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ARTICLES

OVERVIEW OF NONLINEAR MACROECONOMETRIC EMPIRICAL MODELS

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A survey of nonlinear multivariate macro empirical models is attempted. Although theory may suggest that nonlinearity is to be expected, empirical studies have difficulty in discovering strong consistent effects. Regime switching techniques appear to be the most successful and evidence of nonlinearity is most found for interest rates. Most of the studies emphasize model fitting rather than model evaluation, which limits their usefulness.

Keywords: Nonlinear Models, Multivariate Models, Vector Autoregressive Model, Error Correction Model, Aggregation

1. INTRODUCTION

Macroeconomics is concerned with the dynamic relationships between a specific set of variables that influence the macroeconomy or are aggregates that measure that economy. The appropriate econometric techniques for the study of such relationships are clearly those developed for multivariate time series. A great deal of the microeconomic theory that forms the basis for macro theory, and also most direct macro theory without micro foundations, is nonlinear, sometimes suggesting a nonlinear parametric form but often without giving a specific form. One might therefore expect to find many examples of interesting nonlinear models in the literature, but this seems not to be the case. An extensive, although by no means complete, survey of the macro literature over the past 5 years or so was conducted with the help of my student, Namwon Hyung. Some papers using nonlinear specifications were found but they remain very much in the minority compared to the number of papers using linear (or log-linear) specifications. There is little evidence that there are empirical macroeconomists who have found strong evidence of nonlinearity in their data are seeking help from time-series econometricians for analysis of such data. Rather, it seems that there is either little nonlinearity in macro data or it is subtle and sophisticated methods are needed to extract it.

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There certainly are a number of papers in the literature, by econometricians, selling new specifications or techniques, and most of these papers include an empirical example in which the technique is found to be somewhat superior to a simple linear alternative model. A possible reason for this apparent weak nonlinearity in macroeconomics is the use of aggregation, both temporal and cross-sectional, as discussed in Section 2. It is possible that other manipulations to the data, such as seasonal adjustments, also reduce nonlinearity, but this is not considered here.

To keep this discussion manageable, I consider only nonlinearity in mean, not topics such as frequency domain techniques, chaos models, nonparametric modeling methods, higher moments including conditional variance, quantile regressions, and high-frequency data (where evidence of nonlinearity is found in finance). Many of these topics would require a separate paper of their own; others are of little relevance. A few particular topics, such as structural breaks and smooth-transition models, were left out of this survey because others will be providing expert discussions about them later in this issue. Teräsvirta (1998) provides a recent account and an application of smooth-transition models.

2. AGGREGATION

I will generally only consider multivariate relationships, that is, models relating two or more variables using one or (possibly) more equations. Occasionally, I will present univariate results when no useful multivariate ones are available.

The aggregation of nonlinear models has been considered by Granger and Lee (1999) largely by Monte Carlo simulations. A variety of univariate and bivariate models were used to generate a series of 200 terms, a test of linearity was applied with a null of linearity, and 10,000 replications were used to find out how many times the null was rejected. Then, an aggregation was performed, so that, for the cross-sectional case, 20 series were added to form the aggregate, and then the test was applied to this sum. The percentage rejection of the aggregate could then be compared to the individual (nonaggregated) series. Four different tests were used for the comparison, including the familiar RESET and a neural network test that had performed well in an earlier comparison experiment by Lee et al. (1993).

If these series were independent, then aggregation resulted in considerable loss in nonlinearity, in the sense that the tests could no longer find evidence of it. If the input shock to the ith series was of the form

$$e_{it} = c_i z_t + \varepsilon_{it},$$

where the ε_{it} are all independent series and z_t is a "common-factor" white noise, then if z_t represented 50% of the variance of e_{it}, on average, some nonlinearity remained after aggregating over 20 units; if z_t represented 90% of the variance, then much of the nonlinearity remained, but there was a distinct reduction. For real macro aggregates, the sums would be over millions of items, and so, these effects would be emphasized. The importance of common factors in linear aggregation

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had been emphasized previously by Granger (1987) and applied and extended by Forni and Lippi (1997).

The simulation also considered temporal aggregation, in which values were summed over time and then observed, as well as systematic samples, for which a series was generated over one time interval (say a week) but observed over a wider interval (say a quarter). In all instances, nonlinearity was weakened by this type of information loss.

The general conclusion is that aggregation in its various forms is inclined to reduce nonlinearity, but that it might survive in data that are measured over relatively short time intervals, that are the outcomes of markets (e.g., interest rates, some prices) rather than aggregates, or that are aggregates with important common factors (e.g., policy variables, tax rates, possibly money supply). We did not expect to find nonlinearity everywhere, but there remained plausible pockets of existence.

An interesting development is presented by Stock and Watson (1999) and Chen et al. (1999). They considered only univariate series, but for 215 U.S. macro variables, measured monthly, and found that a variety of univariate neural network models do not outperform a simple AR(4) model in a forecasting test. However, a combination of all the neural network models clearly beat the AR(4) model. The combination used equal weights after "trimming" poor forecasts. The results suggest that nonlinearity can be squeezed out of univariate models.

3. SIMPLE MODEL SPECIFICATIONS

There are many nonlinear functional forms that can be used as the basis for the specification of an empirical model; Zellner and Ryu (1998) provide a number of examples based on production, cost, and returns-to-scale functions. However, several authors take a simpler viewpoint, starting with a linear model and adding only an occasional nonlinear term to investigate the possible presence of nonlinearity. For example, Devarajan et al. (1996) considered the 5-year-forward moving average of per-capita real GDP for a panel of countries explained by $X =$ ratio of current expenditure to total expenditure and $Y =$ ratio of capital expenditure to total expenditure, plus other explanatory variables, and they use a specification including either X, X^2, and Y or X, Y, and Y^2. Both of the squared terms had significant t-values. Similarly, Edwards (1998) studied total factor productivity growth by using a linear specification and mixtures of 10 explanatory variables and then explored possible nonlinearities by adding squared values of 3 of them, with "mixed results." This approach is very similar to that used in the specification of models in cross-sectional analysis, where any nonlinearity is approximated in a simple way but often is not explored in depth. Of course, many nonlinear models nest a linear one.

A further popular and slightly more sophisticated nonlinear model used threshold specifications, so that the set of parameters would change value conditional on the size of some variable. Thus, a linear model could have parameters with one set of values if the capacity index was under 90% and a different set if it was over 90%. Thus, they varied over the business cycle and the model fell into two regimes.

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For example, Laroque and Rebault (1995) considered a fairly simple model for the inventory cycle with two regimes corresponding to excess demand or excess supply. The resulting estimated model looks promising, but does not outperform a simple linear AR(2) specification. Tsay (1998) tackled the more complicated problem of testing and modeling threshold models for a vector of series and applied the proposed methods to a pair of interest rates, with the 3-month moving spread between the rates acting as the switching variable. Three regimes were obtained and significant differences were found between the regimes.

4. VECTOR AUTOREGRESSIVE AND ERROR-CORRECTION MODELS

The classical structural system of equations, as developed at the Cowles Foundation, frequently contained nonlinear elements. When linear, its reduced form is a vector autoregressive model with no simultaneous terms or with these terms placed in an arbitrary triangular form. In recent years, large-scale structural models continue to be used and to be updated in most economies and are joined via trade and capital flows in Project LINK. However, little academic research has been published on the testing of economic hypotheses, consideration of alternative policies, and comparison of forecasts involving these systems. Most work has used linear specification for VAR's; an excellent recent survey of this area is by Lütkepohl (2000). There appears to be no systematic methodology for introducing nonlinear terms into the VAR. Clearly, it is possible to take a standard specification and replace some linear components with nonlinear forms, such as quadratics, but whether this is done as an exploratory device or in response to theoretical considerations is not clear. Lütkepohl (1991), in a 540-page book on multivariate time series, has just 2 pages on nonlinear state-space models. This is a promising class of models that requires further development and an application to macro data.

A class of models related to VAR's, in which nonlinearity has been successfully inserted, are the error-correction models. In the linear bivariate form, this is

$$\Delta X_t = m_1 + \gamma_1 Z_{t1} + \sum_{j=1}^{P} \alpha_{1j} \Delta X_{tj} + \sum_{j=1}^{P} \beta_{1j} \Delta Y_{tj} + e_{xt},$$

$$\Delta Y_t = m_2 + \gamma_2 Z_{t1} + \sum_{j=1}^{P} \alpha_{2j} \Delta X_{tj} + \sum_{j=1}^{P} \beta_{2j} \Delta Y_{tj} + e_{yt},$$

where X_t, Y_t are I(1) processes, $Z_t = X_t - AY_t$ is I(0) and so $(1, A)$ is the cointegration vector, e_{xt}, e_{yt} are zero-mean white-noise series (or Martingale differences), and there is an important constraint that $|\gamma_1| + |\gamma_2| \neq 0$, so that at least one of the γ terms is nonzero. The nonlinear error-correction (NLEC) model replaces $\gamma_1 Z_t, \gamma_2 Z_t$ with $\gamma_1(Z_t)$, $\gamma_2(Z_t)$ where $\gamma_1(\,)$, $\gamma_2(\,)$ are appropriate functions.

To illustrate how such an NLEC can arise, consider the following generating mechanism:

$$W_t = W_{t-1} + \varepsilon_t,$$

$$Z_t = f(Z_{t-1}) + \eta_t,$$

where ε_t, η_t are each zero-mean white noises and are unobserved components, with $W_t \sim \mathrm{I}(1)$, $Z_t \sim \mathrm{I}(0)$, so that $f(Z)$ is bounded and $|f(Z)| < 1$, for all $|Z| > m$, some positive m. Let X_t, Y_t be a pair of observed series given by

$$X_t = AW_t + C_1 Z_t,$$

$$Y_t = W_t + C_2 Z_t,$$

where C_1, C_2 obey the constraint $C_1 - C_2 A = 1$. For simplicity, only the case $A \neq 0$ will be considered. It is seen that a consequence of the constraint on the C's gives

$$Z_t = X_t + AY_t,$$

so that X_t, Y_t are both I(1), having a common stochastic trend W_t but are linearly cointegrated, having a linear combination, Z_t, which is I(0). Differencing the equations for X_t, Y_t and after a little algebra one gets

$$\Delta X_t = C_1[f(Z_{t1})Z_{t1}] + A\varepsilon_t + C_1\eta_t,$$

$$\Delta Y_t = C_2[f(Z_{t1})Z_{t1}] + \varepsilon_t + C_2\eta_t,$$

which are simple NLEC forms, with no lagged ΔX, ΔY terms and with the nonlinear term in Z_{t1} the same in each equation. This construct, and its generalizations, has been considered by Granger (1996) and partially by Granger and Swanson (1996). It is seen that the form of the nonlinearity in the error-correction model here is directly related to the nonlinearity in the generating mechanism of the error-correcting series Z_t. If $Z_t = 0$ is an attractor for the system, the strength of the pull may be largest when $|Z|$ is large than when Z is near zero. This would correspond to $f(Z)$ being small for $|Z|$ large but $f(Z)$ could be large, even approaching 1 as Z becomes small. Another alternative is that the pull toward the attractor could be different in magnitude on one side of the attractor than on the other, so that $1 \geq f(Z) > f(-Z)$, say for $Z \geq 0$, and this will mean that the error-correction term will enter the NLEC model asymmetrically. The construct discussed here, which is not completely general, also suggests constraints on the nonlinear forms that can be used in NLEC models, as previously discussed by Granger and Haldrup (1997).

NLEC models were originally introduced by Escribano (1987). The concept has been considerably generalized by Escribano and Mira (1998), to also include nonlinear cointegration. The asymmetric NLEC has been recently discussed and applied by Hong and Lee (1997). They found evidence of asymmetry both for U.S. data on short-term and long-term interest rates and for Korean data on consumption and income. The evidence was particularly strong for this second set of data. Granger and Lee (1989) found asymmetries in cointegrations involving sales, production, and inventories. Granger and Escribano (1998) used a variety of nonlinear forms when considering the cointegration relationship between gold and silver prices. Evidence of nonlinearity was found in-sample but less for out-of-sample, and so, forecast performance was not improved.

A further popular and successful form involved threshold NLEC models. Balke and Fomby (1997) considered a pair of interest rates with the spread as the error-correction term which itself follows a threshold autoregression. Z_t is I(1) within some region near to the attractor and only becomes I(0) as it switches to a distant regime. The published paper considers just the underlying theory, a testing procedure, and a simulation, but the original working paper contained an interesting empirical example. Peel and Taylor (1998) consider a similar specification in a trivariate EC model. The variables used are

$$\phi_t = 400\left(\frac{F_t - S_t}{S_t}\right),$$

where S_t, F_t are the spot and future exchange rates, i_t and i_t^* are the domestic and foreign interest rates on some asset; weekly data for the period January 7, 1922, through March 21, 1925, are used, with a total of 168 observations. The cointegration has $\delta_t = \phi_t i_t + i_t^*$ and the threshold NLEC model takes the form

$$\Delta \boldsymbol{X}_t = \boldsymbol{A}_1 \Delta \boldsymbol{X}_{t1} + \boldsymbol{e}_{1t} \quad \text{if } |\delta_{t1}| < k \tag{1}$$

and

$$\Delta \boldsymbol{X}_t = \boldsymbol{A}_2 \Delta \boldsymbol{X}_{t1} + \gamma\delta_t + \boldsymbol{e}_{2t} \quad \text{if } |\delta_{t1}| \geq k, \tag{2}$$

where $\boldsymbol{X}' = (i, i^*, \phi)$. As with Balke and Fomby (1997), δ_t is found to be a random walk inside the band (1) but a stationary AR(1),

$$\delta_t = \underset{(0.03)}{0.89}\,\delta_{t1} + \varepsilon_t,$$

in the outside region.

There also exists a number of single-equation models using nonlinear error-correction terms, for example, Hendry (1984), Hendry and Ericsson (1991), and Ericsson et al. (1998).

5. IMPULSE RESPONSE AND ASYMMETRIC SHOCKS

A problem with the VAR model is that it contains many coefficients and is thus rather difficult to interpret. Sims (1980) introduced the idea of impulse responses to help both with interpretation and also potentially with the policy usefulness of the model, conditioned on the specification being correct. It tracks the effects of a unit

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impulse imposed on one variable through the system onto future values of that variable and also other variables into the future. The technique requires a one-to-one identification with a vector of shocks $\boldsymbol{\varepsilon}_t$ to the vector of variables $\boldsymbol{X}_t$ and then converts the VAR model into a vector moving average,

$$\boldsymbol{X}_t = \boldsymbol{C}(B)\boldsymbol{\varepsilon}_t,$$

where $C_{ij,k}$ is the coefficient on the kth lag of the jth ε component in the ith equation. Now, if ε_{jt} is replaced by $\varepsilon_{jt} + \delta_t$, which is shock of size δ_t at time t but no similar shock at other times, one generates a new set of present and future values of $\boldsymbol{X}$, denoted by $\boldsymbol{X}_{t+k}(\delta)$. It is seen that

$$X_{ij,\,t+k}(\delta) X_{ij,\,t+k} = C_{ij,\,k}\delta_t,$$

so that the unit impulse response for the jth component of $\boldsymbol{X}$ k steps in the future is C_{jk}, which is a nonrandom quantity, if the original VAR model is assumed to be known with certainty, which in practice is not correct.

As an illustration of what happens with a simple nonlinear form, consider the univariate bilinear model

$$X_t = \beta X_{t-2}\varepsilon_{t-1} + \varepsilon_t$$

and define $\Delta_k(\delta) = X_{t+k}(\delta)X_{t+k}$, where there is an impulse of δ at time t but none at other times. Some simple algebra gives

$$\Delta_1(\delta) = \beta X_{t-1}\delta$$

$$\Delta_2(\delta) = \beta\delta\varepsilon_{t-1}$$

$$\Delta_3(\delta) = \beta^2 X_{t-1}\delta\varepsilon_{t+2}$$

$$\Delta_4(\delta) = \beta^2\delta\varepsilon_{t+3}\varepsilon_{t+1}$$

$$\Delta_k(\delta) = \beta\varepsilon_{t+k-1}\Delta_{k-2}(\delta) \text{ in general.}$$

It is seen that even a simple model produces complicated, stochastic forms for these impulse responses. Giving the expected value of the response, that is, the mean, does not capture the distributional aspect of the impulse responses. In this example, the responses are linear in the size of the impulse, δ, but this generally will not be true for most nonlinear models.

For any given nonlinear model with a vector input $\boldsymbol{\varepsilon}_t$ and the one-period impulse, $\delta_s = \delta$ at $s = t$; $\delta_s = 0$; $s \neq t$. $\Delta_k(\delta)$ will be stochastic, depending on both future values of $\boldsymbol{\varepsilon}_t$ and on $\boldsymbol{X}_t$. Most authors have considered the expected impulse such as $E[X_{t+k}(\delta)\, X_{t+k}|X_{tj},\, j > 0]$ [in Potter (2000)] or this quantity divided by ε_t, called the normalized impulse in response by Boswijk and Franses (1996). "Persistence" is determined by the behavior of this quantity as k get large. The obvious

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multivariate version of this definition, with the X_{tj}, $j \geq 0$ term replaced by a large proper multivariate information set, is discussed by Koop et al. (1996). Estimation of these quantities is difficult and compares with that of multistep forecasting from nonlinear models.

As an application, those authors considered an interesting nonlinear bivariate model for U.S. output and unemployment rates. The two variables to be explained are 100 times log real GDP growth (ΔX_t) and the unemployment rate (U_t) for the period 1952 to 1973. Three regimes are defined, a corridor regime in which growth rate is "normal," a ceiling regime in which the economy is "overheating," and a floor regime in which output growth has been low. An overheating variable OH_t is given by

$$OH_t = C_t(OH_{t-1} + \Delta X_t rc),$$

where rc is a given constant and C_t is 1 if ΔX_t and ΔX_{t1} are both greater than rc, zero otherwise. A second variable, designed to measure the current depth of the recession is CDR_t, given by

$$\begin{aligned} CDR_t &= (\Delta X r f) F_t \quad \text{if } F_{t-1} = 0, \\ &= (CDR_{t1} + Y_t) F_t \quad \text{if } F_{t-1} = 1, \end{aligned}$$

where $F_t = 1$ means that the floor regime is active, $F_t = 0$ indicates that it is not, at time t, where

$$\begin{aligned} F_t &= I(\Delta X < rf) \quad \text{if } F_{t-1} = 0, \\ &= I(CDR_{t1} + \Delta X_t < 0) \quad \text{if } F_{t-1} = 1, \end{aligned}$$

where $I\,(Z < 0)$ is the indicator function, so that $I\,(Z < 0) = 1$ if the condition holds, and zero otherwise. Note that no strict ceiling or floor is operating because these ideas are being applied to growth rates rather than to the values of levels.

The nonlinear model that is built is a VAR in ΔX, U but with terms in CDR_{t-1}, OH_{t-1} in each equation. Results of the estimation of this model (rather than impulse responses) are presented in terms of the traditional linear response (expectation) and the distribution of generalized responses, for various lead times. The distributions appear to be symmetric, visually similar to the Gaussian, with variance that slowly increases with lag for output but quickly for unemployment. It is difficult to evaluate the economic usefulness of such results.

Macroeconomists have paid a great deal of attention to the asymmetric effects of shocks. In the short run, one can ask if the shocks from one equation enter another equation in a system positively or negatively in terms of significance. Rhee and Rich (1995) and Karras (1996) used nonlinear moving-average forms but in a way

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that can be directly estimated. For example, if m_t is money, y_t is output (both growth rates), p_t is inflation, O_t is oil price, one can build a model

$$m_t \text{ on lags of } m_t, y_t \text{ producing residual } u_t$$

and then

$$p_t \text{ on lags of } p_t, O_t, u_t^+, u_t, \text{ with residual } v_t$$

where

$$\begin{aligned} u_t^+ &= u_t \quad \text{if } u_t > 0 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

and $u_t^- = u_t - u_t^+$. With this model, Karras investigated annual data from 38 countries (1951–1990) and finds apparent significant evidence of nonsymmetry in 15 of the 38 equations for a money–output system and for 12 of the 38 equations for a money–price system. Rhee and Rich (1995) found that if they use quarterly U.S. data (1961–1990), there is evidence of nonsymmetry with constant parameters on a simple lag of "other equation shocks," but if these parameters are allowed to be time varying with an inflation expectations term, the effect becomes much more significant. Pagan (1984) has warned about potential econometric problems in using estimated residuals from one equation as regressors in another equation, but these warnings are not being given any weight. The ability of the models achieved to forecast is not evaluated in either of these papers.

A somewhat different equation that has been considered is whether positive shocks are more or less persistent than negative ones. Hess and Iwata (1997) suggest that the finding by Beaudry and Koop (1993) that there is a difference may be due to problems with the test used. Using data from the G7 countries, Hess and Iwata consider a unvariate model

$$y_t = \alpha_0 + \alpha_1 y_{t1} + \alpha_2 y_{t1} + \beta\,(\max j \leq t1 y_j y_{t1}) + \varepsilon_t + \theta\varepsilon_{t1}$$

where y_t is the log level of GNP and the max term represents the distance between the current max of the y series and the value of y_{t1}, thus possibly measuring the extent of the present cyclical depression. It is shown that nonstandard distribution for the t-statistics on the coefficients of this equation will occur, so that "spurious relationships" can be found. When corrected for these problems and applied to the G7 countries, evidence of asymmetry is found for the United States, the United Kingdom, and France.

6. REGIME SWITCHINGS AND TIME-VARYING PARAMETERS

If one considers a simple model

$$Y_t = f(X_t, \theta) + \varepsilon_t \tag{3}$$

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relating a pair of series X_t, Y_t, then there are a variety of models available to allow for the parameters to be time-changing. If they vary continuously, so that θ is replaced by θ_t in (3), then one has a standard time-varying parameter model, with the parameters being estimated by the Kalman filter for a linear model, or an extended Kalman filter for a locally linearized model. If θ changes value only occasionally, then the structural-break models are appropriate and if θ switches between just a few possible values, then a regime-switching model can be used. All of these techniques have been used with macroeconomic data in recent years.

An example of time-varying parameters is given by Bacchieta and Gerlach (1997) who consider growth in log consumption in terms of growth in disposable income, consumer credit, and the borrowing/lending wedge for five countries (United States, Canada, United Kingdom, Japan, and France). Evidence is presented that a varying-parameter model is superior to a fixed-parameter model. Sarno and Taylor (1998) use a parametric, nonlinear TVP model to relate growth in consumption to growth in income, real interest rates r_t, and an index of financial liberalization (κ_t) in the model

$$\Delta C_t = \lambda_t \Delta Y_t + (1\lambda_t)\sigma r_{t1} + \varepsilon_t,$$

where

$$\lambda_t = \left[1\phi_0 + \phi_1\kappa_t + \phi_2\kappa_t^b\right].$$

When UK data are used, ϕ_0 is found to be insignificant, ϕ_1 and ϕ_2 are jointly significant, σ is significant, and b is estimated as 3.128 (SE of 0.327). It is thus suggested that evidence of nonlinearity is found.

There have been many applications of Hamilton's (1989) Markov-switching regime-changing models, in which there are two or more regimes, with a specific set of probabilities for staying in a regime or for switching. Recently, the possibility of these probabilities being time-varying has been considered. The regimes have been linked with changes in political regimes and types of policies and with phases of the business cycle. An interesting example is provided by Diebold and Rudebusch (1996). A two-regime time-varying probability switching model was fitted to the Composition Coincidence Index and the major components of that index. They found that that nonlinear model was preferred significantly over a linear, nonswitching model. Hamilton's (1989) paper only allowed for the mean of the process to change with regime. It is natural to allow all parameters, such as those of an autoregressive model or of some nonlinear model, to vary from one regime to another. Lindgren (1978) discusses the background theory for such a model.

Ang and Bekaert (1998) consider a regime-switching VAR model for three short interest rates and three spreads (long–short) using monthly data from the United States, the United Kingdom, and Germany for 1972 to 1996. There is also evidence that multiple regimes can be helpful in explaining short-term rates. Further, univariate switching models are less effective than multicountry models and ones involving other variables such as spreads. I have found that causality can change with regime, and thus is nonlinear.

Raun and Silva (1995) consider a four-state Markov switching regime with either high or low growth in each of output growth and inflation for the United States, the United Kingdom, Japan, and Germany. The correlation between growth and inflation is found to vary by regime and thus over different groups of years depending on what regimes occur. Other users of regime-switching models include Feldstein and Stock (1996), who ask if using time-varying weights on components of a monetary aggregate can produce a better indicator of GNP. Ayuso et al. (1998) and Kaminsky and Lewis (1996) considered Spanish inflation and foreign exchange interventions as signals of monetary policy. Clearly this particular nonlinear modeling technique is both popular and appears to be successful.

An alternative form of nonlinear model that can produce regime switching is the min-max, or M-M model [see Granger and Hyung (1999)], given by

$$x_{t+1} = \max(\alpha x_t + a, \beta y_t + b) + \varepsilon_{t+1},$$
$$y_{t+1} = \min(\gamma x_t + c, \delta y_t + d) + \eta_{t+1},$$

where ε and η are i.i.d., zero mean. It is found that x_t, y_t may not reject the null hypothesis of linearity when univariate tests are used but will do so strongly when a bivariate test is used (six different tests were used, including a neural network test). The special case $\alpha = \beta = \gamma = \delta = 1$ produces individual nonlinear I(1) processes, with a complicated cointegration structure. An application to a pair of interest rates shows that the spread has a threshold-switching form of stationarity similar to that found by Balke and Fomby (1994).

7. COMMON NONLINEAR FACTORS

The idea that several variables all have a property because they share a factor that has that property is often used; cointegration is an example. Anderson and Vahid (1998) ask the question for a vector of series and test for common nonlinear components. A general method of moments test is considered, with specific form of the tests available for particular types of nonlinearity, such as threshold autoregressions (TAR) or smooth-transition autoregressions (STAR), or alternatives for unspecific forms using a neural network test. In their first application, it is suggested that there is a common asymmetry in the business cycles of Canada and the United States. The second example finds an LSTAR (logistic STAR) common factor for U.S. output, consumption, and investment, thus accounting for their nonlinearity according to tests.

As a general warning, note that Balke and Fomby (1994) found from studying a number of single series that, after removing outliers, “much of the evidence of nonlinearity is eliminated.” The impact of outliers on some univariate nonlinear models is discussed by Van Dijk (1999). See also the discussion by Van Dijk and Franses (1997).

8. FORECASTING

Given a reduced-form nonlinear model, one-step least-squares forecasting is simple, assuming that the model is correctly specified and will continue unchanged into the future. However, multistep forecasts are much more difficult, as discussed by Granger and Teräsvirta (1993, ch. 8) and elsewhere. For example, consider the simple bivariate model,

$$Y_t = g(X_{t1}) + \varepsilon_t,$$

where

$$X_t = \alpha X_{t1} + e_t,$$

and where both ε_t, e_t are i.i.d., zero mean. The one-step forecast is just $f^Y_{t,1} = g(X_t)$ but the two-step forecast is given by

$$f^Y_{t,2} = \int_{\infty}^{\infty} g\left(f^X_{t,1} + Z\right) d\Phi(Z),$$

where $\Phi(Z)$ is the distribution function of e_t. Thus, a good two-step forecast depends upon a well-specified function $g(\ )$ as well as the distribution function Φ. If either are incorrectly specified, a suboptimal forecast will result. For multistep forecasts, complicated multiple integrals have to be determined.

An alternative procedure that has pragmatic advantages, although it is theoretically less appealing, is to build separate nonlinear models for different step sizes so that the sequence of models $Y_{t+h} = g_h(X_t) + e_{t,h}$, with $h = 1, 2, 3, \ldots,$ are constructed and used to produce h-step forecasts. This procedure was used by Swanson and White (1995) to assess the usefulness of neural network models to forecast interest rates. The same authors, in a pair of papers [Swanson and White (1995), (1997)], use the same models to forecast a number of macro variables. The artificial neural network model used takes the form

$$Y_t = \boldsymbol{\beta}'\boldsymbol{X}_t + \sum_{j=1}^{q} \lambda_j G(\gamma'_j \boldsymbol{X}_t) + \varepsilon_t,$$

where $G(Z)$ is the logistic cumulative distribution function $G(Z) = 1/(1 + \exp(-Z))$ and $\boldsymbol{X}$ is the vector of explanatory variables. The interest-rate data were spot and futures, end-of-month U.S. Treasury bill rates, and forecasts of 1- to 5-month horizons are considered. The nonlinear models did not beat a set of linear models in the postsample using a mean-squared error (MSE) criterion but did well on other criteria. The authors declared the technique to be promising. The 1997 paper considered nine major quarterly macro variables for the period 1960:1 to 1993:3. Three evaluation measures were used: MSE; mean absolute error; and absolute error divided by actual minus one. Table 1 shows whether the linear or the neural net (nonlinear) model performs best under all three criteria; "(linear)" means best under two of the three criteria. A variety of experiences can be

Table 1. Comparison of linear and nonlinear models

Criterion	One-step horizon	Four-step horizon
Unemployment rate	linear	nonlinear
Corporate bond yield	linear	linear
Industrial production	(linear)	linear
GNP, nominal	nonlinear	nonlinear
Profits after taxes	nonlinear	linear
GNP, real	nonlinear	(linear)
Consumption	linear	nonlinear
Change, business inventories	(linear)	(nonlinear)
Exports	linear	nonlinear

seen—between alternative models, between horizons, and across variables. Only bonds are clearly linear over both horizons, and nominal GNP appears to be nonlinear over both.

A further example of using neural network models for forecasting is presented by Kuan and Liu (1995), who consider five European daily exchange rates against the U.S. dollar, a variety of specifications, two evaluation procedures, root MSE, and the proportion of directions that are forecast correctly. Again, a mixed set of results were obtained: On some occasions, an ARMA model outforecast the neural net model; on other occasions, the reverse situation occurred. There does appear to be evidence of some nonlinearity in these data sets, at least for some time periods.

9. CONCLUSIONS

It appears that nonlinearity is not a strong feature of macro relationships but this incomplete survey does suggest that subtle forms may be found in some situations by some modeling technique. Regime-switching appears to be a successful technique and interest rates are often found to apparently contain nonlinear elements.

The vast majority of the studies involve univariate or bivariate models; trivariate and larger systems are rarely discussed. For example, Stanca (1999) applied several univariate tests to five major macro Italian series with both quarterly and annual data, and found some evidence of nonlinearity, particularly of asymmetry in the business cycle. The comparisons are usually between a linear and a specific nonlinear formulation; two nonlinear models are hardly ever compared. An exception is Jansen and Oh (1999), who compared a depth-of-recession model, a STAR model, and a linear model for U.S. GNP both in and out of sample, and found the depth-of-recession model to be better, also in a univariate situation. Other examples are given by Clements and Krolzig (1998), Sarantis (1999), Montgomery et al. (1998), and Acemoglu and Scott (1994).

In fact, a major weakness in most (but not all) of the studies in the survey is their evaluation phase. It is well known that nonlinear models are inclined to overfit the data, and so, a postsample forecasting evaluation is recommended. A problem encountered in practice is that the events that emphasize nonlinearity may occur infrequently, so that a long postsampling period is necessary. This is particularly true in recent years in those countries having few business cycles, for example. It is unclear how many of the results discussed in the survey would survive a comprehensive postsample evaluation exercise, except those specifically dealing with forecasting.

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