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The effects of risk aversion and density of contribution on comparisons of administrative charges in individual account pension systems

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Abstract

This paper studies the effects of risk aversion and density of contribution (DoC) on comparisons of proportional charges on flow (contributions) and balance (assets) during the accumulation phase of a defined-contribution pension plan in a system of individual retirement accounts. If the participant's degree of risk aversion increases and both charges yield the same expected terminal wealth, then the charge on balance improves with respect to the charge on flow when performing comparisons that examine the ratio between the resulting expected utilities of terminal wealth. When this methodology is applied to the Peruvian Private Pension System, empirical results demonstrate that the aforementioned result also holds for arbitrary charges on flow and balance and that the effect of DoC on these comparisons is nearly negligible for most of the assessed scenarios.

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1 Introduction

The asset allocation, performance and risk of a defined-contribution (DC) pension plan during its accumulation and decumulation phases have received considerable interest in the literature since DC plans became viable alternatives to defined-benefit (DB) plans.[1] Two important characteristics of a DC plan are that affiliates

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[1] The following is by no means a thorough or complete literature review; however, we want to mention different examples addressing the issues in DC pension funds. For example, Blake *et al.* (2001), using different models for asset returns and portfolio strategies, estimate the value-at-risk of the pension ratio for a DC pension fund. Poterba *et al.* (2005) calculate the expected utility of retirement wealth for different investment strategies and assumptions. Devolder *et al.* (2003) derive several optimal portfolio strategies for different types of utility functions assuming the risky asset follows a GBM. Gao (2009) provides a similar analysis but under a constant elasticity variance (CEV) process. The efficiency of the mean-variance portfolio selection in a DC pension plan is studied in Vigna (2014). Haberman and Vigna (2002) consider the downside risk of an optimal asset allocation strategy derived from a discrete-time dynamic programming approach. Salary risk and inflation risk were incorporated by Battocchio and Menoncin (2004) and Han and Hung (2012) when maximizing the expected utility of terminal

(participants) bear the risk associated with fluctuations in asset values and that imposed administrative charges have direct and significant effects on the terminal wealth of the corresponding individual retirement accounts (IAs).[2] Substantial attention has been devoted to these charges, particularly in countries that have partially or completely transformed their public DB pension systems into individual capitalization systems.[3] Furthermore, high charges in IA systems are a primary target of criticism; these charges discourage participation, damage systems' reputations, reduce future pensions and increase future costs for the government in cases involving a guaranteed minimum pension (James *et al.*, 2001 and Whitehouse, 2001).

The objective of this paper is to compare the outcome when fees are levied proportional to flow (or as a percentage of the affiliate's salary) with the outcome when fees are proportional to assets (the balance in the IA).[4] These two types of administrative charges are the most common fees in IA pension systems (Kritzer *et al.*, 2011).[5] This study was partly motivated by the most recent reform of the Peruvian Private Pension System (PPS), which occurred in 2012 and required participants to choose one of the two aforementioned types of fees. With the objective of evaluating the likely impact of this reform and similar reforms on the welfare of participants in IA pension systems, we construct a model to compare both types of fees and analyze how these fees affect welfare when interacting with both risk aversion and the stability of contributions

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wealth. Battocchio *et al.* (2007) and Yang and Huang (2009) include longevity risk in the optimal asset allocation of a DC plan; expected utility was used as an objective for the former, and deviation of terminal wealth with respect to a predetermined target for the latter. Stochastic lifestyling under terminal utility with habit formation is identified and compared with other strategies in Cairns *et al.* (2006). Finally, readers interested in the analysis of optimal allocation during the decumulation phase can be referred to, among others, Blake *et al.* (2003), Gerrard *et al.* (2004), Horneff *et al.* (2008) and Gerrard *et al.* (2006).

[2] Devesa-Carpio *et al.* (2003) consider the charge scheme adopted by the IA system to be very important because the fund accumulation process is exponential and targeted for long horizons. For example, Murthi *et al.* (2001) estimate that in the UK, over 40% of the IA's value is dissipated through fees and charges, whereas Whitehouse (2001) determines that a levy of 1% of assets adds up to nearly 20% of the final pension value.

[3] The most familiar and documented example is Chile. The reader can find the primary aspects of this reform in Arrau *et al.* (1993), Diamond and Valdés-Prieto (1994), Edwards (1998), Arenas de Mesa and Mesa-Lago (2006). An analysis of the Peruvian pension system's reform and its current state can be found in Marthans and Stok (2013). Queisser (1998), Sinha (2000), Kay and Kritzer (2001), Mesa-Lago (2006) and Kritzer *et al.* (2011) provide good references for the reform, situation and perspective of pension systems in Latin America. Chlon *et al.* (1999), Chlon (2000) and Holzmann *et al.* (2003) provide information about the reforms in some European countries.

[4] Queisser (1998) considers the charge on flow to be more advantageous for the PFA in the initial stages of the system, and although the charge on balance aligns the PFA's objectives in terms of increasing the fund's profitability, it tends to be more expensive in the long-run as personal accounts grow in size. Moreover, Shah (1997) mentions that the charge on flow generates distortions and undesirable tendencies by, e.g., promoting high start-up costs for the PFAs, discouraging competition in the system and generating losses for older affiliates.

[5] Analyses and comparisons of administrative charges across different countries can be found in James *et al.* (2001), Whitehouse (2001), Gómez-Hernández and Stewart (2008), Corvera *et al.* (2006), Tapia and Yermo (2008) and Devesa-Carpio *et al.* (2003). Moreover, Sinha (2001), Masías and Sánchez (2007) and Martínez and Murcia (2008) analyze in detail (although there have been some modifications) the administrative charges in Mexico, Peru and Colombia, respectively. Finally, Medina Giacomozzi *et al.* (2013) study the effect of administrative charges in the profitability of Chilean pension funds.

over an affiliate's working life. Such stability can be measured by density of contribution (DoC), which is defined as the ratio between the periods during which contributions are paid or credited and the total number of potential contribution periods.[6,7]

As noted in the literature review, the optimal asset allocation of DC funds is primarily studied using an expected utility maximization criterion; however, when administrative charges are compared, it is customary to utilize procedures or techniques that are based either on an IA's expected terminal wealth or on the assumption of risk-neutral preferences. Therefore, we close this literature gap by introducing risk aversion into comparisons of fees on flow and balance. To perform these comparisons, we use the ratio between the expected utility of terminal wealth generated with a charge on balance and the corresponding expected utility for a charge on flow. Assuming that both charges yield the same expected terminal wealth and the expected DoC remains constant during the accumulation phase, we prove that the more risk-averse the affiliate is, the more the charge on balance improves with respect to the charge on flow. We also compare the fee schemes using the ratio between expected value and the standard deviation of terminal wealth and demonstrate that the charge on balance always generates a better ratio when the contribution stream is uninterrupted. However, if DoC is introduced into this final scenario, then the growth rate of the IA must be greater than the growth in contributions (salaries) to achieve the same theoretical result.

We apply the proposed methodology in comparisons of the fees charged to the PPS's participants. For a constant charge of 1% on balances (as a ratio of total fees to total assets of the system) and a charge on flow equal to 16.2% of every contribution (the PPS's average fee), a representative participant with an accumulation period of 45 years and a high degree of risk aversion could obtain an 8% greater certainty equivalent for terminal wealth when fees are levied on flow instead of balance. In a case involving a low degree of risk aversion, this difference is reduced to 4%. However, if the accumulation phase is only 15 years, the charge on balance generates a certainty equivalent 10% greater than the certainty equivalent for the charge on flow, and the effect of risk aversion becomes almost negligible. Moreover, for all examined degrees of risk aversion, assuming fixed charges as specified above, the charge on flow is preferable for participants younger than 28 years of age, whereas the charge on balance is preferable for affiliates older than 34 years of age. Finally, DoC does not constitute an important variable in these comparisons because certainty equivalent ratios remain approximately constant despite dramatic changes in DoC.

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[6] DoC has an important effect on the replacement rate and wealth at the end of the accumulation phase. McGillivray (2001) determines (under mild assumptions consisting of wage growth of 2%, interest earnings of 4%, and 40 years of contribution) that the replacement rate of a DC fund under a 100% DoC will be 50%; i.e., the life annuity will be 50% of the wages corresponding to the 40th year. However, with an 80% DoC, the replacement rate can be as low as 37% depending on the interruption profile. The characteristics and determinants of DoC in a private security system are studied in Arenas de Mesa *et al.* (2004).[7] Additionally, its effect on the replacement rate for several countries in Central and Latin America and its implications for pension design can be found in Durán and Pena (2011) and Valdés-Prieto (2008), respectively.

[7] The paper draws conclusions about the Chilean case, but the methodology and some conclusions can be extended to other DC pension systems, particularly in Latin America.

The remainder of the paper proceeds as follows. Section 2 introduces a methodology for mathematically representing charges on balance and flow and the interruption of contributions in a DC pension plan. Section 3 presents and analyzes the ratios used for comparisons of the aforementioned administrative charges. Section 4 discusses the application of the methodology to the Peruvian PPS, and Section 5 draws conclusions.

2 Methodology

Consider $i \in \mathbb{N}$ and $T \in \mathbb{N}^+$ such that $0 \leq i \leq T-1$, where i represents a particular month and T is the number of months before the affiliate's retirement, i.e., the length of her accumulation phase. We assume that the share value, V, of a representative pension fund managed by a Pension Fund Administrator (PFA) at time $t \in \mathbb{R}^+$ (expressed in months) satisfies the following stochastic differential equation (SDE):

$$dV(t) = \mu V(t)dt + \sigma V(t)dB(t), \qquad V(0) = V_0, \tag{1}$$

where μ is the monthly growth rate of the share value, σ is the monthly volatility of its log-returns, V_0 is the initial share value, and B is a one-dimensional standard Brownian motion. The SDE in (1) is a common specification to model asset values, and it is heavily utilized in the stochastic control of DC pension funds, as mentioned in the introduction. Next, we describe in detail the charges on flow and balance using a structure similar to those considered in Shah (1997), Diamond (2000), Blake and Board (2000), Whitehouse (2001), Devesa-Carpio *et al.* (2003) and Gómez-Hernández and Stewart (2008).

2.1 Charge on balance

Let $\delta > 0$ be the monthly charge on balance[8] expressed in continuous time, and let $\mathcal{W}_T = \{W_i \mid W_i > 0,\ 0 \leq i \leq T-1\}$ be the affiliate's contribution stream. Then, at the beginning of month i the affiliate contributes an amount W_i to her individual account.[9] If the share value, V, is normalized to the unit in month i, then contribution W_i is equivalent to the same number of shares. For $t \geq i$, and considering the SDE in (1), month i's contribution will evolve according to the following geometric Brownian motion (GBM):

$$W_s^i(t) = W_i e^{(\mu-\delta-(\sigma^2/2))(t-i)+\sigma(B(t)-B(i))}, \qquad i \leq t \leq T. \tag{2}$$

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[8] It is also known as a charge on assets or on stock, and we will use the subscript 's' to identify this type of charge. In general, the charge on balance is applied as a percentage of the value of the assets under management in the affiliate's individual account. Additionally, a constant δ might imply that the pension system has achieved 'maturity' with respect to this type of charge.

[9] $\mathcal{W}_T$ could be interpreted as the sequence of representative contributions for homogeneous groups of affiliates sharing a common accumulation horizon T. For example, it is possible to determine $\mathcal{W}_T$ from a certain wage projection depending on an average growth rate that is a function of, e.g., age, gender and education level.

It is in the affiliate's interest to determine the value of her IA at the end of the accumulation period. We denote the final wealth as $W_s(T)$, and it corresponds to the sum of the terminal values of all contributions made according to (2). Consequently,

$$W_s(T) = \sum_{i=0}^{T-1} W_s^i(T), \tag{3}$$

where all processes W_s^i are driven by the same source of uncertainty B in SDE (1).

2.2 Charge on flow

Let $\alpha > 0$ be the charge on flow.[10] If the affiliate makes a contribution W_i in month i, then the charge she will pay to the PFA (at the moment the contribution is made) will be equal to $C_i = W_i(1 - e^{-\alpha})$. Considering that C_i is not discounted from W_i (which is the most common case in DC pension funds), it could have been invested in the fund, so it is possible to express contribution W_i as $e^{-\alpha} W_i$ to adjust for the opportunity cost of C_i. Under this assumption, the adjusted contribution in month i, W_f^i, will evolve based on the following GBM:

$$W_f^i(t) = W_i e^{-\alpha} e^{(\mu - (\sigma^2/2))(t-i) + \sigma(B(t) - B(i))}, \quad i \leq t \leq T. \tag{4}$$

The affiliate considers it important to compute her final wealth adjusted for the charge on flow. If we denote this amount as $W_f(T)$, we obtain

$$W_f(T) = \sum_{i=0}^{T-1} W_f^i(T). \tag{5}$$

Recall that $W_f(T)$ represents not the affiliate's true wealth at the end of the accumulation phase but her final wealth adjusted for the opportunity cost generated by the charge on flow. The terminal wealth in her individual account will be equal to $e^{\alpha} W_f(T)$. Consequently, only when the opportunity cost is considered can variables $W_f(T)$ and $W_s(T)$ be compared.

2.3 Interruption in contributions

There is a possibility that for some month i, the affiliate will not be able to contribute an amount W_i to her IA. To represent such interruptions, we introduce sequence $\mathcal{P}_T = \{p_i \mid p_i \in [0, 1], 0 \leq i \leq T - 1\}$ and a stochastic process $Z = \{Z_i, \ 0 \leq i \leq T - 1\}$, independent of process B, such that Z is a sequence of independent Bernoulli (p_i) random variables with at least one i^* such that $p_{i^*} > 0$. If $Z_i = 0$, then there is no contribution in period i, which occurs with probability $1 - p_i$. Note that process Z is one of the simplest ways to introduce interruptions in $\mathcal{W}_T$. We also define

$$\widehat{W}_s(T) = \sum_{i=0}^{T-1} Z_i W_s^i(T), \tag{6}$$

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[10] It is also known as a charge on contribution, and we will use the subscript '*f*' to identify this type of charge. Additionally, this charge is applied as a percentage of the affiliate's salary or contributions.

$$\widehat{W}_f(T) = \sum_{i=0}^{T-1} Z_i W_f^i(T), \tag{7}$$

where $W_s^i(T)$ and $W_f^i(T)$ are given by (2) and (4), respectively. Both $\widehat{W}_s(T)$ and $\widehat{W}_f(T)$ represent the adjusted terminal wealth (under balance and flow charges) when interruptions are introduced by means of the stochastic process Z.

If we compute the expected values of $\widehat{W}_s(T)$ and $\widehat{W}_f(T)$, we have

$$\mathbb{E}[\widehat{W}_s(T)] = \sum_{i=0}^{T-1} \mathbb{E}[Z_i W_s^i(T)] = e^{(\mu-\delta)T} \sum_{i=0}^{T-1} p_i W_i e^{-(\mu-\delta)i}, \tag{8}$$

$$\mathbb{E}[\widehat{W}_f(T)] = \sum_{i=0}^{T-1} \mathbb{E}[Z_i W_f^i(T)] = e^{-\alpha+\mu T} \sum_{i=0}^{T-1} p_i W_i e^{-\mu i}. \tag{9}$$

To obtain (8) and (9), we used the independence of Z_i with respect to $W_s^i(T)$ and $W_f^i(T)$. Moreover, the variances of $\widehat{W}_s(T)$ and $\widehat{W}_f(T)$ can be determined using the following proposition.

Proposition 2.1. (Variance of adjusted terminal wealth).
Under interruption process Z, the variances of $\widehat{W}_s(T)$ and $\widehat{W}_f(T)$ in (6) and (7) are

$$\begin{aligned}\mathrm{Var}(\widehat{W}_s(T)) = &\sum_{i=0}^{T-1}\sum_{j=0}^{T-1} p_i p_j W_i W_j e^{(\mu-\delta)(T-i+T-j)}\left(e^{\sigma^2(T-\max\{i,j\})} - 1\right)\\ &+ \sum_{i=0}^{T-1} p_i(1-p_i) W_i^2 e^{(2(\mu-\delta)+\sigma^2)(T-i)},\end{aligned} \tag{10}$$

$$\begin{aligned}\mathrm{Var}(\widehat{W}_f(T)) = &\sum_{i=0}^{T-1}\sum_{j=0}^{T-1} p_i p_j W_i W_j e^{-2\alpha+\mu(T-i+T-j)}\left(e^{\sigma^2(T-\max\{i,j\})} - 1\right)\\ &+ \sum_{i=0}^{T-1} p_i(1-p_i) W_i^2 e^{-2\alpha+(2\mu+\sigma^2)(T-i)}.\end{aligned} \tag{11}$$

Proof. See Appendix A.1 (online).

The variances in the absence of interruptions, $\mathrm{Var}(W_s(T))$ and $\mathrm{Var}(W_f(T))$, can be found considering $p_i = 1$ for all i in (10) and (11). If we assume $p_i = p$ for all $0 \leq i \leq T-1$, then the affiliate has the same probability of contributing each month, and p can be interpreted as the affiliate's DoC.

2.4 Risk factors

So far, we have considered two main risk factors: the return on assets given by SDE (1) and the interruption in contributions given by the stochastic process Z defined in Section 2.3. We have not considered randomness in the level of contributions, even though they are often expressed as a percentage of wages and are indexed to inflation. Moreover, in a market with stochastic returns, it is well known that a correlation exists between inflation rates and returns.[11] Consequently, introducing randomness to returns but not to contributions could lead to false conclusions because both depend on inflation. We could work in nominal terms imposing a stochastic process to $\mathcal{W}_T$ with some correlation with process V; however, we preferred to work with a deterministic sequence of contributions $\mathcal{W}_T$; to account for inflation, we are considering and calibrating both returns (given by process V) and contributions in real terms.

2.5 Terminology

Throughout this section, we have used terminology related to DC pension plans in Peru and other countries in Latin America with private pension systems – more specifically, the cases of Chile, Colombia and Mexico. However, there is a possibility that either the definitions or the methodology might be opaque to an audience familiar with DC institutions in other countries. In an effort to add clarity to the paper, we will try to relate our definitions to those for 401(k) plans in the USA.

The employer serves as the plan sponsor for the 401(k) and hires a firm, the plan vendor (it may be a mutual fund company, a brokerage firm or an insurance company), to administer the plan and its investments. In our methodology, the PFA acts as the plan vendor but with the difference that it is never selected by the employer but instead it is chosen by the employee. Therefore, the figure of the plan sponsor does not exist in our case. Once an employee voluntarily signs up for a 401(k), she becomes a participant (or what we call an 'affiliate'), and then she decides how much to contribute to her individual account. In our framework, participation in the DC plan is mandatory and the rate of contribution is fixed as a percentage of the employee's monthly wage.[12] Typically, a 401(k) participant cannot withdraw her money before reaching 59.5 years of age, and early withdrawals are usually subject to a penalty. For example, in the Peruvian PPS, it is not possible to withdraw any part of the individual fund before reaching 65 years of age (the common retirement age); there are options for early retirement after the age of 50 but only in cases of prolonged unemployment.[13]

Plan vendors in the USA usually offer a wide array of investment alternatives to participants; on the contrary, Latin American PFAs generally offer limited choices. As an example, Peruvian PFAs manage only three funds – high, medium (default

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[11] It is well documented that expected inflation, unexpected inflation, and changes in expected inflation are all negatively related to stock returns. See Fama and Schwert (1977) and Geske and Roll (1983). However, Boudoukh and Richardson (1993) found evidence suggesting that long-horizon nominal stock returns are positively related to both ex-ante and ex-post long-term inflation.

[12] In 401(k) plans, there is a maximum amount that can be contributed in a year, and in some cases, the employer matches the employee's contributions. Additionally, these plans have tax advantages because contributions are discounted from the employee's paycheck before taxes are deducted. On the contrary, in Latin American DC funds, there is no contribution limit; contributions are neither matched by the employer nor tax-deductible.

[13] If the employee leaves the company when she is in a 401(k) plan, she has the following options: roll the money over either to an Individual Retirement Account (IRA) or to the new employer's 401(k) plan, leave the money in the original plan (if applicable) or cash out and take the money as a distribution. In our case, the individual account is always attached to the PFA, and the affiliate does not have the option to cash the money out when leaving or changing companies; however, she is free to move her individual account to another PFA at any time.

option) and low risk – and the affiliate decides in which to invest her contributions. To cover the expenses of providing a 401(k) plan,[14] fees are paid by the plan itself, the employer, and/or the plan participants. These fees can be levied based on the number of participants, the amount of assets, or as a fixed dollar amount for the plan as a whole. In Latin American DC funds, the administrative charges introduced in Sections 2.1 and 2.2 cover the aforementioned expenses and are levied by the PFA but are paid only by the affiliate; i.e., the employer does not pay any fee or charge to the PFA. Finally, it is common that the PFA establishes a unique fee (either on balance or flow) for all its affiliates, so the fee structure is simpler than that of 401 (k) plans.

3 Comparing charges on balance and flow

A particular affiliate wants to determine or assess the suitability of the administrative charges by contrasting $\widehat{W}_s(T)$ and $\widehat{W}_f(T)$. The comparison could be performed using, among other forms, expected values, ratios of expected value to standard deviation, and expected utilities. We proceed to describe each of the three methods in detail.

3.1 Expected terminal wealth

If a comparison is performed using the expected value of adjusted terminal wealth, then we can define

$$\mathrm{RE}_{sf} = \frac{\mathbb{E}[\widehat{W}_s(T)]}{\mathbb{E}[\widehat{W}_f(T)]}, \tag{12}$$

where $\mathbb{E}[\widehat{W}_s(T)]$ and $\mathbb{E}[\widehat{W}_f(T)]$ are given by expressions (8) and (9), respectively. In this criterion, if $\mathrm{RE}_{sf} > 1$, the charge on balance will be preferred. If $\mathrm{RE}_{sf} < 1$, then the charge on flow will be preferred. This criterion will be adequate for a risk-neutral affiliate who only cares about adjusted terminal wealth. The next proposition studies the effect of the growth rate μ on RE_{sf}.

Proposition 3.1. (*Derivative of* RE_{sf} *with respect to* μ).
In process Z, we assume there exists $i^{**} \neq i^*$ such that $p_{i^{**}} \in (0, 1]$ and i^* was defined in Section 2.3. If we consider RE_{sf} in (12) as a function of the growth rate μ, then for every μ and $T > 1$:

$$\frac{\partial \mathrm{RE}_{sf}(\mu)}{\partial \mu} < 0. \tag{13}$$

Proof. See Appendix A.2 (online).

Proposition 3.1 shows that higher growth rates benefit the charge on flow relative to the charge on balance, or more intuitively, if the mean return increases, then the fund

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[14] Many types of services are required to operate a 401(k) plan including administrative services (e.g., recordkeeping and transaction processing), participant-focused services (e.g., participant communication, education, or advice), regulatory and compliance services (e.g., plan documentation services; consulting, accounting and audit services, and legal advice) and investment management.

tends to become more important. Thus, not surprisingly, fees on balance are less interesting to the affiliate. Note that result (13) is independent of the structure of sequences $\mathcal{W}_T$ and $\mathcal{P}_T$. Moreover, if $p_i = p$ for all i with $p \in (0, 1]$, RE_{sf} will be independent of the interruption process Z and the DoC, p. Next, we introduce the equivalent charge on balance for a risk-neutral affiliate.

Definition 3.1. (*Risk-neutral equivalent charge on balance*).

Given a set of parameters $\mathbf{I} = \{T, \alpha, \mu, \sigma^2, \mathcal{W}_T, \mathcal{P}_T\}$, we define the risk-neutral equivalent charge on balance, $\delta_{\mathrm{I}}^*(\mathbf{I})$, as the value of δ such that $\mathrm{RE}_{sf} = 1$ under scenario $\mathbf{I}$.

For example, if we want to show the explicit dependence of $\delta_{\mathrm{I}}^*(\mathbf{I})$ with respect to T, α, or both, we can use $\delta_{\mathrm{I}}^*(T)$, $\delta_{\mathrm{I}}^*(\alpha)$ and $\delta_{\mathrm{I}}^*(T, \alpha)$, respectively. Additionally, we will use δ_{I}^* when we refer to this equivalent charge in a general situation. Note that $\delta_{\mathrm{I}}^*(T = 1, \alpha) = \alpha$ and $\partial_\alpha \delta_{\mathrm{I}}^*(T, \alpha) > 0$ for any scenario $\mathbf{I}$, and by Proposition 3.1 we have $\partial_\mu \delta_{\mathrm{I}}^*(T, \mu) < 0$ for $T > 1$. Finally, because sequences $\mathcal{W}_T$ and $\mathcal{P}_T$ are assumed to be unstructured, we are not able to infer, for example, that $\delta_{\mathrm{I}}^*(T)$ is a decreasing function in T.

3.2 Ratio of expected value to the standard deviation of terminal wealth

It is also possible to determine the convenience of one of the two schemes using the ratio of expected terminal wealth (adjusted for charges) to its corresponding standard deviation. For the charges considered, the ratios are

$$\mathrm{H}_s = \frac{\mathbb{E}[\widehat{W}_s(T)]}{\sqrt{\mathrm{Var}(\widehat{W}_s(T))}} \quad \text{and} \quad \mathrm{H}_f = \frac{\mathbb{E}[\widehat{W}_f(T)]}{\sqrt{\mathrm{Var}(\widehat{W}_f(T))}}, \tag{14}$$

where $\mathbb{E}[\widehat{W}_s(T)]$ and $\mathbb{E}[\widehat{W}_f(T)]$ are given by (8) and (9), and $\mathrm{Var}(\widehat{W}_s(T))$ and $\mathrm{Var}(\widehat{W}_f(T))$ are given by (10) and (11), respectively. With this criterion, if a particular set of parameters satisfies $\mathrm{H}_s > \mathrm{H}_f$, then the charge based on assets will be preferred. The next proposition establishes the conditions for process Z such that $\mathrm{H}_s > \mathrm{H}_f$ holds.

Proposition 3.2. (*Derivative of* H_f *with respect to* μ *under no interruptions*).
In process Z, we assume $p_i = 1$ for all i, and we consider H_f in (14) as a function of the growth rate μ. Then, for any μ and $T > 1$ we have

$$\frac{\partial \mathrm{H}_f(\mu)}{\partial \mu} < 0. \tag{15}$$

Proof. See Appendix A.3 (online).

Expression (15) is equivalent to $\mathrm{H}_s > \mathrm{H}_f$ for any common set of parameters and $T > 1$ because $\mathrm{H}_f > 0$, $\delta > 0$, and α cancels in H_f. Consequently, the charge on balance, in the absence of interruptions, always generates a better ratio H than that for the charge on flow. Proposition 3.2 also establishes that the standard deviation of the adjusted terminal wealth grows faster than its expected value as the fund's growth rate increases. If we assume $p_i = p \in (0,1]$ for all i and express the ratios in (14) as a function of the DoC, then we have $\mathrm{H}_s(p)$ and $\mathrm{H}_f(p)$, and their partial derivatives satisfy $\partial_p \mathrm{H}_s(p) > 0$ and $\partial_p \mathrm{H}_f(p) > 0$. Hence, increments in the probability of contributing improve the corresponding ratios. The next proposition gives conditions under which $\mathrm{H}_s(p) > \mathrm{H}_f(p)$ holds when $0 < p < 1$.

Proposition 3.3. (*Effect of DoCs on* H_s *and* H_f).
If $\mu - \delta + \sigma^2 > 0$, $T > 1$, $W_{i+1} \leq W_i$ and $p_i = p$ for all $0 \leq i \leq T-1$ with $p \in (0,1]$, then $\mathrm{H}_s(p) > \mathrm{H}_f(p)$ for all p.

Proof. See Appendix A.4 (online).

We can assume that $\mathcal{W}_T$ satisfies the corresponding exponential growth model

$$W_i = W_0 e^{\beta i}, \quad \beta \in \mathbb{R}, \quad W_0 > 0 \quad \text{and} \quad 0 \leq i \leq T-1. \tag{16}$$

Then, β is the monthly real growth rate of contributions, and the initial contribution is equal to W_0. Note that in this model[15] we have $W_i > 0$ for all i. The next corollary studies Proposition 3.3 when the contributions follow model (16).

Corollary 3.1 (*Proposition under the exponential growth model of contributions*).
If $\mu - \delta + \sigma^2 > \beta$, $p \in (0,1]$, $T > 1$, W_i as in (16) and $p_i = p$ for all $0 \leq i \leq T-1$, then $\mathrm{H}_s(p) > \mathrm{H}_f(p)$ for all p.

Proof. See Appendix A.5 (online).

Corollary 3.1 states that if the fund's net growth rate, $\mu - \delta$, is greater than the growth in contributions, β, then the charge on balance will always be preferred (using the ratio of expected value to standard deviation criterion) even in the presence of interruptions.

3.3 Expected utility of terminal wealth

We assume the affiliate exhibits a Constant Relative Risk Aversion (CRRA) utility function given by

$$U(\widehat{W}) = \frac{\widehat{W}^{1-\gamma}}{1-\gamma}, \tag{17}$$

where $\widehat{W} > 0$ is a realization of adjusted terminal wealth, and $\gamma > 0$ measures the degree of relative risk aversion.[16] To determine the most appropriate charge scheme, the

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[15] Alternative models for $\mathcal{W}_T$ can be found in the literature. For example, Devesa-Carpio and Vidal (1997) propose a model in which real contributions increase with age until reaching a maximum, and from this point onwards they gradually decrease. Carriere and Shand (1998) assume that real contributions increase with age but at a decreasing rate because the merit factor decreases as time passes. Devesa-Carpio *et al.* (2003) propose a polynomial function to model each contribution as a function of age.

[16] If $\gamma = 1$, then $U(\widehat{W}) = \ln(\widehat{W})$. The CRRA utility is a common specification in pension fund research. See, for example, Boulier *et al.* (2001), Blake *et al.* (2003), Deelstra *et al.* (2003), Poterba *et al.* (2005), De Jong (2008), Gao (2009), Siegmann (2011), etc.

affiliate compares the expected utilities $\mathbb{E}[U(\widehat{W}_s(T))]$ and $\mathbb{E}[U(\widehat{W}_f(T))]$, or their corresponding certainty equivalents, $\mathrm{CE}[\widehat{W}_s(T)]$ and $\mathrm{CE}[\widehat{W}_f(T)]$, using the following expressions:

$$\Upsilon = \frac{\mathbb{E}[U(\widehat{W}_s(T))]}{\mathbb{E}[U(\widehat{W}_f(T))]}, \quad \text{and} \quad \Delta\mathrm{CE}_{sf} = \frac{\mathrm{CE}[\widehat{W}_s(T)]}{\mathrm{CE}[\widehat{W}_f(T)]} - 1. \tag{18}$$

Under the expected utility criterion, if $\Delta\mathrm{CE}_{sf} > 0$, the charge on balance will be preferred. If $\Delta\mathrm{CE}_{sf} < 0$, then the charge on flow will be preferred. However, if she uses Υ for the comparison, then different cases need to be considered. For example, $\gamma > 1$ implies $U(\widehat{W}) < 0$, so the charge on balance will be preferred if $\Upsilon < 1$. On the contrary, $\gamma < 1$ implies $U(\widehat{W}) > 0$, and the charge on balance will be preferred if $\Upsilon > 1$. An interesting feature of utility (17) is that it preserves indifferences and preferences when multiplied by a positive constant factor. For example, if $\mathcal{W}_T$ satisfies (16), then the ratios in (18) become independent of the initial contribution.

Closed-form expressions for expected utilities and certainty equivalents in (18) are not available, and although simulation is a good technique for obtaining the corresponding estimators, the error involved in such computations (due to the embedded stochastic processes and the complicated relationship of the variables) is likely to obscure the analysis or reduce its scope. An alternative approach consists of constructing a Taylor series expansion of (17) around the expected value of adjusted terminal wealth, $\mathbb{E}[\widehat{W}]$. Expressing $U(\widehat{W})$ for some $\widehat{W}$, we obtain

$$U(\widehat{W}) = \sum_{j=0}^{\infty} U^{(j)}(\mathbb{E}[\widehat{W}]) \frac{(\widehat{W} - \mathbb{E}[\widehat{W}])^j}{j!}, \tag{19}$$

where $U^{(j)}(\mathbb{E}[\widehat{W}])$ is the jth derivative of the utility function at $\mathbb{E}[\widehat{W}]$. Loistl (1976) showed that (19) converges for the CRRA utility when $0 < \widehat{W} < 2\mathbb{E}[\widehat{W}]$. Because we have derived closed-form expressions for the first and second moments of $\widehat{W}$, we find it interesting to use the second-order approximation of (17), U_{MV}, given by

$$U_{\mathrm{MV}}(\widehat{W}) = \frac{1}{\mathbb{E}[\widehat{W}]^{\gamma}} \left(\frac{\mathbb{E}[\widehat{W}]}{1-\gamma} + (\widehat{W} - \mathbb{E}[\widehat{W}]) - \frac{\gamma}{2} \frac{(\widehat{W} - \mathbb{E}[\widehat{W}])^2}{\mathbb{E}[\widehat{W}]} \right), \quad \gamma \neq 1. \tag{20}$$

Equation (20) generates the following expectations of U_{MV} for the charges on balance and flow:[17]

$$\mathbb{E}[U_{\mathrm{MV}}(\widehat{W}_s(T))] = \frac{1}{\mathbb{E}[\widehat{W}_s(T)]^{\gamma}} \left(\frac{\mathbb{E}[\widehat{W}_s(T)]}{1-\gamma} - \frac{\gamma}{2} \frac{\mathrm{Var}(\widehat{W}_s(T))}{\mathbb{E}[\widehat{W}_s(T)]} \right), \tag{22}$$

$$\mathbb{E}[U_{\mathrm{MV}}(\widehat{W}_f(T))] = \frac{1}{\mathbb{E}[\widehat{W}_f(T)]^{\gamma}} \left(\frac{\mathbb{E}[\widehat{W}_f(T)]}{1-\gamma} - \frac{\gamma}{2} \frac{\mathrm{Var}(\widehat{W}_f(T))}{\mathbb{E}[\widehat{W}_f(T)]} \right). \tag{23}$$

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[17] If $\gamma = 1$ and $\widehat{W}$ is either $\widehat{W}_s(T)$ or $\widehat{W}_s(T)$, then

$$\mathbb{E}[U_{\mathrm{MV}}(\widehat{W})] = \ln(\mathbb{E}[\widehat{W}]) - \frac{1}{2} \frac{\mathrm{Var}(\widehat{W})}{\mathbb{E}[\widehat{W}]^2}. \tag{21}$$

Then, we define Υ_{MV} to be the ratio of $\mathbb{E}[U_{\mathrm{MV}}(\widehat{W}_s(T))]$ to $\mathbb{E}[U_{\mathrm{MV}}(\widehat{W}_f(T))]$, and we express it as a function of γ, $\mathcal{P}_T$ and δ. Assuming $p_i = p \in (0,1]$ in process Z, we obtain

$$\Upsilon_{\mathrm{MV}}(\gamma, p, \delta) = \frac{\mathbb{E}[U_{\mathrm{MV}}(\widehat{W}_s(T))]}{\mathbb{E}[U_{\mathrm{MV}}(\widehat{W}_f(T))]}. \tag{24}$$

We expect (22) and (23) to be good approximations of the expected utility.[18] Hence, Υ_{MV} would exhibit a behavior similar to that of Υ. Additionally, we have not considered Υ or Υ_{MV} to be function of α because $\partial_\alpha \Upsilon = (1-\gamma)\Upsilon$ and $\partial_\alpha \Upsilon_{\mathrm{MV}} = (1-\gamma)\Upsilon_{\mathrm{MV}}$ for $\gamma \neq 1$. Then, the charge on flow will always become less attractive with respect to the charge on balance as α increases.

To interpret Υ_{MV} unambiguously, we will assume $\gamma > 1$ so $U_{\mathrm{MV}}(\widehat{W}_s(T))$ and $U_{\mathrm{MV}}(\widehat{W}_f(T))$ yield negative expectations. Therefore, the affiliate will prefer the charge on balance when $\Upsilon_{\mathrm{MV}} < 1$, and she will prefer that on flow when $\Upsilon_{\mathrm{MV}} > 1$. The next proposition analyzes the behavior of Υ_{MV} with respect to the charge on balance and the degree of relative risk aversion.

Proposition 3.4. (*Effect of the charge on balance and risk aversion on* Υ_{MV}).
Let $\gamma > 1$, $T > 1$, $p_i = p \in (0,1]$ in process Z, and let $\delta_{\mathrm{I}}^* = \delta_{\mathrm{I}}^*(\mathbf{I})$ be the risk-neutral equivalent charge on balance of Definition 3.1, where $\mathbf{I} = \{T, \alpha, \mu, \sigma^2, \mathcal{W}_T\}$. The following statements hold:

1. If both $\sigma^2 \leq \frac{1}{T}\ln(2p)$ and
$$\mathbb{E}[\widehat{W}_s(T)^2] \geq \left(1 + \frac{2}{\gamma^2}\right)\mathbb{E}[\widehat{W}_s(T)]^2 \tag{25}$$
hold, then $\partial_\delta \Upsilon_{\mathrm{MV}}(\gamma, p, \delta) > 0$.
2. If $\mu - \delta_{\mathrm{I}}^* + \sigma^2 > 0$ and $W_{i+1} \leq W_i$ for all $0 \leq i \leq T-1$, then $\Upsilon_{\mathrm{MV}}(\gamma, p, \delta_{\mathrm{I}}^*) < 1$ and $\partial_\gamma \Upsilon_{\mathrm{MV}}(\gamma, p, \delta_{\mathrm{I}}^*) < 0$.
3. If $\mathcal{W}_T$ is given by (16) and $\mu - \delta_{\mathrm{I}}^* + \sigma^2 > \beta$, then $\Upsilon_{\mathrm{MV}}(\gamma, p, \delta_{\mathrm{I}}^*) < 1$ and $\partial_\gamma \Upsilon_{\mathrm{MV}}(\gamma, p, \delta_{\mathrm{I}}^*) < 0$.

Proof. See Appendix A.6 (online)[19]

Statement (1) establishes conditions under which Υ_{MV} is an increasing function of δ, i.e., the charge on flow improves with respect to the charge on balance as δ increases.

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[18] Hlawitschka (1994) presented empirical evidence and concluded that even when the Taylor series of expected utility diverges, the second-order expansion provides an excellent approximation of the expected utility (in terms of the Spearman rank coefficient of correlation) even in the case of random variables with realizations that frequently fall outside the convergence interval of (19).

[19] If $\gamma < 1$ and the following condition

$$\mathbb{E}[\widehat{W}^2] < \left(1 + \frac{2}{\gamma(1-\gamma)}\right)\mathbb{E}[\widehat{W}]^2 \tag{26}$$

holds for $\widehat{W}$ equal to either $\widehat{W}_s(T)$ or $\widehat{W}_f(T)$, then $\mathbb{E}[U_{\mathrm{MV}}(\widehat{W}_s(T))]$ and $\mathbb{E}[U_{\mathrm{MV}}(\widehat{W}_f(T))]$ are positive, and Proposition 3.4 can be extended. Condition $\sigma^2 \leq (1/T)\ln(2p)$ will imply $\partial_\delta \Upsilon_{\mathrm{MV}}(\gamma, p, \delta) < 0$. Furthermore, $\Upsilon_{\mathrm{MV}}(\gamma, p, \delta_{\mathrm{I}}^*) > 1$ and $\partial_\gamma \Upsilon_{\mathrm{MV}}(\gamma, p, \delta_{\mathrm{I}}^*) > 0$ (for $\gamma < 0.5$) under the assumptions of the last two claims of Proposition 3.4. For more detail, refer to the proof in Appendix A.6.

Statements (2) and (3) give conditions under which the charge on balance is preferred or under which the latter improves with respect to the charge on flow as risk aversion increases. Because the aforementioned conditions are the same as those in Proposition 3.3, the intuition behind those results is that the risk (standard deviation) is greater for the charge on flow; and, therefore the more risk-averse the affiliate is, the more the charge on balance improves with respect to that on flow. Statement (2) is more general than (3) because it requires $\mu + \sigma^2 > \delta_{\mathrm{I}}^*$ and a decreasing sequence of contributions. On the contrary, claim (3) requires a growth rate for contributions, β, not exceeding $\mu - \delta_{\mathrm{I}}^* + \sigma^2$. Note that we have assumed $\delta = \delta_{\mathrm{I}}^*$, so arbitrary choices for δ cannot guarantee that (2) and (3) will hold. The effect of DoC, p, on Υ_{MV}, i.e., the sign of $\partial_p \Upsilon_{\mathrm{MV}}(\gamma, p, \delta_{\mathrm{I}}^*) > 0$, cannot be determined even under assumptions such as those in Proposition 3.4; it will be studied empirically in Section 4.

4 Application to the Peruvian PPS

In this section, we present an application of the proposed methodology to the Peruvian PPS. This application is relevant because the PPS is going through an important reform exactly 20 years after its creation. Part of the reform consists of replacing the charge on flow with a charge on balance, and this situation has partially motivated the present research paper.[20]

4.1 Parameters of the model

We are considering a retirement age of 65 years and a moderate scenario that corresponds to the medium-risk (default option) pension fund available for PPS's affiliates. Additionally, we fix the charge on flow to $\alpha = 0.1761$. This value is related to the PPS's average charge as of December 2013, which was $f = 1.615\%$ of the affiliate's salary under a constant contribution rate of 10%, i.e., $\alpha = -\ln(1 - 10f)$.

We also need to determine parameters μ and σ for process V given by (1). For the moderate scenario, the monthly volatility is $\sigma_M = 2.511\%$ estimated from the daily real log-returns of the PPS's Type 2 (medium-risk) funds. For calibration purposes, we have considered the daily real returns of Integra's[21] Type 2 fund from the period 01/02/2009 to 05/30/2013. Additionally, we have assumed an expected real return of 5.00% per year. Based on GBM's properties, we have $r_M = \mu_M - 0.5\sigma_M^2$, where r_M is the expected monthly real return expressed in continuous time. After adequate transformations, we obtain $\mu_M = 0.44\%$.

The monthly sequence of real contributions, $\mathcal{W}_T$, is assumed such that

$$W_{i+1} = (1 + \tau_i) W_i, \quad W_0 > 0, \quad \tau_i \in \mathbb{R} \ntrianglelefteq \quad \text{and} \quad i = 0, \ldots, T - 1, \qquad (27)$$

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[20] Peruvian Law No. 29903 contains the main aspects of the reform. One is that affiliates will migrate to a mixed charge that has a 10-year transient flow component, and from year 10 onwards the charge will be only on balance. The reform also includes a bidding mechanism on charges to allocate new affiliates and norms to incorporate independent workers.

[21] Integra PFA is the most important PFA in Peru either in terms of the number of affiliates and the monetary value of managed funds.

where factors τ_i are considered to be the sum of the growth along the salary curve plus a component related to productivity growth. Additionally, τ_i depends on three characteristics: the affiliate's gender, education level and age. The calibration details are presented in Appendix B (online); however, it is important to mention that for young affiliates, the average growth factors fluctuate between 2.5% and 3.5% per year.

Although the independence assumptions are strong, the interruption process Z is flexible enough because the sequence of likelihoods p_i can reflect different contribution profiles for affiliates. For the numerical experiments, we have considered its simplest specification: $p_i = p$ for all i. The goal behind this assumption is to show, in the most direct form, the effect of p on the comparison between both charge schemes. We consider three scenarios for the DoC: $p = 1.00$ (non-interrupted stream of contributions), $p = 0.51$ (the PPS average), and $p = 0.18$ (the average for affiliates less than 21 years of age).

4.2 Numerical results

4.2.1 Risk-neutral equivalent charge on balance: δ_{I}^*

Figure 1 shows the plot of δ_{I}^* (see Definition 3.1) annualized and in percentage form for certain ages and contribution profiles and in a moderate scenario (an expected fund return of 5.00% per year). Detailed values can be found in Table C1 in online Appendix C. In the case of a 32-year-old risk-neutral female affiliate without a college education (*F*/*NC* profile), we have $T = (65 - 32) \times 12 = 396$ months and the risk-neutral equivalent charge on balance is 0.93% per year. This implies that a charge on balance smaller than 0.93% makes such a scheme convenient for this particular risk-neutral affiliate when the charge on flow is $\alpha = 0.1761$. We can observe that δ_{I}^* is strictly increasing in age; i.e., the charge on balance improves with age. This is intuitive because as the accumulation horizon (T) decreases, the fund becomes less important; thus, the fees on balance become more attractive. Note that $\delta_{\mathrm{I}}^* > 0.65\%$ for all profiles and ages in online Table C1 (the value corresponds to a 20-year-old with *F*/*NC* profile), and this level would make the charge on balance preferable for all considered affiliates. Additionally, the differences between δ_{I}^* s for the same age are very small (<0.04% for affiliates younger than 40 years of age), implying that different growth rates in wages generate similar values of δ_{I}^*. Finally, recall that δ_{I}^* is independent of the DoC because p cancels in RE_{sf}.

4.2.2 Percentage difference on certainty equivalent: ΔCE_{sf}

In this section, we study (empirically) the percentage difference, $\Delta\mathrm{CE}_{sf}$, given by (18). For utility (17), closed-form expressions for $\mathrm{CE}[\widehat{W}_s(T)]$ and $\mathrm{CE}[\widehat{W}_f(T)]$ are not available; thus, we will rely on a stochastic simulation to obtain an estimator of $\Delta\mathrm{CE}_{sf}$. Figures 2 and 3 and online Tables C2 and C3 show estimated values of $\Delta\mathrm{CE}_{sf}$ for different values of DoC ($p = 1.00$, $p = 0.51$ and $p = 0.18$) and relative risk aversion, γ. Following Poterba *et al.* (2005), we select $\gamma = 1$ for a low degree of risk aversion, $\gamma = 4$ for a moderate degree of risk aversion, and $\gamma = 8$ for a high degree of risk aversion. In every scenario of Figure 2 and Table C2 (online), the charge on balance, δ, was fixed at its corresponding risk-neutral equivalent charge, δ_{I}^*. In Figure 3 and

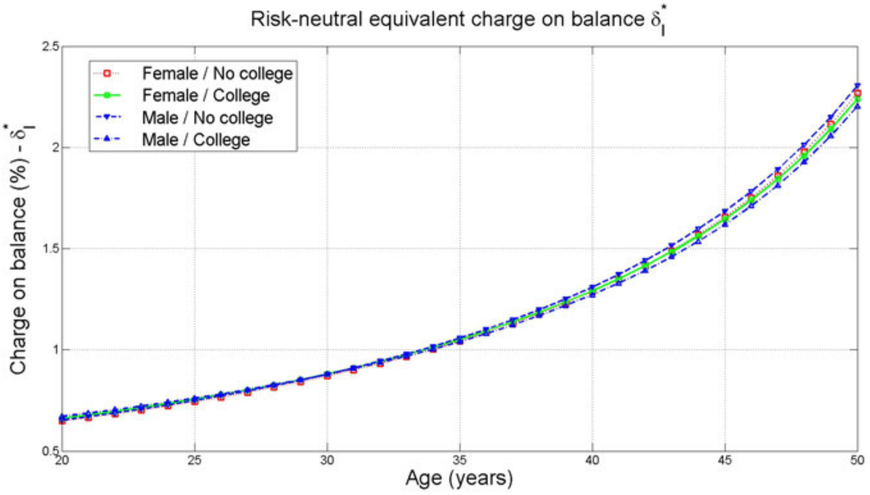

Figure 1. (Colour online) Risk-neutral equivalent charge on balance, δ_I^*, in percentage (%) and annualized for different ages in years (*x*-axis) and gender/education combinations in a moderate scenario (an expected fund return of 5.00% per year). We have assumed $\alpha = 0.1761$ (which corresponds to a charge on flow of 1.615% of the salary with a constant contribution rate of 10%) and a retirement age of 65 years.

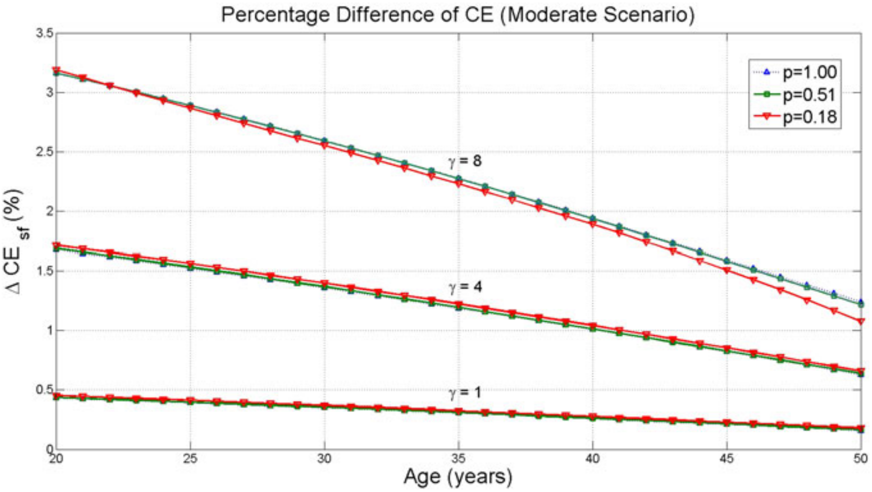

Figure 2. (Colour online) Estimated values of $\Delta \mathrm{CE}_{sf} = \mathrm{CE}[\widehat{W}_s(T)]/\mathrm{CE}[\widehat{W}_f(T)] - 1$ in percentage (%) for different ages in years (*x*-axis), values of relative risk aversion ($\gamma = 1,\ 4,\ 8$) and density of contribution ($p = 1.00,\ 0.51,\ 0.18$) under a moderate scenario (an expected fund return of 5.00% per year) and *F*/*NC* contribution profile. We have also assumed $\delta = \delta_I^*$, $\alpha = 0.1761$ (which corresponds to a charge on flow of 1.615% of salary with a constant contribution rate of 10%) and a retirement age of 65 years.

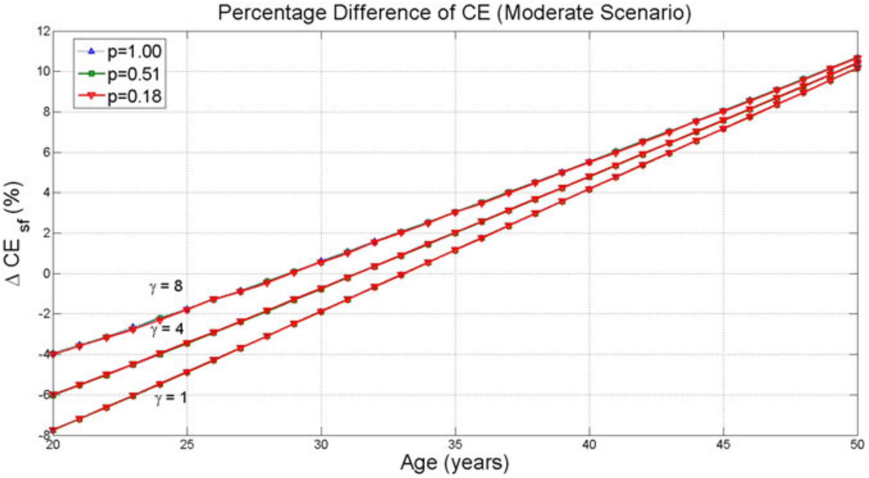

Figure 3. (Colour online) Estimated values of $\Delta\text{CE}_{sf} = \text{CE}[\widehat{W}_s(T)]/\text{CE}[\widehat{W}_f(T)] - 1$ in percentage (%) for different ages in years (x-axis), values of relative risk aversion ($\gamma = 1,\ 4,\ 8$) and density of contribution ($p = 1.00,\ 0.51,\ 0.18$) under a moderate scenario (an expected fund return of 5.00% per year) and *F*/*NC* contribution profile. We have also assumed $\delta = 1.0\%$ per year, $\alpha = 0.1761$ (which corresponds to a charge on flow of 1.615% of salary with a constant contribution rate of 10%) and a retirement age of 65 years.

Table C3, the value of δ was fixed to 1.00% per year (current ratio of total fees to total assets of the system). The number of sample paths of adjusted wealth to estimate ΔCE_{sf} (for every scenario) was determined using the sequential procedure of Kelton and Law (2000) with a relative error of 0.0001 and a confidence level of 99%. Additionally, we report only results for contribution profile *F*/*NC* (see Appendix B online) because the others generate very similar outputs.

From Figure 2 ($\delta = \delta_{\text{I}}^*$), we observe that $\Delta\text{CE}_{sf} > 0$ for every scenario and that γ clearly has a stronger effect on ΔCE_{sf} than p. Moreover, note that as risk aversion increases, ΔCE_{sf} also increases. This fact is in line with the theoretical results of Proposition 3.4. For example, in the case of a 20-year-old *F*/*NC* affiliate, we have $\Delta\text{CE}_{sf} \approx 0.5\%$ for $\gamma = 1$, $\Delta\text{CE}_{sf} \approx 1.7\%$ for $\gamma = 4$ and $\Delta\text{CE}_{sf} \approx 3.2\%$ for $\gamma = 8$. On the contrary, the effect of p on ΔCE_{sf} for a fixed γ is not clear (in the sense that for older affiliates, small values of p benefit the charge on flow, whereas the opposite is true for younger ones), and it is almost negligible in magnitude. Figure 3 considers a fixed $\delta = 1\%$ per year and, as opposed to Figure 2, we can observe that $\Delta\text{CE}_{sf} > 0$ does not hold for every scenario. For example, if $\gamma = 4$, then we have $\Delta\text{CE}_{sf} > 0$ only for affiliates older than 31 years of age, i.e., the charge on balance will be preferable for the aforementioned group of affiliates. The same threshold is approximately equal to 33 years for $\gamma = 1$ and 29 years for $\gamma = 8$. This observation and the fact that ΔCE_{sf} is increasing in γ (for a fixed age and p) shows (again) that the charge on balance improves with respect to that on flow as risk aversion increases. Finally, we can verify the almost null effect that p has on ΔCE_{sf} because for a common γ, the curves in the figure are almost overlapping (the largest difference is <0.1%).

4.2.3 The effect of age on ΔCE_{sf}

In Figure 1, we can observe that δ_{I}^{*} is approximately 1% for age 35, i.e., $\delta_{\mathrm{I}}^{*}(35) \approx 1\%$. For ages $x < 35$ we have $\delta_{\mathrm{I}}^{*}(35) < 1\%$ and for ages $x > 35$ we have $\delta_{\mathrm{I}}^{*}(35) > 1\%$. This implies that a constant charge on balance of 1% is too high for risk-neutral affiliates younger than 35 and too low for affiliates older than 35. Consequently, with $\delta = 1\%$ young affiliates will pay a charge on balance that is too high but old affiliates will pay a charge on balance that is too low. This explains why for young affiliates in Figure 3 $\Delta\mathrm{CE}_{sf}$ is negative (i.e., the charge on flow is better) and for old affiliates not only $\Delta\mathrm{CE}_{sf}$ is positive but it also takes high values, such as 10% for age 50. Also, we can notice from Figures 2 and 3 that the effect of relative risk aversion on $\Delta\mathrm{CE}_{sf}$ decreases as age increases. For example, in Figure 2 the difference in $\Delta\mathrm{CE}_{sf}$ for $\gamma = 8$ and $\gamma = 1$, i.e., $\Delta\mathrm{CE}_{sf}(\gamma = 8) - \Delta\mathrm{CE}_{sf}(\gamma = 1)$, is approximately 2.6% for age 20 and the same difference is 1% for age 50. The same fact also appears in Figure 3 but the corresponding difference is 4% for age 20 and it is almost negligible for age 50. Finally, if age increases, then the effect of DoC on $\Delta\mathrm{CE}_{sf}$ is noticeable only when $\delta = \delta_{\mathrm{I}}^{*}$, risk aversion is high ($\gamma = 8$) and DoC is in its worst scenario ($p = 0.18$).

5 Conclusions

This paper studies how the affiliate's degree of risk aversion and DoC affects the comparison of proportional charges on flow and balance in DC pension systems with individual accounts. Assuming a GBM for the share value of a representative pension fund and using an independent process to model the interruption of contributions, we represent the terminal wealth in the affiliate's account assuming an arbitrary sequence of contributions and fixed values for the corresponding charges. Additionally, we proposed three ways to compare the aforementioned charges: the ratio of expected values of terminal wealth, the ratio of the expected value to the standard deviation of terminal wealth, and the ratio of expected utilities of terminal wealth. We derive theoretical results that explain the behavior of the charges with respect to the key parameters of the model and to the three methods of comparison. It is important to mention that under mild assumptions, the charge on balance improves its performance relative to that on flow as risk aversion increases. On the contrary, the effect of the DoC in the comparison is almost negligible when it is assumed to be constant during the accumulation phase. Finally, it is possible to include many refinements to the proposed methodology that can generate new research articles. For example, we can consider a variable charge on balance (which can be related to the total system's fund size) or work under the assumption of a complete market and provide expressions for an arbitrage-free relationship between the types of charges.

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Supplementary material

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