exists \bar{a}'\, \exists \bar{a}'_1\, \big[|\bar{a}| = |\bar{a}'|,\ \bar{c}, \bar{a}, \bar{a}_1 \leqslant_\beta \bar{c}, \bar{a}', \bar{a}'_1, \text{ and } \bar{c}, \bar{a} \not\Rightarrow \bar{c}, \bar{a}' \big].$$

If $\Rightarrow$ is $\geqslant_\alpha$ then this definition coincides with the one in [1].

Theorem. *Let $\mathfrak{A}$ be a computable α-friendly structure. Suppose that $\Rightarrow$ is a relation on finite tuples in $\mathfrak{A}$ such that*

(a) *$\Rightarrow$ is transitive, i.e., $\bar{a} \Rightarrow \bar{b}$ and $\bar{b} \Rightarrow \bar{c}$ imply $\bar{a} \Rightarrow \bar{c}$;*
(b) *if $g : \mathfrak{A} \to \mathfrak{A}$ is an automorphism then $\bar{a} \Rightarrow g(\bar{a})$ for every $\bar{a}$ in $\mathfrak{A}$.*

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

If the relation $\not\Rightarrow$ is c.e. and for every $\bar{c}$ in $\mathfrak{A}$, we can effectively find $\bar{a}$ s.t. $\mathrm{Free}_{\alpha}^{\Rightarrow}(\bar{a}, \bar{c})$, then there exists a computable sequence $\{\mathfrak{B}_i\}_{i\in\omega}$ of computable presentations of $\mathfrak{A}$ s.t. there is no Δ_{α}^{0} isomorphism between $\mathfrak{B}_i$ and $\mathfrak{B}_j$ for $i \neq j$.

[1] C. J. Ash, *Categoricity in hyperarithmetical degrees*. ***Annals of Pure and Applied Logic***, vol. 34 (1987), no. 1, pp. 1–14.

► SVETLANA ALEKSANDROVA, *Uniformization in the hereditarily finite list superstructure over the real exponential field*.
Novosibirsk State University, 2 Pirogova St. Novosibirsk 630090, Russia.
E-mail: svet-ka@eml.ru.

This work is concerned with the generalized computability theory, as well as properties of the real exponential field. To describe computability we use an approach via definability by $\Sigma-$formulas in hereditarily finite superstructures, which was introduced in [1].

In particular, we establish the uniformization property for $\Sigma-$predicates in the hereditarily finite list superstructure over the real exponential field. (See [2] for the structure's definition).

We shall outline the proof of the following theorem.

Theorem 1. *For any $\Sigma-$predicate P in the hereditarily finite list superstructure over the real exponential field exists a $\Sigma-$function f with the domain $dom(f) = \{x : \exists y\, P(x, y)\}$ and graph $\Gamma_f \subseteq P$.*

As a corollary we obtain existence of an universal $\Sigma-$function in the same structure.

[1] Yu. L. Ershov, ***Definability and computability***, Consultants Bureau, New York-London-Moscow, 1996.

[2] S. S. Goncharov and D. I. Sviridenko, $\Sigma-$*programming*. ***Vychislitelnye Sistemy***, 1985, no. 107, pp. 3–29.

► OLGA ANTONOVA, *Aristotle's conception of demonstration and modern proof theory*.
Department of Philosophy, Catholic University of Toulouse, 31 rue de la Fonderie, 31068 Toulouse, France.
E-mail: olgaantonova73@gmail.com.

The history of modern mathematical proof theory begins with Beweistheorie or Hilbert's proof theory. The mathematical theories such as logicism (Frege, Russell), intuitionism (Brouwer, Heyting), set theory (Cantor, Dedekind) influenced directly the conception of proof and generally modern proof theory. The modern proof theory is based not only on mathematical theories, but also on the philosophical and logical proof theories, such as Aristotle's conception of demonstration. According to Aristotle a demonstration is a 'scientific syllogism', in which the premises are true, first, immediate, more known than the conclusion, prior to the conclusion and causes of the conclusion. Aristotle's theory of demonstration impacted on the development of logic and, in particular, on the philosophical and logical conception of proof. Can we say that Aristotle's conception of demonstration is modern? Is the actual conception of proof really based on Aristotle's conception? The purpose of my talk is to analyze Aristotle's definition of demonstration and compare it with the modern approach to demonstration.

[1] Aristotle, ***Posterior analytics***, (translated by J. Barnes), Oxford University Press, Oxford, 1976.

[2] V. Hendrics et al. (editors), ***Proof theory: History and philosophical significance***, Kluwer, Dordrecht, 2000.

► SERGEI ARTEMOV AND TUDOR PROTOPOPESCU, *An outline of intuitionistic epistemic logic*.
The Graduate Center CUNY, 365 Fifth Avenue, New York, NY 10016, USA.
E-mail: sartemov@gc.cuny.edu.
E-mail: tprotopopescu@gmail.com.

We outline an intuitionistic view of knowledge which maintains the Brouwer–Heyting–Kolmogorov semantics and is consistent with Williamson's suggestion that intuitionistic

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

knowledge is the result of verification and that verifications do not necessarily yield strict proofs. On this view, $A \to \mathbf{K}A$ is valid and $\mathbf{K}A \to A$ is not. The former expresses the constructivity of truth, while the latter demands that verifications yield strict proofs. Unlike in the classical case where

$$\textit{Classical Knowledge} \Rightarrow \textit{Classical Truth}$$

intuitionistically

$$\textit{Intuitionistic Truth} \Rightarrow \textit{Intuitionistic Knowledge}.$$

Consequently we show that $\mathbf{K}A \to A$ is a distinctly classical principle, too strong as the intuitionistic truth condition for knowledge, "false is not known," which can be more adequately expressed by e.g., $\neg(\mathbf{K}A \wedge \neg A)$ or, equivalently, $\neg\mathbf{K}\perp$.

We construct a system of intuitionistic epistemic logic:

$$\mathsf{IEL} = \textit{intuitionistic logic } \mathsf{IPC} + \mathbf{K}(A \to B) \to (\mathbf{K}A \to \mathbf{K}B) + (A \to \mathbf{K}A) + \neg\mathbf{K}\perp,$$

provide a Kripke semantics for it and prove IEL soundness, completeness and the disjunction property.

IEL can be embedded into an extension of S4, S4V, via the Gödel embedding *"box every subformula."* S4V is a bi-modal classical logic consisting of the rules and axioms of S4 for $\Box$ and D for $\mathbf{K}$, with the connecting axiom $\Box A \to \mathbf{K}A$. The soundness of the embedding is proved.

Within the framework of IEL, the knowability paradox is resolved in a constructive manner. Namely, the standard Church–Fitch proof reduces the intuitionistic knowability principle $A \to \Diamond\mathbf{K}A$ to $A \to \neg\neg\mathbf{K}A$, which is an IEL-theorem. Hence the knowability paradox in the domain of IEL disappears since neither of these principles are intuitionistically controversial. We argue that previous attempts to analyze the paradox were insufficiently intuitionistic.

▸ SERIKZHAN BADAEV AND SERGEY GONCHAROV, *Relativized universal numberings*.
Department of Mechanics and Mathematics, Al-Farabi Kazakh National University, 71 Al-Farabi Ave., Almaty 050038, Kazakhstan.
E-mail: Serikzhan.Badaev@kaznu.kz.
Sobolev Institute of Mathematics, 4 Academician Koptyug Ave., Novosibirsk 630090, Russia.
E-mail: S.S.Goncharov@math.nsc.ru.

A numbering ν is called *universal in* a class $C(\mathfrak{F})$ of numberings of a family $\mathfrak{F}$ of sets, if $\nu \in C(\mathfrak{F})$ and every numbering of $C(\mathfrak{F})$ is reducible to ν. In the theory of numberings, a lot is known on universal numberings when $\mathfrak{F}$ is a family of sets lying in a given level of the arithmetical, or hyperarithmetical, or analytical hierarchy, or the hierarchy of Ershov, and $C(\mathfrak{F})$ is as the class of all computable numberings of $\mathfrak{F}$.

Let A be any set of natural numbers. A numbering ν of a family $\mathfrak{F}$ of A-c.e. sets is called *A-computable* if the sequence $\nu(0), \nu(1), \dots$ is uniformly A-c.e. We will be concerned with those families $\mathfrak{F}$ of A-c.e. sets, that posses an A-computable numbering, and we will denote the class of all A-computable numberings of $\mathfrak{F}$ by $C_A(\mathfrak{F})$. W_x^A will stand for the A-c.e. set with Gödel index x.

THEOREM 1. *If there exists an A-computable function g such that, for every x, $W_{g(x)}^A \in \mathfrak{F}$, and $W_x^A = W_{g(x)}^A$ if $W_x^A \in \mathfrak{F}$, then $\mathfrak{F}$ has a universal numbering in $C_A(\mathfrak{F})$.*

THEOREM 2. *If $\emptyset' \leq_T A$ and $\mathfrak{F}$ has a universal numbering in $C_A(\mathfrak{F})$, then $\mathfrak{F}$ is closed under unions of increasing A-computable sequences of sets from $\mathfrak{F}$.*

If $\mathfrak{F}$ contains the least set under inclusion then the condition $\emptyset' \leq_T A$ in Theorem 2 can be omitted.

THEOREM 3. *If $\emptyset' \leq_T A$ then a finite family $\mathfrak{F}$ of A-c.e. sets has a universal numbering in $C_A(\mathfrak{F})$ if and only if $\mathfrak{F}$ contains the least set under inclusion.*

THEOREM 4. *For every set A, there exists an infinite A-computable family $\mathfrak{F}$ of pairwise disjoint A-c.e. sets that has a universal numbering in $C_A(\mathfrak{F})$.*

Theorems 2 and 4 imply that the presence of the least set under inclusion in $\mathfrak{F}$ is neither necessary nor sufficient for an infinite family $\mathfrak{F}$ to have a universal numbering in $C_A(\mathfrak{F})$.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

THEOREM 5. *For every A, there is an infinite family $\mathfrak{F}$ with universal numbering in $C_A(\mathfrak{F})$ such that any infinite subfamily of $\mathfrak{F}$ has no Friedberg numbering.*

▸ NIKOLAY BAZHENOV, *Boolean algebras and degrees of autostability relative to strong constructivizations*.
Sobolev Institute of Mathematics and Novosibirsk State University, 4 Acad. Koptyug Av., Novosibirsk, Russia.
E-mail: nickbazh@yandex.ru.

Let $\mathbf{d}$ be a Turing degree. A computable structure $\mathfrak{A}$ is $\mathbf{d}$-*autostable* if, for every computable structure $\mathfrak{B}$ isomorphic to $\mathfrak{A}$, there exists a $\mathbf{d}$-computable isomorphism from $\mathfrak{A}$ onto $\mathfrak{B}$. A decidable structure $\mathfrak{A}$ is $\mathbf{d}$-*autostable relative to strong constructivizations* if every decidable copy $\mathfrak{B}$ of $\mathfrak{A}$ is $\mathbf{d}$-computably isomorphic to $\mathfrak{A}$.

Let $\mathfrak{A}$ be a computable structure. A Turing degree $\mathbf{d}$ is called the *degree of autostability* of $\mathfrak{A}$ if $\mathbf{d}$ is the least degree such that $\mathfrak{A}$ is $\mathbf{d}$-austostable. A degree $\mathbf{d}$ is the *degree of autostability relative to strong constructivizations* (*degree of SC-autostability*) of a decidable structure $\mathfrak{A}$ if $\mathbf{d}$ is the least degree such that $\mathfrak{A}$ is $\mathbf{d}$-autostable relative to strong constructivizations. Note that here we follow [4] and use the term *degree of autostability* in place of *degree of categoricity*. A great number of works (see, e.g., [1, 2, 3]) uses the term *degree of categoricity*.

THEOREM 1. *Let α be a computable ordinal.*

(1) $\mathbf{0}^{(\alpha)}$ *is the degree of autostability of some computable Boolean algebra*;

(2) $\mathbf{0}^{(\alpha)}$ *is the degree of SC-autostability of some decidable Boolean algebra.*

Using the results of [2], we obtain the following corollaries.

COROLLARY 2. *There exists a decidable Boolean algebra without degree of SC-autostability.*

COROLLARY 3. *The index set of decidable Boolean algebras with degrees of SC-autostability is Π^1_1-complete.*

This work was supported by RFBR (grant 14-01-00376), and by the Grants Council (under RF President) for State Aid of Leading Scientific Schools (grant NSh-860.2014.1).

[1] N. A. BAZHENOV, *Degrees of categoricity for superatomic Boolean algebras*. ***Algebra and Logic***, vol. 52 (2013), no. 3, pp. 179–187.

[2] B. F. CSIMA, J. N. Y. FRANKLIN, AND R. A. SHORE, *Degrees of categoricity and the hyperarithmetic hierarchy*. ***Notre Dame Journal of Formal Logic***, vol. 54 (2013), no. 2, pp. 215–231.

[3] E. B. FOKINA, I. KALIMULLIN, AND R. MILLER, *Degrees of categoricity of computable structures*. ***Archive for Mathematical Logic***, vol. 49 (2010), no. 1, pp. 51–67.

[4] S. S. GONCHAROV, *Degrees of austostability relative to strong constructivizations*. ***Proceedings of the Steklov Institute of Mathematics***, vol. 274 (2011), no. 1, pp. 105–115.

▸ DAVID BELANGER AND RICHARD SHORE, *A non-uniqueness theorem for jumps of principal ideals*.
Department of Mathematics, Cornell University, Ithaca, NY 14853, USA.
E-mail: dbelanger@math.cornell.edu.
E-mail: shore@math.cornell.edu.

We show that for every degree $\mathbf{u}$ REA in $\mathbf{0}'$, there is a pair $\mathbf{a}_0, \mathbf{a}_1$ of distinct r.e. degrees such that $\mathbf{a}_0' = \mathbf{u} = \mathbf{a}_1'$, and such that the set $\{\mathbf{x}' \colon \mathbf{x} \leq \mathbf{a}_0\}$, which consists of all jumps of sets Turing-below $\mathbf{a}_0$, is equal to the corresponding set $\{\mathbf{x}' \colon \mathbf{x} \leq \mathbf{a}_1\}$. This defeats certain approaches to proving the rigidity of the r.e. degrees.

▸ THOMAS BENDA, *Formalizing vagueness as a doxastic, relational concept*.
Institute of Philosophy of Mind, National Yang Ming University, 155 Li-nong St., Sec. 2, Taipei 112, Taiwan.
E-mail: tbenda@ym.edu.tw.

Descriptions and statements about the physical world often involve vague predicates, e.g., “x is red”. It has become a common procedure to assign vague predicates intermediate truth values that are real numbers between 0 and 1. However, there is no satisfactory account what it means to be true to a given degree, which leaves doxastic degrees as the only option.

Furthermore, real numbers provide an almost absurd accuracy as well as a natural metric, where in fact we want to state no more than, say, that x is rather red than not, perhaps redder or less red than some y. That suggests considering vagueness as a relational notion.

A thereby established vagueness relation is a partial order. Advantages of a relational account of vagueness are that vague predicates form a comparatively weak structure without metric and that the well-known problem of higher-order vagueness vanishes.

There is no reason not to implement doxastic degrees on the object language level. Furthermore, with the practice of evaluating vague predicates, relational vagueness should be allowed to depend on perception and epistemic as well as pragmatic context and hence be nonextensional. To set up a requisite formal language, we enclose vagueness predicates in quotation marks and perform their assessment with a background in mind which provides epistemic and pragmatic context. Thus a ternary predicate is introduced, $\mathrm{B'Ax''Ay'b}$, read "I believe, with background b, Ax to at least as high a degree as Ay". Given background b, believing Ax with absolute confidence is formalized as $\mathrm{B'Ax''0{=}0'b}$.

Such a formalization may be applied to conferring values to physical magnitudes which uses approximations and error bars. "The value of a is v" would then be vague as much as "x is red", acknowledging a fuzzy nature of experimental, particularly, macroscopic physics.

► ACHILLES A. BEROS, *A DNC function that computes no effectively bi-immune set*.
Univeristé de Nantes, Bureau 105, Laboratoire LINA, UMR CNRS 6241, UFR de Sciences et Techniques, 2 rue de la Houssinère, BP 92208, 44322 Nantes Cedex 03, France.
E-mail: achilles.beros@univ-nantes.fr.

In *Diagonally Non-Computable Functions and Bi-Immunity* [2], Carl Jockusch and Andrew Lewis-Pye proved that every DNC function computes a bi-immune set. They asked whether every DNC function computes an effectively bi-immune set. Several attempts have been made to solve this problem in the last few years. We construct a DNC function that computes no effectively bi-immune set, thereby answering their question in the negative. We obtain a few corollaries that illustrate how our technique can be applied more broadly.

[1] Carl Jockusch, *Degrees of functions with no fixed points*. ***Logic, Philosophy, and Methodology of Science***, vol. VIII (1989), pp. 191–201.

[2] Carl Jockusch and Andrew Lewis-Pye, *Diagonally non-computable functions and bi-immunity*. ***The Journal of Symbolic Logic***, (to appear).

[3] Antonín Kučera, *An alternative, priority-free, solution to Post's problem*. ***Mathematical Foundations of Computer Science***, vol. 233 (1986), pp. 493–500.

► KONSTANTINOS A. BEROS, *Co-analytic ideals on ω*.
Department of Mathematics, University of North Texas, 1155 Union Circle #311430, Denton, TX 76203-5017, USA.
E-mail: beros@unt.edu.

We consider a variant of the Rudin–Keisler order for ideals on ω and prove the existence of a complete co-analytic ideal with respect this order. The key tool is a parameterization of all co-analytic ideals. We obtain this parameterization via a method which yields a simple proof of Hjorth's 1996 theorem on the existence of a complete co-analytic equivalence relation. Unlike Hjorth's proof, ours does not rely on the use of the effective theory specific to Π^1_1 sets and thus generalizes under PD to other projective classes.

► RAVIL BIKMUKHAMETOV, *On Σ^0_2-initial segments of computable linear orders*.
Institute of Mathematics and Mechanics, Kazan (Volga region) Federal University, 18 Kremlyovskaya St., Russian Federation.
E-mail: ravil.bkm@gmail.com.

In my talk I consider the complexity of initial segments of computable linear orders. In all notations and definitions we shall adhere to [4]. A linear order $\mathcal{L} = (L, <_L)$ is *computable* (*X-computable*) if its domain is a computable (X-computable) set and its ordering relation is a computable (X-computable) relation. A suborder $\mathcal{I}$ of $\mathcal{L}$ is called *an initial segment* of $\mathcal{L}$ if

$$\forall x, y\, [(x <_L y \ \ \&\ \ y \in I) \Rightarrow x \in I]$$

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

M. Raw [3] showed that any Π^0_1-initial segment of a computable linear order has a computable presentation. On the other hand, he constructed a computable linear order with a Π^0_3-initial segment which has no computable copy. R. Coles, R. Downey, and B. Khoussainov [2] showed that there is a computable linear order with a Π^0_2-initial segment which is not isomorphic to any computable linear order. Note that they obtained the previous result using an infinite injury priority method. M. V. Zubkov [5] proved the same result using only finite injury priority method. K. Ambos-Spies, S. B. S. Cooper, and S. Lempp [1] showed that every Σ^0_2-initial segment of any computable linear order has a computable copy. We prove the following theorem which is a supplement to the previous result.

THEOREM 1. *For any computable linear order $\mathcal{L} = (L, <_L)$ without the greatest element and for any set $M \in \Sigma^0_2$ there is a computable linear order $\tilde{\mathcal{L}} = \mathcal{A} + \eta$ such that $\mathcal{A} \cong \mathcal{L}$ and $\mathcal{A} \equiv_T M$.*

Clearly, every computable linear order with the greatest element can only be a computable (i.e., Σ_0-) initial segment. Thus, Σ^0_2-initial segments of computable linear orders contain in total all computable linear orders without the greatest elements and all Σ^0_2-degrees.

[1] K. AMBOS-SPIES, S. B. COOPER, AND S. LEMPP, *Initial segments of recursive linear orders*. ***Order***, vol. 14 (1997), no. 2, pp. 101–105.

[2] R. J. COLES, R. G. DOWNEY, AND B. KHOUSSAINOV, *On initial segments of computable linear orders*. ***Order***, vol. 14 (1997), no. 2, pp. 107–124.

[3] M. J. S. RAW, ***Complexity of automorphisms of recursive linear orders***, Ph.D. Thesis, University of Wisconsin-Madison, 1995.

[4] R. I. SOARE, ***Recursively enumerable sets and degrees***, Springer-Verlag, Berlin, 1987.

[5] M. V. ZUBKOV, *Initial segments of computable linear orders with additional computable predicates*, ***Algebra and Logic***, vol. 48 (2009), no. 5, pp. 564–579.

▸ ALEXANDER C. BLOCK, *A new lower bound for the length of the hierarchy of norms*.
Department of Mathematics, University of Hamburg, Bundesstrasse 55, 20146 Hamburg, Germany.
E-mail: `fmua001@uni-hamburg.de`.

A *norm* is a surjective function from the Baire space $\mathbb{R}$ onto an ordinal. Given two norms φ, ψ we write $\varphi \leq_{\mathrm{N}} \psi$ if φ continuously reduces to ψ. Then $\leq_{\mathrm{N}}$ is a preordering and so passing to the set of corresponding equivalence classes yields a partial order, the *hierarchy of norms*.

Assuming the axiom of determinacy (AD) the hierarchy of norms is a wellorder. The length Σ of the hierarchy of norms was investigated by Löwe in [1]; he determined that $\Sigma \geq \Theta^2$ (where $\Theta := \sup\{\alpha \mid \text{There is a surjection from } \mathbb{R} \text{ onto } \alpha\}$). In his talk "Multiplication in the hierarchy of norms", given at the ASL 2011 North American Meeting in Berkeley, Löwe presented a binary operation $\boxtimes$ on the hierarchy of norms such that for wellchosen norms φ, ψ the ordinal rank of $\varphi \boxtimes \psi$ in the hierarchy of norms is at least as big as the product of the ordinal ranks of φ and ψ, which implies that Σ is closed under ordinal multiplication and so $\Sigma \geq \Theta^\omega$.

In this talk I will note that in fact for wellchosen norms φ, ψ the ordinal rank of $\varphi \boxtimes \psi$ is exactly the product of the ranks of φ and ψ with an intermediate factor of ω_1. Furthermore using a stratification of the hierarchy of norms into initial segments closed under the $\boxtimes$-operation I will show that $\Sigma \geq \Theta^{(\Theta^\Theta)}$.

[1] BENEDIKT LÖWE, *The length of the full hierarchy of norms*. ***Rendiconti del Seminario Matematico dell'Università e del Politecnico di Torino***, vol. 63 (2005), no. 2, pp. 161–168.

▸ WILL BONEY, *Computing the number of types of infinite length*.
Department of Mathematical Sciences, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, Pennsylvania, 15232, USA.
E-mail: `wboney@andrew.cmu.edu`.
URL Address: `http://www.math.cmu.edu/~wboney/`.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

We show that the number of types of sequences of tuples of a fixed length can be calculated from the number of 1-types and the length of the sequences. Specifically, if $\kappa \leq \lambda$, then

$$\sup_{\|M\|=\lambda} |S^{\kappa}(M)| = \left(\sup_{\|M\|=\lambda} |S^{1}(M)|\right)^{\kappa}$$

We show that this holds for any Abstract Elementary Class with λ amalgamation. No such calculation is possible for nonalgebraic types. We introduce a subclass of nonalgebraic types for which the same upper bound holds. We use this to answer a question of Shelah from [1].

[1] Saharon Shelah, ***Classification theory and the number of nonisomorphic models***, 2nd edition, vol. 92, North-Holland, Amsterdam, 1990.

▸ MARIJA BORIČIĆ, *Models for the probabilistic sequent calculus*.
Faculty of Organizational Sciences, University of Belgrade, Jove Ilića 154, 11000 Belgrade, Serbia.
E-mail: marija.boricic@fon.bg.ac.rs.

The usual approach to treating the probability of a sentence leads to a kind of polymodal logic with iterated (or not iterated) probability operators over formulae (see [3]). On the other hand, there were some papers dealing with probabilistic form of inference rules (see [1], [2] and [4]). The sequent calculi present a particular mode of deduction relation analysis. The combination of these concepts, the sentence probability and the deduction relation formalized in a sequent calculus, makes it possible to build up sequent calculus probabilized—the system **LKprob**. Sequents in **LKprob** are of the form $\Gamma \vdash_a^b \Delta$, meaning that 'the probability of provability of $\Gamma \vdash \Delta$ is in interval $[a, b] \cap I$', where I is a finite subset of reals [0, 1].

Let Seq be the set of all sequents of the form $\Gamma \vdash \Delta$. A model for **LKprob** is any mapping $p\colon \mathrm{Seq} \to [0, 1]$ satisfying: (i) $p(A \vdash A) = 1$, for any formula A; (ii) if $p(AB \vdash) = 1$, then $p(\vdash AB) = p(\vdash A) + p(\vdash B)$, for any formulas A and B; (iii) if sequents $\Gamma \vdash \Delta$ and $\Pi \vdash \Lambda$ are equivalent in **LK**, in sense that there are proofs for both sequents $\bigwedge \Gamma \to \bigvee \Delta \vdash \bigwedge \Pi \to \bigvee \Lambda$ and $\bigwedge \Pi \to \bigvee \Lambda \vdash \bigwedge \Gamma \to \bigvee \Delta$ in **LK**, then $p(\Gamma \vdash \Delta) = p(\Pi \vdash \Lambda)$.

We prove that our probabilistic sequent calculus **LKprob** is sound and complete with respect to the models just described.

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[2] T. Hailperin, *Probability logic*. ***Notre Dame Journal of Formal Logic***, vol. 25 (1984), pp. 198–212.

[3] Z. Ognjanović, M. Rašković, and Z. Marković, *Probability logics*, ***Logic in Computer Science***, Zbornik radova 12 (20), Z. Ognjanović (editor), Mathematical Institute SANU, Belgrade, 2009, pp. 35–111.

[4] C. G. Wagner, *Modus tollens probabilized*. ***British Journal for the Philosophy of Science***, vol. 54(2004), no. 4, pp. 747–753.

▸ BRANISLAV BORIČIĆ AND MIRJANA ILIĆ, *An intuitionistic interpretation of classical implication*.
Faculty of Economics, University of Belgrade, Kamenička 6, 11000 Beograd, Serbia.
E-mail: boricic@ekof.bg.ac.rs.
E-mail: mirjanailic@ekof.bg.ac.rs.

A connection between the classical and the Heyting's logic is given by the Glivenko's Theorem: for every propositional formula A, A is classically provable iff $\neg\neg A$ is provable intuitionistically. This theorem can be understood as a possible way of intuitionistic interpretation of the classical reasoning. Embedding of the implicative fragment of classical logic into the implicative fragment of the Heyting's logic was considered by J. P. Seldin [3] and L. C. Pereira, E. H. Haeusler, V. G. Costa, W. Sanz [2]. Seldin's interpretation essentially depends on the presence of conjunction, but the second one is obtained in the pure

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

language of implication. Here we define, in spirit of Kolmogorov's interpretation, a mapping of the pure implicational propositional language enabling to prove the corresponding result. Let $p_1, \dots, p_n$ be a list of all propositional letters occurring in formula $A \to B$ and q any propositional letter not occurring in $A \to B$. Then the image $b(A \to B)$ of $A \to B$ is defined inductively as follows: $b(p) = (p \to q) \to q$, for each $p \in \{p_1, \dots, p_n\}$, and $b(A \to B) = b(A) \to b(B)$. Namely, $b(A \to B)$ is obtained by replacing each occurrence of a propositional letter p in $A \to B$ by $(p \to q) \to q$, where q is a new letter.

Embedding Lemma. For every propositional implicational formula A, A is provable in classical logic iff $b(A)$ is provable in Heyting logic.

This is a part of our paper [1] dealing with an alternative approach to normalization of the implicative fragment of classical logic.

[1] B. Boričić and M. Ilić, *An alternative normalization of the implicative fragment of classical logic*, (to appear).

[2] L. C. Pereira, E. H. Haeusler, V. G. Costa, and W. Sanz, *A new normalization strategy for the implicational fragment of classical propositional logic*. ***Studia Logica***, vol. 96 (2010), no. 1, pp. 95–108.

[3] J. P. Seldin, *Normalization and excluded middle I*. ***Studia Logica***, vol. 48 (1989), no. 2, pp. 193–217.

▸ QUENTIN BROUETTE AND FRANÇOISE POINT, *Differential Galois theory in the class of formally real fields*.

Université de Mons, 20 Place du Parc, 7000 Mons, Belgique.
E-mail: quentin.brouette@gmail.com.
E-mail: point@math.univ-paris-diderot.fr.

Inside the class of formally real fields, we study strongly normal extensions as defined in [1, chap. VI]. Fix L/K a strongly normal extension of formally real differential fields such that the subfield C_K of constant elements of K is real closed.

Let $\mathcal{U}$ be a saturated model of the theory of closed ordered differential fields containing L (see [3]), $\mathcal{U}$ is real closed and for $i^2 = -1$, $\mathcal{U}(i)$ is a model of DCF_0.

We denote $\mathrm{gal}(L/K)$ the group of differential K-automorphisms of L and $\mathrm{Gal}(L/K) := \mathrm{gal}(\langle L, C_{\mathcal{U}}\rangle / \langle K, C_{\mathcal{U}}\rangle)$.

Theorem 1. The group $\mathrm{Gal}(L/K)$, respectively $\mathrm{gal}(L/K)$, is isomorphic to a definable group G in the real closed field $C_{\mathcal{U}}$, respectively C_K.

Under the hypothesis that K is relatively algebraically closed in L, we prove that given any $u \in L \setminus K$, there exists $\sigma \in \mathrm{Gal}(L/K)$ such that $\sigma(u) \neq u$.

Let $K \subseteq E \subseteq L$ be an intermediate differential field extension. As the elements of $\mathrm{Gal}(E/K)$ are not supposed to respect the order induced on $\langle E, C_{\mathcal{U}}\rangle$ by the one of $\mathcal{U}$, they do not need to have an extension in $\mathrm{Gal}(L/K)$. Therefore, we do not get a 1-1 Galois correspondence like in the classical case where C_K is algebraically closed (see [2]).

Let $\mathrm{Aut}(L/K)$ denote the subgroup of elements of $\mathrm{Gal}(L/K)$ that are increasing, let $\eta\colon G \to \mathrm{Gal}(L/K)$ denote a group isomorphism given by Theorem 1 and $\langle L, C_{\mathcal{U}}\rangle^r$ be the real closure of $\langle L, C_{\mathcal{U}}\rangle$ in $\mathcal{U}$.

Proposition 2. Let G_0 be a definable subgroup of G. There is a finite tuple $\bar{d} \in \langle L, C_{\mathcal{U}}\rangle^r$ such that $\eta(G_0) \cap \mathrm{Aut}(L/K)$ is isomorphic (as a group) to $\mathrm{Aut}(L(\bar{d})/K(\bar{d}))$.

[1] E. R. Kolchin, ***Differential algebra and algebraic groups***, Pure and Applied Mathematics, vol. 54. Academic Press, New York-London, 1973.

[2] A. Pillay, *Differential Galois theory I*. ***Illinois Journal of Mathematics***, vol. 42 (1998), no. 4, pp.978–699.

[3] M. Singer, *The model theory of ordered differential fields*. ***The Journal of Symbolic Logic***, vol. 43 (1978), no. 1, pp. 82–91.

► ANAHIT CHUBARYAN, ARMEN MNATSAKANYAN, AND HAKOB NALBANDYAN, *On proof complexities of strong equal modal tautologies*.
Department of Informatics and Applied Mathematics, Yerevan State University, Armenia.
E-mail: achubaryan@ysu.am.
E-mail: arm.mnats@gmail.com.
E-mail: hakob_nalbandyan@yahoo.com.

The research of the lengths of proofs in the systems of propositional calculus is important because of its relation to some of main problems of the common complexity theory. The investigations of proof complexity start for the systems of Classical Propositional Logic (CPL). However, natural real conclusions have constructive character and the most statements of natural and technical languages have modalities (necessary and possible). Therefore the investigation of the proofs complexities is important also for the systems of Intuitionistic Propositional Logic (IPL) and in some cases also for Modal Propositional Logic (MPL). The information about proof complexity in IPL and MPL can be important, in particular, to formalize reasoning about the way programs behave and to express dynamical properties of transitions between states.

The strong equality of tautologies in CPL and IPL, based on the notion of determinative conjunct, was introduced by first coauthor earlier (strong equality implies well-known equality but not vice versa), and the relations between the proof complexities of strong equal tautologies in different proof systems of CPL and IPL are investigated.

By analogy with the notions of determinative conjuncts in CPL, we introduce the same notion for modal tautologies. On the base of introduced modal determinative conjuncts we introduce the notion of strong equality for modal tautologies and compare different measures of proof complexity (size, steps, space and width) for them in some proof systems of MPL. We prove that (1) in some proof systems the strong equal modal tautologies have the same proof complexities and (2) there are such proof systems, in which some measures of proof complexities for strong equal modal tautologies are the same, the other measures differ from each other only by the sizes of tautologies.

Acknowledgment. This work is supported by Grant 13-1B004 of SSC of Government of RA.

► WILLEM CONRADIE AND ANDREW CRAIG, *Algorithmic-algebraic canonicity for mu-calculi*.
Department of Pure and Applied Mathematics, University of Johannesburg, Kingsway, Auckland Park 2006, South Africa.
E-mail: wconradie@uj.ac.za.
E-mail: acraig@uj.ac.za.

The correspondence and completeness of logics with fixed point operators has been the subject of recent research (see [1], [2]). These works aim to develop a Sahlqvist-like theory for their respective fixed point settings. That is, they identify classes of formulas which are preserved under canonical extensions and have first-order frame correspondents.

We prove that the members of a certain class of intuitionistic mu-formulas are *canonical*, in the sense of [1]. When projected onto the classical case, our class of canonical mu-formulas subsumes the class described in [1]. Our methods use a variation of the algorithm ALBA (Ackermann Lemma Based Algorithm) developed in [3]. We show that all mu-inequalities that can be successfully processed by our algorithm, μ^*-ALBA, are canonical. Formulas are interpreted on a bounded distributive lattice $\mathbf{A}$ with additional operations. The canonical extension of $\mathbf{A}$, denoted $\mathbf{A}^\delta$, is a complete lattice in which the completely join-irreducible elements ($J^\infty(\mathbf{A}^\delta)$) are join-dense, and the completely meet-irreducible elements ($M^\infty(\mathbf{A}^\delta)$) are meet-dense. An *admissible valuation* takes all propositional variables to elements of $\mathbf{A}$. The algorithm aims to "purify" an inequality $\alpha \leqslant \beta$ by rewriting it as a (set of) pure (quasi-)inequalities. A pure quasi-inequality has no occurrences of propositional variables; only special variables whose interpretations range over $J^\infty(\mathbf{A}^\delta) \cup M^\infty(\mathbf{A}^\delta)$ are present. The fact that admissible and ordinary validity coincide for pure inequalities is the lynchpin for proving canonicity.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

The proof of the soundness of the rules of the algorithm μ^*-ALBA rests on the order-topological properties of formulas (term functions) of the μ-calculus.

[1] N. Bezhanishvili and I. Hodkinson, *Sahlqvist theorem for modal fixed point logic*. ***Theoretical Computer Science***, vol. 424 (2012), pp. 1–19.

[2] W. Conradie, Y. Fomatati, A. Palmigiano, and S. Sourabh, *Algorithmic correspondence for intuitionistic modal mu-calculus*. ***Theoretical Computer Science***, to appear.

[3] W. Conradie and A. Palmigiano, *Algorithmic correspondence and canonicity for distributive modal logic*. ***Annals of Pure and Applied Logic***, vol. 163 (2012), pp. 338–376.

▸ WILLEM CONRADIE AND CLAUDETTE ROBINSON, *Hybrid extensions of S4 with the finite model property*.
Department of Mathematics, University of Johannesburg, Kingsway, Auckland Park, 2006, South Africa.
E-mail: wconradie@uj.ac.za.
E-mail: claudetter@uj.ac.za.

In [1] R. A. Bull characterized a class of axiomatic extensions of the modal logic S4 (the logic of the class of transitive and symmetric Kripke frames) with the finite model property. This result takes the form of a syntactic characterization of a class of formulas that may be added as axioms to S4, somewhat in the spirit of Sahlqvist's famous result in modal correspondence theory. Hybrid logics (see e.g., [2]) expand the syntax of modal logic by adding special variables, known as nominals, which are always interpreted as singletons in models and thus act as names for the states at which they hold. Additional syntactic machinery which capitalizes on the naming power of the nominals, like the satisfaction operator $@_i \phi$ or the universal modality, is often added. This makes hybrid languages significantly more expressive than their modal cousins, while retaining their good computational behaviour. In this talk we show how to extend Bull's result to three hybrid languages. The proofs we offer are algebraic and serve to illustrate the usefulness of the new algebraic semantics for hybrid logics recently introduced by the authors. Bull's proof makes essential use of the algebraic property of 'well-connectedness' which is equivalent, in the dual relational semantics, to the ability to take generated submodels. Since the truth of hybrid languages is not invariant under generated submodels, the generalization to hybrid logic is not straight-forward.

[1] R. A. Bull, *A class of extensions of the modal system S4 with the finite model property*. ***Zeitschrift für mathematische Logik und Grundlagen der Mathematik***, vol. 11 (1965), pp. 127–132.

[2] George Gargov and Valentin Goranko, *Modal logic with names*. ***Journal of Philosophical Logic***, vol. 22 (1993), no. 6, pp. 607–636.

▸ ANDRÉS CORDÓN-FRANCO AND F. FÉLIX LARA-MARTÍN, Π_1-*induction axioms vs* Π_1-*induction rules: Some conservation results*.
Department of Computer Science and Artificial Intelligence, University of Seville, Facultad de Matemáticas, C/ Tarfia s/n, Sevilla, Spain.
E-mail: acordon@us.es.
E-mail: fflara@us.es.

As proved independently by Mints, Adamowicz-Bigorajska, Kaye and Ratajczyk, if a Π_2-sentence θ is derived (over the base theory $I\Delta_0$) using m instances of parameter-free Σ_1-induction axiom scheme then θ can also be derived using at most m (nested) applications of Σ_1-induction rule. If θ is a Π_1-sentence then a similar result for Π_1-induction can be proved by exploiting the equivalence between Local Σ_2-reflection and the parameter-free Π_1-induction axiom scheme, $I\Pi_1^-$ (see [1]). However, due to the use of Local Reflection principles, the base theory used in this result must extend at least $I\Delta_0 + \exp$ and, as far as we know, no similar results for $I\Pi_1^-$ are known over plain $I\Delta_0$.

In this work we address this question. Working over $I\Delta_0$, we obtain a number of conservation results relating the number of instances of $I\Pi_1^-$ needed to derive a sentence θ, and the number and depth of *nested* applications of Π_1-induction rule needed in a derivation

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

of θ. Several formulations of Π_1-induction rule are considered in correspondence with the quantifier complexity of the sentence θ.

Our approach is model-theoretic and uses theories of *Local Induction* as a basic tool.

Partially supported by grant MTM2011-26840 Ministerio de Ciencia e Innovación, Spanish Government.

[1] L. D. Beklemishev, *Reflection principles and provability algebras in formal arithmetic*. ***Russian Mathematical Surveys***, vol. 60 (1999), pp. 197–268.

▸ VALERIA DE PAIVA AND LUIZ CARLOS PEREIRA, *An intuitive semantics for Full Intuitionistic Linear Logic*.
Nuance Comms, 1198 East Arques Avenue, Sunnyvale, CA 94805, USA.
E-mail: valeria.depaiva@gmail.com.
Dept. Filosofia, PUC-Rio, Rio de Janeiro, Brazil.
E-mail: luiz@inf.puc-rio.br.

This work describes an *intuitive* semantics in the style of Girard's well-known cigarette vending machine for Full Intuitionistic Linear Logic. Full Intuitionistic Linear Logic (FILL) was introduced by Hyland and de Paiva [1] as arising from its categorical semantics, while hinting at its independent interest as a framework for forms of parallelism in Functional Programming. The systems FILL and its intuitionistic counterpart FIL [2] show that the constructive character of logical systems is not given by syntactic size restrictions on sequent calculus, but comes about by explaining connectives in terms of intensional constructions/operations/transformations on derivations of the system. This seems to us the central message of the Brouwer–Heyting–Kolmogorov (BHK) interpretation and also of the Curry–Howard isomorphism, which we take as guiding criteria for our mathematical logic investigations. This work also aims to explain to the mythical man-on-the street what Full Intuitionistic Linear Logic is about. We were pressed on the point that, elegance of categorical constructions and esthetic criteria on proof systems notwithstanding, one should always be able to say what our logical operations mean in common words, when describing a new logical system like FILL. Initially we had no intuitive explanation for the multiplicative disjunction 'par', which now seems more understandable in terms of interactions with a 'stock-keeping' system.

[1] Martin Hyland and Valeria de Paiva, *Full Intuitionistic Linear Logic (extended abstract)*. ***Annals of Pure Applied Logic***, vol. 64 (1993), no. 3, pp. 273–291.

[2] Valeria de Paiva and Luiz Carlos Pereira, *A short note on Intuitionistic Propositional Logic with multiple conclusions*. ***Manuscrito—Revista Internacional Filosofia, Campinas***, vol. 28 (2005), pp. 317–329.

▸ BRUNO DINIS AND IMME VAN DEN BERG, *An axiomatic approach to modelling of orders of magnitude*.
CMAF, University of Lisbon, Faculdade de Ciencias, Universidade de Lisboa, Campo Grande, Edificio C6, Gabinete 6.2.18, P-1749-016 Lisboa, Portugal.
E-mail: bruno.salsa@gmail.com.
Mathematics Department, University of Evora, Colegio Luis Antonio Verney, Rua Romao Ramalho, 59, 7000-671 Evora, Portugal.
E-mail: ivdb@uevora.pt.

Many arguments deal informally with orders of magnitude of numbers. If one tries to maintain the intrinsic vagueness of orders of magnitude—they should be bounded, but stable under at least some additions—, they cannot be formalized with ordinary real numbers, due to the Archimedean property and Dedekind completion. Still there is the functional approach through Oh's and oh's and more generally Van der Corput's neutrices [1], both have some operational shortcomings.

Nonstandard Analysis disposes of a natural example of order of magnitude: the (external) set of infinitesimals is bounded and closed under addition [5, 6]. Adopting the terminology of Van der Corput, we call a *neutrix* an additive convex subgroup of the nonstandard reals. An *external number* is the set-theoretic sum of a nonstandard real and a neutrix.

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The external numbers capture the imprecise boundaries of informal orders of magnitude and permit algebraic operations which go beyond the calculus of the Oh's and oh's [2]. This external calculus happens to be based more on semigroup operations than group operations, but happens to be fairly operational in concrete cases and allows for total order with a generalized form of Dedekind completion [3].

Based on joint work with Imme van den Berg, we discuss an axiomatics for the calculus of neutrices and external numbers, trying to do justice to the vagueness of orders of magnitude. In particular we consider foundational problems which appear due to the fact that some axioms are necessarily of second order, and the fact that the external calculus exceeds existing foundations for external sets [4].

The first author acknowledges the FCT grant SFRH/BPD/97436/2013. Both authors acknowledge FCT project PEst-OE/MAT/UIO117/2014.

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► JAN DOBROWOLSKI, *Topologies on Polish structures*.
Uniwersytet Wrocławski, pl. Uniwersytecki 1, Wroclaw 50-137, Poland.
E-mail: `dobrowol@math.uni.wroc.pl`.

In [1], the following definition was introduced.

Definition 1. A Polish structure is a pair (X, G), where G is a Polish group acting faithfully on a set X so that the stabilizers of all singletons are closed subgroups of G. We say that (X, G) is small if for every $n < \omega$, there are only countably many orbits on X^n under the action of G.

Notice that, in the above definition, it is not required that X is a topological space. I will discuss some issues concerning existence of topologies on X satisfying some natural conditions. Special attention will be given to the case in which X carries a structure of a group (i.e., (X, G) is a Polish group structure).

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► MARINA DORZHIEVA, *Computable numberings in analytical hierarchy*.
Novosibirsk State University, 2 Pirogova Street, Novosibirsk, Russia.
E-mail: `dm-3004@inbox.ru`.

We investigate minimal enumerations in analytical hierarchy. Enumeration $\nu \in Com^1_{n+1}$ is called minimal, if for every $\mu \in Com^1_{n+1}$ such that $\mu \leq \nu$, performed $\nu \equiv \mu$. One of the most important minimal numberings is Friedbergs numbering. Owings showed in [1] that there is no Π^1_1-computable Friedberg enumeration of all Π^1_1-sets using metarecursion theory. This result is obtained in classic computability theory for higher levels of analytical hierarchy:

Theorem 1.

(1) *There are infinite minimal numberings of an infinite family S of Π^1_{n+1}-sets.*
(2) *There is no a Π^1_{n+1}-computable Friedberg enumeration of all Π^1_{n+1}-sets.*

This work was supported by RFBR (grant 14-01-31278).

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► BENEDICT EASTAUGH, *Computational reverse mathematics and foundational analysis*.
Department of Philosophy, University of Bristol, Cotham House, Bristol BS6 6JL, UK.
E-mail: benedict@eastaugh.net.
URL Address: http://extralogical.net.

Reverse mathematics studies which natural subsystems of second order arithmetic are equivalent to key theorems of ordinary or non-set-theoretic mathematics. The main philosophical application of reverse mathematics proposed thus far is *foundational analysis*, which explores the limits of various weak foundations for mathematics in a formally precise manner. Richard Shore [1, 2] proposes an alternative framework in which to conduct reverse mathematics, called *computational reverse mathematics*. The formal content of his proposal amounts to restricting our attention to ω-models of RCA_0 when we prove implications and equivalences in reverse mathematics.

Despite some attractive features, computational reverse mathematics is inappropriate for foundational analysis, for two major reasons. Firstly, the computable entailment relation employed in computational reverse mathematics does not preserve justification for all of the relevant foundational theories, particularly a partial realisation of Hilbert's programme due to Simpson [3].

Secondly, computable entailment is a Π^1_1-complete relation, and hence employing it commits one to theoretical resources which outstrip those acceptable to the stronger foundational programmes such as predicativism and predicative reductionism. This argument can be formalised within second order arithmetic, making it accessible to partisans of foundational frameworks such as predicativism. In doing so we show that the existence of the set of sentences which are computably entailed is equivalent over ACA_0 to Π^1_1 comprehension.

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► CHRISTIAN ESPÍNDOLA, *Semantic completeness of first order theories in constructive reverse mathematics*.
Department of Mathematics, Stockholm University, Roslagsv 101 hus 5–6 (10691) Stockholm, Sweden.
E-mail: espindola@math.su.se.

We introduce a general notion of semantic structure for first-order theories, covering a variety of constructions such as Tarski and Kripke semantics, and prove that, over Zermelo Fraenkel set theory (ZF), the completeness of such semantics is equivalent to the Boolean Prime Ideal theorem (BPI). In particular, we deduce that the completeness of that type of semantics for nonclassical theories is unprovable in intuitionistic Zermelo Fraenkel set theory IZF ([4]). Using results of Joyal ([2]) and McCarty ([3]), we conclude also that the completeness of Kripke semantics is equivalent, over IZF, to the Law of Excluded Middle plus BPI. By results in [1], none of these two principles imply each others, and so this gives the exact strength of Kripke completeness theorem in the sense of constructive reverse mathematics.

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▸ MARAT FAIZRAHMANOV AND ISKANDER KALIMULLIN, *Limitwise monotonic sets of reals*.
Institute of Mathematics and Mechanics, Kazan (Volga Region) Federal University, Kremlyovskaya 18, Russian Federation.
E-mail: marat.faizrahmanov@gmail.com.

We extend the limitwise monotonicity notion to the case of arbitrary computable linear ordering to get a set which is limitwise monotonic precisely in the noncomputable degrees. Also we get a series of connected nonuniformity results to obtain new examples of nonuniformly equivalent families of computable sets with the same enumeration degree spectrum.

▸ HADI FARAHANI AND HIROAKIRA ONO, *Predicate Glivenko theorems and substructural aspects of negative translations*.
Department of Computer Sciences, Shahid Beheshti University, Evin, Tehran, Iran.
E-mail: h_farahani@sbu.ac.ir.
Research Center for Integrated Science, Japan Advanced Institute of Science and Technology, Nomi, Ishikawa, 923-1292, Japan.
E-mail: ono@jaist.ac.jp.

In [2], the second author has developed a proof-theoretic approach to Glivenko theorems for substructural propositional logics. In the present talk, by using the same techniques, we will extend them for substructural predicate logics. It will be pointed out that in this extension, the following *double negation shift* scheme (DNS) plays an essential role.

$$(\mathrm{DNS}): \forall x \neg\neg\varphi(x) \to \neg\neg\forall x \varphi(x)$$

Among others, the following is shown, where QFLe and $\mathrm{QFLe}^{\dagger}$ are predicate extensions of FLe and $\mathrm{FLe}^{\dagger}$, respectively (see [2]). The Glivenko theorem holds for $\mathrm{QFLe}^{\dagger}+(\mathrm{DNS})$ relative to classical predicate logic. Moreover, this logic is the weakest one among predicate logics over QFLe for which the Glivenko theorem holds relative to classical predicate logic. Then we will study negative translations of substructural predicate logics by using the same approach. We introduce a negative translation, called extended Kuroda translation and over QFLe it will be shown that standard negative translations like Kolmogorov translation and Gödel–Gentzen translation are equivalent to our extended Kuroda translation. Thus, we will give a clearer unified understanding of these negative translations by substructural point of view.

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▸ DAVID FERNÁNDEZ-DUQUE AND JOOST J. JOOSTEN, *Provability logics and proof-theoretic ordinals*.
Instituto Tecnológico Autónomo de México, Mexico.
E-mail: david.fernandez@itam.mx.
Universitat de Barcelona, Spain.
E-mail: jjoosten@ub.edu.

A recent approach by Beklemishev uses provability logics to represent reflection principles in formal theories and uses said principles to calibrate a theory's consistency strength [1]. There are several benefits to this approach, including semi-finitary consistency proofs and independent combinatorial statements.

A key ingredient is Japaridze's polymodal provability logic GLP_ω [4]. In order to study stronger theories one needs to go beyond GLP_ω to the logics GLP_Λ, where Λ is an arbitrary ordinal. These logics have for each ordinal $\xi < \Lambda$ a modality $\langle \xi \rangle$. Proof theoretic ordinals below Γ_0 may be represented in the closed fragment of GLP_Λ *worms* therein [2, 3].

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

Worms are iterated consistency statements of the form $\langle \xi_n \rangle \dots \langle \xi_1 \rangle \top$ and are well-ordered by their consistency strength.

We present a calculus for computing the order types of worms and compare the resulting ordinal representation system with standard systems based on Veblen functions. We will also discuss how larger ordinals arising from impredicative proof theory may be represented within provability logics.

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▸ HARTRY FIELD AND HARVEY LEDERMAN, *Prospects for a naïve theory of classes*.
Department of Philosophy, New York University, New York 10003, USA.
Department of Philosophy, University of Birmingham, Birmingham, UK.
E-mail: `hartry.field@nyu.edu`.
URL Address: `http://philosophy.fas.nyu.edu/object/hartryfield`.
Faculty of Philosophy, University of Oxford, Oxford, UK.
E-mail: `harvey.lederman@philosophy.ox.ac.uk`.
URL Address: `http://users.ox.ac.uk/~hert2388/`.

We examine the prospects for a naïve theory of classes, in which full "naïve" comprehension and an extensionality rule are maintained by weakening the background logic. Without extensionality, proving naïve comprehension consistent is formally analogous to proving naïve truth consistent, and in recent years much progress has been made on the latter question. But there is no natural analog for extensionality in the case of truth, so the question arises whether these logics for reasoning about truth can also be shown consistent with a form of extensionality. In a series of papers, and in his 2006 book ([2]), Ross Brady has presented various theories of naïve classes. We begin by providing a simpler, more accessible version of Brady's proof of the consistency of these theories. Our new presentation of Brady then makes it easy to see how Brady's result can be generalized to apply to certain logics which have a modal-like semantics given using four-valued, as opposed to three-valued worlds. (These include some logics from [1].) These "new" logics have a significant advantage over Brady's original: they validate a weakening rule (indeed, a weakening axiom) for a noncontraposable conditional. Since these laws are crucial if the conditional is to be used for restricted quantification, this is a substantial improvement.

Still, we do not think even these logics are satisfactory. The noncontraposable conditional which validates weakening in these logics is not the conditional of the extensionality rule. But there's strong intuitive motivation for the conditional in the extensionality rule to validate weakening. Otherwise, there will be "sets" which contain everything, but which are not extensionally equivalent. While Brady's logics (and the four-valued generalizations) deliver a form of extensionality, in the absence of weakening the formal rule does not capture the intuitive notion of extensionality.

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▸ ANDREY FROLOV, Δ^0_2-*spectra of linear orderings*.
Department of Mathematics and Mechanics, Kazan Federal University, 18 Kremlyovskaya St., Kazan, Russia.
E-mail: `Andrey.Frolov@kpfu.ru`.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

In [2], for any $n \geq 2$, it was constructed a linear ordering L such that the spectrum $Sp(L)$ contains exactly all non-low$_n$ degrees. Recall, the *spectrum* of a linear ordering L is the class $Sp(L) = \{\deg_T(R) \mid R \cong L\}$.

R. Miller [3] constructed a linear ordering whose Δ^0_2-spectrum contains exactly all nonlow$_0$ Δ^0_2-degrees, i.e., all nonzero Δ^0_2-degrees. The Δ^0_2-*spectrum* of linear ordering L is the class $Sp(L)^{\Delta^0_2} = \{\deg_T(R) \in \Delta^0_2 \mid R \cong L\} = Sp(L) \cap \Delta^0_2$.

The author [1] constructed a linear ordering whose Δ^0_2-spectrum contains exactly all nonlow$_1$ Δ^0_2-degrees.

In [2], for any $n \geq 2$, it was constructed a linear ordering L such that $Sp(L)$ contains exactly all high$_n$ degrees. Also in [2] it was remarked that there does not exist a linear orderings L such that $Sp(L)$ is exactly all high$_n$ degrees for $n \in \{0, 1\}$.

Theorem 1. *There exists a linear ordering L such that* $Sp^{\Delta^0_2}(L) = \{\mathbf{0}'\}$. *In other words,* Δ^0_2-*spectrum of L contains exactly all high*$_0$ Δ^0_2-*degrees.*

Theorem 2. *There exists a linear ordering whose* Δ^0_2-*spectrum contains exactly all high*$_1$ Δ^0_2-*degrees.*

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▸ ANDREY FROLOV AND MAXIM ZUBKOV, *On categoricity of scattered linear orders*.
N. I. Lobachevsky Institute of Mathematics and Mechanics, Kazan Federal University, Kremlevskaya 18, Kazan, Russia.
E-mail: andrey.frolov@kpfu.ru.
E-mail: maxim.zubkov@kpfu.ru.

We consider the categoricity of countable scattered linear orders. Recall that linear order is *scattered* if it has no dense suborder. A computable linear order L is *computably* (Δ^0_n-, *resp.*) *categorical* if for every computable copy L' of L there is a computable (Δ^0_n-, resp.) isomorphism between L' and L. J. Remmel [3], S. Goncharov, V. Dzgoev [1] obtained the description of computably categorical linear orders. Namely, they proved that a computable linear order is computably categorical if and only if it contains finitely many pairs of successors. Ch. McCoy [2] obtained the description of Δ^0_2-categorical computable linear order with additional conditions. We proved that if L is a computable scattered linear order such that L is a finite sum of scattered orders of rank n then L is Δ^0_{2n}-categorical. The definition of rank of scattered linear orders can be fined in [4].

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▸ HAO-CHENG FU, *A defense of information economy principle*.
Department of Philosophy, Chinese Culture University, No. 55, Hwa Kang Rd., Yang-Ming-Shan, Taipei 11114, Taiwan.
E-mail: fuhaocheng@gmail.com.

In our ordinary life it is inevitable for everyone has to adjust one's own belief state in light of new information when the new information is inconsistent with his belief state. Some philosophers such as Quine and *AGM* (Alchourrón et al.) suggested that the loss of information value should be minimized as possible whenever one confronts the inconsistency

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

and the principle in belief revision theory is usually so-called information economy principle (*IEP* for short). Furthermore, Gärdenfors has constructed a model who recommended the idea of epistemic entrenchment to this model to explain why *IEP* works. But Rott casted some doubts on *IEP* due to the postulates of epitemic entrenchment proposed by Gärdenfors sometimes failed to realize the features of nonmonotonic reasoning, i.e., it is possible that one might keep the less entrenched beliefs rather than the more ones in the process of belief change. In this paper, I want to present a game-theoretic framework to reconstruct the notion of epistemic entrenchment to avoid the challenges from Rott and prove that *IEP* is still available to be the norm to estimate the process of belief change.

Keywords: belief change, information economy principle, epistemic entrenchment, game theory.

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► VALENTIN GORANKO, *On the almost sure validities in the finite in some fragments of monadic second-order logic*.
Department of Applied Mathematics and Computer Science, Technical University of Denmark, Richard Petersens Plads, Bld. 324, Lyngby, Denmark.
E-mail: vfgo@dtu.dk.

This work builds on the well-known *0-1 law* for the asymptotic probabilities of first-order definable properties of finite graphs (in general, relational structures). Fagin's proof of this result is based on a transfer between almost sure properties in finite graphs and true properties of the *countable random graph* (aka, Rado graph).

Both the transfer theorem and the 0-1 law hold in some nontrivial extensions of first-order logic (e.g., with fixed point operators) but fail in others, notably in most natural fragments of monadic second-order (MSO) and even for modal logic formulae, in terms of frame validity. The question we study here is how to characterise—axiomatically or model-theoretically—the set of *almost surely valid in the finite* formulae of MSO, i.e., those with asymptotic probability 1. This question applies likewise to every logical language where truth on finite structures is well-defined. The set of almost sure validities in the finite of a given logical language is a well-defined logical theory, containing all validities of that language and closed under all sound finitary rules of inference. Beyond that, very little is known about these theories in cases where the transfer theorem fails.

In this work we initiate a study of the theories of almost sure validity in modal logic and in the Π^1_1 and Σ^1_1 fragments of MSO on binary relational structures, aiming at obtaining explicit logical characterisations of these theories. We provide such partial characterisations in terms of characteristic formulae stating almost sure existence (for Σ^1_1) or nonexistence (for Π^1_1) of bounded morphisms to special target finite graphs. Identifying explicitly the set of such finite graphs that generate almost surely valid characteristic formulae seems a quite difficult problem, to which we so far only provide some partial answers and conjectures.

► JEROEN P. GOUDSMIT, *Characterising logics through their admissible rules*.
Department of Philosophy, Utrecht University, Janskerkhof 13, The Netherlands.
E-mail: J.P.Goudsmit@uu.nl.
URL Address: http://jeroengoudsmit.com.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

The admissible rules of a logic are those rules under which the set of its theorems are closed. Many nonclassical logics have interesting admissible rules, and these admissible rules carry quite a bit of information. In fact, some logics can even be completely characterised by way of their admissible rules.

Consider the lattice of intermediate logics, that is, the lattice of consistent extensions of propositional intuitionistic logic. The disjunction property is an example of an admissible rule. It is well-known that there are continuum many logics among that lattice that enjoy the disjunction property, Medvedev's logic and Kreisel–Putnam logic to name but two. It was shown by Maksimova [4] that Medvedev's logic can be characterised as the maximal intermediate logic above Kreisel–Putnam logic with the disjunction property.

The intuitionistic propositional logic itself can also be characterised as the maximal intermediate logic that admits a particular set of rules, as has been independently confirmed by Skura [5] and Iemhoff [3]. The rules they employed arose independently in the work of Citkin [1] and Visser [6]. These rules can be stratified in a natural way with respect to admissibility in the logics of bounded branching, also known as the Gabbay–de Jongh logics [2]. We present a characterisation of each of these logics as being the maximal intermediate logic admitting a particular strata of the aforementioned rules.

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► OLEG GRIGORIEV, *Two formalisms for a logic of generalized truth values*.
Faculty of Philosophy, Chair of Logic, Moscow State University, Leninskie Gory, Russia.
E-mail: grig@philos.msu.ru.

This report concerns to the problem of constructing tableau-based proof procedure for a logic of generalized truth values [2, 3].

Generalized truth values are based on the two 'sorts' of truth, ontological (we denote it as 't') and epistemic ones ('1'). They constitute a four-element lattice with natural set theoretical order and familiar binary operations: $L = (\{\varnothing, \{1\}, \{t\}, \{t, 1\}\}, \subseteq, \cap, \cup)$.

One of the most interesting feature of this structure is a definition of the unary operations. We introduce two of them: $-_t$ sends $\varnothing$ to $\{t\}$ and back, $\{1\}$ to $\{t, 1\}$ and back, while $-_1$ switches between $\varnothing$ and 1, and between $\{t\}$ and $\{t, 1\}$. This semantic structure gives rise to a propositional logic based on the language over $\{\wedge, \vee, \neg_t, \neg_1\}$ with classical binary operation and two nonclassical negation-style connectives. It is worth mention that combinations $\neg_t\neg_1$ or $\neg_t\neg_1$ behave exactly like boolean negation.

For a definition of a semantic consequence relation there are several candidates, each of its own interest. We choose the simplest and most natural one: $A \models B$ iff the value of A is a subset os the value of B.

We propose two different tableau-style formalisation for a logic which captures a syntactical analogue of semantic logical consequence relation. One of them is more or less 'traditional' and resembles tableau systems for relevant logic FDE [1]. Another one is appropriate for designing a proof search procedure and based on well known KE formalism [1].

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▸ ZALÁN GYENIS, *Interpolation property in homogeneous structures*.
MTA Rényi Institute of Mathematics, 13–15 Reáltanoda utca, Budapest, Hungary.
E-mail: gyz@renyi.hu.

Formula interpolation and related problems have been intensively studied in the literature of algebraic logic. It turned out that interpolation properties of different logics are strongly related to various amalgamation properties of certain classes of algebras.

In this talk we introduce interpolation property for models of first order logic. Informally, a model has the interpolation property if Craig's interpolation theorem holds within the model. It turns out that this local version of Craig's interpolation is equivalent to a local version of Robinson's joint consistency theorem.

The relationship between amalgamation and interpolation property is also studied. New kind of amalgamation property, the so called prescribed amalgamation property is defined. We study connections between this new amalgamation property and the traditional ones.

Finally, a correspondence theorem is established between universal homogeneous structures $\mathcal{M}$ of finite binary signature and classes of finite structures which embed into $\mathcal{M}$ having this new amalgamation property. Particularly, we prove that Fraïssé limits of several classes serve as examples for models that have the interpolation property.

▸ MIHA E. HABIČ, *Restricting Martin's axiom to a ccc ground model*.
Mathematics Department, CUNY Graduate Center, 365 Fifth Avenue, New York, NY 10016, USA.
E-mail: mhabic@gc.cuny.edu.

We introduce a variant of Martin's axiom, called the grounded Martin's axiom or grMA. This principle asserts that the universe is a ccc forcing extension and that MA holds for posets from the ground model. The new axiom, which emerges naturally from the analysis of the Solovay–Tennenbaum proof of the consistency of MA, is shown to have many of the desirable properties of the weaker fragments of MA. In particular, we show that grMA is consistent with a singular continuum and also that it is consistent with the left side of Cichoń's diagram collapsing to ω_1. We also show that grMA is better behaved than MA when adding generic reals. Specifically, grMA is preserved under adding a Cohen real and holds after adding a random real to a model of MA.

▸ SHERWOOD HACHTMAN, *Unraveling $\Sigma^0_\alpha(\Pi^1_1)$-Determinacy*.
Department of Mathematics, University of California at Los Angeles, Los Angeles, CA, 90095, USA.
E-mail: shac@math.ucla.edu.

In parallel with the Borel hierarchy, one can define the levels $\Sigma^0_\alpha(\Pi^1_1)$ ($\alpha < \omega_1$) of the Borel-on-coanalytic hierarchy by starting with Π^1_1 in place of the class Δ^0_1 of clopen sets. In this talk, we consider the consistency strength of determinacy for infinite perfect-information games with payoff in $\Sigma^0_\alpha(\Pi^1_1)$. This has been computed exactly for $\alpha = 0, 1$, by Martin, Harrington, and J. Simms. For $\alpha > 1$, dual results of Steel [2] and Neeman [1] have shown the strength to reside within a very narrow range in the region of a measurable cardinal κ of largest possible Mitchell order $o(\kappa)$. However, an exact equiconsistency had yet to be isolated.

We have recently completed work pinpointing the determinacy strength of levels of the Borel hierarchy of the form $\Sigma^0_{1+\alpha+3}$, showing a level-by-level correspondence between these and a family of natural Π_1 reflection principles. Combining our techniques with those of [1] and [2], we can characterize the strength of $\Sigma^0_{1+\alpha+3}(\Pi^1_1)$-DET in terms of inner models with measurable cardinals. In particular, $\Sigma^0_4(\Pi^1_1)$-DET is equivalent to the existence of a mouse satisfying $(\exists\kappa)o(\kappa) = \kappa^{++}$ plus the schema that each true Π_1 statement with parameters in $P^2(\kappa)$ reflects to an admissible set containing $P(\kappa)$.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

We will also discuss progress on calculating the strength of $\Sigma^0_2(\Pi^1_1)$-DET, relating this to Mitchell's hierarchy of weak repeat point measures.

[1] ITAY NEEMAN, *Unraveling Π^1_1 sets, revisited*. ***Israel Journal of Mathematics***, vol. 152 (2006), pp. 181–203.

[2] JOHN R. STEEL, *Determinacy in the Mitchell models*. ***Annals of Mathematical Logic***, vol. 22 (1982), no. 2, pp. 109–125.

▸ MATTHEW HARRISON-TRAINOR, *Degree spectra of relations on a cone*.
Logic and the Methodology of Science, University of California, Berkeley, 2440 Bancroft Way, Berkeley, CA 94720, USA.
E-mail: matthew.h-t@berkeley.edu.

We consider structures $\mathcal{A}$ with an additional relation R. We say that two relations R and S on structures $\mathcal{A}$ and $\mathcal{B}$ respectively have the same (relativised) degree spectrum if, for sets C on a cone above $\mathbf{d}$,

$$\{R^{\tilde{\mathcal{A}}} \oplus C : \tilde{\mathcal{A}} \cong \mathcal{A} \text{ and } \tilde{\mathcal{A}} \leq_T C\} = \{S^{\tilde{\mathcal{B}}} \oplus C : \tilde{\mathcal{B}} \cong \mathcal{B} \text{ and } \tilde{\mathcal{B}} \leq_T C\}.$$

Using determinacy, these degree spectra are partially ordered. Many classes of degrees which relativise, such as the Σ^0_α degrees or α-CEA degrees, are degree spectra. This is a notion which captures solely the model-theoretic properties of the relation R. We will advocate for the naturality of this viewpoint by recasting existing results in this new language, giving new results, and putting forward new questions. Existing results of Harizanov in [3] show that there are two minimal degree spectra, the computable sets and the c.e. sets. In [1] and [2], Ash and Knight considered whether Harizanov's results could be generalised. We give a partial positive answer by showing that any degree spectrum which contains a non-Δ^0_2 degree contains all of the 2-CEA degrees. We also give an example of two incomparable degree spectra.

[1] C. J. ASH AND J. F. KNIGHT, *Possible degrees in recursive copies I*. ***Annals of Pure and Applied Logic***, vol. 75 (1995), no. 3, pp. 215–221.

[2] ———, *Possible degrees in recursive copies II*. ***Annals of Pure and Applied Logic***, vol. 87 (1997), no. 2, pp. 151–165.

[3] V. S. HARIZANOV, *Some effects of Ash–Nerode and other decidability conditions on degree spectra*. ***Annals of Pure and Applied Logic***, vol. 55 (1991), no. 1, pp. 51–65.

▸ NADJA HEMPEL, *Around n-dependent fields*.
217 Avenue Roger Salengro, Veilleurbanne, 69100, France.
E-mail: hempel@math.univ-lyon1.fr.

The notion of n-dependent theories introduced by Shelah is a natural generalization of dependent or more frequently called NIP theories. They form a proper hierarchy of first order theories in which the case n equals to 1 coincides which NIP theories.

In my talk, I give an overview about algebraic extensions of fields defined in structures with certain properties (superstable, stable, NIP, etc.). For instance, infinite NIP fields of positive characteristic are known to be Artin–Schreier closed. I extend this result to the wider class of infinite n-dependent fields for any natural number n and present some applications to valued fields defined in this setting. Secondly, I show that nonseparable closed pseudo-algebraically closed (PAC) fields have the n-independence property for all natural numbers n which is already known for the independence property (n equal to 1) due to Duret. Hence, nonseparable closed PAC fields lie outside of the hierarchy of n-dependent fields.

▸ ASSYLBEK ISSAKHOV, ***Ideals without minimal numberings in the Rogers semilattice***.
Department of Mechanics and Mathematics, Al-Farabi Kazakh National University, 71 Al-Farabi Ave., Almaty 050038, Kazakhstan.
E-mail: asylissakhov@mail.ru.

It is well known many infinite families of c.e. sets whose Rogers semilattice contains an ideal without minimal elements, for instance, the family of all c.e. sets, [3]. Moreover, there exists a computable family of c.e. sets whose Rogers semilattice has no minimal elements at

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

all, [1]. In opposite to the case of the families of c.e. sets, for every computable numbering α of an infinite family $\mathfrak{F}$ of computable functions, there is a Friedberg numbering of $\mathfrak{F}$ which is reducible to α, [3]. This means that the Rogers semilattice of any computable family of total functions from level 1 of the arithmetical hierarchy contains no ideal without minimal elements.

We study computable families of total functions of any level of the Kleene–Mostowski hierarchy above level 1 and try to find elementary properties of Rogers semilattices that are different from the properties of Rogers semilattices for the families of computable functions.

THEOREM 1. *For every n, there exists a Σ^0_{n+2}-computable family of total functions whose Rogers semilattice contains an ideal without minimal elements.*

Note that every Rogers semilattice of a Σ^0_{n+2}-computable family $\mathfrak{F}$ contains the least element if $\mathfrak{F}$ is finite, [3], and infinitely many minimal elements, otherwise, [2].

Theorem 1 is based on the following criterion that extends the criterion for minimal numbering from [1].

THEOREM 2. *Let α be a numbering of an arbitrary set $\mathcal{S}$. Then there is no minimal numbering of $\mathcal{S}$ that is reducible to α if and only if, for every c.e. set W, if $\alpha(W) = \mathcal{S}$ then there exists a c.e. set V such that $\alpha(V) = \mathcal{S}$ and, for every positive equivalence ε, either $\varepsilon \restriction W \nsubseteq \theta_\alpha$ or $W \nsubseteq [V]_\varepsilon$.*

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[2] S. A. BADAEV AND S. S. GONCHAROV, *Rogers semilattices of families of arithmetic sets*. ***Algebra and Logic***, vol. 40 (2001), no. 5, pp. 283–291.

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► GRZEGORZ JAGIELLA, *Definable topological dynamics and real Lie groups*.
Instytut Matematyczny, Uniwersytet Wrocławski, pl. Grunwaldzki 2/4, 50-247, Wrocław, Poland.
E-mail: grzegorz.jagiella@math.uni.wroc.pl.

Methods of topological dynamics have been introduced to model theory by Newelski in [3] and since then saw further development in that field by other authors. Given a model M with all types over M definable and a definable group G, we consider the category of definable flows. This category has a universal object $S_G(M)$, the space of types in G over M. It is shown that the Ellis semigroup of this flow is isomorphic to $S_G(M)$ itself. It can be considered as a model-theoretic equivalent to βG, the large compactification of G.

In the talk I will describe the results from [2] that give a description of definable topological dynamics of a large class of groups interpretable in an o-minimal expansion of the field of reals along with their universal covers interpreted in a certain two-sorted structure. The results provide a wide range of counterexamples to a question by Newelski whether the Ellis group of the universal definable G-flow is isomorphic to G/G^{00} and generalize methods from [1] that provided a particular counterexample.

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[2] G. JAGIELLA, *Definable topological dynamics and real Lie groups*, preprint.

[3] L. NEWELSKI, *Topological dynamics of definable group actions*. ***The Journal of Symbolic Logic***, vol. 74 (2009), no. 1, pp. 50–72.

► ANTONIS KAKAS, FRANCESCA TONI, AND PAOLO MANCARELLA, *Argumentation Logic*.
Department of Computer Science, University of Cyprus, 1 University Avenue 2109 Aglantzia, Cyprus.
E-mail: antonis@ucy.ac.cy.
Department of Computing, Imperial College, London, UK.
E-mail: f.toni@imperial.ac.uk.
Department of Computer Science, University of Pisa, Pisa, Italy.
E-mail: paolo@di.unipi.it.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

Born out of the need to formalize common sense in Artificial Intelligence (AI), Argumentation Logic (AL) brings together the syllogistic roots of logic with recent argumentation theory [1] from AI to propose a new logic based on argumentation.

Argumentation Logic is purely proof theoretic defined via a criterium of *acceptability of arguments* [3]. Arguments in AL are sets of propositional formulae with the acceptability of an argument ensuring that the argument can defend against any other argument that is inconsistent with it, under a given propositional theory. AL can be linked to Natural Deduction allowing us to reformulate Propositional Logic (PL) in terms of argumentation and to show that, under certain conditions, AL and PL are equivalent. AL separates proofs into direct and indirect ones, the latter being through the use of a restricted form of Reductio ad Absurdum (RAA) where the (direct) derivation of the inconsistency must depend on the hypothesis posed when we apply the RAA rule [4].

As such AL is able to isolate inconsistencies in the given theory and to behave agnostically to them. This gives AL as a conservative paraconsistent [5] extension of PL that does not trivialize in the presence of inconsistency. The logic then captures in a single framework defeasible reasoning and its synthesis with the strict form of reasoning in classical logic. The interpretation of implication in AL is different from that of material implication, closer to that of default rules but where proof by contradiction can be applied with them. AL has recently formed the basis to formalize psychological theories of story comprehension [2].

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► ISKANDER KALIMULLIN AND DAMIR ZAINETDINOV, *On limitwise monotonic reducibility of* Σ^0_2*-sets*.

N. I. Lobachevsky Institute of Mathematics and Mechanics, Kazan Federal University, 18 Kremlyovskaya St., Kazan, Russian Federation.

E-mail: `ikalimul@gmail.com`.

E-mail: `damir.zh@mail.ru`.

One of the directions of research in modern computability theory focus on studying properties of limitwise monotonic functions and limitwise monotonic sets.

I. Kalimullin and V. Puzarenko [2] introduced the concept of reducibility on families of subsets of natural numbers, which is consistent with Σ-definability on admissible sets. Let $\mathcal{F}_A$ denote the families of initial segments $\{\{x \mid x < n\} \mid n \in A\}$. Accordingly to [2], we define the notion of *limitwise monotonic reducibility* of sets as a Σ-reducibility of the corresponding initial segments, namely $A \leqslant_{lm} B \iff \mathcal{F}_A \leqslant_\Sigma \mathcal{F}_B$.

Let $A \equiv_{lm} B$ if $A \leqslant_{lm} B$ and $B \leqslant_{lm} A$. The *limitwise monotonic degree* (also called lm-degree) of A is $\deg(A) = \{B \colon B \equiv_{lm} A\}$. Let $\mathbf{S}_{lm}$ denote the class of all lm-degrees of Σ^0_2 sets. The degrees $\mathbf{S}_{lm}$ form a partially ordered set under the relation $\deg(A) \leqslant \deg(B)$ iff $A \leqslant_{lm} B$.

We prove the following theorems.

Theorem 1. *There exist infinite* Σ^0_2*-sets* A *and* B *such that* $A \nleqslant_{lm} B$ *and* $B \nleqslant_{lm} A$.

Theorem 2. *Every countable partial order can be embedded into* $\mathbf{S}_{lm}$.

Theorem 3 (jointly with M. Faizrahmanov). *There is no maximal element in* $\mathbf{S}_{lm}$.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

The research is supported by the grant of the President of the Russian Federation for state support of young Russian scientists—doctors MD-4838.2013.1.

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► AHMAD KARIMI AND SAEED SALEHI, *A universal diagonal schema by fixed-points and Yablo's paradox*.

Department of Mathematics, Tarbiat Modares University, Tehran, Iran.
E-mail: ahmad.m.karimi@gmail.com.
Department of Mathematics, University of Tabriz, Tabriz, Iran.
E-mail: salehipour@tabrizu.ac.ir.

In 1906, Russell [5] showed that all the known set-theoretic paradoxes (till then) had a common form. In 1969, Lawvere [3] used the language of category theory to achieve a deeper unification, embracing not only the set-theoretic paradoxes but incompleteness phenomena as well. To be precise, Lawvere gave a common form to Cantor's theorem about power sets, Russell's paradox, Tarski's theorem on the undefinability of truth, and Gödel's first incompleteness theorem. In 2003, Yanofsky [7] extended Lawvere's ideas using straightforward set-theoretic language and proposed a universal schema for diagonalization based on Cantor's theorem. In this universal schema for diagonalization, the existence of a certain (diagonalized-out and contradictory) object implies the existence of a fixed-point for a certain function. He showed how self-referential paradoxes, incompleteness, and fixed-point theorems all emerge from the single generalized form of Cantor's theorem. Yanofsky extended Lawvere's analysis to include the Liar paradox, the paradoxes of Grelling and Richard, Turing's halting problem, an oracle version of the P=?NP problem, time travel paradoxes, Parikh sentences, Löb's Paradox and Rice's theorem. In this talk, we fit more theorems in the universal schema of diagonalization, such as Euclid's theorem on the infinitude of the primes, and new proofs of Boolos [1] for Cantor's theorem on the nonequinumerosity of a set with its powerset. We also show the existence of Ackermann-like functions (which dominate a given set of functions such as primitive recursive functions) using the schema. Furthermore, we formalize a reading of Yablo's paradox [6], the most challenging paradox in the recent years, in the framework of Linear Temporal Logic (LTL [2]) and the diagonal schema, and show how Yablo's paradox involves circularity by presenting it in the framework of LTL. All in all, we turn Yablo's paradox into a genuine mathematico-logical theorem. This is the first time that Yablo's paradox becomes a (new) theorem in mathematics and logic. We also show that Priest's [4] inclosure schema can fit in our universal diagonal/fixed-point schema. The inclosure schema was used by Priest for arguing for the self-referentiality of Yablo's sequence of sentences, in which no sentence directly refers to itself but the whole sequence does so.

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▸ NURLAN KOGABAEV, *The isomorphism problem for computable projective planes*.
Sobolev Institute of Mathematics and Novosibirsk State University, Koptyug Prospect 4, Novosibirsk 630090, Russia.
E-mail: kogabaev@math.nsc.ru.

Estimating the complexity of the isomorphism problem for some class K of structures is one of the approaches to obtain classification theorems for computable structures in K. It is widely assumed that K has a computable classification if the isomorphism problem in K is hyperarithmetical.

For a class K of structures, closed under isomorphism, the *isomorphism problem* is the set

$$E(K) = \{\langle a, b\rangle \mid \mathcal{A}_a, \mathcal{A}_b \in K \text{ and } \mathcal{A}_a \cong \mathcal{A}_b\},$$

where $\mathcal{A}_a$ is the computable structure with computable index a.

If the set of all indices for computable members of K is hyperarithmetical, then $E(K)$ is Σ^1_1. Several classes are well-known to have maximally complicated isomorphism problems. $E(K)$ is Σ^1_1-complete under m-reducibility for each of the following classes: undirected graphs, linear orders, trees, Boolean algebras, distributive lattices, Abelian p-groups, nilpotent groups, semigroups, rings, fields, real closed fields, etc.

In the present paper we estimate the complexity of the isomorphism problem for familiar classes of projective planes and obtain the following results.

Theorem. $E(K)$ *is* Σ^1_1*-complete for the following classes* K:
(1) *pappian projective planes*;
(2) *desarguesian projective planes*;
(3) *arbitrary projective planes*.

This work was supported by RFBR (grants 14-01-00376-a and 13-01-91001-FWF-a).

▸ BEIBUT KULPESHOV, *Some remarks on* $\aleph_0$*-categorical weakly circularly minimal structures*.
Department of Information Systems and Mathematical Modelling, International Information Technology University, 34 A Manas str./8 A Zhandosov str., 050040, Almaty, Kazakhstan.
E-mail: b.kulpeshov@iitu.kz.

The notion of *circular minimality* has been introduced and originally studied by D. Macpherson and C. Steinhorn in [4]. Here we continue studying the notion of *weak circular minimality* being its generalisation.

A *circular* order relation is described by a ternary relation K satisfying the following conditions:
(co1) $\forall x\forall y\forall z(K(x, y, z) \to K(y, z, x))$;
(co2) $\forall x\forall y\forall z(K(x, y, z) \wedge K(y, x, z) \Leftrightarrow x = y \vee y = z \vee z = x)$;
(co3) $\forall x\forall y\forall z(K(x, y, z) \to \forall t[K(x, y, t) \vee K(t, y, z)])$;
(co4) $\forall x\forall y\forall z(K(x, y, z) \vee K(y, x, z))$.

A set A of a circularly ordered structure M is said to be *convex* if for any $a, b \in A$ the following holds: for any $c \in M$ with $K(a, c, b)$ we have $c \in A$ or for any $c \in M$ with $K(b, c, a)$ we have $c \in A$. A circularly ordered structure $M = \langle M, K, \dots\rangle$ is *weakly circularly minimal* if any definable (with parameters) subset of M is a finite union of convex sets [3]. Any weakly o-minimal structure is weakly circularly minimal, but the inverse is not true in general. Some of interesting examples of weakly circularly minimal structures that are not weakly o-minimal were studied in [3, 1, 2].

In [3]–[2] $\aleph_0$-categorical 1-transitive weakly circularly minimal structures have been studied, and was obtained their description up to binarity. Here we discuss some properties of $\aleph_0$-categorical weakly circularly minimal structures that are not 1-transitive. In particular, we study a behaviour of 2-formulas in such structures.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

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► RUTGER KUYPER, *Effective genericity and differentiable functions*.
Department of Mathematics, Radboud University Nijmegen, P.O. Box 9010, 6500 GL Nijmegen, the Netherlands.
E-mail: r.kuyper@math.ru.nl.

Recently, connections between differentiability and various notions of effective randomness have been studied. These results are typically of the form "$x \in [0,1]$ is random if and only if every function $f \in \mathcal{C}$ is differentiable at x," where $\mathcal{C}$ is some subclass of the computable functions; for example, Brattka, Miller and Nies [1] gave such characterisations for computable and Martin-Löf randomness.

In this talk we will present a complementary result for effective genericity. More precisely, our result says that $x \in [0,1]$ is 1-generic if and only if every differentiable computable function has continuous derivative at x. This result can be seen as an effectivisation of a result by Bruckner and Leonard [2].

This talk is based on joint work with Sebastiaan Terwijn [3].

[1] V. Brattka, J. S. Miller, and A. Nies, *Randomness and differentiability*, arXiv:1104.4465 [math.LO], 2011.

[2] A. M. Bruckner and J. L. Leonard, *Derivatives*. ***The American Mathematical Monthly***, vol. 73 (1966), no. 4, pp. 24–56.

[3] R. Kuyper and S. A. Terwijn, *Effective genericity and differentiability*, submitted.

► JUI-LIN LEE, *Explosiveness, model existence, and incompatible paraconsistencies*.
Center for General Education and Department of Computer Science & Information Engineering, National Formosa University, No. 64, Wunhua Rd., Huwei Township, Yunlin County 632, Taiwan.
E-mail: jlleelogician@gmail.com.

In this talk we present that the general concept of formal inconsistencies can be well-developed for any given semantics $\models$ (no matter it is truth functional or not). Note that the concept negation is not a necessary part in our treatment. In this theory of formal inconsistencies, there are two important concepts, model existence property (i.e., w.r.t. the given inconsistency, every consistent set has a model with respect to $\models$) and explosiveness property (i.e., w.r.t. the given inconsistency, every inconsistent set is also absolutely inconsistent). Now given a semantics $\models$, it will generate a set of inconsistencies, say, $Ins_{\models} = \{I_i, \dots\}$. If a $\models$-sound proof system L has both model existence property and explosiveness for some inconsistency $I \in Ins_{\models}$, then all inconsistencies in $Ins_{\models}$ are provably equivalent in L.

Then it is natural to ask, for the classical semantics, whether there are incompactible paraconsistencies in the following sense, i.e., are there two inconsistencies I_1, I_2 (generated from classical semantics) such that there are classically sound proof systems L_1, L_2 such that in L_1 it has I_1 model existence and I_2 explosiveness but not I_1 explosiveness and not I_2 model existence. And in L_2 it has I_2 model existence and I_1 explosiveness but not I_2 explosiveness and not I_1 model existence. We will prove that the answer is positive, which shows that there are incompatible paraconsistencies.

Keywords: model existence, explosiveness, paraconsistency.

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▸ LAURENŢIU LEUŞTEAN, *Effective results on the asymptotic behavior of nonexpansive iterations*.

Simion Stoilow Institute of Mathematics of the Romanian Academy, 21 Calea Griviţei, 010702, Bucharest, Romania.
E-mail: laurentiu.leustean@imar.ro.

This talk reports on an application of proof mining to the asymptotic behavior of Ishikawa iterations for nonexpansive mappings [4, 3]. *Proof mining* is a paradigm of research concerned with the extraction, using proof-theoretic methods, of finitary content from mathematical proofs. This research direction can be related to Terence Tao's proposal [6] of *hard analysis*, based on finitary arguments, instead of the infinitary ones from *soft analysis*.

We present uniform effective rates of asymptotic regularity for the Ishikawa iteration associated to nonexpansive self-mappings of convex subsets of uniformly convex Busemann geodesic space. We show that these results are obtained by a logical analysis of an asymptotic regularity proof due to Tan and Xu [5], consisting of two main steps: the first one with a classical proof, analyzed using the combination of monotone functional interpretation and negative translation, while the second one has a constructive proof, analyzed more directly using monotone modified realizability. As a consequence, our results are guaranteed by a combination of logical metatheorems for classical and semi-intuitionistic systems, proved by Gerhardy and Kohlenbach [1, 2] for different classes of spaces and adapted to uniformly convex Busemann spaces in [4].

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▸ ROBERT LUBARSKY AND NORMAN PERLMUTTER, *Elementary epimorphisms between models of set theory*.

Department of Mathematical Sciences, Florida Atlantic University, 777 Glades Rd., Boca Raton, FL 33431, USA.
E-mail: Lubarsky.Robert@comcast.net.
E-mail: NLPerlmutter@gmail.com.

Rothmaler [3] defined an elementary epimorphism $f\colon M \to N$ (between model-theoretic structures in some language) to be a homomorphism such that, for every formula ϕ in the language with parameters $n_1, \ldots, n_k$ from N true in N, there are f-pre-images $m_1, \ldots, m_k$ of the n_i's such that $\phi(m_1, \ldots, m_k)$ holds in M. Here we investigate elementary epimorphisms between models of set theory, as well as the restricted notion of a Γ-elementary epimorphism, by which ϕ is restricted to a set Γ. We show that the only Π_1-elementary epimorphisms between models of ZF are isomorphisms. That result seems to be optimal, in that any of the obvious weakenings of the hypotheses allow for nontrivial such epimorphisms. For instance, there are nontrivial Σ_1-elementary epimorphisms. Also, using a result of Caicedo [1], there are nontrivial (full) elementary epimorphisms between models of ZFC^-, which is ZFC without Power Set. Furthermore, we study the inverse system induced by the last example, and its inverse limit. Inverse limits do not always exist, and even when they do they might not be the entire thread class [2], but in this case it is.

[1] Andrés Eduardo Caicedo, *Real-valued measurable cardinals and well-orderings of the reals*, ***Set Theory: Centre de Recerca Matemàtica Barcelona, 2003–2004*** (Joan Bagaria and Steve Todorcevic, editors), Birkhäuser, Basel, 2006, pp. 83–120.

[2] Norman Lewis Perlmutter, ***Inverse limits of models of set theory and the large cardinal hierarchy near a high-jump cardinal***, PhD dissertation, CUNY Graduate Center, Department of Mathematics, May 2013, `http://boolesrings.org/perlmutter/files/2013/05/Dissertation.pdf`.

[3] Philipp Rothmaler, *Elementary epimorphisms*. ***The Journal of Symbolic Logic***, vol. 70 (2005), no. 2, pp. 473–488.

▶ ALBERTO MARCONE, *Reverse mathematics of WQOs and Noetherian spaces*.
Dipartimento di Matematica e Informatica, Università di Udine, 33100 Udine, Italy.
E-mail: `alberto.marcone@uniud.it`.
URL Address: `http://users.dimi.uniud.it/~alberto.marcone/`.

If $(Q, \leq_Q)$ is a quasi-order we can equip Q with several topologies. We are interested in the Alexandroff topology $A(Q)$ (the closed sets are exactly the downward closed subsets of Q) and the upper topology $u(Q)$ (the downward closures of finite subsets of Q are a basis for the closed sets). $A(Q)$ and $u(Q)$ are (except in trivial situations) not T_1, yet they reflect several features of the quasi-order. For example, $(Q, \leq_Q)$ is a well quasi-order (WQO: well-founded and with no infinite antichains) if and only if $A(Q)$ is Noetherian (all open sets are compact or, equivalently, there is no strictly descending chain of closed sets). Moreover, if $(Q, \leq_Q)$ is WQO then $u(Q)$ is Noetherian.

Given the quasi-order $(Q, \leq_Q)$, we consider two natural quasi-orders on the powerset $\mathcal{P}(Q)$:

$$A \leq^\flat B \iff \forall a \in A\, \exists b \in B\; a \leq_Q b;$$
$$A \leq^\sharp B \iff \forall b \in B\, \exists a \in A\; a \leq_Q b.$$

We write $\mathcal{P}^\flat(Q)$ and $\mathcal{P}^\sharp(Q)$ for the resulting quasi-orders, and $\mathcal{P}^\flat_f(Q)$ and $\mathcal{P}^\sharp_f(Q)$ for their restrictions to the collection of finite subsets of Q.

Goubault-Larrecq proved that if $(Q, \leq_Q)$ is WQO then $u(\mathcal{P}^\flat(Q))$ and $u(\mathcal{P}^\sharp_f(Q))$ are Noetherian, even though $\mathcal{P}^\flat(Q)$ and $\mathcal{P}^\sharp_f(Q)$ are not always WQOs.

We study these theorems and some of their consequences from the viewpoint of reverse mathematics, proving for example:

- over RCA_0, ACA_0 is equivalent to each of "if $(Q, \leq_Q)$ is WQO then $u(\mathcal{P}^\flat(Q))$ is Noetherian", and "if $(Q, \leq_Q)$ is WQO then $A(\mathcal{P}^\flat_f(Q))$ is Noetherian";
- ACA_0 proves "if $(Q, \leq_Q)$ is WQO then $u(\mathcal{P}^\sharp_f(Q))$ is Noetherian", yet WKL_0 does not.

This work in progress is joint with Emanuele Frittaion, Matthew Hendtlass, Paul Shafer, and Jeroen Van der Meeren.

▶ ALBA MASSOLO AND LUIS URTUBEY, *Modelling inference in fiction*.
Escuela de Filosofía, FFyH, Universidad Nacional de Córdoba, Haya de la Torre y Medina Allende, Ciudad Universitaria, Córdoba, Argentina / Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET).
E-mail: `albamassolo@gmail.com`.
Escuela de Filosofía, FFyH, Universidad Nacional de Córdoba, Haya de la Torre y Medina Allende, Ciudad Universitaria, Córdoba, Argentina.
E-mail: `luis.urtubey@gmail.com`.

As it is widely-known, fiction became a serious problem for several classical conceptions closed related to philosophy of logic (J. Woods, 2006). This was mainly due to some of the leading features of reasoning in fiction. Firstly, inference in fiction involves reasoning with incomplete information. Stories describe their characters, places, and events only in an incomplete way. Due to the fact that stories are composed by a finite set of sentences, a large amount of information about them remains unknown. Secondly, inference in fiction also involves reasoning with inconsistent information. Inconsistencies can emerge from two sources. On the one hand, information belonging to a fiction contradicts reality in many aspects. On the other hand, some stories are based on a contradiction or contain inconsistent information. This is the case of stories in which contradictions are an essential part of their plots.

In order to cope with the aforementioned features of reasoning in fiction, we propose a semantic approach of fiction based on an intuitionistic modal system. The semantic model is an adaptation of the multiple-expert semantics developed by Melvin Fitting in 1992. Firstly, we consider a propositional language to represent fictional information formally. That propositional language is interpreted in an intuitionistic modal semantics that involves two different perspectives and a partial valuation. On the one hand, these two perspectives make it possible to distinguish two sources of information involved in reasoning in fiction, i.e., fiction and reality. On the other hand, the partial valuation makes it possible to deal with incomplete information. A relation of logical consequence is defined in order to distinguish between valid and invalid inferences within the fictional context. Finally, we explore different proof-theoretical alternatives in order to characterize a deductive system for this semantic approach.

[1] Melvin Fitting, *Many-valued modal logic II*. ***Fundamenta Informaticae***, vol. 17 (1992), no. 4, pp. 55–73.

[2] John Woods, *Fictions and their Logic*, ***Philosophy of Logic***, (Dale Jacquette, editor), Elsevier, Amsterdam, 2006, pp. 1061–1126.

► MICHAEL MCINERNEY, *Integer-valued randomness and degrees.*,
School of Mathematics, Statistics and Operations Research, Victoria University of Wellington, P.O. Box 600, Wellington, New Zealand.
E-mail: michael.mcinerney@msor.vuw.ac.nz.

Analysing betting strategies where only integer values are allowed, perhaps for a given set F, gives an interesting variant on algorithmic randomness where category and measure intersect. We build on earlier work of Bienvenu, Stephan, and Teutsch, and study reals random in this sense, and their intricate relationship with the c.e. degrees.

This is joint work with George Barmpalias and Rod Downey.

[1] Laurent Bienvenu, Frank Stephan, and Jason Teutsch, *How powerful are integer-valued martingales?*. ***Theory of Computing Systems***, vol. 51 (2010), no. 3, pp. 330–351.

► NADAV MEIR, *On various strengthenings of the notion of indivisibility*.
Department of Mathematics, Ben Gurion University of the Negev, P.O.B. 653 Be'er Sheva 84105, Israel.
E-mail: mein@math.bgu.ac.il.
URL Address: http://www.math.bgu.ac.il/~mein.

A structure $\mathcal{M}$ in a first order language $\mathcal{L}$ is *indivisible* if for every colouring of its universe M in two colours, there is a monochromatic substructure $\mathcal{M}' \subseteq \mathcal{M}$ such that $\mathcal{M}' \cong \mathcal{M}$. Additionally, we say that $\mathcal{M}$ is *symmetrically indivisible* if $\mathcal{M}'$ can be chosen to be *symmetrically embedded* in $\mathcal{M}$ (That is, every automorphism of $\mathcal{M}'$ can be can be extended to an automorphism of $\mathcal{M}$), and that $\mathcal{M}$ is *elementarily indivisible* if $\mathcal{M}'$ can be chosen to be an elementary substructure.

The notion of indivisibility is a long-studied subject. We will present these strengthenings of the notion, examples and some basic properties. We will define a new "product" of structures which preserves these notions and use is to answer some questions presented in [1] regarding the properties and interaction between these notions.

[1] Assaf Hasson, Menachem Kojman, and Alf Onshuus, *On symmetric indivisibility of countable structures*, ***Model theoretic methods in finite combinatorics*** (Martin Grohe and Johann A. Makowsky, editors), American Mathematical Society, 2011, pp.417–452.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

► JOSÉ M. MÉNDEZ, GEMMA ROBLES, AND FRANCISCO SALTO, *Blocking the routes to triviality with depth relevance*.
Universidad de Salamanca. Edificio FES, Campus Unamuno, 37007, Salamanca, Spain.
E-mail: sefus@usal.es.
URL Address: http://sites.google.com/site/sefusmendez.
Dpto. de Psicología, Sociología y Filosofía, Universidad de León, Campus Vegazana, s/n, 24071, León, Spain.
E-mail: gemmarobles@gmail.com.
URL Address: http://grobv.unileon.es.
Dpto. de Psicología, Sociología y Filosofía, Universidad de León, Campus Vegazana, s/n, 24071, León, Spain.
E-mail: francisco.salto@unileon.es.

The depth relevance condition (drc) is a strengthening of the variable-sharing property. A logic S has the drc if A and B share at least a propositional variable at the same depth in all theorems of the form $A \to B$ (cf. [1]). Logics with the drc have been used for defining nontrivial strong naïve set theories. In [3], "the class of implication formulas known to trivialize NC" is recorded. (NC abbreviates "naïve comprehension"; cf. [3], p. 435.) The aim of this paper is to show how to invalidate any member in this class by using "weak relevant model structures" (cf. [2]). Weak relevant model structures only verify logics with the drc.

Acknowledgements. Work supported by research project FFI2011-28494, financed by the Spanish Ministry of Economy and Competitiveness. G. Robles is supported by Program Ramón y Cajal of the Spanish Ministry of Economy and Competitiveness.

[1] R. T. Brady, ***Universal Logic***, CSLI, Stanford, CA, 2006.

[2] G. Robles and J. M. Méndez, *Generalizing the depth relevance condition. Deep relevant logics not included in R-Mingle*. ***Notre Dame Journal of Formal Logic***, vol. 55 (2014), pp. 107–127.

[3] S. Rogerson and G. Restall, *Routes to triviality*. ***Journal of Philosophical Logic***, vol. 33 (2006), pp. 421–436.

► OMER MERMELSTEIN, *Reducts of simple (non-collapsed) Fraïssé–Hrushovski constructions*.
Department of Mathematics, Ben-Gurion University of the Negev, Beer-Sheva, Israel.
E-mail: omermerm@math.bgu.ac.il.

Fraïssé–Hrushovski constructions were first introduced by Hrushovski as a method for constructing strongly minimal sets that do not fit within Zilber's trichotomy conjecture. The construction can be seen as a two-step process where first a rank ω structure is constructed from a countable amalgamation class, using a variation of a Fraïssé limit construction, and then the structure is "collapsed" to a strongly minimal substructure.

In this talk we acquaint ourselves with the rank ω, noncollapsed version of the construction and its associated combinatorial geometry, and provide a general method of showing that one simple Fraïssé–Hrushovski construction is a (proper) reduct of another Fraïssé–Hrushovski construction.

► RUSSELL MILLER, JENNIFER PARK, BJORN POONEN, HANS SCHOUTENS, AND ALEXANDRA SHLAPENTOKH, *Coding graphs into fields*.
Mathematics Department, Queens College and CUNY Graduate Center, 65-30 Kissena Blvd., Queens, NY 11367, USA.
E-mail: Russell.Miller@qc.cuny.edu.
URL Address: qcpages.qc.cuny.edu/~rmiller.
Mathematics Department, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139, USA.
Mathematics Department, New York City College of Technology, 300 Jay Street, Brooklyn, NY 11201, USA.
Mathematics Department, East Carolina University, East Fifth Street, Greenville, NC 27858, USA.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

It is well established that the class of countable symmetric irreflexive graphs is complete in computable model theory: every countable structure in a finite language can be coded into a graph in such a way that the graph has the same spectrum, the same computable dimension, and the same categoricity spectrum as the original structure, and shares most other known computable-model-theoretic properties of the original structure as well. In 2002, Hirschfeldt, Khoussainov, Shore, and Slinko collected related results and proved more, showing that many other classes of countable structures are complete in the same sense. On the other hand, classes such as linear orders, Boolean algebras, trees, and abelian groups are all known not to be complete in this way. We address the most obvious class for which this question was still open, by giving a coding of graphs into countable fields in such a way as to preserve all of these properties.

▸ RYSZARD MIREK, *Natural deduction in renaissance geometry*.
Institute of Logic, Pedagogical University of Krakow, ul. Podchorazych 2, 30 084 Krakow, Poland.
E-mail: mirek.r@poczta.fm.

Moritz Cantor was so impressed by the achievements of Piero della Francesca in mathematics and geometry that devoted him in his *Vorlesungen uber Geschichte der Mathematik* far more attention than to any other contemporary algebraicist. In Francesca's treatise *De prospectiva pingendi* we find the advanced geometrical exercises presented in the form of propositions. For instance, in Book 1, Proposition 8, he shows that the perspective images of orthogonals converge to a point. Proposition 12 shows how to draw in perspective a surface of undefined shape, which is located in profile as a straight line. The task is to find the image of a line perpendicular to the picture plane. But the most interesting is Proposition 13 that shows how to "degrade" a square and, more precisely the sides of the square. It is obvious that most of these propositions are used in the paintings of Francesca.

The purpose of the study is to describe these results in the form of logical system *EF*. Generally, the logical language is six sorted, with sorts for points, lines, circles, segments, angles, and areas. As proofs it is possible to employ the method of natural deduction. The aim is to demonstrate that such a method is the most useful for the presentation of the geometric proofs of Francesca, taking into account also the importance of *diagrams* within them.

▸ ARMEN MNATSAKANYAN, *The relation between the graphs structures and proof complexity of corresponding Tseitin graph tautologies*.
Department of Informatics and Applied Mathematics, Yerevan State University, Armenia.
E-mail: arm.mnats@gmail.com.

There are many well known examples of tautologies, which require exponential proof complexities in weak systems. Some of them are graph-based formulas introduced by Tseitin in [1]. As Tseitin graph tautologies, constructed on the base of different graphs, have different proof complexities, it is interesting to investigate the relation between the structure of graphs and proof complexities of corresponding Tseitin graph tautologies. In [2] A. Urquhart constructed the sequence of graphs such that the formulas based on them are hard examples for Resolution. We describe two sufficient properties of graphs G_n on n vertices such that the formulas based on them have exponential Resolution proof steps. The network style graphs of Tseitin's formulas and graphs of Urquhart are examples of graphs with mentioned properties. If at least one of these properties is not valid for any graph, then the corresponding formula has polynomial bounded resolution refutation.

Acknowledgment. This work is supported by Grant 13-1B004 of SSC of Government of RA.

[1] G. S. Tseitin, *On the complexity of derivation in propositional calculus*. ***Studies in constructive mathematics and mathematical logic***, vol. 2 (1970), pp. 115–125.

[2] A. Urquhart, *Hard examples for resolution*. ***Journal of the Association for Computing Machinery***, vol. 34 (1987), pp. 209–219.

▶ ANDREY MOROZOV, AIZHAN SATEKBAYEVA, AND JAMALBEK TUSSUPOV, *On the existential interpretability of structures*.
Sobolev Institute of Mathematics SB RAS, Koptyug Ave 4, Novosibirsk, Russia.
E-mail: `morozov@math.nsc.ru`.
Gumilyov Eurasian National University, Pushkin str. 11, Astana, Kazakhstan.
E-mail: `satekbayeva@gmail.com`.
E-mail: `tussupov@mail.ru`.

We study the $\exists$-interpretability of constructive structures of finite predicate signatures. This definition is motivated by a kind of effective interpretability of abstract databases and leads to a good natural translation of $\exists$-queries.

The following definition is a restricted variant of the standard well-known definition of interpretability of structures:

Definition. Let $\mathfrak{A}_0$ and $\mathfrak{A}_1$ be two structures of finite predicate signatures and let $\langle P_1, \dots, P_k\rangle$ be the signature of $\mathfrak{A}_0$. We say that $\mathfrak{A}_0$ has a $\exists$-interpretation in $\mathfrak{A}_1$ if there exist

- $n \in \omega$ and a finite tuple of parameters $\bar{p} \in \mathfrak{A}_1$,
- $\exists$-formula $U(\bar{x}, \bar{y})$, $|\bar{x}| = n$,
- $\exists$-formulas $E^+(\bar{x}_0, \bar{x}_1, \bar{y})$ and $E^-(\bar{x}_0, \bar{x}_1, \bar{y})$ such that $|\bar{x}_0| = |\bar{x}_1| = n$,
- $\exists$-formulas $P^+(\bar{x}_1, \dots, \bar{x}_m, \bar{y})$ and $P^-(\bar{x}_1, \dots, \bar{x}_m, \bar{y})$, for each predicate symbol P of the signature of $\mathfrak{A}_0$, where m is the arity of P with the property $|\bar{x}_1| = \dots = |\bar{x}_m| = n$,

such that

1. The set $(U^{\mathfrak{A}_1}(\bar{x}))^2$ is a disjunct union of the sets $\{\langle \bar{x}_0, \bar{x}_1\rangle \mid \mathfrak{A}_1 \models E^{\varepsilon}(\bar{x}_0, \bar{x}_1, \bar{p})\}$, $\varepsilon \in \{+, -\}$.
2. For any m-ary predicate symbol P of the signature of $\mathfrak{A}_0$, the set $(U^{\mathfrak{A}_1}(\bar{x}))^m$ is a disjunct union of the sets $\{\langle \bar{x}_0, \dots, \bar{x}_m\rangle \mid \mathfrak{A}_1 \models P^{\varepsilon}(\bar{x}_0, \dots, \bar{x}_m, \bar{p})\}$, $\varepsilon \in \{+, -\}$.
3. Let $\widehat{P}_i = \{\langle \bar{x}_1, \dots, \bar{x}_m\rangle \mid \mathfrak{A}_1 \models P^+(x_1, \dots, \bar{x}_m, \bar{p})\}$, ??? $i = 1, \dots, k$. Then the relation $E = \{\langle \bar{x}_0, \bar{x}_1\rangle \mid \mathfrak{A}_1 \models E^+(\bar{x}_0, \bar{x}_1, \bar{p})\}$ is a congruence on $\mathfrak{B} = \langle U^{\mathfrak{A}_1}(\bar{x}), \widehat{P}_1, \dots, \widehat{P}_k\rangle$ and the quotient algebra $\mathfrak{B}/E$ is isomorphic to $\mathfrak{A}_0$.

Theorem.

1. *The $\exists$-interpretability generates an upper semilattice $\mathcal{L}_{\exists}$ in which computable structures form a principal ideal $\mathcal{L}^0_{\exists}$; in particular, there exists a universal computable structure, i.e., a computable structure that $\exists$-interprets any computable structure.*
2. *Any finite partial order is embeddable into $\mathcal{L}^0_{\exists}$.*

▶ WILMARI MORTON AND CLINT VAN ALTEN, *Canonical extensions and prime filter completions of poset expansions*.
Department of Mathematics, University of Johannesburg, PO Box 524, Auckland Park 2006, South Africa.
E-mail: `wmorton@uj.ac.za`.
School of Computer Science, University of the Witwatersrand, Johannesburg, Private Bag 3, Wits 2050, South Africa.
E-mail: `clint.vanalten@wits.ac.za`.

The algebraic models of substructural logics are residuated ordered algebras [2]. Embedding a residuated ordered algebra into a complete algebra of the same class has many applications in logic, e.g., the canonical extension is used to obtain relational semantics for nonclassical logics [1].

The underlying sets of the algebraic structures of interest are often partially ordered. The canonical extensions of posets have been studied in [1, 2]. Upon closer inspection it can be seen that the completions in [1] and [2] are generally different. Both use a construction, first appearing in [3], based on a Galois connection between sets of filters and ideals, however, the choice of filters differs.

We investigate the construction from [3] for various choices of filters and ideals, consider the extension of operations defined on the posets and focus on some specific properties of completions obtained via this construction. Next we present a construction for completions of posets that makes use of the prime filters of the posets. We show that the completion obtained via this second construction is isomorphic to the former for a particular choice of filters.

[1] J. M. DUNN, M. GEHRKE, AND A. PALMIGIANO, *Canonical extensions and relational completeness of some substructural logics*. ***The Journal of Symbolic Logic***, vol. 70 (2005), no. 3, pp. 713–740.

[2] N. GALATOS, P. JIPSEN, T. KOWALSKI, AND H. ONO, ***An algebraic glimpse at substructural logics***, Studies in Logic and the Foundations of Mathematics, vol. 151, Elsevier, Amsterdam, 2007.

[3] W. R. TUNNICLIFFE, *The completion of a partially ordered set with respect to a polarization*. ***Proceedings of the London Mathematical Society***, vol. 28 (1974), no. 3, pp. 13–27.

▸ JOACHIM MUELLER-THEYS, *Metalogical extensions—Part II: First-order consequences and Gödel*.
Kurpfalzstr. 53, 69226 Nußloch bei Heidelberg, Germany.
E-mail: `Mueller-Theys@gmx.de`.

The aim is conservative extension of $\Phi\, seq\, \phi$ ($seq \in \{\models, \vdash\}$) to *metalogical consequence* $\Phi\, seq^{\Box}\, a$ such that, specifically: $\Phi\, seq^{\Box}\, \Box\phi$ iff $\Phi\, seq\, \phi$, non $\Phi\, seq\, \phi$ implies $\Phi\, seq^{\Box}\, \neg\Box\phi$, and $\Phi\, seq^{\Box}\, \neg\Box\phi$ implies non $\Phi\, seq\, \phi$ if Φ is consistent.

We will define *metalogical satisfaction* and *semantic consequence* such that $\mathfrak{M}, V \mid\models \Phi\Box a$ iff $\Phi \mid\models a$, and we give the evident calculus *QNI*: a if a is a tautology, $\forall x\, \phi(x) \to \phi(t)$ if t free for x in ϕ, $x \equiv x$, $\phi(x) \wedge x \equiv y \to \phi(y)$, $\Box T$; $a, a \to \beta/\beta$, $\phi \to \psi/\phi \to \forall x\psi$ if $x \notin \mathrm{fv}\phi$, $a \leftrightarrow \beta/\forall xa \leftrightarrow \forall x\beta$, $a \leftrightarrow \beta/\Box a \leftrightarrow \Box\beta$, whence $\Phi \Vdash a$:iff $\Phi \cup \{\neg\Box\phi \colon \Phi \nvdash \phi\} \vdash_{\mathrm{QNI}} a$.

Successive reduction $\mathrm{r}^{\Phi}a$ will be our method to proceed. Thereby we will establish that *there is only one* $seq^{\Box}$. $\Phi\mid\models a$ iff $\Phi \models a$ will follow. $\Phi\, seq^{\Box}\, a$ implies $\Phi\, seq^{\Box}\, \Box a$, non $\Phi\, seq^{\Box}\, a$ implies $\Phi\, seq^{\Box}\, \neg\Box a$. $\Phi\, seq^{\Box}\, \Box a \to a$, $\neg\Box a \to \Box\neg\Box a$, $\Box a \wedge \Box(a \to \beta) \to \Box\beta$. $seq^{\Box}$ does not produce Gödel formulae: naturally, a displays itself, and for every consistent Φ and for all a, non $\Phi\, seq^{\Box}\, a \leftrightarrow \neg\Box a$. In addition, e.g., $\Phi\, seq^{\Box}\, \neg\Box\bot$, and non $\Phi\, seq^{\Box}\, \neg\Box\neg\Box\bot$ (if Φ consistent).

Immanent attempts cipher ϕ by $\langle\phi\rangle$ (with respect to some Gödelisation) and try to reflect provability or truth by means of formulae $\iota = \iota(x)$. $seq^{\Box}$, uniquely achieving complete representation (transcendently, so to speak), yields the *soundness criterion*: $\Phi\, seq\, I_\iota(a)$ must imply $\Phi\, seq^{\Box}\, a$, whereby the translation $I_\iota \colon \mathrm{L}^{\Box} \to \mathrm{L}$ is inductively defined with $I_\iota(\Box a) := \iota(\langle I_\iota(a)\rangle)$. However, if Φ is *sufficiently strong and consistent, then* Φ *is not soundly representable immanently*. Proof: By assumption, $\Phi\, seq\, \sigma_0 \leftrightarrow \neg\iota(\langle\sigma_0\rangle)$ for any ι. Let $a_0 := \sigma_0 \leftrightarrow \neg\Box\sigma_0$. Then $\Phi\, seq\, I_\iota(a_0)$, but non $\Phi\, seq^{\Box}\, a_0$.—Sound representation of metalogic within arithmetics is impossible. Among other things, the 2nd Incompleteness Theorem must be doubted.

Mathematization would have been unthinkable without Wilfried Buchholz.

▸ ELENA NOGINA, *On Explicit-Implicit Reflection Principles*.
BMCC, Department of Mathematics, City University of New York, 199 Chambers Street, New York, NY 10007, USA.
E-mail: `e.nogina@gmail.com`.

We study reflection principles of Peano Arithmetic PA based on both proof and provability predicates (cf. [1, 2]). Let P be a propositional letter and each of $Q_1, Q_2, \ldots, Q_m$ is either '$\Box$' standing for provability in PA ([2]), or 'u:' standing for '*u is a proof of … in PA*' ([1]), u is a fresh proof variable. Then the formula

$$Q_1 Q_2 \ldots Q_m P \to P$$

is called *generator*, and the set of all its arithmetical instances is the *reflection principle* corresponding to this generator. We will refer to reflection principles using their generators. It is immediate that all reflection principles without explicit proofs ($Q_i = \Box$ for all i) are equivalent to the local reflection principle $\Box P \to P$. All $\Box$-free reflection principles are provable in PA and hence equivalent to $u\colon P \to P$. Mixing explicit proofs and provability yields infinitely many new reflection principles.

THEOREM 1. *Any reflection principle in* PA *is equivalent to either* $\Box P \to P$ *or* $\Box^k u\colon P \to P$ *for some* $k \geq 0$.

THEOREM 2. *Reflection principles constitute a non-collapsing hierarchy with respect to their deductive strength*

$$[u\colon P \to P] < [\Box u\colon P \to P] < [\Box^2 u\colon P \to P] < \cdots < [\Box P \to P].$$

The proof essentially relies on the Gödel–Löb–Artëmov logic GLA introduced in [3].

[1] S. ARTEMOV, *Explicit provability and constructive semantics*, this BULLETIN, vol. 7 (2001), no. 1, pp. 1–36.

[2] G. BOOLOS, ***The logic of provability***, Cambridge University Press, Cambridge, 1993.

[3] E. NOGINA, *On logic of proofs and provability*, this BULLETIN, vol. 12 (2006), no. 2, pp. 356.

► CYRUS F. NOURANI, *More on completion with Horn filters*.
180 Stuart, 19072, SFU Burnaby, 94105, Canada.
E-mail: acdmkrd@gmail.com.

Let LP, be the positive fragment obtained from the Kiesler fragment. On a subsequent paper to ASL-SLK, the author wrote that CH is not necessary to prove the proposition that every formula on the presentation P is completable with a companion closure T*. Without CH we can prove that for Horn presentations. Let us abbreviate Rasiowa–Sikorski Lemma as RSL and positive fragment consistency as PFC, respective. Now we can state the following proposition on: Define the category LP, to be the category with objects positive fragments and arrows the subfoumual preorder on formulas.

THEOREM. *PFC+RSL implies that every positive Horn presentation is completable on a Horn PFC theory.*

[1] NOURANI *Positive realizability on Horn filters*, ***Logic Colloquium 2008***, www.lc08.iam.unibe.ch.

► SERGEY OSPICHEV, *Computable numberings in Ershov hierarchy*.
Sobolev Institute of Mathematics and Novosibirsk State University, Novosibirsk, Russia.
E-mail: ospichev@gmail.com.

Study the cardinality and the structure of Rogers semilattices of families of sets in different hierarchies is one of the main questions in numbering theory [2]. Here we concentrate our interest on Rogers semilattices in Ershov hierarchy [1]. The talk will cover some recent results from this field.

In work are proven

THEOREM 1. *For any nonzero ordinal notation* a *there is* $\mathcal{S}$, *infinite family of* Σ^{-1}_a*-sets, with only one minimal numbering.*

THEOREM 2. *For any nonzero ordinal notation* a *there is* $\mathcal{S}$, *infinite family of* Σ^{-1}_a*-sets, without minimal and principal numberings.*

Supported by the Grants Council (under RF President) for State Aid of Leading Scientific Schools (grant NSh-860.2014.1).

[1] M. M. ARSLANOV, ***Ershov hierarchy***, Kazan State University, Kazan, 2007.

[2] S. S. GONCHAROV AND S. BADAEV, *Theory of numberings, open problems*, ***Contemporary Mathematics***, vol. 257, pp. 23–38.

► FEDOR PAKHOMOV, *Ordinal notations and fundamental sequences in Caucal hierarchy*.
Steklov Mathematical Institute, Gubkina str. 8, 119991 Moscow, Russian Federation.
E-mail: pakhfn@gmail.com.

The Caucal hierarchy of infinite graphs with colored edges is a wide class of graphs with decidable monadic theories [1]. Graphs from this hierarchy can be considered as structures with finite number of binary relations. It is known that the exact upper bound for order types of the well-orderings that lie in this class is ε_0 [2]. Actually, any well-ordering from

Caucal hierarchy can be used as a constructive ordinal notation system. We investigate systems of fundamental sequences for that well-orderings and the corresponding fast-growing hierarchies of computable functions.

For a well-ordering $(A, <_A)$ we can determine a system of fundamental sequences $\lambda[n]$ by a relation $Cs(x, y)$ such that

$$Cs(\alpha, \beta) \iff \alpha \text{ is a limit point of } <_A \text{ and } \beta = \alpha[n], \text{ for some } n.$$

Our principal result is that for a well-ordering with a pair of Schmidt-coherent fundamental sequences $(A, <_A, Cs_1, Cs_2)$ from Caucal hierarchy the corresponding fast-growing hierarchies $f^1_\alpha(x)$ and $f^2_\alpha(x)$ are equivalent in the following sense: for all $\alpha <_A \beta$ we have $f^1_\beta(n) > f^2_\alpha(n)$ and $f^1_\beta(n) > f^2_\alpha(n)$, for all large enough n (Schmidt-coherence is a classical condition that implies that functions from fast-growing hierarchy are strictly increasing [3]). We show that any two well-orderings with Schmidt-coherent systems of fundamental sequences from Caucal hierarchy of the same order type $< \omega^\omega$ give rise to the equivalent fast-growing hierarchies. We also prove that it is possible to extend a graph with a well-ordering from Caucal hierarchy by a Schmidt-coheren system of fundamental sequences for the well-ordering in such a way that the resulting graph will lie in Caucal hierarchy.

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[2] Braud Laurent and Arnaud Carayol, ***Linear orders in the pushdown hierarchy***, Lecture Notes in Computer Science, (Samson Abramsky, et al., editors), vol. 6199, Springer, Berlin Heidelberg, 2010, pp. 88–99.

[3] Diana Schmidt, *Built-up systems of fundamental sequences and hierarchies of number-theoretic functions*. ***Archive for Mathematical Logic***, vol. 18 (1976), pp. 47–53.

► PAOLO PISTONE, *Type equations and second order logic*.
Department of Philosophy, Universitá Roma Tre, Via Ostiense 234, 00144, Rome, Italy/ I2M, Aix-Marseille Université, Campus de Luminy, Case 907 13288 Marseille Cedex 9, France.
E-mail: paolo.pistone@uniroma3.it.

The aim of this talk is to propose a constructive understanding of second order logic: it is argued that a better grasp of the functional content of the comprehension rule comes from the consideration of inference rules independently of logical correctness; the situation is analogous to that of computation, whose proper functional description imposes to consider nonterminating (i.e., "wrong") algorithms.

The Curry–Howard correspondence allows indeed a shift from the question of provability (within a formal system) to that of typability for pure lambda terms, representing for instance recursive functions. By relying on well-known results on type inference, an equational description, independent of type systems, of the predicates required to build proofs of totality is presented: one no more focuses on what one can prove by means of a certain package of rules, but rather on what the rules needed to prove a certain formula must be like, at the level of their functional description.

This might look a bit weird at first glance: by applying this technique it is possible, in principle, to construct second order proofs of totality for all *partial* recursive functions! The assumption that every system of equations for a predicate *defines* a predicate is indeed equivalent to a naïve comprehension axiom.

The focus on typability conditions exposes a different point of view on the phenomenon of incompleteness: the lack of the relevant "diagonal" or "limit " proof is indeed explained by the lack of the relevant "diagonal" or "limit" predicates. On the other hand, on the basis of a characterization of the solvability of type equations by means of recursive techniques, it is conjectured that such a "naïve" approach to second order proofs is "complete" in the following sense: all total recursive functions are provably total in some consistent subsystem of the whole (violently inconsistent) system of equational types.

▶ DENIS PONOMARYOV, *The algorithmic complexity of decomposability in fragments of first-order logic*.
Institute of Artificial Intelligence, University of Ulm, 27 James-Franck-Ring 89069, Germany.
A. P. Ershov Inst. of Informatics Systems, 6 Lavrentyev av., 630090 Novosibirsk, Russia.
E-mail: ponom@iis.nsk.su.

Definition 1. Let $\mathcal{T}$ be a theory and $\Delta \subseteq \texttt{sig}(\mathcal{T})$ be a subsignature. The theory $\mathcal{T}$ is called Δ-decomposable if there exist theories $\mathcal{S}_1$ and $\mathcal{S}_2$ such that:
(1) $\texttt{sig}(\mathcal{S}_1) \cap \texttt{sig}(\mathcal{S}_2) = \Delta$ and $\texttt{sig}(\mathcal{S}_1) \neq \Delta \neq \texttt{sig}(\mathcal{S}_2)$;
(2) $\texttt{sig}(\mathcal{S}_1) \cup \texttt{sig}(\mathcal{S}_2) = \texttt{sig}(\mathcal{T})$ and $\mathcal{T}$ is equivalent to $\mathcal{S}_1 \cup \mathcal{S}_2$.
The theories $\mathcal{S}_1$ and $\mathcal{S}_2$ are called Δ-decomposition components of $\mathcal{T}$.

We consider the algorithmic complexity of the following problems.

Let Σ and $\Delta \subseteq \Sigma$ be finite signatures. The Δ-*decomposability problem* for signature Σ is the set of indices of pairs $\langle \mathcal{T}, \Delta \rangle$, where $\mathcal{T}$ is a finite Δ-decomposable theory in signature Σ. In other words, this is the problem to decide whether a given finite set of sentences in signature Σ is Δ-decomposable. We also consider the problem of deciding whether a finite theory $\mathcal{T}$ in a finite signature Σ given by a partition $\{\sigma_1, \sigma_2, \Delta\}$ is Δ-decomposable into some components in signatures $\sigma_1 \cup \Delta$ and $\sigma_2 \cup \Delta$, respectively. We refer to this as the problem to decide whether a given theory $\mathcal{T}$ is Δ-*decomposable with a partition* $\{\sigma_1, \sigma_2\}$.

The algorithmic complexity of the Δ-decomposability problem has been studied in various calculi, ranging from expressive fragments of first-order logic [3] to classical propositional [1] and description logics [2]. The results suggested that the complexity of decomposability coincides with the complexity of entailment in the underlying logic. Although this observation was not too surprising (since, the definition of decomposability contains the logical equivalence), a general method for proving this claim was missing. We describe a method for proving that the complexity of deciding decomposability coincides with the complexity of entailment in fragments of first-order logic. We illustrate this method by showing the complexity of decomposability in signature fragments of first-order logic, i.e., those which are obtained by putting restrictions on signature.

We call a finite signature σ *complex* if it contains at least one binary predicate, or a function of arity $\geqslant 2$, or at least two unary functions.

Theorem 2. (1) *For any complex signature σ, there exists a finite extension $\Sigma \supseteq \sigma$ such that the $\varnothing$-decomposability problem for Σ is undecidable.* (2) *For a finite signature Σ consisting of unary predicates and constants it is coNEXPTIME-complete to decide whether a finite theory in signature Σ is Δ-decomposable with a given partition $\{\sigma_1, \sigma_2\}$.*

An extended version of the abstract containing proofs is available at:

http://persons.iis.nsk.su/en/person/ponom/papers

[1] P. Emelyanov and D. Ponomaryov, *The complexity of AND-decomposition of boolean formulas*, manuscript submitted, 2014, available at http://persons.iis.nsk.su/en/person/ponom/papers.

[2] B. Konev, C. Lutz, D. Ponomaryov, and F. Wolter, *Decomposing description logic ontologies*, ***Proceedings of the twelfth international conference on the principles of knowledge representation and reasoning*** (***KR 2010***) (Toronto, Canada), http://www.csc.liv.ac.uk/~frank/publ/fulldecomp.pdf.

[3] A. Morozov and D. Ponomaryov, *On decidability of the decomposability problem for finite theories*. ***Siberian Mathematical Journal***, vol. 51 (2010), no. 4, pp. 667–674, http://link.springer.com/article/10.1007/s11202-010-0068-6.

▶ GEMMA ROBLES, *A Routley–Meyer semantics for Gödel 3-valued logic G3*.
Dpto. de Psicología, Sociología y Filosofía, Universidad de León, Campus Vegazana, s/n, 24071, León, Spain.
E-mail: gemmarobles@gmail.com.
URL Address: http://grobv.unileon.es.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

Gödel 3-valued logic G3 is the strongest of the Gödel many-valued logics introduced in [1]. Although the Routley–Meyer semantics (RM-semantics) was defined for interpreting relevant logics in the early seventies of the last century (cf. [4]), it was soon found out to be suitable for characterizing a wide family of logics regardless of their being relevant or not, due to its malleability. Still, a necessary condition for a logic S to be characterized by the RM-semantics is that Routley and Meyer's basic positive logic B_+ is included in S (cf. [4]). The aim of this paper is to provide an RM-semantics for G3 once this logic has been axiomatized as an extension of B_+ (cf. [2], [3]).

Acknowledgments. Work supported by research project FFI2011-28494, financed by the Spanish Ministry of Economy and Competitiveness. G. Robles is supported by Program Ramón y Cajal of the Spanish Ministry of Economy and Competitiveness.

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[2] G. Robles, *A Routely–Meyer semantics for Gödel 3-valued logic and its paraconsistent counterpart*. ***Logica Universalis***, vol. 7 (2013), pp. 507–532.

[3] ———, *A simple Henkin-style completeness proof for Gödel 3-valued logic G3*. ***Logic and Logical Philosophy***, DOI: 10.12775/LLP.2014.001, 2014.

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▸ SAEED SALEHI, *A characterization for diagonalized-out objects*.
Department of Mathematics, University of Tabriz, 51666–17766 Tabriz, Iran.
E-mail: `root@saeedsalehi.ir`.
E-mail: `salehipour@tabrizu.ac.ir`.
URL Address: `http://saeedsalehi.ir/`.

Cantor's Diagonal Argument came out of his third proof for the uncountability of the set of real numbers (see e.g., [2]). Unlike the first and second proofs, the diagonal argument can also show the nonequinumerosity of a set with its powerset. In modern terms the proof is as follows: for a function $F\colon A \to \mathcal{P}(A)$, where $\mathcal{P}(A) = \{B \mid B \subseteq A\}$ is the powerset of A, the anti-diagonal set $D_F = \{a \in A \mid a \notin F(a)\}$ is not in the range of F because if it were, say $D_F = F(\alpha)$, then $\alpha \in D_F \leftrightarrow \alpha \notin F(\alpha) \leftrightarrow \alpha \notin D_F$ contradiction. This argument shows up also in Russell's Paradox, the set of sets which do not contain themselves, $R = \{x \mid x \notin x\}$, and in Turing's non-recursively-enumerable set $\overline{K} = \{n \in \mathbb{N} \mid n \notin W_n\}$ where W_n is the domain of the n^{th} recursive function φ_n (i.e., $W_n = \{x \in \mathbb{N} \mid \exists y\colon \varphi_n(x) = y\}$) by which one can show the algorithmic unsolvability of the halting problem (of a given algorithm on a given input). There are, in fact, many other instances of the diagonal arguments in wide areas of mathematics from logic and set theory to computability theory and theory of computational complexity.

In this talk, we examine this argument in more detail and discuss some other proofs (e.g., [4, 5]) of Cantor's theorem (on the nonequinumerosity of a set with its powerset). By introducing a generalized diagonal argument, we show that all other proofs should fit in this generalized form, which is roughly as follows: for a function $g\colon A \to A$ the generalized anti-diagonal set $D_F^g = \{g(a) \mid g(a) \notin F(a)\}$ is not in the range of F because if it were, say $D_F^g = F(\alpha)$, then $g(\alpha) \in D_F^g \leftrightarrow g(\alpha) \notin F(\alpha) \leftrightarrow g(\alpha) \notin D_F^g$ contradiction. For the argument to go through, the function g should satisfy some conditions; and we will prove that every subset of A (say $B \subseteq A$) that is not in the range of F (for all $a \in A$, $B \neq F(a)$ holds) should somehow be in this generalized anti-diagonal form ($B \cap g[A] = D_F^g$) for some suitable function g which satisfies those conditions; cf. [1, 3]. We will argue that this provides a characterization for diagonal proofs and indeed characterizes the objects whose existence are proved by a kind of diagonal(izing out) argument.

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[2] John Franks, *Cantor's other proofs that $\mathbb{R}$ is uncountable*. ***Mathematics Magazine***, vol. 83 (2010), no. 4, pp. 283–289.

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[4] Natarajan Raja, *A negation-free proof of Cantor's theorem*. ***Notre Dame Journal of Formal Logic***, vol. 46 (2005), no. 2, pp. 231–233.

[5] ———, *Yet another proof of Cantor's theorem*, ***Dimensions of logical concepts*** (Jean-Yves Báziau and Alexandre Costa-Leite, editors), Coleção CLE, Volume 54, Campinas, Brazil, 2009, pp. 209–217.

▶ LUCA SAN MAURO, *Towards a theory of computably enumerable graphs*.
Scuola Normale Superiore, Piazza dei Cavalieri 7, Pisa, Italia.
E-mail: `luca.sanmauro@sns.it`.

In recent literature, the theory of computably enumerable equivalence relations (ceers) has been widely investigated (see, for instance, [1], [3]). One of the most fruitful approaches is to study them considering the degree structure generated by the following reducibility: Given two ceers R and S, we say that R is reducible to S $(R < S)$ if there is a computable function f s.t., for every x, y, $x \, R \, y \Leftrightarrow f(x) \, S \, f(y)$.

In this talk, we propose to make use of this reducibility within a more general context than that of ceers, namely in the study of (simply undirected) c.e. graphs. Our presentation is divided in two parts.

Firstly, we focus on computable graphs. While the theory of computable equivalence relations is quite trivial ([1]), in this context the situation is more intricate. We provide a partial characterization for the computable case.

Secondly, we move to universal graphs. Let U be defined as follows: $e \, U \, i \Leftrightarrow e \in W_i \vee i \in W_e$. We prove that, for any c.e. graph G, $G < U$.

More generally, recall that there is a unique random graph RG s.t. every countable graph G can be embedded as an induced subgraph of RG ([2]). This fact depends on a specific property $(*)$ of RG (see ([2]) for the definition of $(*)$). Hence, it is natural to ask for some analogue of $(*)$ in our context—specially after noticing that $(*)$ fails for U. We discuss several candidates for this role.

[1] U. Andrews, S. Lempp, J. S. Miller, K. M. Ng, L. San Mauro, and A. Sorbi, *Universal computably enumerable equivalence relations*. ***The Journal of Symbolic Logic***, to appear.

[2] P. Cameron, *The random graph*, ***The Mathematics of Paul Erdös, II***, 2nd edition (R. L. Graham, J. Nešetřil and S. Butler, editors), Springer, Berlin, 2013, pp. 353–378.

[3] S. Gao and P. Gerdes, *Computably enumerable equivalence relations*. ***Studia Logica***, vol. 67 (2001), no. 1, pp. 27–59.

▶ SAM SANDERS, *Reverse Mathematics, more explicitly*.
Department of Mathematics, Ghent University, Bldg. S22, Krijgslaan 281, B9000 Gent, Belgium.
Munich Center for Mathematical Philosophy, LMU Munich, Germany.
E-mail: `sasander@cage.ugent.be`.

The program *Reverse Mathematics* ([4]) can be viewed as a classification of theorems of ordinary, i.e., nonset theoretical, mathematics from the point of view of *computability*. Working in Kohlenbach's *higher-order Reverse Mathematics* ([3]), we study an alternative classification of theorems of ordinary mathematics, namely based on the central tenet of Feferman's *Explicit Mathematics* ([1, 2]) that *a proof of existence of an object is converted into a procedure to compute said object*. Nonstandard Analysis is used in an essential way.

Our preliminary classification gives rise to the *Explicit Mathematics theme* (EMT). Intuitively speaking, the EMT states a standard object with certain properties can be computed by a functional if and only if this object *merely exists classically* with the same *nonstandard* properties. Besides theorems of classical mathematics, we also consider intuitionistic objects, like the fan functional ([3, p. 293]).

Acknowledgment. This research is generously sponsored by the John Templeton Foundation and the Alexander Von Humboldt Foundation.

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[2] ———, *Constructive theories of functions and classes*, ***Logic Colloquium 1978, Bergen***, Studies in Logic and the Foundations of Mathematics, vol. 97, North-Holland, Amsterdam, 1979, pp. 159–224.

[3] U. Kohlenbach, *Higher-order Reverse Mathematics*, ***Reverse Mathematics 2001***, Lecture Notes in Logic, vol. 21, Association for Symbolic Logic, La Jolla, CA, 2005, pp. 281–295.

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► ANDREY SARIEV, *Definability of* $\mathbf{0}'$ *in the structure of the* ω*-enumeration degrees*.
Faculty of Mathematics and Computer Science, Sofia University, 5 James Bourchier Blvd., 1164 Sofia, Bulgaria.
E-mail: acsariev@gmail.com.

In this paper we find a first order formula which defines the first jump of the least element in the structure of ω-enumeration degrees.

[1] I. N. Soskov, *The* ω*-enumeration degrees*. ***Journal of Logic and Computation***, to appear.

[2] I. N. Soskov and H. Ganchev, *The jump operator on the* ω*-enumeration degrees*. ***Annals of Pure and Applied Logic***, to appear.

► NOAH SCHWEBER, *Computability in generic extensions*.
University of California, Berkeley, Berkeley, CA 94270, USA.
E-mail: schweber@math.berkeley.edu.

In this talk we will explore connections between computable structure theory and generic extensions of the set-theoretic universe, V. Recall the definition of *Muchnik reducibility* for countable structures: $\mathcal{A} \leq_w \mathcal{B}$ if every copy of $\mathcal{B}$ computes a copy of $\mathcal{A}$. We will begin by introducing the notion of *generic Muchnik reducibility*, $\leq_w^*$: we say $\mathcal{A} \leq_w^* \mathcal{B}$ for uncountable structures $\mathcal{A}, \mathcal{B}$ if $\mathcal{A} \leq_w \mathcal{B}$ in some (=every) generic extension $V[G]$ in which $\mathcal{A}$ and $\mathcal{B}$ are both countable. We will discuss the basic properties and give some examples of generic Muchnik nonreducibilities among natural uncountable structures.

We will then turn our attention to *generic presentability*. Roughly speaking, an object $\mathcal{X}$ is generically presentable if a "copy" of $\mathcal{X}$, up to the appropriate equivalence relation, exists in every generic extension of the universe by some fixed forcing notion. Solovay [3] showed that all generically presentable *sets* (up to equality) already exist in the ground model; we will investigate the situation for *countable structures* (up to isomorphism) and *infinitary formulas* (up to semantic equivalence). We will present two Solovay-type results (and some consequences): (1) any structure generically presentable by a forcing not making ω_2 countable has a copy in V, and (2) (under CH) any structure generically presentable by a forcing not collapsing ω_1 has a *countable* copy in V. Time permitting, we will discuss a contrasting result coming from work by Laskowski and Shelah [2].

This is joint work with Julia Knight and Antonio Montalban [1].

[1] Julia Knight, Antonio Montalbán, and Noah Schweber, *Computable structures in generic extensions*, in preparation.

[2] M. C. Laskowski and S. Shelah, *On the existence of atomic models*. ***The Journal of Symbolic Logic***, http://dx.doi.org/10.2307/2275137, vol. 58 (1993), no. 4, pp. 1189–1194.

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► PAUL SHAFER, *Every nonzero honest elementary degree has the cupping property*.
Department of Mathematics, Ghent University, Krijgslaan 281 S22, B-9000 Ghent, Belgium.
E-mail: paul.shafer@ugent.be.
URL Address: http://cage.ugent.be/~pshafer/.

If $a < b$ are elements of a lattice, then we say that *a cups to b* if there is a $c < b$ such that $a \cup c = b$. In [1], Kristiansen proves that if $\mathbf{a} <_{\mathrm{E}} \mathbf{b}$ in the lattice of honest elementary degrees and $\mathbf{a}$ is significantly above $\mathbf{0}$ (that is, there is a function elementary in $\mathbf{a}$ that majorizes every elementary function), then $\mathbf{a}$ cups to $\mathbf{b}$. We improve this result by relaxing the restriction that $\mathbf{a}$ is significantly above $\mathbf{0}$ to simply that $\mathbf{a}$ is nonzero: if $\mathbf{a}$ and $\mathbf{b}$ are honest elementary degrees with $\mathbf{0} <_{\mathrm{E}} \mathbf{a} <_{\mathrm{E}} \mathbf{b}$, then $\mathbf{a}$ cups to $\mathbf{b}$. This answers a question in [2].

[1] Lars Kristiansen, *Subrecursive degrees and fragments of Peano arithmetic*, ***Archive for Mathematical Logic***, vol. 40 (2001), no. 5, pp. 365–397.

[2] Lars Kristiansen, Jan-Christoph Schlage-Puchta, and Andreas Weiermann, *Streamlined subrecursive degree theory*. ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 6, pp. 698–716.

▶ ALEXANDRA SOSKOVA, *Degree spectra of sequences of structures*.
Faculty of Mathematics and Informatics, Sofia university, 5 James Bourchier blvd, 1164 Sofia, Bulgaria.
E-mail: asoskova@fmi.uni-sofia.bg.

There is a close parallel between classical computability and the effective definability on abstract structures. For example, the Σ^0_{n+1} sets correspond to the sets definable by means of computable infinitary Σ_{n+1} formulae on a structure $\mathfrak{A}$. In his last paper, Soskov gives an analogue for abstract structures of Ash's reducibilities between sets of natural numbers and sequences of sets of natural numbers. He shows that for every sequence of structures $\vec{\mathfrak{A}}$, there exists a structure $\mathfrak{M}$ such that the sequences that are ω-enumeration reducible to $\vec{\mathfrak{A}}$ coincide with the c.e. in $\mathfrak{M}$ sequences. He generalizes the method of Marker's extensions for a sequence of structures. Soskov demonstrates that for any sequence of structures its Marker's extension codes the elements of the sequence so that the n-th structure of the sequence appears positively at the n-th level of the definability hierarchy. The results provide a general method given a sequence of structures to construct a structure with n-th jump spectrum contained in the spectrum of the n-th member of the sequence. As an application a structure with spectrum consisting of the Turing degrees which are non-low$_n$ for all $n < \omega$ is obtained. Soskov shows also an embedding of the ω-enumeration degrees into the Muchnik degrees generated by spectra of structures.

We apply these results and generalize the notion of degree spectrum with respect to an infinite sequence of structures $\vec{\mathfrak{A}}$ in two ways as Joint spectra of $\vec{\mathfrak{A}}$ and Relative spectra of $\vec{\mathfrak{A}}$. We study the set of all lower bounds of the generalized notions in terms of enumeration and ω-enumeration reducibility.

The work is supported by Sofia University Science Fund.

▶ VLADIMIR STEPANOV, *Truth theory for logic of self-reference statements as a quaternion structure*.
Dorodnicyn Computing Centre of RAS, Vavilov str. 40, Moscow, 119333, Russia.
E-mail: vlast@ccas.ru.

Let $P(x)$ be a predicate formula of a fragment of the type-free second-oder language without $\forall$- and $\exists$-quantors, in which predicates can take other predicate as arguments. Let $P(x)$ be constructed by $\leftrightarrow$ $\neg$ from atomic predicate $Tr(x)$, which satisfies Tarsky axiom:

$$Tr(x) \leftrightarrow x.$$

The self-reference might be expressed with the help of the fixed-point axiom. As for us, for the same aim we would use the quantor of self-reference $\mathsf{S}x$ combined with the axiom of self-reference [1]:

$$\mathsf{S}xP(x) \leftrightarrow P(\mathsf{S}xP(x)).$$

The logic which there are only those formulas which contain biconditional $(\leftrightarrow)$ and negation $(\sim)$ is the three Cartesian direct power of classical propositional logic C^2. The characteristic matrix of that logic is

$$\underline{\boldsymbol{M}_{\mathbf{8}}^{c}} = (\boldsymbol{M}_2^c)^3 = \langle \{\boldsymbol{T}, \boldsymbol{V}, \boldsymbol{A}, \boldsymbol{K}, \sim\boldsymbol{K}, \sim\boldsymbol{A}, \sim\boldsymbol{V}, \sim\boldsymbol{T}\}, \sim, \underline{\leftrightarrow}, \{\boldsymbol{T}\}\rangle.$$

Here $\boldsymbol{T}$ = true, $\boldsymbol{V}$ = truthteller, $\boldsymbol{A}$ = liar, $\boldsymbol{K} = (\boldsymbol{V} \underline{\leftrightarrow} \boldsymbol{A})$. In thus certain multiple-valued logic $\underline{\boldsymbol{M}}^c_8$ the truth table for connection of biconditional ($\underline{\leftrightarrow}$) represents the Cayley table for the Klein four group (see below).

$\underline{\leftrightarrow}$	$\boldsymbol{T}$	$\boldsymbol{V}$	$\boldsymbol{A}$	$\boldsymbol{K}$
$\boldsymbol{T}$	$\boldsymbol{T}$	$\boldsymbol{V}$	$\boldsymbol{A}$	$\boldsymbol{K}$
$\boldsymbol{V}$	$\boldsymbol{V}$	$\boldsymbol{T}$	$\boldsymbol{K}$	$\boldsymbol{A}$
$\boldsymbol{A}$	$\boldsymbol{A}$	$\boldsymbol{K}$	$\boldsymbol{T}$	$\boldsymbol{V}$
$\boldsymbol{K}$	$\boldsymbol{K}$	$\boldsymbol{A}$	$\boldsymbol{V}$	$\boldsymbol{T}$

The Klein four group

$$\boldsymbol{V}^2 = \boldsymbol{A}^2 = \boldsymbol{K}^2 = \\ = \boldsymbol{VAK} = \boldsymbol{T}$$

are replaced with

$$\boldsymbol{V}^2 = \boldsymbol{A}^2 = \boldsymbol{K}^2 = \\ = \boldsymbol{VAK} = \sim \boldsymbol{T}$$

$\leftrightarrow_Q$	$\boldsymbol{T}$	$\boldsymbol{V}$	$\boldsymbol{A}$	$\boldsymbol{K}$
$\boldsymbol{T}$	$\boldsymbol{T}$	$\boldsymbol{V}$	$\boldsymbol{A}$	$\boldsymbol{K}$
$\boldsymbol{V}$	$\boldsymbol{V}$	$\sim \boldsymbol{T}$	$\boldsymbol{K}$	$\sim \boldsymbol{A}$
$\boldsymbol{A}$	$\boldsymbol{A}$	$\sim \boldsymbol{K}$	$\sim \boldsymbol{T}$	$\boldsymbol{V}$
$\boldsymbol{K}$	$\boldsymbol{K}$	$\boldsymbol{A}$	$\sim \boldsymbol{V}$	$\sim \boldsymbol{T}$

The quaternion group

Thus received the quaternion group allows us to make the following hypothesis:

The Quaternion Hypothesis. We postulate that truth space of self-reference statements is a quaternion structure, so that the units $\{\boldsymbol{V}, \boldsymbol{A}, \boldsymbol{K}\}$ represent dimensions of truth space of properly self-reference statements , while the scalar $\boldsymbol{T}$ represents a classical statements, and the space units obey the product rules given by W. R. Hamilton in 1843. This property we try to use for recording estimates of logical formulas in the form of a quaternion: $\boldsymbol{Q} = a_0\boldsymbol{T} + a_1\boldsymbol{V} + a_2\boldsymbol{A} + a_3\boldsymbol{K}$. Here $a_0 \div a_3$ take the values 1, $\sim$, 0, which means that the component may be positive or negative occurrence, or may not have it all.

[1] V. Stepanov, *Many-valued logics for dynamical semantics of the atomic self-reference statements*, this Bulletin, vol. 18 (2012), no. 3, pp. 475–476.

► ALEXEY STUKACHEV, ***Dynamic logic on approximation spaces***.
Novosibirsk State University, Pirogova str. 2, Novosibirsk, 630090, Russia; Sobolev Institute of Mathematics, Acad. Koptyug avenue 4, Novosibirsk, 630090, Russia.
E-mail: aistu@math.nsc.ru.

We present recent results on a version of dynamic logic [2, 4, 5] suitable to describe properties of approximation spaces [1, 3, 6], with the set of finite (compact) elements considered as a structure (typical example is the set of rational numbers within the set of real numbers). We consider the case when this structure generates on a whole approximation space an induced structure in a way definable in dynamic logic. One of the natural questions is to describe properties (model-theoretic, effective, etc.) of structures induced this way.

We apply this general technique to the topics studied in [7, 8].

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[7] A. I. Stukachev, ***Effective Model Theory: an Approach via Σ-Definability***, Lecture Notes in Logic, vol. 41, 2013, pp. 164–197.

[8] ———, ***On processes and structures***, Lecture Notes in Computer Science, vol. 7921, 2013, pp. 393–402.

► MAKOTO TATSUTA AND WEI-NGAN CHIN, *Completeness of second-order separation logic for program verification*.
National Institute of Informatics, 2-1-2 Hitotsubashi, 101-8430 Tokyo, Japan.
E-mail: tatsuta@nii.ac.jp.
Department of Computer Science, National University of Singapore, Singapore.
E-mail: chinwn@comp.nus.edu.sg.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

This paper extends the separation logic given in [2] to second-order logic and investigates the system. Assertions are extended by $X(e, \ldots, e)$ with a second-order variable X and second-order universal quantification $\forall XA$. Since higher-order separation logic has been actively studied, for example, in [1], this system is interesting.

Since the system has the inference rule

$$\frac{\{A_1\}P\{B_1\}}{\{A\}P\{B\}}\ (conseq) \qquad (A \to A_1 \text{ true}, B_1 \to B \text{ true})$$

the completeness is relative completeness with respect to true assertions in the standard model.

The expressiveness theorem is proved by extending [3] to second-order logic. In particular, the heapcode translation is extended as follows:

$$\mathrm{HEval}_{X(\vec{t})}(m) = X(\vec{t}, m),$$
$$\mathrm{HEval}_{\forall XA}(m) = \forall X \mathrm{HEval}_A(m).$$

Expressiveness Theorem. For every program P and assertion A, there is a formula W such that for any store s, any heap h, and any second-order assignment σ, W is true at (s, h) with σ if and only if (s, h, σ) is in the weakest precondition for P and A.

Completeness Theorem. If $\{A\}P\{B\}$ is true in the standard model, then $\{A\}P\{B\}$ is provable in the system.

[1] W. N. Chin, C. David, H. H. Nguyen, and S. Qin, *Automated verification of shape, size and bag properties via user-defined predicates in separation logic*. ***Science of Computer Programming***, vol. 77 (2012), no. 9, pp. 1006–1036.

[2] J. C. Reynolds, *Separation logic: A logic for shared mutable data structures*. ***Proceedings of LICS 2002***, (2002), pp. 55–74.

[3] M. Tatsuta, W. N. Chin, and M. F. Al Ameen, *Completeness of pointer program verification by separation logic*. ***Proceeding of SEFM 2009***, (2009), pp. 179–188.

► HSING-CHIEN TSAI, *Finite inseparability of elementary theories based on connection*.
Department of Philosophy, National Chung Cheng University, 168 University Road, Min-Hsiung Township, Chia-yi County 621, Taiwan.
E-mail: pythc@ccu.edu.tw.

Consider a first-order language L. For any L-formula α, let $\#\alpha$ stand for the Gödel number of α. An L-theory T is finitely inseparable if and only if there is a recursive function f such that for any two disjoint recursively enumerable sets A and B such that $\{\#\alpha\colon \alpha$ is a valid sentence in $L\} \subseteq A$ and $\{\#\alpha\colon \alpha$ is an L-sentence refuted by some finite model of $T\} \subseteq B$, $f(a, b) \notin A \cup B$, where a and b are indices of A and B respectively. It is easy to see that finite inseparability implies undecidability and the former is strictly stronger than the latter. Let C be a binary predicate and I will show the finite inseparability of the theory axiomatized by the following three axioms: (1) $\forall x Cxx$; (2) $\forall x \forall y(Cxy \to Cyx)$; (3) $\forall x \forall y((x \neq y \wedge Cxy) \to \exists z(Cxz \wedge \neg Cyz))$. Making use of the said result, I will also show the finite inseparability of the theory axiomatized by (1), (2), (4) $\forall x \forall y(\forall z(Cxz \leftrightarrow Cyz) \to x = y)$ and (5) for any formula α, $\exists x\alpha \to \exists y \forall z(Cyz \leftrightarrow \exists u(\alpha \wedge Cuz))$. The foregoing theory contains exactly the mereological part and the quasi-Boolean part of Clarke's system. There is still one more part of Clarke's system, that is, the quasi-topological part. It is still unknown whether the full Clarke's system is finitely inseparable or not. However, such a system does have finite models and some of them are of a peculiar kind. Based on this observation, I conjecture that the full Clarke's system is also finitely inseparable.

Keywords: decidability, undecidability, finite inseparability, mereology, mereotopology.

► TOSHIMICHI USUBA, *Reflection principle of list-chromatic number of graphs*.
Organization of Advanced Science and Technology, Kobe University, Rokko-dai 1-1, Nada, Kobe 657-8501, Japan.
E-mail: usuba@people.kobe-u.ac.jp.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

Let $G = \langle V, \mathcal{E}\rangle$ be a simple graph, that is, V is a nonempty set of vertexes and $\mathcal{E} \subseteq [V]^2$ is a set of edges. The *list chromatic number of* G, $\mathrm{List}(G)$, is the minimal (finite or infinite) cardinal κ such that for every function F on V with $|F(x)| = \kappa$ for $x \in V$, there is a function f on V satisfying that $f(x) \in F(x)$ and if $x\mathcal{E}y$ then $f(x) \neq f(y)$. The *coloring number of* G, $\mathrm{Col}(G)$, is the minimal (finite or infinite) cardinal κ such that there is a well-ordering $\lhd$ on V such that $|\{y \in V : y \lhd x, y\mathcal{E}x\}| < \kappa$ for every $x \in V$. It is known that $\mathrm{List}(G) \leq \mathrm{Col}(G) \leq |V|$.

The *reflection principle of coloring number of graphs*, RP(Col), is the assertion that every graph with uncountable coloring number has a subgraph of size $\aleph_1$ with uncountable coloring number. This principle was studied in [1] and [3], and it was appeared that this principle is a very strong large cardinal property. On the other hand, Komjáth [4] showed the consistency of the statement that $\mathrm{Col}(G) = \mathrm{List}(G)$ for every graph G with infinite coloring number. Using his result, Fuchino and Sakai [2] proved that the standard model with RP(Col) also satisfies the *reflection principle of list-chromatic number of graphs*, RP(List), which assets that every graph with uncountable list-chromatic number has a subgraph of size $\aleph_1$ with uncountable list-chromatic number. They also constructed a model in which RP(Col) holds but RP(List) fails. These results suggest the natural question: Does RP(List) imply RP(Col)?

In this talk, we prove the following consistency results, which show that RP(List) does not imply RP(Col), and the bounded version of RP(List) is not a large cardinal property:

1. Suppose GCH. Let λ be a cardinal $> \omega_1$. Then there is a poset which preserves all cardinals, and forces that "RP(List) restricted to graphs of size $\leq \lambda$ holds".
2. Relative to a certain large cardinal assumption, it is consistent that RP(List) holds but RP(Col) fails.

[1] S. Fuchino, *Remarks on the coloring number of graphs*. ***RIMS Kôkyûroku***, vol. 1754 (2011), pp. 6–16.

[2] S. Fuchino and H. Sakai, *On reflection and non-reflection of countable list-chromatic number of graphs*. ***RIMS Kôkyûroku***, vol.1790 (2012), pp. 31–44.

[3] S. Fuchino, H. Sakai, L. Soukop, and T. Usuba, *More about the Fodor-type reflection principle*, preprint,

[4] P. Komjáth, *The list-chromatic number of infinite graphs*. ***Israel Journal of Mathematics***, vol. 196 (2013), no. 1, pp. 67–94.

► JEROEN VAN DER MEEREN, *The maximal order type of the trees with the gap-embeddability relation*.
Department of Mathematics, Ghent University, Krijgslaan 281 S22, B 9000 Gent, Belgium.
E-mail: jvdm@cage.ugent.be.

In 1985, Harvey Friedman [1] introduced a new kind of embeddability relation between finite labeled rooted trees, namely the gap-embeddability relation. Under this embeddability relation, the set of finite rooted trees with labels bounded by a fixed natural number n is a well-partial-ordering. The well-partial-orderedness of these trees (if we put a universal quantifier $\forall n$ in front) gives rise to a statement not provable in $\Pi^1_1\text{–}CA_0$.

There are still some open questions left about these famous well-partial-orderings. For example, what is the maximal order type of these sets of trees with the gap-embeddability relation? The maximal order type of a well-partial-ordering is an important characteristic of that well-partial-ordering and it captures in some sense its strength. In this talk, I will discuss some new recent developments concerning this topic.

[1] S. G. Simpson, *Nonprovability of certain combinatorial properties of finite trees*, ***Harvey Friedman's research on the foundations of mathematics***, Studies in Logic and the Foundation of Mathematics, (L. A. Harrington, M. D. Morley, A. Scedrov, and S. G. Simpson, editors), Elsevier, Amsterdam, 1985, pp. 87–117.

► SEBASTIEN VASEY, *Indiscernible extraction and Morley sequences*.
Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, USA.
E-mail: sebv@cmu.edu.
URL Address: http://math.cmu.edu/~svasey/.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

We present a new proof of the existence of Morley sequences in simple theories. We avoid using the Erdős–Rado theorem and instead use Ramsey's theorem. The proof shows that the basic theory of forking in simple theories can be developed inside $\langle H((2^{2^{|T|}})^+), \in\rangle$ without using the axiom of replacement, answering a question of Grossberg, Iovino and Lessmann, as well as a question of Baldwin.

► STEFAN V. VATEV, *Embedding the ω-enumeration degrees into the Muchnik degrees generated by spectra of structures*.
Faculty of Mathematics and Informatics, Sofia University, 5 James Bourchier blvd., 1164, Sofia, Bulgaria.
E-mail: stefanv@fmi.uni-sofia.bg.

For an infinite sequence of sets $\mathcal{R} = \{R_n\}_{n\in\omega}$ and a set X, we write $\mathcal{R} \leq_{c.e.} X$ if for every n, R_n is computably enumerable in $X^{(n)}$, uniformly in n. Soskov [4] considered the following redicibility between sequences of sets

$$\mathcal{R} \leq_\omega \mathcal{P} \text{ iff } (\forall X \subseteq \mathbb{N})[\mathcal{P} \leq_{c.e.} X \Rightarrow \mathcal{R} \leq_{c.e.} X].$$

This reducibility naturally induces an equivalence relation, whose equivalence classes are called ω-enumeration degrees. They form an upper semi-lattice, which have been extensively studied by a number of researchers at Sofia University over the past decade.

In this talk we discuss how to encode an infinite sequence of sets $\mathcal{R}$ into a single countable structure $\mathcal{N}_\mathcal{R}$, preferably in a finite language, such that the Turing degree spectrum of $\mathcal{N}_\mathcal{R}$ is the set

$$Sp(\mathcal{N}_\mathcal{R}) = \{d_T(X) \mid \mathcal{R} \text{ is c.e. in } X\}.$$

We present two such methods. The first one was studied by Soskov [3] and is based on the so-called Marker's extensions [2]. The other approach is based on the idea of coding each set R_n by a sequence of pairs of computable structures [1]. We conclude that for any two infinite sequences of sets $\mathcal{R}$ and $\mathcal{P}$ we can build countable structures $\mathcal{N}_\mathcal{R}$ and $\mathcal{N}_\mathcal{P}$ such that

$$\mathcal{R} \leq_\omega \mathcal{P} \iff Sp(\mathcal{N}_\mathcal{P}) \subseteq Sp(\mathcal{N}_\mathcal{R}).$$

In other words, the ω-enumeration degrees are embeddable into the Muchnik degrees generated by spectra of structures.

[1] C. J. Ash and J. F. Knight, *Pairs of recursive structures*. ***Annals of Pure and Applied Logic***, vol. 46, (1990), pp. 211–234.

[2] David Marker, *Non Σ_n axiomatizable almost strongly minimal theories*. ***The Journal of Symbolic Logic***, vol. 54, (1989), no. 3, pp. 921–927.

[3] Ivan N. Soskov, *Effective properties of Marker's extensions*. ***Journal of Logic and Computation***, vol. 23 (2013), no. 6, pp. 1335–1367.

[4] ———, *The ω-enumeration degrees*. ***Journal of Logic and Computation***, vol. 17 (2007), pp. 1193–1217.

► PAULO VELOSO, SHEILA VELOSO, AND MARIO BENEVIDES, *On graph calculus approach to modalities*.
Programa de Engenharia de Sistemas e Computação; COPPE, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil.
E-mail: pasveloso@gmail.com.
Departamento de Engenharia de Sistemas e Computação; Fac. Eng., Universidade Estadual do Rio de Janeiro, Rio de Janeiro, Brazil.
E-mail: sheila@cos.ufrj.br.
Programa de Engenharia de Sistemas e Computação; COPPE and Inst. Matemática, Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil.
E-mail: mario@cos.ufrj.br.

We introduce a graphical approach to modalities. We employ formal systems where graphs are expressions that can be manipulated so as to mirror reasoning at the semantical level. This visual approach is flexible and modular providing decision procedures for several normal logics. Promising cases are the application of this approach to PDL for structured data [2] and to memory logics [1].

[1] C. Areces, S. Figueira, and S. Mera, *Completeness results for memory logics*, Lecture Notes in Computer Science, vol. 5407, pp. 16–30, Springer, 2009.

[2] P. Veloso, S. Veloso, and M. Benevides, *PDL for structured data: A graph-calculus approach*. ***Journal of the IGPL***, to appear.

▶ ROGER VILLEMAIRE, *An ordinal rank characterising when Forth suffices*.
Department of Computer Science, UQAM, CP 8888 succ. centre-ville, Montreal, Canada.
E-mail: `villemaire.roger@uqam.ca`.
URL Address: `http://intra.info.uqam.ca/personnels/Members/villemaire_r`.

In the original proof that countable dense linear orders are isomorphic, Cantor maps elements in a single direction, contrary to the now common back-and-forth method. He then relies on specific properties of dense linear orders to show that his mapping is indeed onto and hence an isomorphism. This map construction method have been named *Forth* by P. J. Cameron, who, settling a question of A. Mathias, constructed an $\aleph_0$-categorical structure for which Forth fails to yield an onto mapping. In [1] Cameron considered homogeneous structures, for which the Forth construction always build an onto mapping (*Forth suffice* in his terminology). In particular he gave a necessary condition for Forth to suffice. McLeish [2] introduced another necessary condition, more general that Cameron's, but still not sufficient.

This talk will present a necessary and sufficient condition for Forth to suffice in terms of a new ordinal rank. We will emphasise that the rank is derived from a combination of a smallest and a greatest fixpoint (of monotone operators), while McLeish implicitly used a single fixpoint. We will also highlight the existence of homogeneous structures for all possible countable ordinal ranks, with a construction using unions of wreath powers.

[1] P. J. Cameron, ***Oligomorphic permutation groups***, London Mathematical Society Lecture Note Series, Cambridge University Press, Cambridge, 1990.

[2] S. J. McLeish, *The forth part of the back and forth map in countable homogeneous structures*. ***The Journal of Symbolic Logic***, vol. 62 (1997), no. 3, pp. 873–890.

▶ ANTONIO VINCENZI, ***On the logical use of implicit contradictions***.
via Belvedere 171, Albissola Mare, 17012 Italy.
E-mail: `antoniovincenzi_000fastwebnet.it`.

The basic idea is that (assuming that the logic languages are not rigid) the counterexamples of the *Robinson* property can be considered as an implicit generalization of the usual antinomian contradictions. Since the Robinson property is very rare, these contradictions are not pathological. On the other hand, they can be used in some generalizations of the *'by absurdum'* strategy that concern properties more_subtile than the truth of a statement.

Mathematically, the use of implicit contradictions has a positive impact on *Abstract Model Theory*. For this consider pairs $(\mathcal{L}, \mathsf{ST})$'s in which $\mathcal{L}$ is a model-theoretic logic and ST is its underlying set-theory (see [2] and [1], respectively) and work in a context where these contradictions can be solved by the relative form $\mathrm{ROB}((\mathcal{L}, \mathsf{ST}), (\mathcal{L}^+, \mathsf{ST}^+))$ of the Robinson property and where *Robinson* $=$ *Interpolation* $+$ *Compactness*.

Then, assuming that a logic operation is *formally pure* if it cannot self-referentially negate itself, the counterexamples of *Interpolation* can be characterized by the following

Purity Theorem. $(\mathcal{L}, \mathsf{ST})$ *has Interpolation iff the* $(\mathcal{L}, \mathsf{ST})$*-proofs are formally pure.*

Instead, the counterexamples of the *Compactness* can be characterized by the results related to the following

Compactification Conjecture. *If* $[\lambda, \lambda]$-$\mathrm{COMP}(\mathcal{L}, \mathsf{ST})$ *fails then there is a set-theory* $\mathsf{ST}^+ = \mathsf{ST} +$ *strong axiom(s) in which 'cofinality* λ*' is absolute and* $[\lambda, \lambda]$-$\mathrm{COMP}((\mathcal{L}, \mathsf{ST}), (\mathcal{L}, \mathsf{ST}^+))$ *holds.*

Metamathematically, since the pure proofs can be formalized by *Gentzen-style proof-systems that do not introduce new symbols*, the first result is a technical specification of the purity

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

aim of Proof Theory related to the complexity of proof systems. The second kind of results is a technical instrument for studying the *interaction between logics and set-theoretic universes*.

Philosophically, implicit contradictions, being nonpathological, solvable and incompatible with pure formalization are good ingredients for a mathematical description of the *Hegel's Dialectic Logic*.

[1] J. Barwise, ***Admissible sets and structure***, Springer, Berlin, 1975.

[2] J. Barwise and S. Feferman, ***Model-theoretic logics***, Springer, Berlin, 1985.

► ALEXEY G. VLADIMIROV, *Some partial conservativity properties for Intuitionistic Set Theory with the principle UP*.
Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, 119991, Moscow, GSP-1, 1 Leninskiye Gory, Main Building, Russia.
E-mail: `a.g.vladimirov@mail.ru`.

Let $\mathbb{ZFI}2C$ be usual intuitionistic Zermelo–Fraenkel set theory in two-sorted language (where sort 0 is for natural numbers, and sort 1 is for sets).

Axioms and rules of the system are: all usual axioms and rules of intuitionistic predicate logic, intuitionistic arithmetic, and all usual proper axioms and schemes of Zermelo–Fraenkel set theory for variables of sort 1, including schemes Separation, Transfinite Induction as Regularity, and Collection as Substitution.

It is well-known that both $\mathbb{ZFI}2C$ and $\mathbb{ZFI}2C + DCS$ (where DCS is a well-known principle Double Complement of Sets) have some important properties of effectivity: disjunction property DP, numerical existence property and also that the Markov Rule, the Church Rule, and the Uniformization Rule are admissible in it. Such collection of existence properties shows that these theories are sufficiently constructive theories.

On the other hand, $\mathbb{ZFI}2C + DCS$ contains the classical theory $\mathbb{ZF}2$ (i.e., $\mathbb{ZFI}2C + LEM$) in the sense of Gödel's negative translation. Moreover, a lot of important mathematical reasons may be formalized in $\mathbb{ZFI}2C + DCS$, so, we can formalize and decide in it a lot of informal problems about transformation of a classical reason into intuitionistical proof and extraction of a description of a mathematical object from some proof of it's existence.

So, $\mathbb{ZFI}2C + DCS$ can be considered as a basic system of Explicit Set Theory. We can extend it by a well-known intuitionistic principles, such that Markov Principle M, Extended Church ECT, and the Uniformization Principle UP.

The author has proved that both $\mathbb{ZFI}2C + DCS + M + ECT$ and $\mathbb{ZFI}2C + DCS + M$ havethe same effectivity properties as $\mathbb{ZFI}2C$.

The author has also proved that $\mathbb{ZFI}2C + DCS + M + ECT$ is conservative over the theory $\mathbb{ZFI}2C + DCS + M$ w.r.t. class of all formulae of kind $\forall a \exists b \vartheta(a; b)$, where $\vartheta(a; b)$ is an arithmetical negative (in the usual sense) formula. We also have that $\mathbb{ZFI}2C + M + ECT$ is conservative over the theory $\mathbb{ZFI}2C + M$ w.r.t. the same class (AEN).

The Principle UP: $\forall x\, \exists a\, \psi(x; a) \to \exists a\, \forall x\, \psi(x; a)$ is a well-known specifical intuitionistic principle.

In the article we prove that $\mathbb{ZFI}2C + DCS + M + CT + UP$ is conservative over the theory $\mathbb{ZFI}2C + DCS + M$ w.r.t. class AEN, and that $\mathbb{ZFI}2C + M + ECT$ is conservative over the theory $\mathbb{ZFI}2C + M$ w.r.t. class AEN.

We also prove that the theories $\mathbb{ZFI}2C + DCS + M + CT + UP$, $\mathbb{ZFI}2C + DCS + M + UP$, $\mathbb{ZFI}2C + DCS + UP$, and $\mathbb{ZFI}2C + UP$ have the same effectivity properties as $\mathbb{ZFI}2C$ and $\mathbb{ZFI}2C + DCS$.

► LINDA BROWN WESTRICK, *A computability approach to three hierarchies*.
Department of Mathematics, University of California-Berkeley, 970 Evans Hall, Berkeley, CA 94720, USA.
E-mail: `westrick@math.berkeley.edu`.

We analyze the computable part of three hierarchies from analysis and set theory. The hierarchies are those induced by the Cantor–Bendixson rank, the differentiability rank of Kechris and Woodin, and the Denjoy rank. Our goal is to identify the descriptive complexity of the initial segments of these hierarchies. For example, we show that for each recursive

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

ordinal $\alpha > 0$, the set of Turing indices of computable $C[0,1]$ functions that are differentiable with rank at most α is $\Pi_{2\alpha+1}$-complete. Similar results hold for the other hierarchies. Underlying of all the results is a combinatorial theorem about trees. We will present the theorem and its connection to the results.

▶ MITKO YANCHEV, *Complexity of generalized grading with inverse relations and intersection of relations*.
Faculty of Mathematics and Informatics, Sofia University, 5 James Bourchier Blvd., 1164 Sofia, Bulgaria.
E-mail: `yanchev@fmi.uni-sofia.bg`.

The language of Graded Modal Logic (GML, Kit Fine, 1972) is an extension of the classical propositional modal language with counting (or *grading*) modal operators $\Diamond_n$, for $n \geq 0$, which have purely quantitative meaning. S. Tobies proves (Tobies, 2000) that the satisfiability problem for the graded modal language is PSPACE-complete.

The language of Majority Logic (MJL, Pacuit and Salame, 2004) augments the graded modal language with some qualitative capabilities. Two extra unary modal operators, M and W, are added. In Kripke models $M\varphi$ says that more than half of all accessible worlds satisfy φ, what represents the simplest case of *rational grading*.

The language of Presburger Modal Logic (PML, Demri and Lugiez, 2006) is a many-relational modal language with independent relations, having the so-called *presburger constraints*, which can express both integer and rational grading. Demri and Lugiez show that the satisfiability for the PML language is PSPACE-complete, what strengthens the main result of Tobies, and answers the open question about MJL.

At that time a generalization of modal operators for rational grading in the spirit of the majority operators is given (Tinchev and Y., 2006), and it is used in the language of Generalized Graded Modal Logic (GGML, Tinchev and Y., 2010). New unary grading operators are considered, M^r and W^r, where r is a rational number in $(0,1)$. These operators distinguish the part of accessible worlds having some property.

The generalized rational grading operators are expressible by presburger constraints, so the PSPACE completeness of the satisfiability for the generalized graded modal language is a consequence of that for PML. On the other hand an independent proof using a specific technique for exploring the complexity of rational grading is given (Y., 2011). The presence of separate integer and rational grading operators, and the use of the technique developed for the latter allow following a common way for obtaining complexity results as in less, so in more expressive languages with rational grading. In particular, complexity results—from polynomial to PSPACE—for a range of description logics, syntactic analogs of fragments of GGML, are obtained (Y., 2012, 2013).

In this talk we consider many-relational generalized graded modal language adopting *inverse relations* and *intersection of relations*. Rational grading operators are $\lceil\sigma\rceil^r$ and $\lfloor\sigma\rfloor^r$, where σ is an intersection of (possibly inverse) relations. We show that the satisfiability problem for this expressive modal language with generalized grading keeps the PSPACE complexity.

▶ AIBAT YESHKEYEV, *On Jonsson sets and some of their properties*.
Faculty of Mathematics and Information Technologies of Karaganda State University, The Institution of Applied Mathematics, University str., 28, building 2, Kazakhstan.
E-mail: `aibat.kz@gmail.com`.

Let L is a countable language of first order. Let T—Jonsson perfect theory complete for existential sentences in the language L and its semantic model is a C.

We say that a set X—Σ-definable if it is definable by some existential formula.

(a) The set X is called Jonsson in theory T, if it satisfies the following properties:
  1. X is Σ-definable subset of C;
  2. $Dcl(X)$ is the support of some existentially closed submodel of C.

(b) The set X is called algebraically Jonsson in theory T, if it satisfies the following properties:

1. X is Σ-definable subset of C;
2. $Acl(X)$ is the support of some existentially closed submodel of C.

Using these definitions of the Jonsson sets we can get relatively invariant properties of the similarity of the Jonsson theories on arbitrary subsets of the semantic model.

We say that two sets are Jonsson (equivalent, categorical, syntactically similar, semantically similar) to each other, respectively, if will be (Jonsson equivalent, categorical, stable, similar syntactically, semantically similar) their corresponding theories of the models, which are obtained by the corresponding closures of these sets.

For example: two Jonsson sets syntactically similar to each other, if syntactically similar the theories obtained as their respective closures. In the case when obtained theories will be not Jonsson theories, we will consider correspondingly syntactically similarity [1] of the elementary theories of existentially closed models which are closures of these sets.

If $\forall\exists$-consequences of arbitrary theories are Jonsson theories, in this case we can consider the Jonsson fragment of such theories and we will try to build results for them in the Jonsson's technic manner. As part of these newly introduced definitions, consider and try to describe the Jonsson strongly minimal set. This in turn will lead to a series of new formulations of the problem, such as a refinement regarding both kinds (countable, uncoutable) of the categoricity under this newly introduced subjects.

All undefined concepts about Jonsson theories in this thesis can be found in [2].

[1] T. G. Mustafin, *On similarities of complete theories*, ***Logic Colloquium '90: proceedings of the Annual European Summer Meeting of the Association for Symbolic Logic*** (Finland, July 15–22), 1990, pp. 259–265.

[2] A. R. Yeshkeyev, ***Jonsson Theories***, Publisher of the Karaganda State University, Karaganda, 2009.

▶ PEDRO ZAMBRANO AND ANDRÉS VILLAVECES, *Uniqueness of limit models in metric abstract elementary classes under categoricity and some consequences in domination and orthogonality of Galois types*.

Departamento de Matemáticas, Universidad Nacional de Colombia, AK 30 45-03 111321 Bogota, Colombia.
E-mail: `phzambranor@unal.edu.co`.
E-mail: `avillavecesn@unal.edu.co`.

Abstract Elementary Classes (AECs) corresponds to an abstract framework for studying non first order axiomatizable classes of structures. In [2], Grossberg, VanDieren and Villaveces studied uniqueness of limit models as a weak notion of superstability in AECs.

In [3], Hirvonen and Hyttinen gave an abstract setting similar to AECs to study classes of metric structures which are not axiomatizable in continuous logic [1], called Metric Abstract Elementary Classes (MAECs).

In this work, we will talk about a study of a metric version of limit models as a weak version of superstability in categorical MAECs [5], and some consequences of uniqueness of limit models in domination, orthogonality and parallelism of Galois types ([4]).

[1] I. Ben-Yaacov, A. Berenstein, C. W. Henson, and A. Usvyatsov, *Model theory for metric structures*, ***Model theory with applications to algebra and analysis*** (Z. Chatzidakis, D. Macpherson, A. Pillay, and A. Wilkie, editors), vol. 2, Cambridge University Press, Cambridge, 2008, pp. 315–427.

[2] R. Grossberg, M. VanDieren, and A. Villaveces, *Uniqueness of limit models in classes with amalgamation*, submitted.

[3] Å. Hirvonen and T. Hyttinen, *Categoricity in homogeneous complete metric spaces*, ***Archive for Mathematical Logic***, vol. 48 (2009), pp. 269–322.

[4] A. Villaveces and P. Zambrano, *Around independence and domination in metric Abstract Elementary Classes, under uniqueness of limit models*, ***Mathematical Logic Quarterly***, 2014, accepted.

[5] ———, *Limit models in metric Abstract Elementary Classes: The categorical case*, ***Mathematical Logic Quarterly***, 2013, submitted.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

Abstracts of talks presented by title

▶ HOVHANNES BOLIBEKYAN, *On the compactness theorem in many valued logics*.
Department of Informatics and Applied Mathematics, Yerevan State University, 1 A. Manoogian st., 0025, Yerevan, Armenia.
E-mail: `bolibekhov@ysu.am`.

Nowadays many-valued logics occupy new areas of computer science. Being extensively used in various areas, theoretical investigations of different properties in such logics is a challenging area of research [1]. Firstly it is worth mentioning that axiomatic systems for many valued logics are not well developed. Secondly many notions are not naturally extended in many valued logics from already existing analogues of classical or other "well-developed" nonclassical logics.

One of the key properties to characterize first order logic is compactness. We formulate an analogue of classical compactness theorem for arbitrary N-valued logic. To prove it overloading operators are constructed.

[1] R. Hahnle and G. Escalada-Imaz, *Deduction in many-valued logics: A survey*. ***Mathware & Soft Computing***, vol. iv (1997), no. 2, pp. 69–97.

▶ JOHN CORCORAN, *Teaching course-of-values induction*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: `corcoran@buffalo.edu`.

Let P be a property that belongs to every number whose predecessors all have it.

Clearly, P could belong to every number: if P belongs to every number, then—*a-fortiori*—P belongs to every number whose predecessors all have it.

Is the converse true? Is it the case that if P belongs to every number whose predecessors all have it, then P belongs to every number? *A-fortiori* reasoning is often nonreversible.

Does P belong to zero? It does *if* P belongs to all of zero's predecessors. No number precedes zero. *A-fortiori*, no number precedes zero but does not have P. Thus—vacuously—P belongs to all of zero's predecessors. Thus—by hypothesis—P belongs to zero.

What else can we determine about any property that—like P—belongs to every number whose predecessors all have it? Does it belong to one? Of course, since zero is the only predecessor of one. Continuing, zero and one are the only predecessors of two and they both have P. Thus two has P. By this kind of bootstrapping, we see that for any given number x, P belongs to x.

Thus, the above converse *is* true: If P is a property that belongs to every number whose predecessors all have it, then P belongs to every number. This is the *course-of-values induction principle* CVIP, also called—more revealingly—the *cumulative induction principle* CIP.

There are other ways of stating CIP or its logical equivalents.

Every property that belongs to every number whose predecessors all have it belongs to every number.

In order for a property to belong to every number, it is sufficient for it to belong to every number whose predecessors all have it.

In order for a property to belong to every number whose predecessors all have it, it is necessary for it to belong to every number.

CIP in symbols: $\forall P\,[(\forall x\,(\forall y\,(y < x \to Py) \to Px) \to \forall x\, Px]$

▶ JOHN CORCORAN AND JOSÉ MIGUEL SAGÜILLO, *Teaching independence of proposition sets*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: `corcoran@buffalo.edu`.
Logic, University of Santiago de Compostela, Santiago 15782, Spain.
E-mail: `josemiguel.saguillo@usc.es`.

In this lesson, 'independent' expresses a property of sets [of propositions] as in 'Gödel's Axiom-Set is independent'. As such, it resembles the words 'consistent', 'categorical', etc.

https://doi.org/10.1017/bsl.2015.3 Published online by Cambridge University Press

The abstract treats only two of several senses of the adjective 'independent': *Propositionally Independent* [PropInd] and *Informationally Independent* [InfoInd]. PropInd refers to propositions per se and dates from the 1890s; InfoInd refers to information in propositions and dates from the 1990s.

This lecture builds on [1] and lectures abstracted in this BULLETIN, vol. 15 (2009), pp. 244–245 and vol. 16 (2010), pp. 436–437, and p. 443. Examples employ the 1931 Gödel Axiom-Set GAX: the Zero Axiom, the Successor Axiom, and the Induction Axiom [1, pp. 13f].

A set is *propositionally independent* iff no member proposition follows from the rest. A set that is non-PropInd is redundant *itself*: it has an excess member deletable without losing information.

A set is *informationally independent* iff no information is repeated (shared between two of its members), i.e., no non-tautological consequence of one member follows from another. A set that is non-InfoInd might not be redundant *itself* but it has a *member* that is redundant: a member that has excess information.

For example, if {A, B, C} is InfoInd, then the [logically] equivalent set {(A&C), (B&C)} is not redundant *itself*—neither member can be deleted without loss of information—but either *member* is redundant: the C can be dropped from one. {A, (B&C)} and {(A&C), B} are both equivalent to {A&C, B&C}.

InfoInd is neither necessary nor sufficient for PropInd. $\{0 = 0\}$ is InfoInd but not PropInd. The Gödel Axiom-Set GAX is PropInd but not InfoInd, as shown in [1] where an InfoInd equivalent to GAX is constructed.

[1] JOHN CORCORAN, *Information recovery problems*. ***Theoria***, vol. 10 (1995), pp. 55–78.

▸ JOHN CORCORAN AND KEVIN TRACY, *Aristotle's third logic: Deduction*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: `corcoran@buffalo.edu`.

Consider the following four sentence schemata: distinct English common nouns replace placeholders.

(Every * No * Some * Not-every) S is a P.

Every instance sentence has two terms: "subject" and "predicate".

Aristotle constructed a "first logic" before constructing the familiar "syllogistic" or "second logic" [1]. Syllogistic argument constituents—premises and conclusions—are expressible using such sentences. We propose a "third logic" Aristotle could have constructed next—using three and four-noun sentences with restrictive relative clauses: 'that is an [. . .]'.

Every S that is an R is an M.
Every R that is an M is a P.
―――――――――――――――
Every R that is an S is a P.

Instances of this "five-term" argument schema cannot be seen to be valid using the second logic. However, they can be seen to be valid using rules Aristotle could accept: expanding his rules of deduction—"conversions" and "perfect syllogisms"—could produce the following deduction schema, using notation from [2].

1. Every S that is an R is an M.
2. Every R that is an M is a P.
? Every R that is an S is a P.

3. Every S that is an R is an M that is an R.	(1) Restriction Repetition
4. Every M that is an R is a P.	(2) Subject-Restriction Conversion
5. Every S that is an R is a P.	(3, 4) Subject-Restriction Barbara
6. Every R that is an S is a P.	(5) Subject-Restriction Conversion

QED

This begins a series of lectures treating Aristotle's third logic.

[1] John Corcoran, *Completeness of an ancient logic*. ***The Journal of Symbolic Logic***, vol. 37 (1972), pp. 692–702.

[2] ———, *Aristotle's demonstrative logic*. ***History and Philosophy of Logic***, vol. 30 (2009), pp. 1–20.

▸ REINHARD KAHLE, *Hilbert vindicated.*
CENTRIA, CMA, and DM, FCT, Universidade Nova de Lisboa, P-2829-516 Caparica, Portugal.
E-mail: kahle@mat.uc.pt.

In this talk, we will review Hilbert's philosophical viewpoints first in a "pre-Gödelian" perspective and then from a "post-Gödelian" point of view.

Based on the textual evidence, including unpublished lecture notes of Hilbert available at the Mathematical Institute in Göttingen, it is easy to argue that the characterization of Hilbert as naive formalist is totally misleading. The formalistic attitude should rather be considered as a tactical move of Hilbert to save "Cantor's paradise". By reconstructing the philosophical master plan behind *Hilbert's programme*, Fraenkel's report that "one could almost characterize him [Hilbert] as an intuitionist" should not even come as a surprise.

Gödel's incompleteness theorems, however, show that Hilbert's programme cannot be carried out in the intended manner. As a consequence, one has to take a new stand to the available philosophical alternatives. We will argue that the current set-theoretic foundation of mathematics—although never explicitely advocated by Hilbert himself—is more in line with his original position than its modern competitors.

Research supported by the Portuguese Science Foundations FCT, projects *Hilbert's Legacy in the Philosophy of Mathematics*, PTDC/FIL-FCI/109991/2009, and *The Notion of Mathematical Proof*, PTDC/MHC-FIL/5363/2012.

▸ STEVEN LINDELL AND SCOTT WEINSTEIN, *An elementary definition for tree-width.*
Department of Computer Science, Haverford College, Haverford, PA 19041, USA.
E-mail: slindell@haverford.edu.
Department of Philosophy, University of Pennsylvania, Philadelphia PA 19104, USA.
E-mail: weinstein@cis.upenn.edu.

We introduce a new combinatorial parameter which naturally generalizes the notion of vertex separation number from linear layouts of graphs to layouts which are tree-like, and use this to show that the tree-width of a graph is a simple property of its normal trees—tree-like partial orders of the vertices which induce acyclic orientations of the edges. As a consequence, every graph admits a normal tree decomposition situated on its nodes which preserves its tree-width. Moreover, for graphs of fixed tree-width, this is elementary—there is a sentence of first-order logic which confirms if a given partially ordered graph determines a normal tree decomposition of width k.

Our normal form is based on a generalization of normal spanning trees which are central to graph theory [1]. We say a partial order is tree-like if it has a unique minimal element, and for every element, its set of predecessors forms a chain. We refer to these chains as branches of the directed tree determined by the cover diagram. An order is *normal* for an undirected graph G if it is a tree-like partial order of the vertices in which each edge parallels a branch of the tree. Entirely analogous to the role of a linear order in situating a path-width preserving path decomposition [2], we use a normal partial order to situate a tree-width preserving tree decomposition, which we call a normal tree decomposition.

[1] Reinhard Diestel, ***Graph theory***, 4th edition, Springer, 2010, (corrected electronic edition 2012).

[2] Nancy Kinnersley, *The vertex separation number of a graph equals its path-width*. ***Information Processing Letters***, vol. 42 (1992), no. 6, pp. 345–350.