domains are subsequently reduced to Fredholm integral equations of the first kind on the domain surface, which are then solved by means of the Fredholm Alternative. The regularity of the solutions of these integral equations is also discussed.

In the last part of the book, the theory developed for general strongly elliptic systems is illustrated in application to the Laplace equation, the Helmholtz equation and the equilibrium equations of linear elasticity in the homogeneous and isotropic case. Some technical analytic details are gathered together in three appendices.

The mathematical treatment is rigorous and the presentation is clear and easy to follow. The book is a good source of information for scientists and engineers interested in learning about boundary integral equation techniques, and in the development of boundary element methods for the approximate solution of a wide class of linear boundary value problems.

C. CONSTANDA

KUSRAEV, A. G. *Dominated operators* (Mathematics and its Applications, vol. 519, Kluwer, 2000), xiii+446 pp., 0 792 36485 6 (hardback), £120.

In spite of the widening contacts between mathematicians in the former Soviet Union (fSU) and the western world, there remain many legacies of the decades of separate development of mathematics in the two regions. One of these is that research in some disciplines has developed in rather different directions. One such field is that of vector and Banach lattices and operators on them. As far back as 1935–1936 Kantorovich introduced the notion of a *lattice-normed space*, which generalizes the notion of a real normed space to consider vector spaces endowed with a 'norm' taking values in a vector lattice rather than the reals. If we take $E = \mathbb{R}$, then we obtain the classical (real) normed spaces, while taking $X = E$, with the lattice modulus taking the role of the norm, gives another class of examples. The role of bounded operators in this setting is played by the so-called *dominated operators*, T, defined by the requirement that the 'norm' of Tx is dominated by the result of applying S to the 'norm' of x for all $x \in X$, where S is a positive linear operator on E. In the case of normed spaces this notion coincides with that of the traditionally bounded operators, while, when $X = E$, this is precisely the class of regular operators (those which can be written as the difference of two positive operators). Professor Kusraev has researched into these and related notions for around 20 years and to a large extent the work under review may be seen as a summary of his research over that period, while at the same time setting it into context. This work provides a convenient opportunity to find out what has been happening in the fSU in this field, one which has been almost completely neglected outside the fSU.

The work starts with a chapter on *Boolean Algebras and Vector Lattices*, which is explicitly presented as a summary of needed results and to establish notation which, in a field where even western literature often disagrees as to terminology and where the fSU usually employs completely different terminology, is vital. After a chapter on *Lattice-Normed Spaces*, a chapter on *Positive Operators* sets the scene for the basic chapter on *Dominated Operators*. There follow chapters on two particular concrete classes of operators, *Disjointness Preserving Operators* and *Integral Operators*, and their relationship with the class of dominated operators. If the vector lattice E is actually a normed lattice, then one can norm (in the traditional sense) a lattice-normed space by composing the norm on E with the lattice norm on X. This allows a generalized abstract setting in which to study *Operators in Spaces with Mixed Norm*, which is the topic of the seventh chapter. Topics included here include various kinds of summing operators and Kaplansky–Hilbert modules. The final chapter is on *Applications of Boolean-Valued Analysis* in this area (an appendix provides an introduction to Boolean-valued models for those unfamiliar with this field).

This book must be regarded as being for those with a serious interest in vector lattice and regular operator theory, and even they will find much of it rather hard going. The first chapter contains few proofs, as is to be expected from its intended function, but references to proofs could be much more complete. For example, Theorem 1.5.3, which formulates several equivalences of order continuity of the norm (including one involving lateral σ-continuity of the norm, which is not in most standard texts), cites four works for the proof, of which three are books, with no indication of where to look in each for the proof or of whether these are alternative sources of a proof or whether they need to be combined. While the author mentions alternative terms for many notions defined, he actually works with an uneasy mixture of western and fSU terms, mainly using western ones but adopting terms like 'fragment' for what is normally referred to as a 'component', apparently because the Russian word for 'component' is used for what the west would call a 'band'. Occasionally notions are used in examples before being formally defined: for example, *orthomorphisms* are first defined on p. 109 but are referred to in an example on p. 50. The English in the work is not perfect, but is good enough that no serious ambiguity should be found and there were no faults that I could find with the typesetting of symbols.

To my mind, though, the most serious defect in this work is the lack of examples. There are, to be sure, many examples of lattice-normed spaces in § 2.3, and the chapter on integral operators abounds with conditions for operators to be dominated. What is missing is examples to show what the results proved in this book amount to in concrete cases and to show that there are new results, of independent interest, to be obtained from this approach. Nor is there any indication as to whether Kantorovich's early applications to approximate solutions of functional equations have been followed up at all. Indeed, a cursory scan through *MathSciNet* suggests that, outside the Russian school of which Kusraev is part, there is little current interest in this area.

For those with a serious interest in vector lattices, though, there are some gems to be gleaned from this work. One example is the proof that there are non-discrete universally complete vector lattices on which every band preserving operator is order bounded. As Gutman's original proof of this depended on Boolean-valued analysis, this, completely standard, exposition is very welcome. Another is the work on extending positive operators to Maharam (interval preserving order continuous) operators. Here, though, as in many other parts of this work, even the enthusiast may well wish that the work had been presented in a rather less general setting.

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