A GUIDE TO LOGICAL PLURALISM FOR NON-LOGICIANS

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There exists today an infinite variety (literally) of formal logics, different systems with incompatible properties. One way to explain the existence of many logics is to posit logical pluralism*: to suppose that there is in fact more than one correct logic. In this selective guide – designed for non-logicians – we look at some examples of how different logics come into apparent conflict with each other. Then we look at different ways of understanding the idea of logical pluralism. Our question throughout will be a basic one: does logical pluralism have any limits?*

Logic, as a discipline, is like any other human enquiry: full of controversy and uncertainty. For any 'law of logic' there is at least one reasonable logician who has challenged it. There exists today an infinite variety (literally) of formal logics, different systems with incompatible properties – see for example Priest (2008). What does this mean for logic as a subject? One way to explain the existence of many logics is to posit *logical pluralism*: to suppose that there is in fact more than one correct logic. Logical pluralism stands in contradiction to the historically prominent conception of validity, according to which there is only One True Logic. 'There is no core of universally accepted logical principles', writes Timothy Williamson. 'Nevertheless, logic seems to have come of age as a mature science' (Williamson 2014: 212).

In this selective guide – designed for non-logicians – we look at some examples of how different logics come into apparent conflict with each other. Then we look at different ways of understanding the idea of logical pluralism, from the basic idea of tolerance, to more sophisticated ideas

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about logical consequence and mathematical models. Our question throughout will be a basic one: does logical pluralism have any limits?

I. The Idea of Logical Pluralism

Logic was conceived of by Aristotle as a tool: rules for debates, and a means of *criticism* (Bobzien 2014). Different tools are appropriate for different jobs. An axe is good for chopping down trees, a chainsaw is better; a scalpel is good for heart surgery, but terrible for cutting down trees; while axes and chainsaws alike are bad for surgery. Why think there is One True Tool for all cutting jobs? And so, why think that there is One True Tool for all reasoning jobs? Or so goes one informal motivation for logical pluralism – the thesis that there is more than one correct notion of logical consequence. An entrée into pluralism is to allow that different logics might be better and worse for different legitimate uses.

Logic is the theory of valid arguments. What makes an argument valid? People disagree. There is, indisputably, a plurality of *logicians*. The pluralistic question is, are these logicians debating a topic with one right answer, or is there a way in which more than one might be right? When all the details are in, and two debaters are at a deadlock ('But the conclusion doesn't follow!' / 'Yes it does!'), the pluralist urges that we should not automatically assume that there must be a loser. At the very least, we miss something important about the debate unless we consider the possibility that it is not a zero sum game.

That's one way to see the idea, anyway. But how well does the idea of logic as a tool – an organ of criticism – speak for logical pluralism? Well, it makes understanding logical pluralism itself rather difficult. When we are having a disagreement, and all else fails, we might have thought that at least we can look to *logic*. We might disagree about values, about how to interpret data, or about how to form

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and implement policy, but can't we always at least say, 'Well, regardless of your opinion, if we do A then we can't avoid B. That's just a matter of logic?' Pluralism in logic tentatively suggests that the answer can be 'no'.

This raises an immediate foundational problem. If we can disagree about logic, it is very difficult to arbitrate disagreements *about logic itself* (Williamson 2014). A successful dialogue requires some common ground; logical pluralism raises the spectre that in some cases, no such ground exists.[1] For example, replying to an invitation to join a debate about a law of logic (see below), David Lewis writes (Lewis 2004: 176):

> I'm sorry; I decline to contribute to your proposed book about the 'debate' over the law of non-contradiction. My feeling is that since this debate instantly reaches deadlock, there's really nothing much to say about it.

We will look at a few disputes in logic. Then we will see if there is a way to reach any kind of pluralistic perspective about them.

II. Logical Controversies

Pluralism or no, logic itself is no different from any other area of human inquiry, in that, as a body of theory, it is open to rational disagreement. Theoretical logic was more or less invented at a distinct point in time, and has been revised and argued ever since, especially since the late nineteenth century, when it became a branch of mathematics. Put simply, then, pluralism is steeped in humility: it seems implausible to suppose that anyone right now has got it completely right, any more than we should suppose a current scientific theory to be 'final'.

But humility is only a start – after all, our current theory could be *defeasible*, open to revision, while still:

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(a) being agreed upon by most serious practitioners, at least on important points, and

(b) aiming at (and ever better approximating) a single final theory.

The idea of logical pluralism really gets its bite if neither (a) nor (b) is a good description of the current state of logical research. Pluralism is an attempt to make sense of the persistence of strong disagreement in the logical community. We will see this disagreement with a quick and selective survey.

1. Classical logic: Between (roughly) the 1850s and the 1930s, a great deal of research by Boole, Frege, Russell, Hilbert and Gödel went into developing what is now, anachronistically, called classical logic.[2] (It is anachronistic because the logic developed by Aristotle, the syllogistic, gives a different account of validity from the classical one.) Classical model theory, in the schools of Tarski and Robinson, flourished in the mid-twentieth century, buttressed by compelling writing by influential philosophers such as Quine (1970); and the 'classical' tradition continues strongly today.

Nevertheless, during roughly the same time frame, alternative ideas about logic were explored. For our purposes, we focus on the fact that classical logic is committed to

CONSISTENT BIVALENCE: every proposition is either true or false, and not both

Then *non-classical* logics (or at least the sort that concern us) are those that do not endorse, in one way or another, the principle of consistent bivalence. There may be propositions that have underdetermined truth; there may be propositions that have overdetermined truth. In the argot (due originally to Kit Fine), these are *gaps* and *gluts*.

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2. Gappy logics: Non-exhaustive or *paracomplete* logics are those that challenge the claim that *every proposition is either true or false.* This is the law of *tertium non datur,* the law of the excluded middle (LEM): it says that there is no third option. This law has a great deal of intuitive plausibility; e.g. you either ate eggs for breakfast this morning, or you didn't, and there seems to be no other way to go. Sophisticated bits of mathematical reasoning rest on the LEM. For example, *reductio ad absurdum*: we wish to show that *p* is true, by showing that *p* cannot be false – thus presupposing that, if *p* is not false, it must be true.

And yet, the LEM has come under a good deal of criticism. For a start, consider that egg you either ate for breakfast, or did not. What if you only took one bite of egg? What if you ate only egg whites? Or meringue? What if you ate an egg at 11:55 a.m., which is really too late for breakfast? In these cases, a definite 'yes' or 'no' answer becomes harder to defend.

Around 1920, the Polish logician Łukasciewicz, following Aristotle, put forward non-bivalent logics, in response to questions about time and freedom. What is the truth-value of the sentence *'You will eat eggs for breakfast tomorrow'*? If it is either true or false, it seems, the future has already been decided, and you have no choice; by contrast, if the future is open, not yet decided, then perhaps '*p* or not *p*' does not hold for all sentences. Similarly, in the 1920s the Dutch mathematician Brouwer argued that mathematical questions that have not been decided yet similarly should not be treated as having a definitive truth-value. A rich body of theory in paracomplete logic and mathematics has developed, showing how much rationality does (and does not) depend on unrestricted assumption of the LEM (Dummett 1977).

3. Glutty logics: Non-exclusive or *paraconsistent* logics challenge the idea that *no proposition can be both true and false*. This impossibility is expressed in the law of *ex falso quodlibet* (EFQ), that *if* a proposition were both true and false,

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then *every* proposition would be true. EFQ says that there are no true contradictions, in the same way that when a politician says 'If the polls are right, I'll eat my hat', he means that the polls are wrong.[3] Of course, not every proposition is true – suggesting otherwise is the logician's equivalent of hat-eating. We'll return at the end of Section III below to some further reasons why a contradiction might imply everything.

EFQ is an entrenched part of classical logic, but it gives rise to some counterintuitive reasoning, like:

1. Quantum mechanics is our best experimentally confirmed theory, so it is true.
2. Quantum mechanics is at odds with Einstein's relativity, which is true; so QM is false.
   Therefore,
3. Vatican City is the largest country on Earth.

The premises are inconsistent. Regardless of their truth, they seem to have no bearing on the conclusion. Some logicians think that an argument can only be valid if the premises have something to do with the conclusion. In EFQ, the conclusion could be any sentence at all – which does not look very much like good reasoning after all.

In 1948 the Polish logician Jaśkowski considered situations where conflicting information is present, and constructed a formal system that can tolerate some inconsistency without absurdity. In the 1950s, the Brazilian mathematician da Costa constructed systems now called 'logics of formal inconsistency'. And in the 1970s, logicians based in Australia developed systems in which EFQ not only fails, but in which some contradictions are taken as *true*. See Priest et al. (2017).

III. Logic and logicians

We have just seen examples of logics in which some foundational assumptions – roughly, of completeness and

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consistency – are relaxed. There are more assumptions that can be questioned; see Priest (2008). For us, the point is that logics can and have been challenged. So what are these disputes really about?

The mere fact that there are people who investigate and even believe in some theory does not imply any serious kind of pluralism. The mere fact that there is at least one person who disputes a point *p* does not make *p* contentious. The fact that, for a time, some astronomers studied the planet Vulcan, does not make Vulcan exist. And, if that's right, then the mere fact that there are different sorts of logic*ians*, does not *eo ipso* imply much about log*ics*. One cannot point out the existence (or not) of a logic in the way that one can point to the non-existence of a planet Vulcan – even if one can point to Vulcanologists (cf. Beziau 2002). Logics are theoretical entities. They may be normative theories of how one ought to reason, or descriptions of laws of how to preserve truth from premises to conclusions, or of the reasoning of ideal agents, or the like. In each case, there are no experiments one can run to check a law of logic.

A note on empirical data: one *can* run experiments on how people actually reason, but that is different: the Wason selection task (Wason 1977) shows that fewer than 10% of people are able to apply principles of reasoning consistently. This does not reflect on the correctness of those principles of reasoning; this reflects on human psychology. Keeping logic distinct from a description of actual human reasoning goes back to Frege, who called the latter 'psychologism'.

Logicians must work in the interface between theory and intuition. We have an initial, unreflective sense of what counts as 'good arguments'. We devise a theory to capture this sense. Then we measure how the theory stacks up, sometimes going back and adjusting the theory to better fit intuition, sometimes abandoning intuition to the theory. Peter Smith (2011: 28) writes,

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> We start with a rather inchoate jumble of intuitions about validity. ... We can sort things out in various directions. Pushing some way along in one direction (and there are other ways we could go, equally well rooted), we get an informal, still somewhat rough-and-ready classical notion of validity-in-virtue-of-form.

Smiley (1998) goes further: the idea of a valid consequence is 'an idea that comes with a history attached to it, and those who blithely appeal to an "intuitive" or "pre-theoretic" idea of consequence are likely to have got hold of just one strand in a string of diverse theories'.

To make things more precise, a logic is at the very least an ordered pair

$$\langle L, \vdash \rangle$$

L is a set of sentences $\{p, q, \ldots\}$. The relation $\vdash$ is to be read as *logical consequence*, which relates some members of L to other members of L. Under this conception, a logic is just *identified* with its consequence relation. Fix the conditions on relation $\vdash$, and you've fixed a logic. And, put this way at least, there really are no constraints on what such a relation can be like. So far, we could devise any relation we want and call it logic.

Two parody examples are 'all' and 'nothing'. The *empty logic* is given by setting the consequence relation equal to the empty set, so that nothing follows from anything. There are no valid inferences. Someone endorsing this would be a 'logical nihilist'. The *trivial logic*, on the other hand, makes everything follow from everything else. All inferences are valid. Someone endorsing it would be a 'logical lunatic'. Neither theory of logic would be very useful, to say nothing of correctness. A decent logic, as a theory of validity, must be something in between.

Where in between? Balance is hard to achieve; theory pushes back against intuition. As an example, it is fairly

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easy to convince people – say, intelligent undergraduates – that aspects of classical logic are not correct. For example, classical logic says, as an instance of EFQ, that all unicorns are purple (since there are no unicorns). That seems counterintuitive. One can start to alter the logical consequence relation, to exclude these counterintuitive results.

But the logically rebellious must tread carefully. One cannot simply gerrymander together a bunch of intuitions and expect the result to count as logic. Rejecting logical assumptions has substantial downstream effects. Consider: isn't the following an example of good logic?

1. Either the rabbit ran down the left path or the right one.
2. But the rabbit didn't run down the left path.
3. So it must have run down the right one.

The ancient logician Chrysippus claimed (Mates 1997: 69) that even a dog, chasing a rabbit, could reason thus, using the principle of

> *DISJUNCTIVE SYLLOGISM*: either *p* or *q*, and not *q*; therefore *p*.

But this inference is disputed, because it allows a proof of EFQ, as was shown by C. I. Lewis (see Priest 2008: 76). For Chrysippus' dog could also reason thus:

1. Suppose the rabbit ran down the left path, and also did not. (Perhaps it was going at a pace that is both running and not running.)
2. Since it ran down the left, it ran down the left *or* the Vatican is the biggest city on earth.
3. But it did not run down the left path.
4. So the Vatican is the world's biggest city.

So a rejection of EFQ also seems to mandate a rejection of disjunctive syllogism. Regardless of whether or not one

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makes this allowance, it is clear that one cannot simply mix and match logical principles at will.

There is a simple way that logical pluralism is true – there are many logicians – and there is a simple way that it is false – they are not all always correct. Altering local logical principles can lead to global effects. So it will take more than intuitions to fix on what a good theory of validity might be. To echo a good question from Hartry Field, 'Can one find a pluralist thesis that is high on the interest scale, and also true?' (Field 2009: 342).

IV. Pluralistic Proposals

Theorizing logical consequence has led in numerous directions, with no clear means of resolving disputes. The pluralist makes a pragmatic suggestion: perhaps rather than mounting the (maybe) insurmountable task of cleaning up this debate, in the quixotic hope of finding the One True Logic, we should embrace the diversity.

The proposals canvassed in this section are not exhaustive of the literature; see Russell (2013).

1. Tolerance Pluralism: A basic, and relatively unimpeachable, claim is that we should make space for intellectual freedom – for the plurality of *logicians* who erect divergent formal systems. Rudolph Carnap put forward perhaps the first modern articulation of pluralism, as a kind of tolerance (Carnap 1959: 52):

> In logic, there are no morals. Everyone is at liberty to build his own logic, i.e., his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments.

Or again (Carnap 1937: xv):

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> Let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols.

In the name of open and fair practice, we should give such logicians space to work.

Not only that, though: we should allow them their chosen epithet, 'logician'. After all, it is not in question that a person is free to explore abstract algebras to their heart's content. The interesting moment is when such a person insists that their algebra is in fact deserving of the name 'logic'. The tolerance pluralist might feel that it would be embarrassingly arrogant to quibble over the word – at least not without good reason. After all, the alternative is to suggest that someone who e.g. takes (p or $\sim p$) to fail has made something like a *typo* in using the '$\sim$' symbol – to suggest that the deviant logician, unbeknownst to him, simply does not know what he is talking about. This is the dogmatic line handed down from Quine (1970: 81):

> [N]either party knows what he is talking about. They think they are talking about negation, '$\sim$', 'not'; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form 'p & $\sim p$' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject.

The tolerance pluralist recoils from this kind of conservative name-calling. As David Lewis once put it, 'to suppose that [a non-classicist] mistakes mere terminological difference for profound philosophical disagreement is to accuse him of stupidity far beyond belief' (Lewis 1990: 30).

Nevertheless, perhaps we can constrain the meaning of 'logic' a bit by focusing on some logical vocabulary –

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words like 'and', 'not' and 'implies' upon which there might be some broad theoretical agreement. The disagreement over bivalence, for instance, is based on differences of viewpoint concerning negation; let's see how one might try to fix the meaning of a logical connective. On a syntactic or proof theoretic approach, the meaning of a logical constant is completely given by its introduction and elimination rules. For example, here is how conjunction, &, works, with the turnstile '⊢' representing consequence:

&-Introduction:	**&-Elimination:**
$p, q \vdash p \,\&\, q$	$p \,\&\, q \vdash p$
	$p \,\&\, q \vdash q$

There is one introduction rule, that given p and q, then p & q; and the two elimination rules say that either p or q, given p & q. And that's all there is to know about '&'. In an idea going back to Wittgenstein, there is no point trying to explain the meaning of '&' in terms of other already understood symbols, any more than it is worth giving a definition of the word 'hello'. The best way to show the meaning of the word 'hello' is to demonstrate appropriate and inappropriate uses thereof, and so too with logical vocabulary.

This conception of logical operators lends itself very easily to logical pluralism. (Cf. Restall (2014).) All one need do to specify a genuine consequence relation is to give rules for constants that constitute the desired relation. Any rule seems to determine a constant, and maybe that's all there is to the meaning of a constant. But matters are not so simple. One problem, identified by Prior (1960), is that the following are introduction and elimination rules for a connective called 'TONK':

TONK-Introduction:	**TONK-Elimination**:
$p \vdash p \text{ TONK } q$	$p \text{ TONK } q \vdash q$

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Why is this a problem? Because it seems that we can go from *any* sentence *p* to *any* sentence *q*. Assuming some very basic rules (modus ponens, transitivity), then the purported logical constant 'TONK' leads from some truth like '$1 + 1 = 2$' to an absurdity, like 'it is raining frogs!'. Thus, focusing in on logical constants as governing which logic is correct, and which not, will only get us so far.

And even if one could solve the TONK problem, this conception of logic (as given by rules of usage) does not obviously help with disagreement over more familiar connectives, e.g. the behaviour of negation. For example, are these rules correct?

Negation introduction	**Negation elimination**
p is absurd $\vdash \sim$p	$\sim\sim$p $\vdash p$

The introduction rule requires us to say what counts as 'absurdity'. A natural thought is that all contradictions are absurd. But this is exactly what a glutty, paraconsistent logician might contest.[4] The elimination rule, by turns, is assuming what the paracompletist denies – that there are only two ways things could be. (If you don't want to get married, perhaps you should wait before you go through with it.) Simply stating these rules can't help us decide whether they are good or not.

2. Models as Logic: It would help to know more about logic than just how to introduce and eliminate its connectives. We need to know what '$\vdash$' is. The most developed form of pluralism is to focus on the *semantics* of logics, and to view models *as* logics. This is the approach pioneered by Beall and Restall (2006), and the source of most of the recent literature on the subject.

For a start, almost everyone agrees that when it comes to deductive logic, we want the $\vdash$ relation between premises $p_0, \ldots, p_n$ and conclusion q to *fail* if there is a way for all of the premises to be *true* but the conclusion *false*. This

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has been the missing ingredient in our discussion so far, and now rules out many relational structures. Proposals for interpreting consequence relations are called *semantics.*

The semantic conception of logical consequence goes back to Tarski (1983), who argued that an argument is valid if and only if there is no case in which the premises are true, but not the conclusion. Expanding on this thought, Beall and Restall (2006: 29) put forward the centrepiece of their proposal – that 'the settled core' of logic is the

> *Generalized Tarski Thesis (GTT): an argument is valid(x) if and only if, in every case(x) in which the premises are true, so is the conclusion*

Beall and Restall's idea is there are multiple different admissible instances of GTT. These are obtained by fixing the meaning of '*case(x)*'. The basic idea is that *x* can range over:

- *worlds*, where truth is exclusive and exhaustive
- *situations*, where truth is not exhaustive (gaps)
- *impossible worlds*, where truth is not exclusive (gluts)
- *impossible situations* (gaps and gluts)

By varying the *case(x)*, one thereby gets varying notions of *valid(x)*. For example, *case(situation)* yields *valid(situation)*, a paracomplete logic.

Still following Tarski, any instance of the GTT is still expected to have properties making it a *Tarskian consequence relation*:

$p \vdash p$	(reflexivity)
if $p \vdash q$ and $q \vdash r$ then $p \vdash r$	(transitivity)
if $p \vdash q$ then $p \ \ r \vdash q$	(monotonicity)

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Beall and Restall stipulate the three further conditions on the consequence relations. *Necessity* is the condition that the truth of the premises necessitates the truth of the conclusion: it *cannot be* that the premises are true and the conclusion false. *Normativity* is the condition that if an argument is valid then one 'goes wrong' accepting its premises while rejecting its conclusion. *Formality* is the condition that the validity of the argument depends on its form rather than its content. Any solution for GTT that satisfies these constraints is, according to Beall and Restall, a legitimate logic.

The models-as-logics approach makes 'the' correct logic a matter of purpose and context. Different contexts call for different tools. If one wants to reason about situations – perhaps in order to focus on beliefs, for example, which are not exhaustive (I neither believe nor disbelieve a great many things) – then one should use an appropriate notion of validity. Then the argument from B to Av∼A is invalid, paracompleteness. If the target cases are impossibilities, then the argument from A & ∼A to B is invalid, paraconsistency. In impossible situations, both these arguments and more are invalid. If one wants to solve a Sudoku puzzle, then good old classical consequence is the tool. Solving for *x* in the GTT shows how this can be done in a principled way, without deviating far from the established methodology of formal logic. For criticisms of this approach, see Goddu (2002), Wyatt (2004).

3. Logics as models: One other form of pluralism takes a cue from the philosophy of science, conceiving of logic as a model. A logic is not meant to be a perfect representation of thought, language, or reasoning. Instead, a logic is a *model* of the data, and as a result, it has all of the advantages and limitations that are present in modelling elsewhere (such as in the empirical sciences). One should expect these advantages and limitations to reappear in the study of logic (Cook 2010: 500). 'A formal language displays certain features of natural languages, or idealizations thereof, while simplifying other features'

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(Shapiro 2006: 49). On this view, there can be multiple, incompatible, competing models of the same phenomenon:

> [W]ith mathematical models generally, there is typically no question of 'getting it exactly right'. For a given purpose, there may be bad models – models that are clearly incorrect – and there may be good models, but it is unlikely that one can speak of the one and only correct model. There is almost always a gap between a model and what it is a model of. (Shapiro 2006: 50)

At given stages of scientific development, it may not be appropriate to say that one theory is clearly closer to the truth than another. The view seems to leave open whether or not there is a single 'final theory' at which logical monism again kicks in, or whether different chains of theories can each terminate in equally acceptable final theories.

V. The limits of pluralism?

Pluralism, as a species of tolerance, makes a good deal of room for different consequence relations to lay equal claim to being logics. But is there any limit? Thinking of pluralism about logic as a claim that every 'local' situation has its own sort of logic (as opposed to there being a 'global' logic)

> soon runs into difficulties (as the geographical image suggests) at the boundaries, as to how the local logics impinge upon one another and how they combine. For example, what happens in a boundary area between the localities? In new (unclassified) situations? If one can't guarantee the location (e.g. because consistency isn't provable)? (Routley 1980: 897)

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The models-as-logic and logic-as-models versions of pluralism both presume there is some 'settled core' of logic, albeit with highly mutable edges, but settled nonetheless. Most explicitly, features of Tarskian consequence like *transitivity* are taken to be inalienable. The question now is: can such inalienable features really be claimed in any principled way? Or is the (dare we say?) logical conclusion of pluralism that 'anything goes'?

Indeed, features at the 'settled core' of logic have been called in to question. Strawson (1952: 15) calls reflexivity into doubt. On transitivity, systems without it have been put forward by Smiley (1959) and more recently, Ripley (2013), who forcefully argues that the rule is *not* valid in full generality. Indeed, Ripley (2015) argues that, using a non-transitive logic, anything *does* go. The apparently excrescent rule of TONK, for instance, is only a problem if we assume that consequence is transitive! Relaxing this assumption, which we do on independently motivated grounds, shows that any introduction and elimination rules really do suffice to define a logical constant.

Pluralism is a claim about *logic*, not *logicians*, but pluralists are nevertheless impressed by the existence of (apparently sane) logicians purporting to be studying different things they call logic. Anyone so impressed will have a hard time excluding hyper-deviant logicians. Faced with this challenge, Beall and Restall find their arguments run out:

> What can we say? We hold the line. The given kinds of non-transitive or irreflexive systems of 'logical consequence' are logical by courtesy and family resemblance, where the courtesy is granted via analogy with logics properly so called. Non-transitive or non-reflexive systems of 'entailment' may well model interesting phenomena, but they are not accounts of logical consequence. One must draw the line somewhere and, pending further argument, we (defeasably) draw it where we have. We are pluralists. It does

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> not follow that absolutely anything goes. (Beall and Restall 2006: 91)

It is easy to sympathize with a sense of needing to 'hold the line'. But what principle is behind it?[5] After all, arch-conservatives hold the line better than anyone.
Williamson writes,

> Once we have seen that the contentiousness of logic is radical enough to reach metalogic, we should be suspicious of any attempt to bound logic or metalogic to the insubstantive, the non-ideological. Much though we may long for such a neutral arbiter to discipline philosophical debate, we cannot always have one ... Logic is a science ... Since when was science uncontentious? (Williamson 2014: 230)

Indeed, as with simpler conceptions of science, one might have hoped logic is not subject to pluralism. On one view, logical laws are preconditions for the very possibility of thought; if so, how could they be coherently questioned without immediately devolving into nonsense? On a standard conception, even *God* is subject to the laws of logic, His omnipotence dulled by the logical impossibility of creating a Rock So Heavy He Himself Cannot Lift It. Even devout theists seem to think that it is not God, but logic, that is the final arbiter of what is possible and what is not. And yet, humble pluralism suggests that when reckoning with impossibly heavy rocks, maybe God should not take Himself to be limited to classical logic.

Where does this leave us? As Williamson points out, the issue reaches metalogic, the logic for talking about logic. Here is a stark example: the logical monist claims that there is *one true logic*, and according to Bertrand Russell's celebrated 1905 theory, we should understand this claim – a definite description, 'the one true logic' – as follows:

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There is a logic, and it is unique.

Well, this itself is a claim that can be analysed using the tools of formal logic. Russell himself suggested:

For some x (Lx and for all y (Ly implies $x = y$))

where L is the predicate 'is a logic'. But an implication (p implies q) is defined, for Russell, as 'either $\sim p$, or q'; so the description reads

For some x (Lx and for all y (either $\sim Ly$ or $x = y$))

Suppose that logical monism is correct, in the sense just formalized. What does it entail? Suppose we have two putative logics: a and b, so

La and for all y ($\sim Ly$ or $a = y$)

Lb and for all y ($\sim Ly$ or $b = y$))

It follows that

(La and $\sim La$) or (Lb and $\sim Lb$) or $a = b$

There are three possibilities, two of them contradictory. To conclude that $a = b$, that there is only one logic after all, requires a disjunctive syllogism (see above). According to a glut-tolerant paraconsistent logic, that there really is only *one* logic will follow only if it is *impossible* that something both be a logic and not. To determine a *unique* logic, it cannot be possible that anything both be a logic and not. But it is clearly a difficult matter as to where the cut-off point for being a logic lies; perhaps some systems both are and are not logics, because they bear too much of a family resemblance to exclude, but have too many pathologies to allow?

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With pluralism at the meta-level, while we can speak of *the* one true logic, at least in the sense that Russell suggested, if we do so from a paraconsistent standpoint, logical monism is *compatible* with there being *multiple* one true logics. The pluralist can be a pluralist about pluralisms.

The logical limits of tolerance, at the limits of logic itself, are only beginning to be explored. Even theologians say that God bows his head to logic. Logicians are another matter.

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Notes

[1] For relations to relativism, see Cook (2010: 493).

[2] For an excellent collection of primary sources from this amazing period, see van Heijenoort (1967).

[3] Lib Dem former leader Paddy Ashdown said to the BBC, 'If this exit poll is right, Andrew, I will publicly eat my hat on your programme' (*The Telegraph*, 8 May 2015). It was right, but he did not.

[4] An absurdity operator, $\perp$, can be given use rules: it has no introduction rule (thankfully), and the elimination is

$$\perp \vdash p$$

for any p. So if $\perp$ is just a contradiction, this would reduce to EFQ.

[5] Beall and Restall are right, that 'some' does not imply 'all'. *That is*, unless one is a metaphysical monist – someone who thinks there is only one object.

References

Beall, J. C. and Restall, Greg (2006) *Logical Pluralism* (Oxford: Oxford University Press).

Beziau, J.-Y. (2002) 'Are Paraconsistent Negations Negations?', in Walter A. Carnielli, Marcelo E. Coniglio, and Itala M. Loffredo D'Ottaviano Marcel (eds.) *Paraconsistency: The Logical Way to the Inconsistent* (New York: Marcel Dekker), 465–86.

https://doi.org/10.1017/S1477175617000239 Published online by Cambridge University Press

Bobzien, Susanne (2014) 'Ancient Logic', in Edward N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy* (Spring 2014 Edition).

Carnap, Rudolph (1937) *The Logical Syntax of Language* (London: Kegan Paul).

Carnap, Rudolph (1959) *The Logical Syntax of Language* (Paterson, NJ: Littlefield, Adams, & Co.).

Cook, Roy (2010) 'Let a Thousand Flowers Bloom: A Tour of Logical Pluralism', *Philosophy Compass* 5/6: 492–504.

Dummett, M. A. E. (1977) *Elements of Intuitionism* (Oxford: Clarendon Press).

Field, Hartry (2009) 'Pluralism in Logic', *Review of Symbolic Logic* 2(2): 342–59.

Goddu, G. C. (2002). 'What exactly is logical pluralism?', *Australasian Journal of Philosophy* 80(2): 218–30.

Lewis, David (1990) 'Allism or Noneism?', *Mind* 99(393): 23–31.

Lewis, David (2004) 'Letters to Beall and Priest', in Graham Priest, J. C. Beall and Bradley Armour-Garb (eds.) *The Law of Non-Contradiction* (Oxford: Oxford University Press), 176–7.

Mates, Benson (1997) *The Skeptic Way* (Oxford: Oxford University Press).

Priest, Graham (2008) *Introduction to Non-Classical Logic* (Cambridge: Cambridge University Press).

Priest, Graham and Tanaka, Koji and Weber, Zach (2017) 'Paraconsistent Logic', in Edward N Zalta (ed.) *Stanford Encyclopedia of Philosophy* (Fall 2017 edition).

Prior, Arthur (1960) 'The Runabout Inference Ticket', *Analysis* 21: 38–9.

Quine, Willard (1970) *Philosophy of Logic* (Englewood Cliffs, NJ: Prentice-Hall).

Restall, Greg (2014) 'Pluralism and Proofs', *Erkenntnis* 79(2): 279–91.

Ripley, David (2013) 'Paradoxes and Failures of Cut', *Australasian Journal of Philosophy* 91(1): 139–64.

Ripley, David (2015) 'Anything Goes', *Topoi* 34(1): 25–36.

Routley, Richard (1980) 'Ultralogic as Universal?', Appendix to *Exploring Meinong's Jungle and Beyond*. Australian National University, 1980.

Russell, Gillian (2013) 'Logical Pluralism', in Edward N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy* (Spring 2014 Edition).

https://doi.org/10.1017/S1477175617000239 Published online by Cambridge University Press

Shapiro, Stewart (2006) *Vagueness in Context* (Oxford: Oxford University Press).

Smiley, Timothy (1959) 'Entailment and Deducibility', *Proceedings of the Aristotelian Society* 53: 233–54.

Smiley, Timothy (1998) 'Conceptions of Consequence', in E. Craig (ed.) *Routledge Encyclopedia of Philosophy* (London: Routledge). Accessed 18 September 2017.

Smith, Peter (2011) 'Squeezing Arguments', *Analysis* 71(1): 22–30.

Strawson, P. F. (1952) *Introduction to Logical Theory* (London: Methuen).

Tarski, Alfred (1983) 'On the Concept of Logical Consequence', John Corcoran (ed.) *Logic, Semantics, Metamathematics*, 2nd edn (Indianapolis, IN: Hackett), 409–20.

van Heijenoort Jean (ed.) (1967) *From Frege to Gödel: A Sourcebook in Mathematical Logic, 1879–1931* (Cambridge, MA: Harvard University Press).

Wason, P. C. (1968) 'Reasoning about a Rule', *Quarterly Journal of Experimental Psychology* 20: 273–81.

Williamson, Timothy (2014) 'Logic, Metalogic and Neutrality', *Erkenntnis* 79(2): 211–31.

Wyatt, N. (2004) 'What are Beall and Restall Pluralists About?', *Australasian Journal of Philosophy* 82: 409–20.

https://doi.org/10.1017/S1477175617000239 Published online by Cambridge University Press