ical operators, together with extraction of roots ("solving by radicals") provides both a focus and a direction for the course. It does not have very demanding prerequisites: a little field and group theory, and a smattering of linear algebra. It seems in fact a convenient application for these topics, reinforcing the students' previous knowledge. Further, by showing that before we can decide whether we can solve a polynomial equation we must define exactly what we mean by "solve", a very useful lesson in precise mathematical modelling is set before the student.

Next, the fact that an "impossibility" proof (of solving by radicals equations with non-soluble Galois group) is produced adds for the student a new dimension to his view of what mathematics can do. It isn't only for proving facts about triangles or vector spaces. It can also be used to show the limitations of a particular constructive technique. Amazing!

Finally, of course, Galois theory has Galois. What a story: the boy genius, the revolutionary politics, the quarrel over a "coquette", the hurriedly written manuscript—"I haven't the time", the duel at dawn, the losses and rejection of his manuscript, its eventual rescue by Liouville,... What other scientific or mathematical topic can boast such a colourful founder?

Garling's book presents Galois theory in a style which is at once readable and compact. The necessary prerequisites are developed in the early chapters only to the extent that they are needed later. The proofs of the lemmas and main theorems are presented in as concrete a manner as possible, without unnecessary abstraction. Yet they seem remarkably short, without the difficulties being glossed over. In fact the approach throughout the book is down-to-earth and concrete. The final chapter, "The calculation of Galois groups", is a good example of this. It contains explicit examples of quintic equations having all possible Galois groups (of orders 5, 10, 20, 60, 120). Also the technique of obtaining information about the Galois group by factoring the polynomial modulo a prime $p$ is explained simply there.

An important definition in Galois theory is that of an extension of a field "by radicals". There are, perhaps surprisingly, several variations of this definition in the literature. Garling (in common with I. Stewart, *Galois theory*, 1973) uses the following one: $L$ is an extension of $K$ by radicals if there are fields $K=K_0\subseteq K_1\subseteq K_2\subseteq\cdots\subseteq K_N=L$, where $K_i=K_{i-1}(\beta_i)$ $(i=1,\ldots,N)$, and some power $\beta_i^{n_i}$ of $\beta_i$ lies in $K_{i-1}$. A polynomial in $K[x]$ is then soluble by radicals if it has all its zeros in some such $L$. According to this definition, however, $x^n-1$ is soluble by radicals simply because the field $L=\mathbb{Q}(\beta_1)$, $\beta_1$ a primitive $n$th root of unity, is obviously an extension by radicals, since $\beta_1^n=1\in\mathbb{Q}$. Thus, in this definition, a primitive $n$th root of unity is regarded as a radical. This definition is simpler than the others, which for instance require that $[K_i:K_{i-1}]=n_i$ (van der Waerden) or that $n_i$ is prime and $K_{i-1}$ already contains a primitive $n_i$th root of unity (W. M. Edwards, *Galois theory*, 1984). It is therefore probably the best one for an undergraduate text. It is not the classical one, however, and for instance renders Gauss's famous expression for the 17th roots of unity using square roots (*Disquisitiones arithmeticae*, Art. 365) irrelevant for solving $x^{17}-1$ by radicals (though not of course for the construction of the regular 17-gon).

Because of its clarity and economy of expression, I can heartily recommend this book as an undergraduate text. It would need, at least for background reading, to be supplemented by a historical perspective on the subject (e.g. from Edwards or Stewart) in order to motivate the subject. All right, we want solutions of polynomial equations, but why solution by *radicals*? Further, since the book presents the subject in a rather closed way, mention would also have to be made of other ways of solving polynomial equations. The same applies to extensions of Galois theory, and to unsolved problems on the subject, which receive only a passing reference in the book. For indeed, as the author says, "Galois theory has a long and distinguished history: nevertheless, many interesting problems remain".

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MATSUMURA, HIDEYUKI, *Commutative ring theory* (Cambridge studies in advanced mathematics 8, Cambridge University Press, 1986), pp. 320, £30.

Professor Matsumura is already well known for his standard text *Commutative Algebra* (Benjamin/Cummings, Reading, Mass., 1969), reissued in 1980 with a number of substantial appendices added. This is a wide-ranging and scholarly work, and an essential reference. However it is rather austere and self-enclosed, with little added in the way of commentary to sketch in background or motivation. The present book, splendidly translated by Miles Reid, is written with a much lighter and more personal touch. There are many stimulating asides on the history of the subject, and on the wider connections of the material with algebraic geometry or within commutative algebra itself; as well, the author does not hesitate to state his own views on what are the highlights of the theory—this may well provoke lively debate! The reader is supplied with plenty of guidance on the direction the material is taking, and there is a goodly number of useful exercises to try (with solutions sketched at the back of the book). As well, there are three short appendices on tensor products, homological algebra and the exterior algebra. Altogether it is a superb text for a beginner at postgraduate level.

The material is cleverly organised so as to present the reader almost immediately with mathematics of substance and to indicate quickly the wide range of algebraic, geometric and homological approaches which gives commutative algebra its allure. Thus, for example, the early discussion of modules includes a proof of Kaplansky's result that a projective module over a local ring is free. Or again, the set of primes at which a finitely generated module can be generated by a fixed number of elements is shown to be an open subset of the spectrum; this indicates the importance of geometrical conceptualization.

Most of the material in the earlier work *Commutative Algebra* appears in the present book, with some of the more technical reaches sketched in or simply left as references. This allows the inclusion of much valuable new material, which makes the book an essential reference for future work. For example there are sections on Rees rings (together with their geometrical interpretation, in particular as a deformation space), on Gorenstein rings, on multiplicity theory and reductions of ideals, on Ratliff's work on catenary rings and on chains of prime ideals, on the theory of higher derivations and on finite free solutions and the Euler characteristic. There is also a more leisurely discussion of the structure theorems for complete local rings, including the description of the regular ramified case in terms of Eisenstein extensions.

As already mentioned, the wider aspects of the material are continually brought in to stimulate the reader. For example, there are brief discussions on normal flatness and the resolution of singularities, on André cohomology and on some of the homological conjectures. (Roberts' recent proof of the Intersection Theorem has now settled the Bass conjecture.) As regards the geometrical side, the section on unique factorization domains is spiced with a description of the interpretation of Picard and class groups in terms of divisors on varieties. Again, the section on valuation rings includes a discussion of the Zariski Riemann surface; very typically, the author takes this opportunity to mention the historical background, *viz.* Zariski's use of this theory in the resolution of singularities in low dimensions.

There are some minor cavils. The basic result on the behaviour of primes under localization could be focused more sharply; as it stands, the aspect concerning uniqueness is somewhat submerged. After the discussion of the Nullstellensatz, the opportunity could have been taken to spell out explicitly the connection between maximal ideals and points (and between prime ideals and varieties); this would have made the geometrical discussions in the text more self-contained. There are a small number of minor misprints. But these are trivial points, set against the authoritative and all-embracing nature of this book. It represents a masterful achievement, and a substantial debt of gratitude is owed to Miles Reid in helping to make this work available to non-readers of Japanese.

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