(3) 38358837677 is the smallest known prime for which the inequality

$$\pi^2(p) < \frac{ep}{\log p}\pi\left(\frac{p}{e}\right)$$

does not hold, where π is the prime-counting function (p. 355). Incidentally, Ramanujan proved that this inequality holds for p sufficiently large.

(4) 130370767029135901 is the 17th horse number. A number n is said to be a *horse number* if, for some k, it represents the number of possible results, accounting for ties, in a race in which k horses participate (p. 381).

The vast majority of the results given are presented simply as facts. Although one would imagine that most of the properties had been obtained with the aid of a computer, it is not in general clear which results had arisen in this manner and which had been derived by way of manual proofs. Although some proofs are provided, they are few and far between.

The target audience is difficult to gauge; maybe, in addition to students with a fascination for number theory, this book might appeal to people interested in recreational mathematics or to those looking for ideas on which to base number-theoretical, possibly computer-aided, investigations. It is extremely well-referenced and could certainly instigate some little explorations into various aspects of number theory.

The statement on the back cover of the book starts with the sentence:

> "Who would have thought that listing the positive integers along with their most remarkable properties could end up being such an engaging and stimulating adventure?"

This is a very good question! Although it is not a book that I could ever see myself buying, I did enjoy rummaging through it in a semi-random fashion, being pulled here and there by way of the various cross references. This is a difficult book to recommend, although if you are genuinely fascinated by the properties of the integers, however obscure they might seem, then you would probably take pleasure in making a rambling journey through its pages. It is the sort of book I will return to every now and again in order to pick out some new number-theoretic gem.

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The finite simple groups, by Robert A. Wilson. Pp. 298. £53.45 (Hardback). 2009. ISBN: 978-1-84800-987-5. (Springer).

In one of her novels [1], Iris Murdoch wrote that the archaic period of early Greek history 'sets a special challenge to the disciplined mind. It is a game with very few pieces, where the skill of the players lies in complicating the rules'. Allowing myself the temerity of altering, or just swapping round, a couple of her words, the same can perhaps be said of the study of the finite simple groups.

Groups are just about the most abstract of mathematical objects and therefore any discovery of their properties empowers us to apply the knowledge to countless different areas. Problems involving the structure of finite groups, and thus their action on other related objects, can often be reduced to problems about finite simple groups. Similar to the primes among the natural numbers, the finite simple groups are the basic building blocks from which the finite groups are made. The reader may have seen such a vague sentence before, but a more precise way of saying it will require the definition for simplicity and the Jordan-Hölder theorem. Anyway, just like the primes, the finite simple groups are of fundamental importance in many different areas of mathematics, and what many have dubbed the *enormous theorem* on the complete classification of finite simple groups has been essentially established. It is tough enough to discover or find the esoteric items for classification, but the seriously difficult part of the game is to prove that they have all been found for classification. Mathematicians rarely exaggerate, and the effort involved in proving the theorem is indeed enormous–we are talking about tens of thousands of pages of published work, by a hundred or so authors in many countries, over several decades.

If abstract objects can be pinned down to an exhaustive list in a classification, and the said objects can be constructed, then the abstractness becomes more concrete. Thus the classification theorem should have widespread applications in many branches of mathematics, perhaps enabling us to solve some problems by studying only finitely many configurations and, although there may still be infinite families of such groups, one may hope that a single method, with small modifications, is enough for the whole family. A huge amount of information about the finite simple groups has been accumulated, and a major challenge is to convey this information to those who wish to apply this knowledge. However, these groups remain difficult to study, which hinders the application of the classification.

The book being reviewed is aimed at advanced undergraduates and graduate students in algebra as well as professional mathematicians and scientists who use groups and want to apply the knowledge which the classification has given us. The main prerequisite is an undergraduate course in group theory up to the level of Sylow's theorems. It is the first text at this level in which all the finite simple groups are treated together, pointing out their connections. Obviously anything even remotely near a complete or proper treatment is totally out of the question. Nevertheless the author succeeds admirably in bringing the groups to life by giving concrete constructions of most of them, thus allowing calculations in them and also in the underlying geometric and algebraic structures.

The text is very well organised. The introduction, which forms the first chapter, contains a brief history and the statement of the classification theorem, together with sections giving remarks on the applications and the proof of the theorem. 'For exposition purposes it is convenient to divide [all the (non-abelian) finite simple groups] into four basic types, namely the alternating, classical, exceptional and sporadic groups.' Indeed they form the four main chapters of the book. There are sections on general linear, sympletic, unitary, and orthogonal groups, in the chapter on classical groups. The ten families of so-called 'exceptional groups of Lie type' are described in the fourth chapter. The approach to them via Lie algebra has been given in the well known text [2] by R. W. Carter, and the author adopts an 'alternative' approach which involves much explicit calculations, thus enabling him to construct the generic covering groups. The 26 sporadic simple groups are then introduced in the last and longest chapter. These exotic groups have names associated with Mathieu, Conway and others, culminating in the Monster. "Finally we describe the six 'pariahs' in Section 5.9 . . . . The behaviour of these six groups is so bizarre that any attempt to describe them ends up looking like a disconnected sequence of unrelated facts. I make no apology for this–it is simply the nature of the subject." Throughout the text the author's commitment to, and interest in, his subject shone through.

The exposition is lean, compact, and succinct without being terse; indeed I would describe it as elegant and as straightforward as it can be given the subject matter. Although most of the subsections are short, the pace is fast, and undivided attention is always required. The author often begins by describing the relevant structure of a group in enough detail to calculate the order of its automorphism group and to prove simplicity of a clearly defined subquotient of this group. Consequently the book may also be useful to a reader who just wants an introduction to a particular group or family of groups. My only criticism is that more suitable examples to illustrate many of the difficult topics could have been given. The first half or so of the book is reasonably self-contained, and if any topic is likely to be new to the undergraduate reader, such as finite fields, quaternions, octonions, Clifford algebras, spin groups, Golay code, there is a concise and precise short section preceding it. However, the reader who wishes to understand thoroughly the material in the book will have to work hard to fill the gaps by doing the tough exercises and follow up the references.

References

1. Iris Murdoch, *The Nice and the Good*, (Penguin, 1968).
2. R. W. Carter, *Simple Groups of Lie Type*, (Wiley, 1972; reprinted 1989).

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Sacred mathematics, by Hidetoshi Fukagawa and Tony Rothman. Pp. 348. £26.95. 2008. ISBN 978-0691127453 (Princeton University Press).

Traditional Japanese mathematics problems, by Hidetoshi Fukagawa and John Rigby. Pp. 191. $50. 2002. ISBN 981-042759X (SCT Publishing, Singapore).

During the 18th and 19th century, a unique form of home-grown mathematical endeavour flourished in Japan, a country which was totally isolated from Western civilisation by self-imposed decrees. Wooden tablets, known as *sangaku*, and elaborated inscribed with beautiful geometrical configurations, were hung in Buddhist temples and Shinto shrines. These problems were part of a tradition of amateur mathematics which involved people – mostly anonymous – from all walks of life.

Hidetoshi Fukagawa is a retired high-school teacher who is a world expert on *sangaku*, and his love for these extraordinary artefacts is manifested in these two collaborations with English mathematicians. The Princeton hardback book is impressively produced, with excellent colour plates of the tablets and paintings by court artists, together with photographs of the temples themselves. It is a pleasure to look at and would grace any coffee table. The first few chapters narrate the history of the tradition and explore the position occupied by mathematics in imperial culture. A fascinating section reproduces parts of the travel diary of Yamaguchi Kansan, who spent six years recording sangaku problems throughout the country.

Readers of the *Gazette* will, however, be mainly interested in the problems themselves. Three chapters of the book are devoted to these, organised in order of difficulty, and there are solutions to most of them. A regular theme in the problems is systems of tangential circles, and an expository chapter outlines the technique of inversion which is a useful tool for tackling these. Many of the problems are, however, very difficult, and the solutions often involve rather unpleasant algebra, coordinate geometry or calculus. I must admit to feeling somewhat frustrated as I tried to work through them and I was also puzzled by the vagueness of some of the descriptions.

The collaboration with John Rigby is a much more modest affair, being a humble paperback with no illustrations and hardly any historical detail. What distinguishes it, however, is the emphasis on mathematics and the quality of the solutions. The