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Effective secondary task execution of redundant manipulators

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SUMMARY

In standard pseudoinverse-based approaches to treat redundant manipulators, the vector of joint increments that corresponds to a desired motion in the space of the secondary task is projected in the Jacobian null space associated with the primary task. In general, this projection may distort the projected vector, so that the secondary task may not adequately be executed. A usual remedy is to rotate the null space projection operator by using a special-purpose weighting matrix. The problem, however, is that this rotation cannot be enforced arbitrarily since it influences the manipulator's performance. In our work we propose an algorithm that is independent on the chosen null space operator and always provides the best attainable motion in the space of the secondary task. Hence, the secondary task is executed more efficiently and the numerical procedure is more robust. A series of numerical experiments confirmed these results.

KEYWORDS: Redundant manipulators; Inverse kinematics; Numerical optimisation; Secondary task

1. INTRODUCTION

A majority of efforts in treating redundancy have been concentrated at the kinematic level of control with respect to different types of criteria. Algorithms were reported that enable robots to avoid obstacles.[1,2] or ill-conditioned configurations.[3–5] Many authors based their approach on the utilisation of the pseudoinverse.[6–8] Some authors developed symbolic solutions to this problem.[9,10] But no symbolic solution can be developed for a general-type redundant manipulator unless certain conditions are met by the mechanism.

The calculation schemes based on the pseudoinverse are procedures of local optimisation.[11] They minimise a weighted norm of joint velocities. Yet, the central point and a distinctive property of various pseudoinverse-based methods is in the determination of the null space projection operator.[12] A proper selection of the null space projection operator provides a secondary motion of the manipulator that respects different criteria.[13,14] Some authors reported results of obtaining global optima with integral-type of criteria,[15,16] others optimised the weighted null space projection operator to avoid instabilities.[17] An alternative to the pseudoinverse-based methods is the extended Jacobian method.[18] In a more recent formulation[19] this algorithm appears to be well suited for more general choices of the secondary criterion. It must be pointed out, however, that different optimum solutions drastically change the manipulator's motion characteristics.[20]

In a standard pseudoinverse-based approach, the vector of joint increments that corresponds to a desired motion in the space of the secondary task is projected in the null space of the Jacobian matrix associated with the primary task. This operation may distort the projected vector, so that the secondary task may not be executed in the best possible way. A usual strategy to overcome the problem is to use the weighted pseudoinverse and optimise the weighting matrix with respect to the requirements of the secondary task. However, the weighting matrix cannot be chosen arbitrarily since it effects the manipulator's motion characteristics. The objective of the present article is to propose a novel formulation which, independently on the chosen null space operator, provides the maximum attainable increment in the range of the secondary task.

2. OUTLOOK OF PSEUDOINVERSE-BASED TECHNIQUE

Let the primary task be to achieve some desired values of task coordinates $\mathbf{p}$ given as a function of joint coordinates $\mathbf{q}$, where vector $\mathbf{p}$ is of dimension m and $\mathbf{q}$ of dimension n. It is expressed in the well known differential form[21]

$$d\mathbf{p}-\mathbf{J}_P d\mathbf{q}=\mathbf{0}, \tag{1}$$

where $\mathbf{J}_P$ is the $m\times n$ Jacobian matrix that incorporates the derivatives $\{\partial\mathbf{p}/\partial\mathbf{q}_i\}$. We assume that $n>m$. The reverse relationship is

$$\mathbf{J}^{+}_{PA}=d\mathbf{p}-d\mathbf{q}=\mathbf{0}, \tag{2}$$

where

$$\mathbf{J}^{+}_{PA}=\mathbf{A}^{-1}\mathbf{J}^{T}_{P}(\mathbf{J}_P\mathbf{A}^{-1}\mathbf{J}^{T}_{P})^{-1} \tag{3}$$

is the so-called weighted pseudoinverse of $\mathbf{J}_P$ whose dimension is $n\times m$, and $\mathbf{A}$ is a positive definite $n\times n$ weighting matrix. The utilisation of the pseudoinverse leads to a minimisation of joint velocities[21] expressed in the quadratic form $\dot{\mathbf{q}}^T\mathbf{A}\dot{\mathbf{q}}$.

Let the secondary task be expressed in the same way as the primary task

$$d\mathbf{s}-\mathbf{J}_S d\mathbf{q}=\mathbf{0}, \tag{4}$$

where the objective is to achieve a $\mathbf{q}$ that corresponds to given values of secondary task coordinates $\mathbf{s}$, and $\mathbf{J}_S$ is the Jacobian that includes derivatives $\{\partial\mathbf{s}/\partial\mathrm{q}_i\}$. If $\mathbf{s}$ is of dimension l, $\mathbf{J}_S$ is of dimension $l\times n$. If $n>l$, we can take

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advantage of the $n \times l$ pseudoinverse $\mathbf{J}_S^+ = \mathbf{J}_S^T(\mathbf{J}_S\mathbf{J}_S^T)^{-1}$ to obtain

$$\mathbf{J}_S^+ \, d\mathbf{s} - d\mathbf{q} = \mathbf{0}. \tag{5}$$

Arrange the increment of joint coordinates in the primary $d\mathbf{q}_P$ and the secondary $d\mathbf{q}_N$ (subordinated) part, so that $d\mathbf{q}_N$ does not produce any change in the primary task coordinates $\mathbf{p}$

$$d\mathbf{q} = d\mathbf{q}_P + d\mathbf{q}_N. \tag{6}$$

By multiplying by $\mathbf{J}_P$ we get

$$d\mathbf{p} = \mathbf{J}_P d\mathbf{q}_P + \mathbf{J}_P d\mathbf{q}_N, \tag{7}$$

where it is requested that

$$\mathbf{J}_P d\mathbf{q}_N = \mathbf{0}. \tag{8}$$

Then

$$d\mathbf{p} = \mathbf{J}_P d\mathbf{q}_P \Rightarrow d\mathbf{q}_P = \mathbf{J}_{PA}^+ d\mathbf{p}. \tag{9}$$

Let $d\mathbf{q}_N$ be given in the following form

$$d\mathbf{q}_N = \mathbf{N}_{PA} d\mathbf{q}_S, \tag{10}$$

where $d\mathbf{q}_S$ is an arbitrary n-dimensional vector of joint increments associated with the secondary task, and matrix $\mathbf{N}_{PA}$ is of dimension $n \times n$. Because of (8) the following must be valid

$$\mathbf{J}_P \mathbf{N}_{PA} = \mathbf{0}. \tag{11}$$

According to Liegois[22] matrix

$$\mathbf{N}_{PA} = \mathbf{I} - \mathbf{J}_{PA}^+ \mathbf{J}_{PA} \tag{12}$$

lies in the null space of $\mathbf{J}_P$. The null space projection operator $\mathbf{N}_{PA}$ is indempotent $\mathbf{N}_{PA}\mathbf{N}_{PA} = \mathbf{N}_{PA}$. It is also Hermitian, $\mathbf{N}_{PA}^T = \mathbf{N}_{PA}$, when $\mathbf{A}$ is a unity matrix. The degree of redundancy is mathematically determined as the rank of the null space projection operator

$$D = \text{rank}\{\mathbf{N}_{PA}\} = n - \text{rank}\{\mathbf{J}_P\}. \tag{13}$$

D is the achievable order of the secondary motion that can in general change in dependence on $\mathbf{q}$ but it does not depend on $\mathbf{A}$ if this is full-rank. We assume that $l > D$.

By substituting (10) into (6) and by multiplying by $\mathbf{J}_S$ we have

$$d\mathbf{s} = \mathbf{J}_S d\mathbf{q}_P + \mathbf{J}_S \mathbf{N}_{PA} d\mathbf{q}_S \Rightarrow d\mathbf{q}_S = (\mathbf{J}_S\mathbf{N}_{PA})^+ (d\mathbf{s} - \mathbf{J}_S d\mathbf{q}_P). \tag{14}$$

An increment in the secondary task coordinates $d\mathbf{s}$ depends on both $d\mathbf{q}_N$ and $d\mathbf{q}_P$, so that it can be separated into

$$d\mathbf{s} = \mathbf{ds}_P + d\mathbf{s}_N, \quad d\mathbf{s}_P = \mathbf{J}_S d\mathbf{q}_P, \quad d\mathbf{s}_N = \mathbf{J}_S d\mathbf{q}_N. \tag{15}$$

If we take into account (6, 10, 15), a complete increment in joint coordinates can be written as follows

$$d\mathbf{q} = \mathbf{J}_{PA}^+ d\mathbf{p} + \mathbf{N}_{PA}(\mathbf{J}_S\mathbf{N}_{PA})^+ (d\mathbf{s} - \mathbf{J}_S\mathbf{J}_{PA}^+ d\mathbf{p}). \tag{16}$$

It is the well know task priority approach where the first part of the joint increment is the particular solution which is associated with the primary task. It is of a higher priority in comparison with the second part which is the homogeneous solution associated with the secondary task.[1,8]

3. RANGE OF SECONDARY TASK SPACE

In accordance to the definition of the manipulability ellipsoids,[23] a sphere $d\mathbf{q}_S^T d\mathbf{q}_S = 1$ produces an ellipsoid in l-dimensional space of $d\mathbf{s}$ whose principal axes are the Eigen vectors of $\mathbf{J}_S\mathbf{J}_S^T$ and their lengths are the related singular values. Even though there is some controversy in its definition and utilisation when different types of task coordinates are treated simultaneously, such as linear and angular velocities,[13,24] the measure of manipulability gives a significant insight in the motion properties of a mechanism, in particular when this is redundant. Since an increment in joint coordinates is referred to as the secondary motion of a redundant manipulator when it does not produce any increment in the primary task coordinates, it is clear that only a part of elements $d\mathbf{s}$ in the manipulability ellipsoid (the space of vectors $d\mathbf{s}_N$) can be accomplished by the secondary motion of the redundant manipulator; the space of $d\mathbf{s}_N$ is the range of the secondary task.

The secondary motion can only be assembled in the null space of the Jacobian matrix $\mathbf{J}_p$ where a vector $d\mathbf{q}_S$ is projected through the null space operator $\mathbf{N}_{PA}$ onto $d\mathbf{q}_N$. Thus, an element on the surface of the sphere $d\mathbf{q}_S^T d\mathbf{q}_S = 1$ is transformed onto an element that lies inside the sphere defined by $d\mathbf{q}_N^T d\mathbf{q}_N \leq 1$. The null space of the Jacobian matrix $\mathbf{J}_p$ is span by the n-dimensional orthonormal Eigen vectors of matrix $\mathbf{N}_{PA}\mathbf{N}_{PA}^T = \mathbf{N}_{PA}$, denoted as $\mathbf{E}_{PA} = (\mathbf{e}_{A1}, \mathbf{e}_{A2}, \ldots, \mathbf{e}_{AD})$. Note that only D Eigen vectors correspond to a non zero singular value of $\mathbf{N}_{PA}$. Thus, any combination $d\mathbf{q}_N$ described as a function of parameters $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_D)^T$

$$d\mathbf{q}_N = \mathbf{E}_{PA}\,\gamma \tag{17}$$

is an element of the surface of the sphere $d\mathbf{q}_N^T d\mathbf{q}_N = 1$ if

$$\gamma_1^2 + \gamma_2^2 + \cdots + \gamma_D^2 = 1. \tag{18}$$

The values of $d\mathbf{s}_N$, as defined in (15), that are functions of $d\mathbf{q}_N$ constrained by (18), form an ellipsoid in the vector space of $d\mathbf{s}$ that can be accomplished by the secondary motion of the manipulator. We could call it the secondary manipulability ellipsoid (Figure 1), whose principal axes are the l-dimensional Eigen vectors of matrix $\mathbf{J}_S\mathbf{N}_{PA}(\mathbf{J}_S\mathbf{N}_{PA})^T$ (denoted by $\mathbf{u}_{A1}, \mathbf{u}_{A2}, \ldots, \mathbf{u}_{AD}$) that correspond to the non zero values of the singular values of matrix $\mathbf{J}_S\mathbf{N}_{PA}$. The non zero singular values are the lengths of the principal axes. We assume here that $\text{rank}\{\mathbf{J}_S\} = l \geq D$, so that $\text{rank}\{\mathbf{J}_S\mathbf{N}_{PA}\} = D$. The secondary manipulability ellipsoid characterises the range of the secondary task bounded by (18). It visualises the potential of a redundant manipulator to solve the

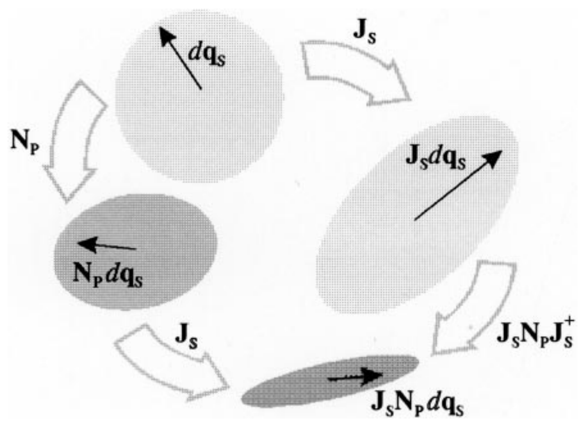

Fig. 1. Manipulability ellipsoid mapped in the null space of the primary task.

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secondary task with no interference with the primary task in a given location $\mathbf{q}$. Some aspects of studying redundancy from a similar viewpoint were reported.[25]

4. BEST NULL SPACE PROJECTION

In relation to the formulation of the range of the secondary task space in the previous section we can state that the vector space of $d\mathbf{s}_N$ that isn't in conflict with the primary task is span by the set of l-dimensional orthonormal Eigen vectors $\mathbf{u}_{A1}, \mathbf{u}_{A2}, \ldots, \mathbf{u}_{AD}$ of matrix $\mathbf{J}_S\mathbf{N}_{PA}(\mathbf{J}_S\mathbf{N}_{PA})^T$. Therefore, it is possible to distinct an arbitrary $d\mathbf{s}_N$ in the range of the secondary task in the following form

$$d\mathbf{s}_N = \mathbf{U}_{PA}\,\boldsymbol{\gamma}. \tag{19}$$

Here

$$\mathbf{U}_{PA} = [\mathbf{u}_{A1} \ldots \mathbf{u}_{AD}] \tag{20}$$

is a $l \times D$ orthogonal matrix (its rows are orthonormal vectors) $\mathbf{U}_{PA}^T\mathbf{U}_{PA} = \mathbf{I}$, and $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_D)^T$ is an arbitrary vector.

To obtain a vector $\boldsymbol{\gamma}$ that produces the desired increment in the secondary task coordinates $d\mathbf{s}$ we must combine expressions (15) and (19). Thus we get

$$\mathbf{U}_{PA}\,\boldsymbol{\gamma} = d\mathbf{s} - d\mathbf{s}_P. \tag{21}$$

By multiplying (21) by $\mathbf{U}_{PA}^T$ and by taking advantage of the orthogonality of $\mathbf{U}_{PA}$ we can extract $\boldsymbol{\gamma}$ explicitly

$$\mathbf{U}_{PA}^T\mathbf{U}_{PA}\,\boldsymbol{\gamma} = \boldsymbol{\gamma} = \mathbf{U}_{PA}^T(d\mathbf{s} - d\mathbf{s}_P). \tag{22}$$

It is then substituted in (19)

$$d\mathbf{s}_N = \mathbf{U}_{PA}\mathbf{U}_{PA}^T(d\mathbf{s} - d\mathbf{s}_P), \tag{23}$$

and from (15)

$$d\mathbf{q}_N = \mathbf{J}_S^+\mathbf{U}_{PA}\mathbf{U}_{PA}^T(d\mathbf{s} - d\mathbf{s}_P). \tag{24}$$

The resultant formula

$$d\mathbf{q} = \mathbf{J}_{PA}^+ d\mathbf{p} + \mathbf{J}_S^+\mathbf{U}_{PA}\mathbf{U}_{PA}^T(d\mathbf{s} - \mathbf{J}_{PA}^+ d\mathbf{p}). \tag{25}$$

is an alternative to (16) and represents the best possible increment in joint coordinates $d\mathbf{q}$ that produces the desired secondary displacement expressed by vector $d\mathbf{s}$.

The following equality

$$\mathbf{U}_{PA}\mathbf{U}_{PA}^T = \mathbf{U}_{PB}\mathbf{U}_{PB}^T, \tag{26}$$

where $\mathbf{A}$ and $\mathbf{B}$ are arbitrary full rank positive definite $n \times n$ weighting matrices as introduced in (3), assures that the secondary part of the joint displacement $\mathbf{J}_S^+\mathbf{U}_{PA}\mathbf{U}_{PA}^T d\mathbf{s}$ does not depend on the selection of the weighting matrix and thus always offers the best attainable solution from the viewpoint of the secondary task. In our opinion, this is the most positive aspect of formulating the inverse kinematics solution in the form of (25) instead of (16). In many control schemes the selection of the weighting matrix is of crucial importance because it provides a desired dynamic property of the manipulator which should not be disturbed by the secondary task. The weighting matrix rotates the null space of the Jacobian in what was termed the effective null space.[17] It is the central point of many attempts to resolve redundancy with solutions processing different characteristics that assist to avoid obstacles and singularities or minimise a given criterion.[12,15,16,26] Lately, a weighted null space projection operator suitable for a weighted joint torque optimisation was reported[17] with the aim to avoid instabilities that arise from unrealisable joint velocities. On the other hand, the proposition in which the weighting matrix is proportiional to the mechanism's inertia matrix,[27] has become the most standard control scheme of redundant robots.

5. STEEPEST DESCENT OPTIMISATION

Assume that the secondary task of a redundant manipulator is to mimimise a quadratic cost function (which is a very reasonable assumption in a vast variety of practical implementations)

$$c = \mathbf{s}^T\mathbf{W}\mathbf{s} \rightarrow \min_{\mathbf{q}}, \tag{27}$$

where $\mathbf{W}$ is a diagonal $l \times l$ full-rank matrix of positive weights w_i, while $d\mathbf{q}_N$ is constrained by the primary task. An iterative procedure is to provide a series of $d\mathbf{q}_N$ that step by step minimise c and do not interfere with the primary task. In a commonly used gradient projection technique to minimise a scalar cost function[13,22] the joint displacement is expressed as

$$d\mathbf{q} = k_P\mathbf{J}_{PA}^+ d\mathbf{p} - k_S\mathbf{N}_{PA}\mathbf{J}_S^+\left\{\frac{\partial c}{\partial s_i}\right\}, \tag{28}$$

which is analogous to (18). Here k_S and k_P are selected in order to assure the convergence of the numerical procedure. The trouble with such an approach is that in general the projection of the gradient in the null space of $\mathbf{J}_p$ rotated by $\mathbf{A}$ many not provide the maximum decrease of the cost function. Hence, the iterative procedure may not be able to locate the desired minimum in an acceptable number of iterations. One remedy is to adequately rotate the null space of $\mathbf{J}_p$ by changing the weighting matrix $\mathbf{A}$. As mentioned in the previous section, this may not be the best way because it interferes with the desired dynamic properties of the manipulator. For this reason we propose to use

$$d\mathbf{q} = k_P\mathbf{J}_{PA}^+ d\mathbf{p} - k_S\mathbf{J}_S^+\mathbf{U}_{PA}\mathbf{U}_{PA}^T\left(\left\{\frac{\partial c}{\partial s_i}\right\} - \mathbf{J}_{PA}^+ d\mathbf{p}\right). \tag{29}$$

which assures the steepest descent optimisation of the quadratic scalar cost function c independently on the chosen weighting matrix $\mathbf{A}$.

6. NUMERICAL EXAMPLE

To illustrate the above concepts, a simple planar mechanism with four parallel revolute joints is used (Figure 2). Vector $\mathbf{q} = (q_1, q_2, q_3, q_4)^T$ includes the joint angles and the primary task coordinates $\mathbf{p} = (x, y)^T$ represent the position vector of the end effector

$$\begin{aligned}
&\theta_i = \theta_{i-1} + q_i, \quad i = 1, \ldots, 4, \quad \theta_0 = 0,\\
&x_i = d_i\cos(\theta_i) + x_{i+1}, \quad i = 4, \ldots, 1, \quad x_5 = 0\\
&y_i = d_i\sin(\theta_i) + y_{i+1}, \quad i = 4, \ldots, 1, \quad y_5 = 0,
\end{aligned} \tag{30}$$

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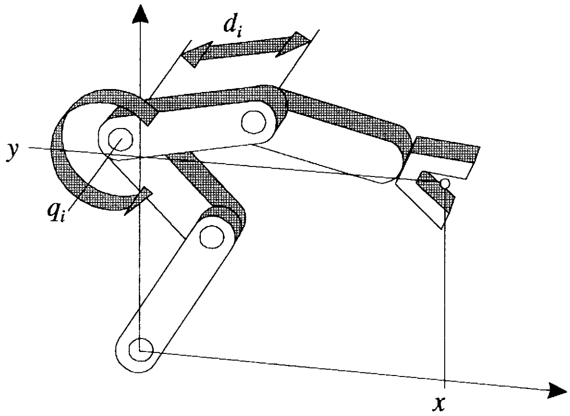

Fig. 2. Planar 4R manipulator.

where $x = x_l$, $y = y_l$, and d_i are the link lengths. The Jacobian and its derivatives are

$$\mathbf{J}_P = \begin{bmatrix} -y_1 & -y_2 & -y_3 & -y_4 \\ x_1 & x_2 & x_3 & x_4 \end{bmatrix},$$

$$\frac{\partial \mathbf{J}_P}{\partial q_1} = -\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{bmatrix}, \frac{\partial \mathbf{J}_P}{\partial q_2} = \begin{bmatrix} x_2 & x_2 & x_3 & x_4 \\ y_2 & y_2 & y_3 & y_4 \end{bmatrix},$$

$$\frac{\partial \mathbf{J}_P}{\partial q_3} = -\begin{bmatrix} x_3 & x_3 & x_3 & x_4 \\ y_3 & y_3 & y_3 & y_4 \end{bmatrix}, \frac{\partial \mathbf{J}_P}{\partial q_4} = -\begin{bmatrix} x_4 & x_4 & x_4 & x_4 \\ y_4 & y_4 & y_4 & y_4 \end{bmatrix}. \quad (31)$$

In the treated example $D = 2$ in non-singular configurations. The secondary task of the manipulator is to minimise the static joint torques $\mathbf{s} = \boldsymbol{\tau} = \mathbf{J}_P^T \mathbf{f}$ produced by an arbitrary finite force vector $\mathbf{f}$ applied to the end effector. The Jacobian related to the secondary task is

$$\mathbf{J}_S = \begin{bmatrix} \frac{\partial \mathbf{J}_P^T}{\partial q_1}\mathbf{f} & \frac{\partial \mathbf{J}_P^T}{\partial q_2}\mathbf{f} & \frac{\partial \mathbf{J}_P^T}{\partial q_3}\mathbf{f} & \frac{\partial \mathbf{J}_P^T}{\partial q_4}\mathbf{f} \end{bmatrix}. \quad (32)$$

In this case, $l = n$ and $\mathbf{J}_S$ is quadratic. The pseudoinverse is then obtained by $\mathbf{J}_S^+ = \mathbf{J}_S^{-1}$. It follows

$$d\boldsymbol{\tau} = \mathbf{J}_S d\mathbf{q}, \quad d\mathbf{q} = \mathbf{J}_S^{-1}\, d\boldsymbol{\tau}. \quad (33)$$

In the described series of numerical experiments, the link lengths $(d_1, d_2, d_3, d_4)^T = (1.0, 0.9, 0.8, 0.7)^T$ were chosen and a force $\mathbf{f} = (0.0, 1.0)^T$ was applied to the end effector. The primary task was to position the end effector in an arbitrary position $\mathbf{p}_0 = (1.2, 0.9)^T$. The secondary task was to minimise the unweighted square norm of joint torques

$$c = \boldsymbol{\tau}^T\boldsymbol{\tau}. \quad (34)$$

Note that at $\mathbf{q}_M = (2.42, -2.29, 0.71, -1.57)^T$, $|\boldsymbol{\tau}| = 2.57$ reaches the absolute maximum of joint torques, and at $\mathbf{q}_m = (0.08, 1.142, 0.24, -3.15)^T$ $|\boldsymbol{\tau}| = 1.22$ is the absolute minimum. We evaluated the following numerical minimisation technique

$$\mathbf{q}^{r+1} = \mathbf{q}^r + d\mathbf{q}^r, \quad (35)$$

where $r = 0, 1, \ldots$ is the number of iterations. The increment in each iteration $d\mathbf{q}^r$ was chosen (a) – in the proposed form established in (29), and (b) – in the standard form given in (28). In both cases we imposed $\mathbf{A} = \mathbf{I}$. The convergence was evaluated in terms of the following norms $e_p = |\mathbf{p}^r - \mathbf{p}_0|$, $e_q = |\mathbf{q}^r - \mathbf{q}_m|$, $e_\tau = |\boldsymbol{\tau}^r|$ normalised by their maximum values.

In the experiment, the initial estimation for the vector of joint coordinates was set to $\mathbf{q}^0 = (2.43, -2.18, 0.70, -1.57) \approx \mathbf{q}_M$. Approach (a) showed much better convergence and numerical stability than (b). The characteristics of the first are presented in Figure 3. The values of norms e_p, e_q, e_τ are shown as functions of the number of iterations when $k_P = 0.5$, $k_S = 0.25$ and $k_P = 0.05$, $k_S = 0.025$ were chosen. In both cases the procedure stabilised at the desired absolute minimum $\mathbf{q}_m$. It is important, however, that the form of function e_q did not depend on chosen constants k_P and k_S. This was the main distinction with respect to approach (b) presented in Figure 4 at $k_P = 0.05$, $k_S = 0.025$, and $k_P = 0.005$, $k_S = 0.0025$. It is evident that by (b) the absolute minimum $\mathbf{q}_m$ was not found. The procedure stabilised at $\mathbf{q}' = (1.61, -2.06, 1.22, -3.84)^T$ and at $\mathbf{q}'' = (1.03, -3.44, 2.89, -6.36)^T$. The obtained results depended on the chosen values of k_P, k_S as is seen from the appearance of function e_q.

A stabile convergence was obtained with smaller values of constants k_P, k_S so that considerably more iterations were needed than by approach (a). Figure 5 shows the manipulator's configurations corresponding to the initial estimation $\mathbf{q}_M$, to minimum $\mathbf{q}_m$, as well as to $\mathbf{q}'$ and $\mathbf{q}''$. In Figure 6, the norm $|\boldsymbol{\tau}'| = |\boldsymbol{\tau} + d\boldsymbol{\tau}|$ where $d\boldsymbol{\tau}$ comes either from (a) or (b), is established depending on the value of constant k_S. The above curves in Figure 6 were calculated at $\mathbf{q}^0$. In accordance to the presented theory, $k_S = 1.0$ provided the maximum decrease of the cost function if we utilised approach (a). The best performance of approach (b) was obtained at $k_S = 0.28$. In contrast to (a), the best k_S in (b) isn't

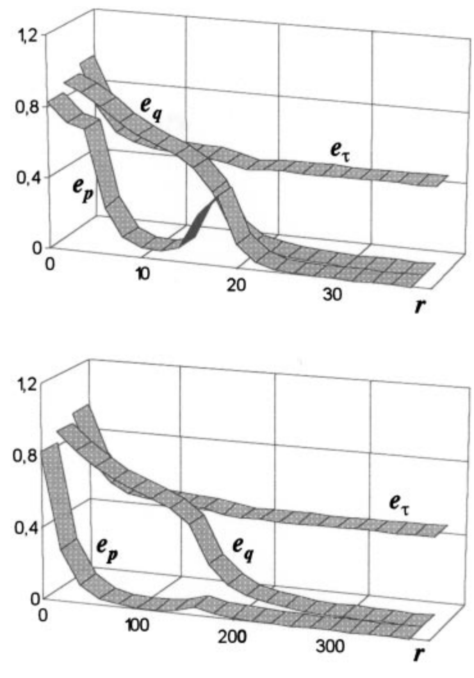

Fig. 3. Convergence of the optimisation procedure when approach (a) is applied.

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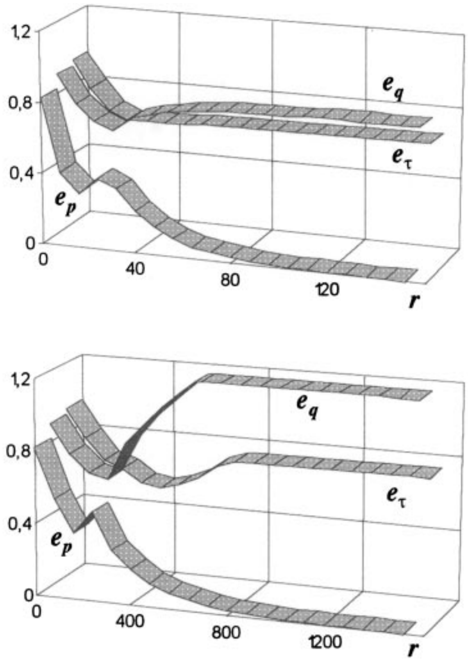

Fig. 4. Convergence of the optimisation procedure when approach (b) is applied.

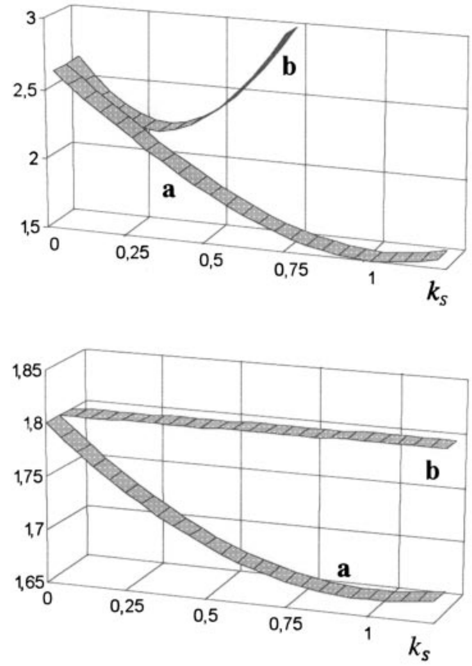

Fig. 6. Absolute increment in joint torques depending on the step size.

known a priori and changes with **q**. Moreover, approach (b) is quite sensitive to k_S. The curves presented below in Figure 6 were calculated at **q**'. In this case, approach (b) failed to provide any decrease of the cost function without regard to the value of k_S, while (a) was still effective and predictable (the best $k_S = 1.0$).

In a series of additional experiments, the end effector positions were changed throughout the workspace and different initial estimations were examined. We confirmed the advantages of the introduced optimisation technique over the standard one. The approach adopted in this work does not guarantee the absolute optimum in all circumstances, but its potential to locate the absolute optimum was consistently greater than that of the standard approach. It was also observed that the convergence was assured in a relatively modest number of iterations. The major criticism to the proposed approach is that in the present form it incorporates the calculation of Eigen vectors of $\mathbf{J}_S\mathbf{N}_P$ in each iteration. However, the singular value decomposition, which provides Eigen vectors, is nowadays quite a frequent operation in advanced kinematic control.[28]

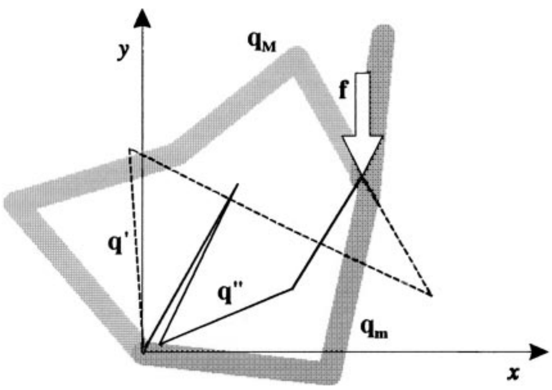

Fig. 5. Initial configuration and final configurations obtained by optimisation.

7. CONCLUSIONS

A standard technique to provide an effective execution of the secondary task of redundant manipulators consists of rotating the null space projection operator by changing the weighting matrix of the pseudoinverse. Unfortunately, this matrix cannot be changed arbitrarily since it influences the manipulator's performance. In our work we propose an approach that is independent on the chosen null space operator. It always provides the best possible execution of the secondary task without interfering with the primary task. Hence, the secondary task is executed more efficiently and the numerical method is more robust.

The paper shows a numerical example with a 4R planar manipulator where the primary task was to position the end effector and the secondary task to minimise the static joint torques. We compared the proposed approach with a standard one. In both we utilised the unweighted pseudo-inverse. The proposed approach showed much faster and stable convergence and its potential to locate the absolute optimum was much greater.

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