J. Plasma Physics (2013), *vol.* 79, *part* 5, *pp.* 859–865. © Cambridge University Press 2013
doi:10.1017/S0022377813000603

Dust-acoustic solitary waves in a dusty plasma with dust of opposite polarity and vortex-like ion distribution

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(Received 28 February 2013; revised 17 April 2013; accepted 26 April 2013; first published online 14 June 2013)

Abstract. The nonlinear propagation of small but finite-amplitude dust-acoustic solitary waves in an unmagnetized, collisionless dusty plasma has been investigated. The fluid model is a generalization to the model of Mamun and Shukla to a more realistic space dusty plasma in different regions of space, viz., cometary tails, mesosphere, and Jupiter's magnetosphere, by considering a four-component dusty plasma consisting of the charged dusty plasma of opposite polarity, isothermal electrons and vortex-like ion distributions in the ambient plasma. A reductive perturbation method was employed to obtain a modified Korteweg–de Vries equation for the first-order potential. The effect of the presence of a positively charged dust fluid, the specific charge ratio μ, the temperature of the positively charged dust fluid, the ratio of constant temperature of free hot ions and the constant temperature of trapped ions, and ion temperature on the soliton properties and dusty grains energy are discussed.

1. Introduction

The dusty plasma is an ionized gas that contains electrons, ions and small micron- or sub-micron-sized extremely massive charged dust grains. In fact, dust and plasma coexist in a wide variety of cosmic and laboratory environments. They are ubiquitous in different parts of our solar system, namely, in planetary rings, in circumsolar dust grains, in the interplanetary medium, in cometary comae and tails, in asteroid zones, in mesosphere and magnetosphere and in interstellar molecular clouds (Horanyi and Mendis 1986; Mendis and Rosenberg 1994; Shukla and Mamun 2002). The charging of dust grains occurs due to a variety of processes (Whipple 1981; Merlino et al. 1998). Dust grains of different sizes can acquire different polarities; large grains become negatively charged and small ones become positively charged (Horanyi and Mendis 1986; Mendis and Rosenberg 1994; Shukla and Mamun 2002; Tasnim et al. 2012). Positively charged dust particles have been observed in different regions of space, viz. cometary tails (Chow et al. 1993; Mendis and Rosenberg 1994) and Jupiter's magnetosphere (Horanyi et al. 1993). There are three principal mechanisms by which a dust grain becomes positively charged (Fortov et al. 1998). Direct evidence is available for the existence of both positively and negatively charged dust particles in the earth's mesosphere (Klumov et al. 2000; Havens et al. 2001; Smiley et al. 2003), as well as in cometary tails and comae (Horanyi and Mendis 1986). Because of the involvement of the charged dust grains in plasmas, different collective processes exist and very rich wave modes can be excited in dusty plasmas such as dust-acoustic (DA) waves (Rao et al. 1990), dust ion acoustic waves (Shukla and Silin 1992; Barkan et al. 1996; Alinejad 2011) and dust-lattice waves (Melandsø 1996; Homann et al. 1997; Sahu and Tribeche 2012a). On the contrary, Schamel et al. (2001) reported the existence of a new class of ultra low-frequency nonlinear mode called dust-Coulomb waves in a dense charge-varying dusty plasma with trapped dust particles. These dust-Coulomb modes exist in a frequency regime much lower than the DA wave regime. The most studied of such modes is the so-called dust-acoustic solitary wave (DAW) which arises due to the restoring force provided by the plasma thermal pressure (electrons and ions) while the inertia is due to the dust mass. The inclusion of the dust charge dynamics effects leads to a considerable increase in the richness and variety of the wave motions which can exist in a plasma. It also affects the nature of particle wave interaction and the possibility of having a trapped ion distribution in the potential well. The presence of trapped ions can significantly modify the wave propagation characteristics in collisionless plasmas, so one considers vortex-like ion distributions in phase space instead of the usual Boltzmann law (Schamel 1982; Schamel et al. 2001). Some recent theoretical studies focused on the effects of ion and electron trapping which is common not only in space plasmas but also in laboratory experiments (Schamel 1972, 1975; Mamun et al. 1996; Shchekinov 1997; Rahman et al. 2011; Alinejad 2012; El-Labany et al. 2012). However, the effects of vortex-like ion distribution on nonlinear DA solitary and shock waves in a strongly coupled dusty plasma or in a magnetized dusty plasma were examined (Mamun 1998; Mamun et al. 2004; Anowar et al. 2009). Mamun and Shukla (2002) considered a dusty plasma model, which consists of positive and negative dust only, and theoretically investigated the properties of linear and nonlinear electrostatic waves in such a dusty plasma. This dusty plasma model is valid only if a complete depletion of the background electrons and ions is possible, and both positive and negative dust fluids are cold. El Wakil et al. (2006) theoretically investigated higher order contributions to nonlinear DA waves that propagate in a mesospheric dusty plasma with a complete depletion of the background (electrons and ions). However, in most space dusty plasma systems, a complete depletion of the background electrons and ions is not possible (Sayed and Mamun 2007; Abdelwahed et al. 2008; Sahu and Tribeche 2012b). A dusty plasma system consisting of electrons, ions, and negative as well as positive dust particles was considered by Mamun (2008). It was found that the presence of additional positive dust component does not only modify the basic properties of solitary structures but also causes the coexistence of positive and negative solitary potential structures. On the contrary, a dusty plasma with opposite polarity dust was also considered in many papers (see Rahman et al. 2008; Mamun and Mannan 2011; Mannan and Mamun 2011). Recently, Akhter et al. (2012) studied the multi-dimensional instability of obliquely propagating DA solitary structures in a magnetized four-component dusty plasma system containing inertial negatively as well as positively charged dust particles, and Boltzmann ions and electrons. They showed that the basic features of such DA solitary structures and their multi-dimensional instability were modified by the presence of opposite polarity dust particles. The study of soliton properties is a common way to further recognize waves in dusty plasmas. The other important way is the study of dust energy (Singh and Honzawa 1993; El-Shewy 2007, 2011; Pakzad 2010). El-Shewy et al. (2011) studied the energy of two-temperature charged dusty grains in a four-component dusty plasma system. Also, the electron acoustic soliton energy in a plasma with superthermal hot electrons has been determined by Pakzad (2012). Finally, the effect of non-thermality of ions on the energy of hot and cold dusty grains in an unmagnetized plasma having electrons, singly charged ions, hot and cold dust grains has been investigated (El wakil et al. 2013). The major topic of this work is to study the effect of the presence of positively charged dust fluid, the specific charge ratio μ, temperature of the positively charged dust fluid, the ratio of constant temperature of free hot ions and the constant temperature of trapped ions, ion temperature on the soliton properties and dusty grains energy in a four-component dusty plasma with massive, micron-sized, positively, negatively dust grains, isothermal electron and non-isothermal ion distributions. The paper is organized as follows. In Sec. 2, the basic equations governing the dusty plasma system and the nonlinear modified Korteweg–de Vries (mKdV) equation for the propagation of DA were derived. The energy of dusty grains is computed in Sec. 3. Finally, some remarks, discussion, and conclusions are given in Secs 4 and 5.

2. Basic equations and the modified KdV equation

Let us consider a four-component dusty plasma with massive, micron-sized, positively, negatively dust grains, isothermal electron and non-isothermal ion distributions. This study is based on the condition that the negative dust particles are much more massive than positive ones (El Wakil et al. 2006). Here, the basic governing equations are follows:

$$\frac{\partial n}{\partial t} + \frac{\partial (n\, u)}{\partial x} = 0, \tag{1.a}$$

$$\mu\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial \phi}{\partial x} + \frac{\sigma_d}{n}\frac{\partial p}{\partial x} = 0, \tag{1.b}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + 3p\frac{\partial u}{\partial x} = 0 \tag{1.c}$$

for positive dust plasma and

$$\frac{\partial N}{\partial t} + \frac{\partial (N\, v)}{\partial x} = 0, \tag{2.a}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} - \frac{\partial \phi}{\partial x} = 0 \tag{2.b}$$

for negative dust plasma.

Equations (1) and (2) are supplemented by Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = N - \mu_1 n + \mu_2 n_e - \mu_3 n_i. \tag{3}$$

In the above equations, n and u are the density and velocity of positively charged dusty grains, N and v are the density and velocity of negatively charged dusty grains, n_e and n_i are the density of electrons and ions, respectively. ϕ and p are the electric potential of dust fluid and the thermal pressure of the positively charged dust fluid, respectively. Here, n and N are normalized by their equilibrium values n_0 and N_0, u and v are normalized by $C_s = \sqrt{\mu} V_T$, $[\mu = Z_n m_1/Z_p m_2, V_T = (Z_p k_B T_i\ /m_1)^{\frac{1}{2}}]$, $Z_p(Z_n)$, represents the number of the positive (negative) charges on the dust grain surface, $m_1(m_2)$ represents the mass of the positive (negative) dust particle, k_B is the Boltzmann constant, T_i is the temperature of the ions, p is normalized by $n_{10} k_B T_p$, where T_p is the temperature of the positively charged dust fluid, and ϕ is normalized by $k_B T_i/e$, x is the space variable normalized by $\lambda_{D1} = (k_B T_i/4\pi N_0 Z_n e^2)^{\frac{1}{2}}$, t is the time variable normalized by $\omega_{p2}^{-1} = (m_2/4\pi N_0 Z_n^2 e^2)^{\frac{1}{2}}$, where $\sigma_d = T_p/T_i Z_p$, $\mu_1 = \frac{n_0 Z_p}{N_0 Z_n}, \mu_2 = \frac{n_{e0}}{N_0 Z_n}$, and $\mu_3 = \frac{n_{i0}}{N_0 Z_n}$. n_i is the ion density employing the vortex-like

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distribution function and given by

$$n_i = 1 - \phi - \frac{4}{3} b(-\phi)^{\frac{3}{2}} + \frac{\phi^2}{2}, \tag{4.a}$$

where b is a constant depending on the temperature parameters of resonant ion (both free and trapped) and is given by

$$b = \pi^{-\frac{1}{2}}(1-\beta), \quad \beta = \frac{T_h}{T_{ht}},$$

where T_h and T_{ht} are respectively the constant temperature of free hot ions and the constant temperature of trapped ions. The term $-\frac{4}{3} b\phi^{\frac{3}{2}}$ in the expansion of n_h represents the contribution of resonant ions to the ion density. n_e is the isothermal electron density and given by

$$n_e = e^{\sigma_i phi}, \tag{4.b}$$

where $\sigma_i = T_i/T_e$. According to the general method of reductive perturbation theory, we introduce the slow stretched coordinates:

$$\tau = \epsilon^{\frac{3}{4}} t, \quad \xi = \epsilon^{\frac{1}{4}}(x - \lambda t), \tag{4.c}$$

where ϵ is a small dimensionless expansion parameter and λ is the speed of DA waves. All physical quantities appearing in (1)–(3) are expanded as power series in ϵ about their equilibrium values as

$$\begin{aligned} n &= 1 + \epsilon n_1 + \epsilon^{\frac{3}{2}} n_2 + \epsilon^2 n_3, \\ u &= \epsilon u_1 + \epsilon^{\frac{3}{2}} u_2 + \epsilon^2 u_3, \\ N &= 1 + \epsilon N_1 + \epsilon^{\frac{3}{2}} N_2 + \epsilon^2 N_3, \\ v &= \epsilon v_1 + \epsilon^{\frac{3}{2}} v_2 + \epsilon^2 v_3, \\ p &= 1 + \epsilon p_1 + \epsilon^{\frac{3}{2}} p_2 + \epsilon^2 p_3, \\ \phi &= \epsilon \phi_1 + \epsilon^{\frac{3}{2}} \phi_2 + \epsilon^2 \phi_3 \,. \end{aligned} \tag{5}$$

In order to derive the mKdV equation, we impose the following boundary conditions:

$$|\xi| \to \infty, \quad n = N = 1, \quad p = 1, \quad u = v = 0, \quad \phi = 0. \tag{6}$$

Substituting (4) and (5) into (1–3) and equating coefficients of like powers of ϵ, one can obtain the following relations from the lowest-order equations in ϵ:

$$\begin{aligned} n_1 &= \frac{(\lambda^2(\mu_3 + \mu_2\sigma_i) - 1)\phi_1}{\lambda^2\mu_1}, \\ N_1 &= \frac{-1}{\lambda^2}\phi_1, \quad u_1 = \frac{(\lambda^2(\mu_3 + \mu_2\sigma_i) - 1)\phi_1}{\lambda\mu_1}, \\ v_1 &= \frac{-1}{\lambda}\phi_1, \quad p_1 = \frac{3(\lambda^2(\mu_3 + \mu_2\sigma_i) - 1)\phi_1}{\lambda^2\mu_1}. \end{aligned} \tag{7}$$

Poisson's equation leads to the linear dispersion relation

$$-\mu\mu_3\lambda^4 - \mu\mu_2\sigma_i\lambda^4 + \mu\lambda^2 + \mu_1\lambda^2 + 3\mu_3\sigma_d\lambda^2 + 3\mu_2\sigma_d\sigma_i\lambda^2 - 3\sigma_d = 0. \tag{8}$$

If we consider the coefficients of $O(\epsilon^2)$, and with the aid of (7), one can get the following set of equations:

$$\begin{aligned} &-\lambda\frac{\partial n_2}{\partial\xi} + \frac{\partial u_2}{\partial\xi} - \left(\frac{1}{\lambda^2} - 1\right)\frac{\partial\phi_1}{\partial\tau} = 0, \\ &\frac{(-1+\lambda^2)}{\lambda}\frac{\partial\phi_1}{\partial\tau} + \sigma_d\frac{\partial p_2}{\partial\xi} - \lambda\mu\frac{\partial u_2}{\partial\xi} + \frac{\partial\phi_2}{\partial\xi} = 0, \\ &-\lambda\frac{\partial p_2}{\partial\xi} + 3\frac{\partial u_2}{\partial\xi} + \frac{3(-1+\lambda^2)}{\lambda^2}\frac{\partial\phi_1}{\partial\tau} = 0 \end{aligned} \tag{9}$$

for positive dust plasma and

$$\begin{aligned} &-\frac{1}{\lambda^2}\frac{\partial\phi_1}{\partial\tau} - \lambda\frac{\partial N_2}{\partial\xi} + \frac{\partial v_2}{\partial\xi} = 0, \\ &-\frac{1}{\lambda}\frac{\partial\phi_1}{\partial\tau} - \lambda\frac{\partial v_2}{\partial\xi} - \frac{\partial\phi_2}{\partial\xi} = 0 \end{aligned} \tag{10}$$

for negative dust plasma and Poisson's equation is

$$\frac{\partial^2\phi_1}{\partial\xi^2} + \mu_1 n_2 - N_2 - \frac{4}{3} b\,\mu_3\sqrt{-\phi_1}\,\phi_1 - \mu_3\phi_2 - \sigma_i\mu_2\phi_2 = 0. \tag{11}$$

By eliminating the second-order perturbed quantities n_2, u_2, N_2, v_2 and ϕ_2 in (9–11), we obtain the following mKdV equation for the first-order perturbed potential:

$$\frac{\partial\phi_1}{\partial\tau} + \mathbf{A}\sqrt{-\phi_1}\,\frac{\partial\phi_1}{\partial\xi} + \mathbf{B}\frac{\partial^3\phi_1}{\partial\xi^3} = 0 \tag{12}$$

with

$$\mathbf{A} = \frac{b\lambda^3\mu_3(3\sigma_d - \lambda^2\mu)}{\lambda^4\mu(\mu_3 + \mu_2\sigma_i) - 3\sigma_d},$$

$$\mathbf{B} = \frac{\lambda^3(\lambda^2\mu - 3\sigma_d)}{2(\lambda^4\mu(\mu_3 + \mu_2\sigma_i) - 3\sigma_d)}.$$

In order to obtain a stationary solution from (12), let us introduce the following traveling variable $\eta = \xi - v\,\tau$, where the parameter v is related to the Mach number $M = v/C_e$. Here, v is the soliton velocity and C_e is the DA velocity. Integrating once with respect to the new variable η and using the appropriate vanishing boundary conditions for $\phi_1(\eta)$ and their derivatives up to second order as $|\eta| \to \infty$, we obtain

$$\frac{d^2\phi_1}{d\eta^2} + \frac{2}{B}\left(\frac{2A}{3}\sqrt{-\phi_1} - v\right)\phi_1 = 0\,. \tag{13}$$

Since A is always negative, therefore the DA waves are rarefactive solitons, the one-soliton solution of (13) is given by

$$\phi_1 = \phi_0\,\mathrm{sech}^4(D\eta), \tag{14}$$

where the soliton amplitude ϕ_0 and the soliton width D^{-1} are given by

$$\phi_0 = -\left(\frac{15v}{8A}\right)^2, \quad D^{-1} = \sqrt{\frac{8B}{v}}. \tag{15}$$

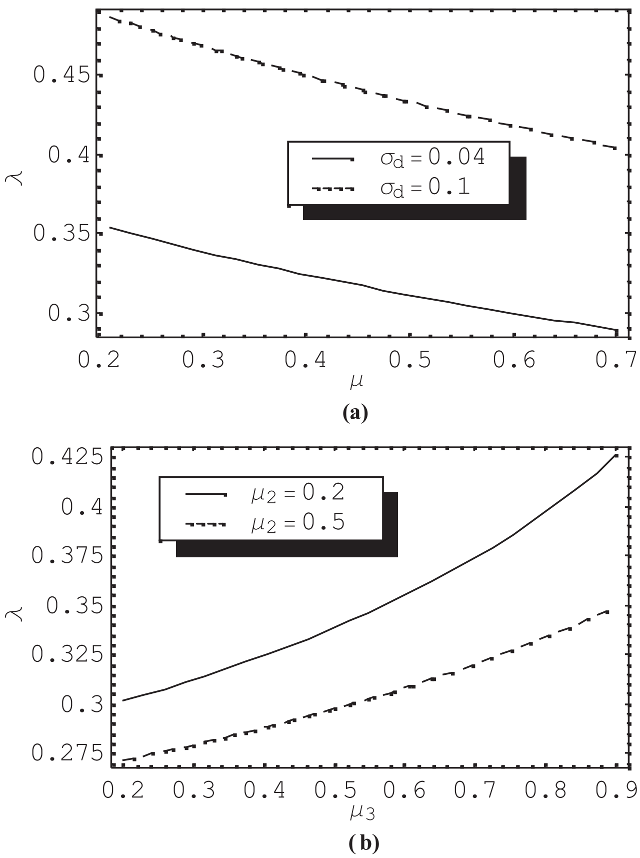

Figure 1. (a) Variation of the phase velocity λ with respect to μ for different values of σ_d for $\beta = -0.5, v = 0.4, \sigma_i = 0.2$, $\mu_2 = 0.2$ and $\mu_3 = 0.5$. (b) Variation of the phase velocity λ with respect to μ_3 for different values of μ_2 for $\beta = -0.5, v = 0.4$, $\sigma_i = 0.2, \sigma_d = 0.04$ and $\mu = 0.3$.

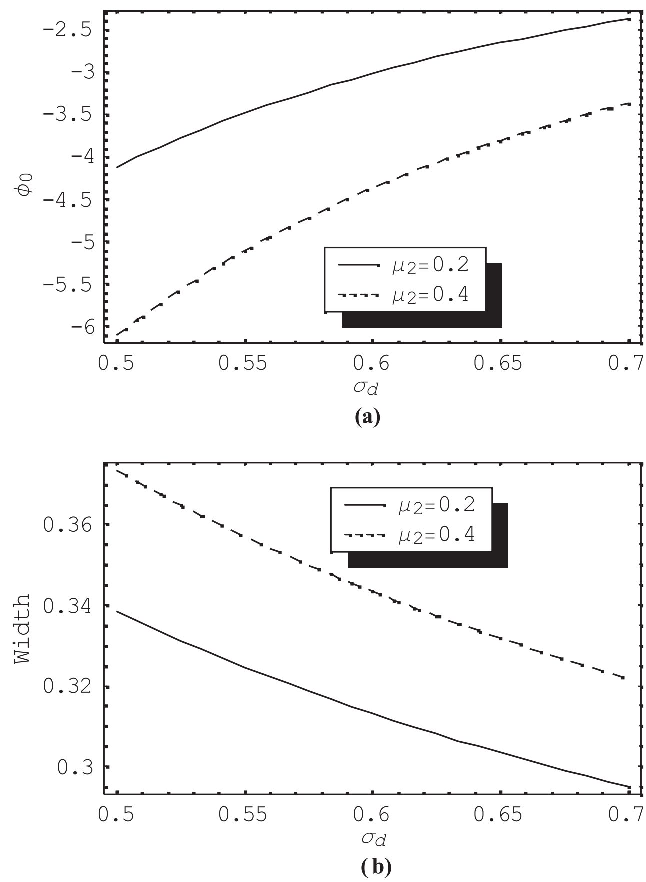

Figure 2. The variation of the soliton amplitude and width with respect to σ_d for different values of μ_2 for $\sigma_i = 0.04$, $\beta = -0.5, v = 0.4, \mu = 0.2$ and $\mu_3 = 0.5$.

3. Dusty grains energy

Inspecting the structures of (7) and (14), one can easily express the first-order perturbed velocities of the positive and negative dust grains as

$$u_1 = \frac{(\lambda^2(\mu_3 + \mu_2\sigma_i) - 1)}{\lambda\mu_1}\phi_0 \operatorname{sech}^4(D\eta),$$
$$v_1 = \frac{-1}{\lambda}\phi_0 \operatorname{sech}^4(D\eta). \tag{16}$$

The energy of positive and negative dusty grains E_p and E_n are obtained according to the integral (Singh and Honzawa 1993)

$$E_p = \int_{-\infty}^{\infty} u_1^2(\eta)\, d\eta, \tag{17}$$

$$E_n = \int_{-\infty}^{\infty} v_1^2(\eta)\, d\eta, \tag{18}$$

where $u_1(\eta), v_1(\eta)$ are the velocities of positive and negative dust grains. Upon introducing (16) into (17) and (18) and performing the integration, we find

$$E_p = \frac{10125}{448b^4\lambda^2\mu_1^2\mu_3^4}(\lambda^2(\mu_3 + \mu_2\sigma_i) - 1)^2 \times \left(\frac{v(\lambda^4\mu(\mu_3 + \mu_2\sigma_i) - 3\sigma_d)}{\lambda^5\mu - 3\lambda^3\sigma_d}\right)^{7/2}, \tag{19}$$

$$E_n = \frac{10125}{448b^4\lambda^2\mu_3^4}\left(\frac{v(\lambda^4\mu(\mu_3 + \mu_2\sigma_i) - 3\sigma_d)}{\lambda^5\mu - 3\lambda^3\sigma_d}\right)^{7/2}. \tag{20}$$

4. Numerical results and discussion

Nonlinear DAWs in an unmagnetized, collisionless dusty plasma consisting of a charged dusty plasma of opposite polarity, isothermal electrons and vortex-like ion distributions in the ambient plasma have been investigated. To make our result physically relevant, numerical calculations were performed referring to typical dusty plasma parameters as given in Havens et al. (2001), Klumov et al. (2000) and Smiley et al. (2003). Generally speaking, the present system supports only rarefactive solitons. However, since one of our motivation was to study the effect of the ratio of temperature of free hot ions to temperature of trapped ions β on the formation of solitary waves, we have studied the effect of some plasma parameters such as μ, σ_d, σ_i, v, β, μ_1,

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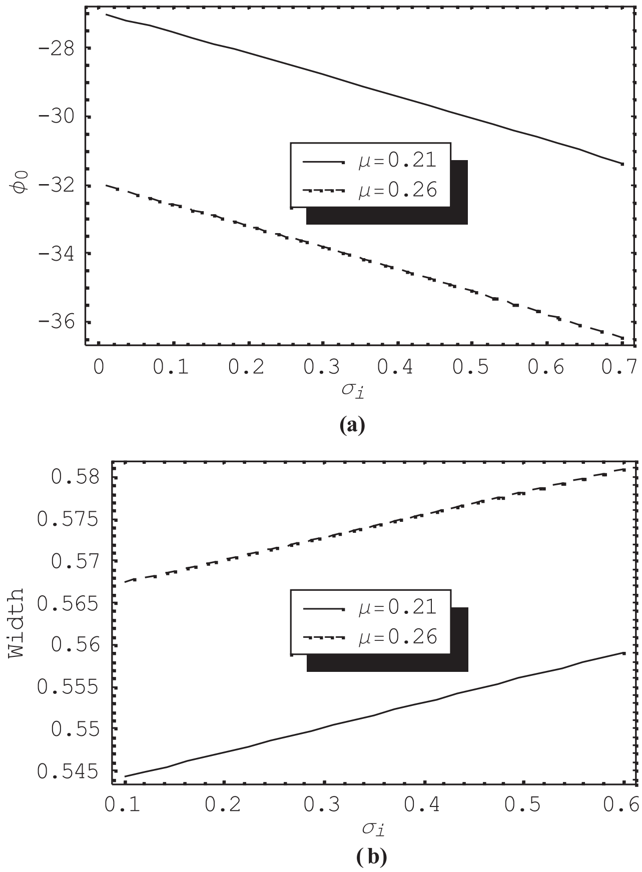

Figure 3. The variation of the soliton amplitude and width with respect to σ_i for different values of μ for $\sigma_d = 0.21$, $\beta = -0.5$, $v = 0.4$, $\mu_2 = 0.2$ and $\mu_3 = 0.5$.

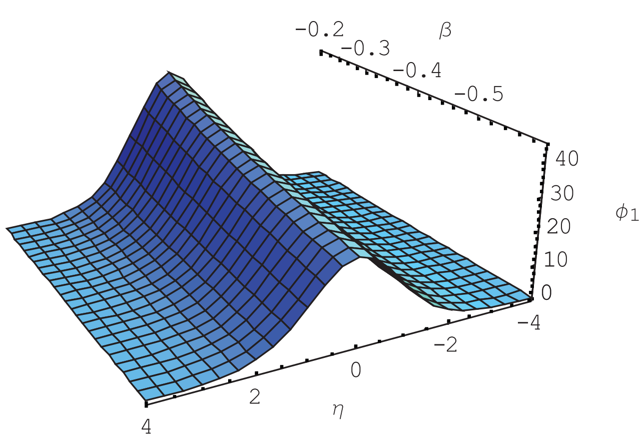

Figure 4. (Colour online) The variation of the soliton amplitude and width with respect to β for $\sigma_i = 0.2, \sigma_d = 0.21$, $v = 0.4$, $\mu = 0.21$ and $\mu_2 = 0.2$, $\mu_3 = 0.5$.

μ_2 and μ_3 on the existence of solitons. For example, the dependence of the phase velocity λ on μ and μ_3 for different values of σ_d and μ_2 is shown in Fig. 1. The basic properties amplitude and width of the small amplitude electrostatic solitary structures are displayed in Figs. 2–4. It is obvious from these figures that the magnitude of the soliton amplitude and width decreases with the increase of σ_d, β, and increases with the increase of σ_i, μ and μ_2. One of the main points of the paper is

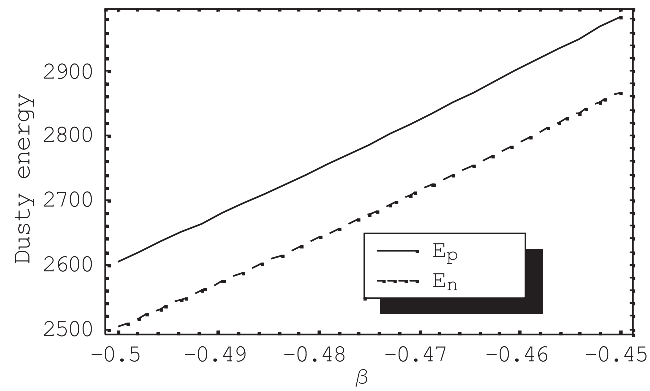

Figure 5. The variation of dusty grains energy E_p and E_n with respect to β for $\sigma_i = 0.2$, $\sigma_d = 0.21$, $v = 0.4$, $\mu = 0.21$ and $\mu_2 = 0.2, \mu_3 = 0.5$.

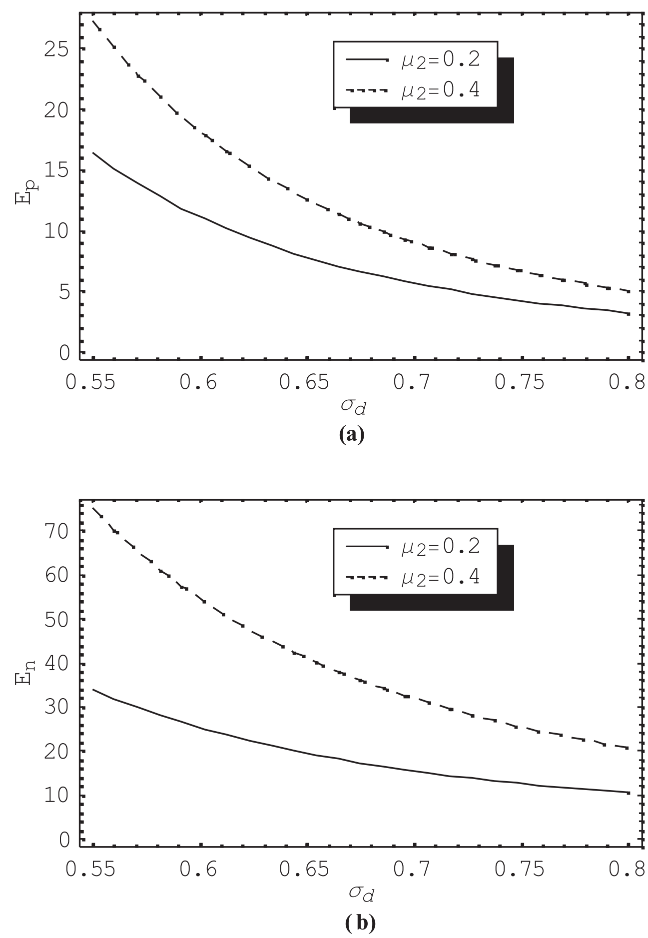

Figure 6. The variation of dusty grains energy E_p and E_n with respect to σ_d for different values of μ_2 for $\sigma_i = 0.04$, $\beta = -0.5$, $v = 0.4$, $\mu = 0.2$ and $\mu_3 = 0.5$.

to examine the dependence of energy of charged dusty grains E_p, E_c on the plasma parameters. It is obvious from Figs. 5–7 that E_p and E_n increase with σ_i, μ, μ_2 and decrease with σ_d, β. In summary, it has been found that the presence of the trapped ions would modify the properties of the DAWs significantly and the results presented here should be useful in understanding salient

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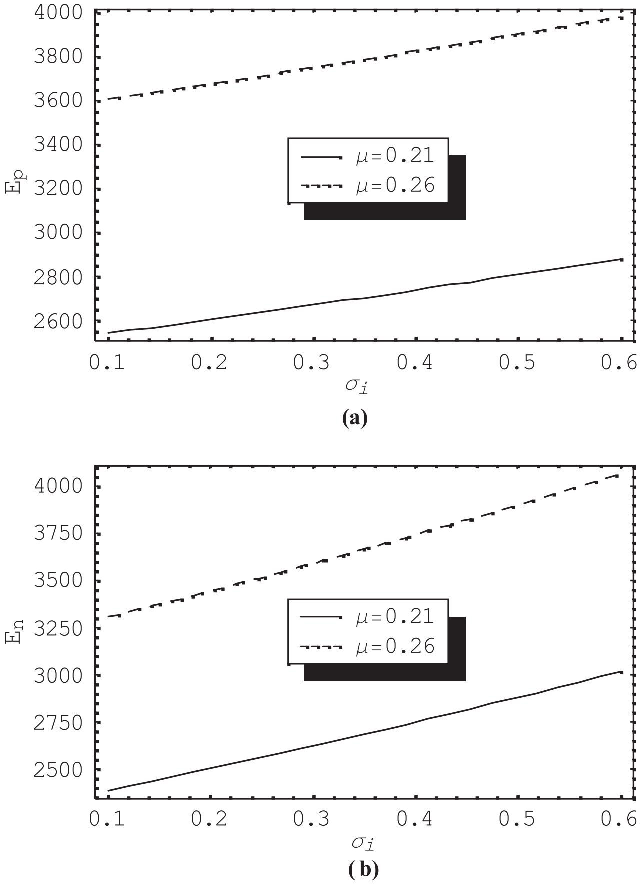

Figure 7. The variation of dusty grains energy E_p and E_n with respect to σ_i for different values of μ for $\sigma_d = 0.21$, $\beta = -0.5, v = 0.4, \mu_2 = 0.2$ and $\mu_3 = 0.5$.

features of localized electrostatic perturbations in space and laboratory plasmas.

5. Conclusion

We have discussed in detail the proper description of the presence of positively charged dust fluid, the ratio of temperature of free hot ions to trapped ions, the specific charge ratio μ, temperature of the positively charged dust fluid in dusty plasma consisting of the charged dusty plasma of opposite polarity, isothermal electrons and vortex-like ion distributions in the ambient plasma. The application of the reductive perturbation theory to the basic set of fluid equations leads to a mKdV equation (12) which describes the nonlinear evolution of DAWs. It is emphasized that the amplitude of DAWs as well as the parametric regime where solitons can exist is sensitive to the positively charged dust fluid parameters and the ratio of temperature of free hot ions to trapped ions. It is interesting to point out that the increase of the temperature of the positively charged dust fluid and the ratio of temperature of free hot ions to trapped ions (the specific charge ratio μ) can lead to the reduction (increase) of the DAW soliton amplitude, width and the energy of charged dusty grains. Generally speaking, it is seen that the soliton amplitude, width and dusty energy were significantly affected by some plasma parameters such as σ_i, v, β, μ_1, μ_2 and μ_3. The ranges ($\sigma-0.1-0.9$, $\mu_2-0.2-0.9$ and $\mu_3-0.3-0.9$) of the dusty plasma parameters used in this numerical analysis are very wide. Therefore, the dusty plasma parameters (viz. σ, μ_2 and μ_3) corresponding to cometary tails (Horanyi and Mendis 1986; Barkan et al. 1996), upper mesosphere (Rao et al. 1990) and Jupiter's magnetosphere (Klumov et al. 2000) are certainly within these ranges. Therefore, the present investigation can help us to identify the origin of charge separation as well as dust coagulation in a plasma containing positive and negative dust.

Acknowledgements

This work was supported by the Deanship of Scientific Research in Salman Bin Abdul-Aziz University, Saudi Arabia under Grants No.92/ت/33.

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