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Fast magnetic field penetration into low resistivity plasma

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Penetration of a magnetic field into plasma that is faster than resistive diffusion can be induced by the Hall electric field in a non-uniform plasma. This mechanism explained successfully the measured velocity of the magnetic field penetration into pulsed plasmas. Major related issues have not yet been resolved. Such is the theoretically predicted, but so far not verified experimentally, high magnetic energy dissipation, as well as the correlation between the directions of the density gradient and of the field penetration.

Key words: magnetized plasmas, plasma dynamics, plasma nonlinear phenomena

1. Introduction

The penetration of a magnetic field into plasma is an important basic process. The rate of magnetic field penetration is expected to be determined by the plasma resistivity. In a common configuration, the magnetic field has one component and the associated current flows in only one direction (perpendicular to the magnetic field). In high-power pulsed plasmas of such configuration, experimental evidence by probes (Weber *et al.* 1984), followed by thorough studies employing spectroscopy at the Weizmann Institute (Sarfaty *et al.* 1995; Shpitalnik *et al.* 1998; Weingarten *et al.* 2001; Arad *et al.* 2003; Doron *et al.* 2004, 2008; Rubinstein *et al.* 2016), showed magnetic field penetration that was much faster than expected by classical resistivity. Theorists at the Kurchatov Institute (Kingsep, Mokhov & Chukbar 1984; Kalda & Kingsep 1989; Kingsep, Chukbar & Yan'kov 1990; Gordeev, Kingsep & Rudakov 1994) and in other research groups (Fruchtman 1991, 1992*a*,*b*; Fruchtman & Gomberoff 1992, 1993; Fruchtman & Rudakov 1992, 1994; Oliver *et al.* 1992; Huba, Grossmann & Ottinger 1994; Swanekamp *et al.* 1996; Fruchtman, Ivanov & Kingsep 1998; Chuvatin, Ivanov & Rudakov 2004; Richardson *et al.* 2013, 2016) explained the measurements by showing that the Hall electric field can induce a fast magnetic field penetration if the plasma is non-uniform, even if the resistivity is small (but not zero). The velocity of the penetration was shown by the theory to be larger than the hydrodynamic velocity of the plasma pushed by magnetic field pressure, when the scale length of the plasma non-uniformity was smaller than the ion skin depth. In such a regime, electron magnetohydrodynamics (EMHD) modelling is usually needed.

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In a configuration in which the magnetic field has more than one non-zero component, and in which magnetic field and current oscillate, the magnetic field can penetrate into the plasma in the EMHD regime as linear or nonlinear whistler waves (Urrutia & Stenzel 1989; Fruchtman & Maron 1991; Degeling, Borg & Boswell 2004; Stenzel, Urrutia & Strohmaier 2006, 2009; Karavaev *et al.* 2010). In particular, whistler waves have been shown to penetrate nonlinearly into an initially unmagnetized plasma (Stenzel *et al.* 2009). The measurements so far indicate that the magnetic field penetrates without noticeable oscillations, and we therefore assume that whistler waves are not dominant in the configuration we address here.

The theory of Hall-induced magnetic field penetration in a non-uniform plasma has been successful in explaining the observed fast velocity of penetration. In addition, various questions that arose about the Hall penetration have been solved theoretically. It was demonstrated how the non-uniformity of the plasma results in the Hall electric field becoming inductive, and not only electrostatic, as the Hall electric field usually is. Although the Hall electric field itself is not dissipative, the Hall theory predicts that a large magnetic energy should be dissipated, as dictated by energy conservation (Fruchtman 1992*a*), inside a current layer that is narrower for a smaller resistivity.

Within the assumptions of the Hall penetration theory, if the resistivity is zero, the magnetic field should obey the frozen-in law, so that the magnetic field cannot penetrate the electron fluid. It has been shown that during the Hall penetration, the magnetic field does penetrate the electron fluid, so that the deviation from the frozen-in law is large, even if the resistivity is low (but not zero). A relation was derived between the deviation from the frozen-in law and the dissipation of magnetic energy per electron along its trajectory (Fruchtman 1992*b*).

Although the discovery and the research of Hall-induced magnetic field penetration enjoyed theoretical success and experimental verification, there are still unanswered questions. The fate of the large dissipated magnetic energy is a major question. If the magnetic field propagates into the plasma, so that the ions acquire a velocity smaller than the velocity of the magnetic field, a large fraction of the dissipated energy should end up in electron kinetic energy. Indeed, a large electron heating is a result of the Hall theory (Fruchtman & Gomberoff 1993; Fruchtman *et al.* 1998). Even if the electrons are not thermalized, the electron kinetic energy should be high. Spectroscopic measurements, however, indicate that the kinetic energy of the electrons accounts only for a small fraction of the dissipated energy (Doron *et al.* 2008). Another important question arises from the prediction of the Hall theory that the magnetic field penetration should rely on the direction of the density non-uniformity, a prediction that is not supported by observations.

In § 2, the magnetic energy dissipation during the magnetic field propagation in a volume is evaluated through flux and energy conservation. The basic Hall-induced magnetic field penetration, described by a travelling wave solution of Burgers equation (Kingsep *et al.* 1984), is described in § 3. The solution is generalized to describe a penetration into plasma that is not necessarily unmagnetized initially. The power partitioning and the rate of magnetic field dissipation in the plasma and near the electrodes are calculated in § 4. According to the theory, the magnetic energy is dissipated during the penetration described here through Joule heating inside a narrow current layer. As measurements (Doron *et al.* 2008) indicate that the kinetic energy of the electrons is much smaller than that predicted by the Hall model, we discuss in § 5 other processes that could explain the power partitioning. We also discuss in § 5 the dependence of the Hall-induced magnetic field penetration on the direction of the density non-uniformity.

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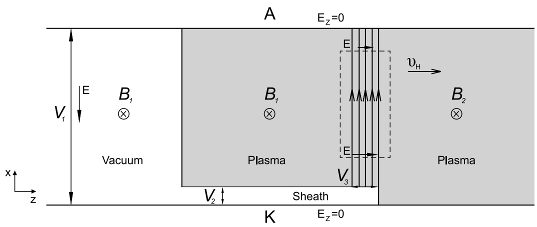

FIGURE 1. Magnetic field penetrates a plasma as a travelling wave with a narrow current channel. The configuration is quite general but has a similarity to the plasma opening switch. The plasma density gradient (not denoted) is in the positive x direction. Shown by arrows is the direction of the electrons motion in the current channel. The sheath size is enlarged. The magnetic flux and energy flow into the plasma along the current channel, in the x direction, perpendicular to the direction of propagation of the travelling wave in the z direction. There is a loop voltage along the dotted rectangle as a result of the non-uniform Hall electric field.

2. Magnetic field propagation and magnetic energy dissipation

In this section we determine the rate of magnetic energy dissipation during magnetic field propagation in the configuration shown in figure 1. The shaded area denotes the presence of a plasma. The current flows along the anode (located at $x=x_0$) to the right, then in the negative x direction (down in the figure) and back to the left along the cathode (located at $x=0$). The electrons flow in the opposite direction, and inside the plasma they flow in the current channel in the anode direction (upward in the figure), their trajectories denoted by vertical arrows. The magnetic field in the plasma and in the vacuum to the left of the current channel is B_1 and to the right of the current channel it is B_2, both B_1 and B_2 pointing into the page, in the negative y direction. Thus, the magnetic field has one non-zero component only. The system variables are uniform in y for a distance that is much longer than x_0 and the length of the plasma in z. This configuration characterizes (although more often in a cylindrical geometry) the plasma opening switch (POS) (Mendel Jr & Goldstein 1977; Weber *et al.* 1987), but the questions addressed, we believe, are of a general interest.

Motivated by the experimental measurements in the POS (Weber *et al.* 1984; Sarfaty *et al.* 1995; Shpitalnik *et al.* 1998; Weingarten *et al.* 2001; Arad *et al.* 2003; Doron *et al.* 2004, 2008; Rubinstein *et al.* 2016), we assume that the magnetic field propagates as a travelling wave to the right with a constant velocity v_B and that B_1 and B_2 are uniform in space and constant in time. In experiments, the magnetic field on the upstream side of the current layer is not exactly uniform due to the finite rise time of the current. We choose a uniform B_1 to simplify the analysis.

In this section, we employ magnetic flux and energy conservation to evaluate the magnetic energy dissipation. In the evaluation, we use only Maxwell's equations for the electromagnetic fields and do not analyse the interaction between the magnetic field and the plasma. We do not specify at this stage the mechanism of magnetic field penetration into the plasma and we will associate the evolution of the magnetic field with Hall penetration only later in the analysis.

Due to the magnetic field propagation, the rate of magnetic flux penetration into the plasma between the electrodes is $(B_1-B_2)x_0v_B$, which, according to Faraday's law, should equal the voltage along a loop that encloses the plasma. The electric field parallel to the electrodes is zero, $E_z=0$. The electric field in the plasma where the magnetic field is B_2 is also zero. The voltage along a loop that encloses the plasma

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therefore equals V_1, the voltage between the electrodes in the vacuum to the left of the plasma. We therefore obtain that

$$V_1 = -\int_0^{x_0} E_x \, \mathrm{d}x = (B_2 - B_1) x_0 v_B \Longrightarrow E_x = (B_1 - B_2) v_B. \tag{2.1}$$

Note that $V_1 > 0$ and $E_x < 0$ (pointing towards the cathode).

We turn now to analysing the energy flow and calculate the total electromagnetic energy that flows from the vacuum to the right. The Poynting flux to the right, $P_z = E_x B_y / \mu_0$ is integrated between the electrodes and we find that the power flow between the electrodes, per unit length in the y direction, is

$$P_{zT} = \int_0^{x_0} \mathrm{d}x P_z = P_z x_0 = \frac{(B_1 - B_2) B_1 v_B x_0}{\mu_0}. \tag{2.2}$$

The rate that magnetic energy is accumulated in the plasma is

$$P_{BT} = \int_0^{x_0} \mathrm{d}x \left(\frac{B_1^2 - B_2^2}{2\mu_0} \right) v_B = \frac{(B_1^2 - B_2^2)}{2\mu_0} v_B x_0. \tag{2.3}$$

The electric energy is much smaller than the magnetic energy, since we assume that v_B is much smaller than c, the speed of light. The difference between the magnetic energy flowing into the plasma and the magnetic energy accumulated in the plasma is the magnetic energy dissipated in the plasma. The rate of magnetic energy dissipation is therefore

$$D_T = P_{zT} - P_{BT} = \frac{(B_1 - B_2)^2 v_B x_0}{2\mu_0}. \tag{2.4}$$

The ratio of power dissipated to the rate of magnetic energy accumulation is

$$r_3 \equiv \frac{D_T}{P_{BT}} = \frac{B_1 - B_2}{B_1 + B_2} = \frac{1 - r}{1 + r}, \tag{2.5}$$

where

$$r \equiv \frac{B_2}{B_1}. \tag{2.6}$$

The dissipated magnetic field energy is smaller when r is larger.

For $B_2 = 0$ (or $r = 0$), there is equipartition of energy. The dissipated electromagnetic energy is equal to the magnetic energy accumulated between the electrodes. This equipartition of energy is not general. When the plasma is initially magnetized, when $r \neq 0$ in our notation, the magnetic field can penetrate as an oscillating wave so that magnetic field energy is not necessarily dissipated. In electromagnetic waves in vacuum, the energy is carried by both electric and magnetic components without dissipation. In plasma, even in slow evolution in which the electric field is small, magnetic field and current reversal, associated with wave oscillations, allow magnetic flux penetration into the volume with no magnetic energy dissipation. Such is the magnetic field penetration during whistler linear and nonlinear wave propagation, mentioned in the introduction. In whistler waves, magnetic energy is carried in more than one wave component (Urrutia & Stenzel 1989; Fruchtman & Maron 1991;

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Degeling *et al.* 2004; Stenzel *et al.* 2006, 2009; Karavaev *et al.* 2010). In particular, as mentioned in the introduction, whistler waves have been shown to penetrate nonlinearly into an initially unmagnetized plasma (Stenzel *et al.* 2009). However, with no field reversal or oscillation, for a penetration in which the magnetic field behind the current layer is uniform, as analysed here, a large magnetic energy dissipation is expected. The energy dissipated is half the energy flowing into the plasma, according to (2.5) for $r = 0$. A finite rise time of the magnetic field may result in a somewhat smaller dissipation (see § II in (Fruchtman 2003)), but the expected dissipation is still large.

3. Hall-induced magnetic penetration

In the previous section we determined the amount of dissipation by applying conservation laws. In this section, we discuss where the dissipated magnetic energy ends.

Rosenbluth showed several decades ago (Rosenbluth 1963) that a collisionless plasma can be specularly reflected by the propagating magnetic field, acquiring a velocity that is twice the velocity of the magnetic piston. In such elastic scattering, the dissipated magnetic energy is converted into directed kinetic energy of the ions and no heat is generated. The ions are reflected by the moving potential hill at the plasma boundary of the width of an electron skin depth. The energy acquired by the collisionless electrons that are reflected by the magnetic field is negligible because of their small mass. This one-dimensional description (1-D) of the specular reflection of the plasma (in the moving frame of the magnetic field) can describe approximately the interaction if the plasma width (in the x direction here) is at least the ion skin depth, since this is the distance in x travelled by the electrons reflected by the magnetic field (Fruchtman & Gomberoff 1993).

As mentioned above, measurements indicate that the magnetic field penetrates into the plasma with a velocity significantly larger than the velocity acquired by the plasma. We therefore present a case that is an opposite limit to specular reflection. The magnetic field is assumed to penetrate the plasma quickly, so that ion motion is small and neglected. The consistency of this assumption should be checked once a solution is derived. The governing equations are the momentum equation for the (assumed) cold electron fluid,

$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} + \eta \boldsymbol{j}, \tag{3.1}$$

where the electron inertia is neglected, Faraday's law

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}, \tag{3.2}$$

and Ampere's law,

$$\mu_0 \boldsymbol{j} = \nabla \times \boldsymbol{B}. \tag{3.3}$$

Here, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields, $\boldsymbol{j}$ is the current density, $\boldsymbol{v}$ is the velocity of the electron fluid and μ_0 is the permeability of free space. Displacement current has been neglected in Ampere's law, therefore the current becomes divergence free, $\nabla \cdot \boldsymbol{j} = 0$. Since ions are motionless, the current is only carried by the electrons,

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$\boldsymbol{j} = -en\boldsymbol{v}$, n is the electron density and e the elementary charge. The current being divergence free results in $\nabla \cdot n\boldsymbol{v} = 0$. From the electron continuity equation,

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n\boldsymbol{v}) = 0, \tag{3.4}$$

so that the (quasi-neutral) plasma density does not vary in time.

The electric field has two terms. The first term on the right-hand side of (3.1) is the Hall electric field, which is non-dissipative, the second term is the resistivity contribution to the electric field.

We now present the travelling wave solution (Kingsep *et al.* 1984). As in the configuration shown in figure 1, we assume a simple geometry in which the magnetic field has one non-zero component only,

$$\boldsymbol{B} = \hat{y}B(z, x). \tag{3.5}$$

Faraday's law is then written as

$$\frac{\partial B}{\partial t} = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \tag{3.6}$$

while Ampere's law becomes

$$\mu_0 j_x = -\frac{\partial B}{\partial z}, \quad \mu_0 j_z = \frac{\partial B}{\partial x}. \tag{3.7a,b}$$

The two components of the electron momentum equation are

$$E_{z,x} = \eta j_{z,x} \mp v_{x,z}B, \tag{3.8}$$

while

$$v_{x,z} = -\frac{j_{x,z}}{en}. \tag{3.9}$$

Let us now assume that the plasma density varies along one direction, say x, only,

$$n = n(x), \tag{3.10}$$

so that

$$\frac{\partial n}{\partial x} > 0. \tag{3.11}$$

The equations are combined to

$$\frac{\partial B}{\partial t} + \frac{1}{e\mu_0}\frac{\partial}{\partial x}\left(\frac{1}{n}\right)B\frac{\partial B}{\partial z} = \frac{\eta}{\mu_0}\left(\frac{\partial^2 B}{\partial z^2} + \frac{\partial^2 B}{\partial x^2}\right). \tag{3.12}$$

Here, the resistivity η was assumed uniform. The nonlinear convective term is due to the Hall electric field. If the plasma density is uniform, the second term vanishes and the magnetic field evolution is determined by the resistivity. The plasma non-uniformity makes the Hall electric field inductive, as will be discussed.

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If $\partial/\partial x(1/n)$ varies slowly with x, there are solutions in which B varies mostly with z, due to the convective term. The second term on the right-hand side of (3.12) is then smaller than the first term on the right-hand side and is therefore neglected. Equation (3.12) is approximated as

$$\frac{\partial B}{\partial t}-\alpha B\frac{\partial B}{\partial z}=\frac{\eta}{\mu_0}\frac{\partial^2 B}{\partial z^2}, \tag{3.13}$$

where

$$\alpha\equiv-\frac{1}{e\mu_0}\frac{\partial}{\partial x}\left(\frac{1}{n}\right)>0. \tag{3.14}$$

Equation (3.13), the governing equation, is the 1-D Burgers equation. The Burgers equation admits a travelling wave solution,

$$B=B(\xi),\quad \xi\equiv z-v_H(x)t, \tag{3.15}$$

for the following boundary conditions:

$$B\longrightarrow B_2,\quad \frac{\partial B}{\partial \xi}\longrightarrow 0\quad \xi\longrightarrow\infty \tag{3.16}$$

$$B\longrightarrow B_1,\quad \frac{\partial B}{\partial \xi}\longrightarrow 0\quad \xi\longrightarrow-\infty \tag{3.17}$$

$$B_1<B_2\leqslant 0. \tag{3.18}$$

Although the case $B_2=0$ is of most interest, we derive here a more general solution. Equation (3.13) with the above boundary conditions is solved as

$$B=B_2+\frac{(B_1-B_2)}{2}\left\{1-\tanh\left[\frac{1}{2d(x)}(z-v_H(x)t)\right]\right\}. \tag{3.19}$$

The velocity of propagation of the magnetic field v_B is the Hall velocity v_H,

$$v_B=v_H\equiv-\alpha\left(\frac{B_1+B_2}{2}\right), \tag{3.20}$$

and the width of the current layer is

$$d=\left(\frac{B_1+B_2}{B_1-B_2}\right)\frac{\eta}{\mu_0 v_H}. \tag{3.21}$$

For simplicity, let us assume that the constant-in-time electron density is

$$n=\frac{n_0x_0}{x_0-x}\quad 0\leqslant x\leqslant x_0. \tag{3.22}$$

In this case,

$$\alpha=\frac{1}{e\mu_0 n_0 x_0}. \tag{3.23}$$

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The 1-D solution is then exact,

$$B = B(z - v_H t), \quad j_z = 0, \tag{3.24a,b}$$

and v_H is constant,

$$v_H = -\frac{B_1 + B_2}{2e\mu_0 n_0 x_0}. \tag{3.25}$$

Note that the electrons carrying the current move from a region where the plasma density is low to a region where the plasma density is high. Such electron motion is a requirement for the magnetic field penetration that is governed by (3.13).

We now assume that (3.19) describes the 1-D propagation of the magnetic field and the current channel in figure 1. The electric field parallel to the electrodes is zero, $E_z = 0$, while inside the current channel E_z is not zero. A narrow sheath is expected to separate the electrode from the quasi-neutral plasma, and across the sheath E_z should vary from zero to its value in the plasma. A model for the sheath has been given in Fruchtman (1992*a*). Here, a model for the sheath is not presented except in deriving properties from conservations laws. The sheath is assumed of a negligible thickness relative to x_0, the distance between the electrodes.

Magnetic flux penetrates into the dotted rectangle in figure 1, which is inside the plasma. Since the vertical sides are in regions of plasma in which the magnetic field is uniform and the current is zero, the electric field there is zero. The inductive voltage in the plasma is due to the axial electric field inside the current layer. Magnetic flux flows into the plasma since the Hall electric field is stronger near the cathode than near the anode. A finite loop voltage exists along the dashed rectangle. The voltage across the horizontal lower side in the figure is larger than the voltage across the horizontal upper side of the rectangle. It is a remarkable property of the Hall-induced magnetic field penetration into a non-uniform plasma that the flow of the magnetic flux is perpendicular to the direction of propagation of the magnetic front. This is opposite to the usual resistive case in which the magnetic flux flow is in the direction of diffusion. The flow of energy is in the direction of the flow of the magnetic flux and is also perpendicular to the direction of propagation of the magnetic front.

In our specific example, the density at the anode was chosen as infinite, so that $v_x(x = x_0) = 0$ and $E_z(x = x_0) = 0$. The loop voltage into the plasma at $x = 0$ (but above the sheath) is

$$V(x=0) = V_3 = -\int_{-\infty}^{\infty} \mathrm{d}z E_z(x=0) = \frac{(B_2^2 - B_1^2)}{2en_0\mu_0} = (B_2 - B_1)x_0 v_H = V_1. \tag{3.26}$$

The inductive voltage between the electrodes in the vacuum to the left of the plasma (due to E_x) falls in the plasma across the current layer (due to E_z).

We note in passing that in the frame of the travelling wave the ions move with velocity $-v_H$ in the z direction and slow down as they climb a potential hill of height V_3 (at $x = 0$). In the case of fast penetration, the ion velocity does not change much in the travelling wave frame, and in the laboratory frame the ions acquire a velocity $v_{ion} \cong e(B_2 - B_1)x_0/m_i$, where m_i is the ion mass. Our analysis is valid if $v_{ion} \ll v_H$, or $2(B_1 - B_2)/(B_2 + B_1) \ll (c/\omega_{pi} x_0)^2$, $\omega_{pi} \equiv (e^2 n_0/\epsilon_0 m_i)^{1/2}$ being the ion skin depth (ϵ_0 is the permittivity of the vacuum).

We turn to completing the estimates of the potential drops. By applying Faraday's law along a vertical line between the electrodes to the left of the current layer (with

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the assumption that the sheath is narrow), we conclude that the voltage between the electrodes along that line (denoted as V_1) should equal the voltage across the current layer in the vicinity of the cathode (denoted as V_2). Therefore,

$$V_1 = V_2 = V_3. \tag{3.27}$$

These relations will be used in the analysis of the energy balance.

4. Energy partitioning

In this section, we examine the partitioning of the electromagnetic energy that flows into the volume between magnetic energy that accumulates in the plasma and energy dissipated through Joule heating. The electromagnetic flows calculated here are in units of power per unit length in y.

Inside the plasma, the magnetic energy flows along the current layer from the cathode to the anode. The flow at the anode out of the plasma is zero (since we chose an infinite density plasma there). The Poynting vector inside the plasma is written as

$$P_x = -\frac{E_z B_y}{\mu_0} = \frac{1}{3en\mu_0^2}\frac{\partial B^3}{\partial z}. \tag{4.1}$$

This flux is zero at the anode and

$$P_{xT} \equiv \int_{-\infty}^{\infty} \mathrm{d}z P_x(z, x=0) = \frac{(B_2^3 - B_1^3)}{3en_0\mu_0^2}, \tag{4.2}$$

at the cathode. The energy that flows into the plasma is a part only of the energy that flows between the electrodes. Using (2.2), we write

$$P_{xT} = \frac{2}{3}\left(\frac{1+r^2+r}{1+r}\right) P_{zT}. \tag{4.3}$$

If the magnetic field is penetrating an unmagnetized plasma, $r = 0$, two-thirds of the energy flow into the plasma, while a third is dissipated in the sheath. The dissipation in the sheath is smaller for a finite r.

The rate of magnetic energy accumulated in the plasma, P_{BxT}, equals P_{BT} calculated above. With the specific expression for $v_B = v_H$, it is written as

$$P_{BxT} = -\frac{(B_1 + B_2)(B_1^2 - B_2^2)}{4en_0\mu_0^2}. \tag{4.4}$$

We now write an expression for the dissipation in the plasma. The difference between the magnetic energy flowing into the plasma and the magnetic field energy accumulated in the plasma is the magnetic field energy dissipated in the plasma. The energy dissipated in the plasma is therefore

$$D_p = P_{xT} - P_{BxT} = \frac{(B_2^3 - B_1^3)}{3en_0\mu_0^2}\left[1 - \frac{3}{4}\frac{(B_1 + B_2)^2}{(B_1^2 + B_2^2 + B_1 B_2)}\right]. \tag{4.5}$$

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Energy is dissipated also in the sheath. This energy has to be the difference between the energy flow through the vacuum and the energy flow along the current channel,

$$D_s = P_{zT} - P_{xT} = \frac{(B_2^2 - B_1^2)B_1}{2en_0\mu_0^2}\left[1 - \frac{2}{3}\frac{(B_1^2 + B_2^2 + B_1B_2)}{B_1(B_1+B_2)}\right]. \tag{4.6}$$

In Fruchtman (1992*a*) these values have been obtained for a particular model for the sheath. Here the partitioning is calculated without specifying a model for the sheath.

We now write various relations between power flows as functions of $r = B_2/B_1$. These are

$$\left.\begin{aligned} r_1 &\equiv \frac{D_p}{P_{zT}} = \frac{2}{3}\frac{(1+r+r^2)}{1+r}\left[1 - \frac{3}{4}\frac{(1+r)^2}{1+r+r^2}\right], \\ r_2 &\equiv \frac{D_s}{P_{zT}} = 1 - \frac{2}{3}\frac{1+r+r^2}{1+r}, \\ r_3 &\equiv \frac{D_s + D_p}{P_{BxT}} = \frac{1-r}{1+r}, \\ r_4 &\equiv \frac{D_p}{P_{xT}} = 1 - \frac{3}{4}\frac{(1+r)^2}{1+r+r^2}, \\ r_5 &\equiv \frac{P_{BxT}}{P_{zT}} = \frac{1+r}{2} = 1 - (r_1 + r_2), \end{aligned}\right\} \tag{4.7}$$

and

$$r_6 \equiv \frac{P_{BxT}}{P_{xT}} = \frac{3}{4}\frac{(1+r)^2}{1+r+r^2} = 1 - r_3. \tag{4.8}$$

The most interesting case in the POS research is penetration into an unmagnetized plasma, $r = 0$. In this case, the energy partitioning is the following

$$r_1 = \frac{1}{6}, \quad r_2 = \frac{1}{3}, \quad r_3 = 1, \quad r_4 = \frac{1}{4}, \quad r_5 = \frac{1}{2}, \quad r_6 = \frac{2}{3}. \tag{4.9a–f}$$

When r is close to unity, as in a weak shock, the dissipation is small. At that limit

$$r_1 = r_2 = r_3 = r_4 = 0, \quad r_5 = r_6 = 1. \tag{4.10a,b}$$

Figure 2 shows r_1, r_2, r_3 and r_5 as functions of r. When the plasma is initially unmagnetized, $r = 0$, the dissipation is large, and the total dissipated magnetic field energy equals the magnetic field energy accumulated in the plasma. There is an equipartition of magnetic field energy between that accumulated and that dissipated in the plasma and in the sheath. The dissipation in the plasma is through Joule heating, while the dissipation in the sheath has not been treated here. The dissipation in the plasma is only a third of the total dissipated energy. In the configuration analysed here, the energy dissipated in the sheath is twice larger than the energy dissipated in the plasma. However, if the energy in the sheath is dissipated in accelerating electrons into the current channel, that dissipated energy should end up in the plasma. When the plasma is magnetized initially, the dissipation is smaller.

In the momentum equation for the electrons, equation (3.1), we used the assumption of a cold electron fluid. This assumption is not consistent with the large energy dissipated through Joule heating if this energy ends up in a high temperature electrons. A self-consistent analysis that included the gradient of the electron pressure in the electron momentum was presented in Fruchtman *et al.* (1998). The velocity of the Hall-induced penetration was shown to be enhanced by the electron heating.

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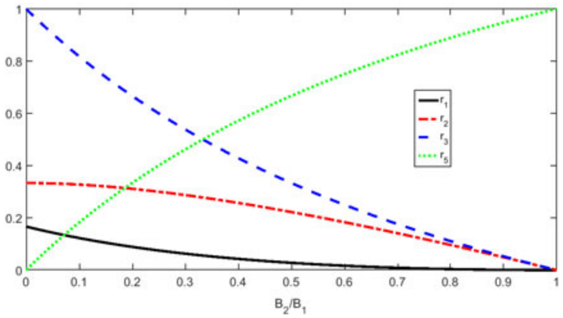

FIGURE 2. The energy partitioning during the Hall-induced magnetic field penetration as a function of B_2/B_1. Shown are the ratio of the energy dissipated in the plasma to the total magnetic energy flux (r_1), the ratio of the energy dissipated in the sheath to the total magnetic energy flux (r_2), the ratio of the total energy dissipated in the plasma and in the sheath to the total magnetic energy accumulated in the plasma (r_3) and the total magnetic energy accumulated in the plasma to the total magnetic energy flux (r_5).

5. Discussion

The theoretical demonstration of a Hall-induced magnetic field penetration provides explanations for important questions raised by the measurements. A mechanism for fast field penetration exists with a velocity that is independent of the resistivity (as long as the resistivity is not too small). Large magnetic energy dissipation that is expected due to energy conservation should occur inside the narrow current layer, even if the resistivity is small.

The fate of the dissipated magnetic field is still not clear. In the case analysed here, where magnetic field penetrates a motionless plasma, the magnetic field energy is dissipated through Joule heating. The electron temperature is thus expected to be high. A calculation based on the dissipated magnetic field energy and the number of electrons in the POS suggests that the electron temperature should be on the keV level. Spectroscopic measurements suggest the presence of electrons in the plasma of 600 eV energy. However, the electron average energy estimated by spectroscopy is less than 200 eV (Doron *et al.* 2008).

In an attempt to explain the low electron temperature, it has been suggested that a large part of the dissipated magnetic energy is carried by the electrons out of the plasma into the anode (Fruchtman & Rudakov 1992). Such replacement of the electrons would make the effective number of electrons larger, so that the energy converted into thermal energy per electron becomes smaller. The electron temperature was estimated to be reduced up to twice due to the electron replacement. Therefore, electron replacement seems to provide a partial explanation to the low temperature.

For the model used here, the electron collision frequency has to be at least the lower hybrid frequency (Fruchtman 1992*b*). If the electron collisionality is smaller, the electron inertia has to be included in the electron momentum equation. Fluid models that included the electron inertia showed magnetic field propagation accompanied by oscillations (Kalda & Kingsep 1989; Fruchtman & Rudakov 1994). Part of the dissipated magnetic energy becomes in this case electron directed energy, instead of thermal energy. Particle simulations show the generation of vortices (Swanekamp *et al.* 1996; Richardson *et al.* 2013, 2016), their existence was also indicated by

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measurements (Shpitalnik *et al.* 1998). Vortices imply a larger magnetic energy and in addition, energy is stored as directed, instead of thermal, electron kinetic energy. Analysing the electron dynamics with a kinetic model or with particle simulations is more accurate than with a fluid picture. However, as said above, whatever the electron energy distribution function is, the experimental results indicate that the electron kinetic energy by itself does not account for the whole dissipated magnetic energy (Doron *et al.* 2008).

The measurements showed that the magnetic field propagates in the plasma (Weber *et al.* 1984; Sarfaty *et al.* 1995; Shpitalnik *et al.* 1998). More recent spectroscopic measurements (Weingarten *et al.* 2001; Rubinstein *et al.* 2016) showed that there is also a significant ion motion. Moreover, they demonstrated a most interesting process; ion separation in a multi-ion species plasma. The magnetic field was found to penetrate the lower charge-to-mass ratio ion plasma and simultaneously to specularly reflect the higher charge-to-mass ratio ion plasma. A large part of magnetic energy is dissipated in this case in the kinetic energy of the reflected ions. Although the temperature of the electrons in the penetrated plasma was expected to be high, the ion reflection does account for a significant part of the dissipated magnetic energy.

In addition to the fate of the dissipated magnetic energy, the dependence of the field penetration on the direction of the density gradient, shown in § 2, is also not clear. Magnetic field penetration is only expected if the current-carrying electrons move in the direction of the density gradient. If the plasma density is constant along the electron trajectory, no field penetration is expected. Moreover, if the current-carrying electrons move in an opposite direction to that of the density gradient, magnetic field expulsion is predicted by the theory (Fruchtman 1991). In experiments so far no correlation was observed between magnetic field penetration and the direction of the plasma density gradient. Various mechanisms are discussed that could make the penetration independent of the initial plasma density distribution. A penetration into an initially uniform density plasma has been theoretically shown (Fruchtman & Rudakov 1992, 1994), in which plasma pushing by the magnetic pressure generates a density gradient that induces the field penetration by the Hall field mechanism. Ion separation modifies the plasma density substantially, perhaps generating sometimes a density gradient that favours magnetic field penetration.

The fate of the dissipated magnetic energy, the mechanism of ion separation and the effect of the direction of magnetic field penetration are issues related to Hall-field-induced magnetic field penetration that call for further research.

Finally, the Hall-induced penetration has been analysed in a laboratory configuration, where a current is driven through a plasma between electrodes. The process of magnetic field penetration is of a general nature and is expected to occur in astrophysical plasmas as well. The Hall effect is often used to describe processes in space plasmas, such as reconnection (Huba & Rudakov 2004; Cassak, Shay & Drake 2005) and tearing modes (Bulanov, Pegoraro & Sakharov 1992; Fruchtman & Strauss 1993; Bian & Vekstein 2007). However, a Hall-induced magnetic field penetration as a travelling wave has not been yet identified theoretically or experimentally in space plasmas. An identification of magnetic field penetration in space plasmas due to the Hall mechanism could contribute to our understanding of processes in astrophysical plasmas and of plasma physics.

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