Acknowledgments

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106.16 Sums of Hex numbers are cubes – a planar Proof without Words

Definition: The sequence of *Hex numbers* $(H_n)_{n \in \mathbb{N}}$ is defined recursively as follows:

$$H_0 = 1 \text{ and then for all } n \in \mathbb{N} : H_{n+1} = H_n + 6 \cdot (n + 1).$$

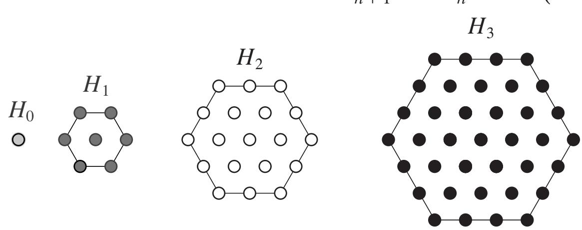

FIGURE 1

Theorem: The partial sums of the sequence of Hex numbers are cubic numbers:

$$\sum_{i=0}^{n} H_i = (n + 1)^3, \text{ for all } n \in \mathbb{N}.$$

In Richard Guy's article *The Strong Law of Small Numbers* [1] one can find a beautifully simple proof for this theorem. The proof is also included in Roger Nelsen's first book *Proofs Without Words* [2]. The idea of that

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proof is to use the Hex numbers in order to actually build a three-dimensional cube. This is done by regarding each Hex number as comprising the three faces at one corner of a cube. The diagram below proves the fact in a different way. It treats the Hex numbers as regular hexagons in the plane and avoids the third dimension by representing the cubic numbers as parallelograms. The proof consists of three steps, in which certain points become rearranged. In order to understand the second and the third steps of the proof, one has to keep in mind that the sum of the first odd numbers is a square number.

Proof: (e.g. for $n = 3$)

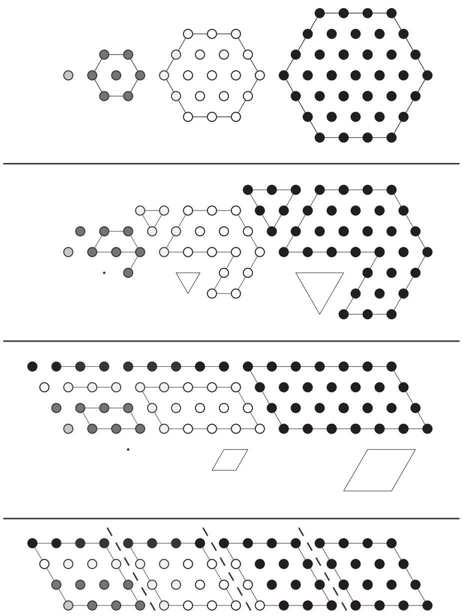

FIGURE 2

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