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Feedback control for object manipulation by a pair of soft tip fingers

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SUMMARY

This paper discusses the problem of stable grasping and object manipulation by a pair of robot fingers when fingertips are covered with soft compressible material and fingers are allowed to incline their last link against the object surface. The area contact between the fingertips and the rigid object surface leads to nonholonomic constraints even for the planar case; however, the variational principle can be applied and the equation of motion is derived as a set of nonlinear differential equations with extra terms of Langrange multipliers incorporating the constraints. The proposed feedback controller is a linear combination of simple feedback control signals each designed for realizing grasp stabilization, regulation of object rotation and regulation of object position respectively. The controller is shown to achieve asymptotic convergence to the desired state at a stable grasping configuration. Simulation results are presented confirming the theoretical findings.

KEYWORDS: Feedback control; Soft tip fingers; Object manipulation; Stable grasping.

1. INTRODUCTION

It has been widely recognized that the realization of human dexterity in grasping and manipulation of various objects by robot hands will have a significant impact in the future of robotic applications and robotic prosthetic devices. It has been argued that the combination of forward facing binocular vision and an opposable thumb was the trigger that eventually turned *homonoid* into *homo*. The hand has been described as the extension of the will. It is true that the act of grasping and manipulating an object, a fundamental activity of daily life, is not a simple process; however, the human learns to perform the task with such skill that it seems to be effortless to the casual observer. The aim of this work is to assist in the understanding of this human ability and to contribute somewhat to the future progress of robotics as well as to hand prosthesis.

In the robotic literature, the problem of dexterous manipulation is formulated around the object which is required to move from one configuration to another. It is often less important to follow a desired object trajectory than it is to achieve grasp stability during object motion. The problem of manipulating an object by fingertips is therefore the determination of the required actuator forces and torques which achieve the desired object configuration with a dynamically stable grasp. In the majority of the published work the control of dexterous manipulation has been decomposed into hierarchical levels of grasp and motion planning at the higher level and real time control at the low level.[1]

At the higher level, a plan is made of the contact locations and finger motions for the desired manipulation based on the choice of the best grasp according to a quality criterion from the space of possible stable grasps. The space of stable grasps is dependent on the contact type and mode. The most commonly used contact types are rigid contacts and point contacts with or without friction. Soft finger contacts have also been considered in some cases. The contact mode depends on whether fingertips are allowed to roll and/or slide upon the surface of the object during manipulation. In this last case, desired contact motion is also planned.

At the low level, suggested control laws have been dominated by model-based approaches which are using the system kinematics and dynamics to linearize and decouple the system with respect to the object motion and internal force. Simple feedback control laws are then utilized to achieve the desired object position and internal force which is determined at the planning level in order to ensure that grasp constraints hold at all times.

The object centered approach of dexterous manipulation requires knowledge of the geometric relations of the fingers-object system, including contact locations and fingertip geometries, in order to develop the kinematic and force relationships between joint, contact and object space. The complexity of these relationships are affected by contact type and mode. Rolling and/or sliding contacts which are usually introduced to enhance manipulation dexterity complicate kinematic relationships. As fingertips are made out of rubber or other soft material the need to consider soft finger contacts has been recognized while point contacts are adequate only when fingertip and object are rigid bodies.[2] Soft finger contacts are usually treated as point contacts but both tangential friction forces and a frictional twist/moment around the contact normal are considered.

During the last fifteen years multi-degree of freedom robotic hands have been built to perform object manipulations.[3] These robotic hands are not restricted in weight, actuator size and computing power to the extent prosthetic hands are. Nevertheless, research in prosthetic hands was based on this technology and started by experimenting with electromyographic control of multifingered hands, force sensory feedback implementation and design of prosthetic hand mechanisms.[4–7] However, prosthetic hand research is at a preliminary level limited not only by the actuator weight and size and the lack of advanced sensing interfaces between the human and artificial part but also by the use of traditional control approaches that cannot yet display the adaptability and versatility of the human hand. As both a human and a robot controller are in the control loop a hierachical structure is suggested in reference [4] according to which the human controller controls the arm, addressing the planning issues of hand preshaping and enclosing motion associated with the object, through proportional myoelectric control, while the robot controller is responsible for maintaining a stable grasp during the task using sensory feedback.

In this work, we address the problem of dexterous object pinching motion by two robotic fingers with soft spherical fingertips made of compressible material by analysing the kinematic and dynamic models of the concerned motion and by proposing simple feedback control signals which obey two essential physical principles identified in references [8–9], that of the passivity relation between each signal and the joint velocities and that of the superposition of all signals that yields a skilled motion of stable grasping and desired manipulation. The problem is similar to the one treated in references [8–10] which concerned the case of fingertips covered with soft noncompressible material but it differs in a number of respects. The material compressibility affects the nature of constraints derived by the tight area contact between the fingertips and the rigid object surface. Constraints are now nonholonomic as opposed to holonomic in references [8–10], although the case treated is constrained to the plane. Fortunately, the nonholonomic constraints belong to a special case for which the variational principle can be applied and therefore the equation of motion is derived as a set of nonlinear differential equations with extra terms of Langrange multipliers incorporating the constraints. Furthermore, the material compressibility affects the contact mode; the rolling of the contacting bodies is now resulting in a pure rolling contact mode, while a combined rolling and sliding contact mode is present for non-compressible soft fingertips as shown in reference [11]. Nevertheless, it is shown that dynamic stable grasping can again be realized. The proposed control signal is a linear combination of feedback control signals each achieving the following tasks: (a) Realize the desired internal force; (b) Balance a pair of moments to stop object rotation; (c) Regulate the object's rotation angle at the desired value; and (d) and (e) Regulate the object's mass center at the desired position along the x and y axis, respectively. The controller's linear superposition structure means that the complexity of learning an overall skilled motion through a number of practices can be significantly reduced from an exponential order to a linear order and this finding may explicate the human ability of becoming skillful at pinching things and manipulating objects successfully.

2. SYSTEM KINEMATICS

In order to simplify the mathematics we assume that motion of the dual fingers is confined to the horizontal plane and is not affected by the gravity force. We further assume that the object is rigid and of a rectangular shape and that the shape of the soft fingertips is spherical with radius r. We also reasonably assume that the pressure distribution in the deformed area of each fingertip i may be represented by a concentrated force at the center point of the area c_i in the direction perpendicular to the object surface and its magnitude is dependent on the maximum displacement Δx_i through a generally unknown or uncertain function $f_i(\Delta x_i)$ which must, however, be a strictly increasing function of Δx_i (Figure 1) for $\Delta x_i > 0$ with $f_i(\Delta x_i) = 0$ for $\Delta x_i \leq 0$.

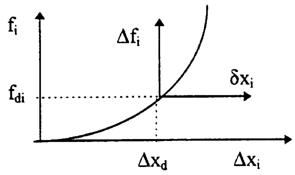

Fig. 1. The non-linear function of the reproducing force.

Let {P} be the inertia frame and {o} the object frame at the center of mass, while $\{t_i\}$ and $\{c_i\}$ are the rigid tip frame and the contact frame for the i finger. The contact frame is attached at c_i, so that its x-axis is along the surface normal pointing inwards the object (Figure 2). Let, $z = [r_o^T \ \ \theta]^T$ where $r_o = [x \ \ y]^T$ is the object position and $\theta \in \Re$ is its orientation angle relative to {P}. Then, the rotation matrix of {o} relative to {P} is $R_o = \begin{bmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{bmatrix}$ where c_θ, s_θ denote the cosine and sine of θ.

Note that $\dot{R}_o R_o^T = \dot{\theta} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. The rightmost matrix that appears here is, in fact, the representation of the cross product operator $\times$ in the two dimensional space. Hence for a scalar $\dot{\theta}$ we can define $\dot{\theta} \times = \dot{\theta} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and for a vector $p = [p_x \ \ p_y]^T$, we define $p \times = [p_y \ -p_x]^T$ which concisely we symbolize by $\hat{p}$. Also, let $r_{oi} = R_o^o r_{oi}$ be the position of c_i from the origin of {o} with ${}^o r_{oi} = [X_{ci} \ \ Y_{ci}]^T$. Left superscripts denote the frame of reference and are omitted for the inertia frame. In the case of the rectangular object $X_{c1} = -\ell/2$ and $X_{c2} = \ell/2$ where ℓ is the object width and Y_{c1}, Y_{c2} are changing with time as the fingers are allowed to incline against the object.

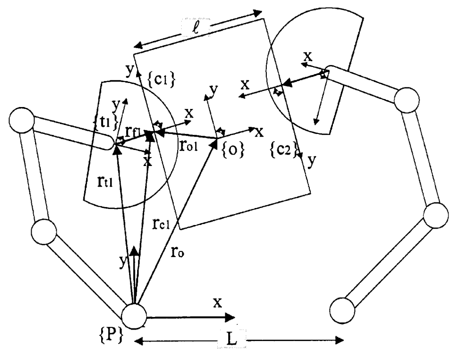

Fig. 2. Dual finger hand with soft fingertips.

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Let $p_i=[r_{ti}^T\ \ \gamma_i]^T$ where r_{ti} is the position of the i'th rigid tip and $\gamma_i=\sum_{j=1}^{n} q_{ji}$ is the orientation angle of $\{t_i\}$ with respect to $\{P\}$. It is also true that if the rotation of $\{t_i\}$ is $R_{ti}=\begin{bmatrix} c_{\gamma i} & -s_{\gamma i} \\ s_{\gamma i} & c_{\gamma i}\end{bmatrix}$ then $\dot{R}_{ti}R_{ti}^T=\dot{\gamma}_i\times$. The rigid fingertip velocity and the joint velocities are related through the Jacobian $J_i\in\Re^{3\times n}$ of the forward kinematics map; i.e. $\dot{p}_i=J_i\dot{q}_i$.

Let vectors r_{ci} and r_{fi} denote the position of c_i from the origin of $\{P\}$ and the rigid tip $\{ti\}$ respectively. Let R_{ci}, denote the rotation matrix of $\{c_i\}$ and note that for the rectangular object $R_{c1}=R_o$ and $R_{c2}=-R_o$. We can thus find that the relative orientation of contact $\{c_i\}$ and rigid tip frames $\{ti\}$ can be expressed by rotation matrices:

$$^{c1}R_{t1}=R_{c1}^TR_{t1}=\begin{bmatrix} c_{\phi 1} & -s_{\phi 1} \\ s_{\phi 1} & c_{\phi 1}\end{bmatrix},\quad \phi_1=\gamma_1-\theta$$

$$^{c2}R_{t2}=R_{c2}^TR_{t2}=\begin{bmatrix} c_{\phi 2} & -s_{\phi 2} \\ s_{\phi 2} & c_{\phi 2}\end{bmatrix},\quad \phi_2=\gamma_2-\theta-\pi$$

where ϕ_1, ϕ_2 denote the angle between the x-axis of $\{c_i\}$ and the x-axis of $\{t_i\}$. Note that $^{ci}\dot{R}_{ti}\ {}^{ci}R_{ti}^T=\dot{\phi}_i\times$.

The relations of contact-object position and velocity can be found by differentiating the position relationship $r_{ci}=r_o+r_{oi}$ and using the fact that the time derivative of $r_{oi}=R_o^o r_{oi}$ is given by $\dot{r}_{oi}=-r_{oi}\times\dot{\theta}+R_o^o\dot{r}_{oi}$:

$$\dot{r}_{ci}=J_{oi}\dot{z}+R_o^o\dot{r}_{oi} \tag{1}$$

where $\qquad J_{oi}=[I_2-\hat{r}_{oi}]\in\Re^{2\times 3}$

and I_2 is the identity matrix of dimension 2.

The relations of contact-finger position and velocity can be found by differentiating the position relationship $r_{ci}=r_{ti}+r_{fi}$ and using the fact that the time derivative of $r_{fi}=R_{ti}^{ti}r_{fi}$ is given by $\dot{r}_{fi}=-r_{fi}\times\dot{\gamma}_i+R_{ti}^{ti}\dot{r}_{fi}$:

$$\dot{r}_{ci}=J_{fi}\dot{p}_i+R_{ti}^{ti}\dot{r}_{fi} \tag{2}$$

where $\qquad J_{fi}=[I_2-\hat{r}_{fi}]\in\Re^{2\times 3}$.

Equating the right hand sides of equations (1) and (2) and expressing $\dot{p}_i$ with respect to the joint velocities $\dot{q}_i$ we obtain the contact velocity constraints:

$$J_{fi}J_i\dot{q}_i=J_{oi}\dot{z}+V_{ri}, \tag{3}$$

where V_{ri} is the relative translational velocity at contact i and is given by:

$$V_{ri}=R_o^o\dot{r}_{oi}-R_{ti}^{ti}\dot{r}_{fi}\,. \tag{4}$$

A compact representation of equation (3) for the two contacts is achieved by stacking them together in a matrix form:

$$J_fJ\dot{q}=J_o\dot{z}+V_r \tag{5}$$

where J_f,J are block diagonals of J_{fi},J_i and

$$J_o=\begin{bmatrix} J_{o1} \\ J_{o2}\end{bmatrix},\ \dot{q}=\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2\end{bmatrix},\ V_r=\begin{bmatrix} V_{r1} \\ V_{r2}\end{bmatrix}.$$

Next, we investigate the contact mode when fingertips are made of soft compressible material. Taking the time derivative of $^{ti}r_{fi}={}^{ci}R_{ti}^T\ {}^{ci}r_{fi}$ we find that $^{ti}\dot{r}_{fi}={}^{ci}\dot{R}_{ti}^T\ {}^{ci}r_{fi}+{}^{ci}R_{ti}^T\ {}^{ci}\dot{r}_{fi}$ which multipled from the left by R_{ti} gives $R_{ti}^{ti}\dot{r}_{fi}=R_{ci}^{ci}r_{fi}\times\dot{\phi}_i+R_{ci}^{ci}\dot{r}_{fi}$. If we now substitute the above equation into equation (4) we find that:

$$V_{ri}=R_o^o\dot{r}_{oi}-R_{ci}^{ci}\hat{r}_{fi}\dot{\phi}_i-R_{ci}^{ci}\dot{r}_{fi}$$

But $\qquad ^{ci}r_{fi}=\begin{bmatrix} 1 \\ 0\end{bmatrix}(r-\Delta x_i)$ and $^{ci}\dot{r}_{fi}=-\begin{bmatrix} 1 \\ 0\end{bmatrix}\Delta\dot{x}_i$.

For the rectangular object we can thus find that:

$$V_{r1}=\begin{bmatrix} -s_\theta \\ c_\theta\end{bmatrix}\{\dot{Y}_{c1}+(r-\Delta x_1)\dot{\phi}_1\}+\begin{bmatrix} c_\theta \\ s_\theta\end{bmatrix}\Delta\dot{x}_1 \tag{6}$$

$$V_{r2}=\begin{bmatrix} -s_\theta \\ c_\theta\end{bmatrix}\{\dot{Y}_{c2}-(r-\Delta x_2)\dot{\phi}_2\}-\begin{bmatrix} c_\theta \\ s_\theta\end{bmatrix}\Delta\dot{x}_2\,.$$

However, as found in the Appendix equation (A.3), $\dot{Y}_{c1}=-(r-\Delta x_1)\dot{\phi}_1$ and $\dot{Y}_{c2}=(r-\Delta x_2)\dot{\phi}_2$ and thus equation (6) becomes:

$$V_{r1}=\begin{bmatrix} c_\theta \\ s_\theta\end{bmatrix}\Delta\dot{x}_1,\ V_{r2}=-\begin{bmatrix} c_\theta \\ s_\theta\end{bmatrix}\Delta\dot{x}_2\,. \tag{7}$$

Hence, the relative velocity concerns a translational velocity along the surface normal which is due to the material deformation. There is no translational velocity along the surface and thus the contact is not in a sliding mode. However, contact points are moving due to the rolling between the contacting bodies of object and fingertip. In fact, if we reasonably assume the lack of bending deformations, the relative rotational velocities of the contacting bodies are:

$$\omega_{ri}\hat{=}\dot{\gamma}_i-\dot{\theta}=\dot{\phi}_i\neq 0.$$

3. SYSTEM DYNAMICS AND PASSIVITY

Let $F_{ci}=R_{ci}^{ci}F_{ci}=R_{ci}\begin{bmatrix} 1 \\ 0\end{bmatrix}f_i(\Delta x_i)$ be the reproducing force applied by the finger along the surface normal. In fact, $F_{c1}=\begin{bmatrix} c_\theta \\ s_\theta\end{bmatrix}f_1(\Delta x_1)$ and $F_{c2}=-\begin{bmatrix} c_\theta \\ s_\theta\end{bmatrix}f_2(\Delta x_2)$ in this case. Then, the corresponding force at the rigid tip is given through the Jacobian transpose $J_{fi}^TF_{ci}=\begin{bmatrix} I_2 \\ -\hat{r}_{fi}^T\end{bmatrix}F_{ci}$. It is easy to prove that, for spherical fingertips, the torque element in the above relation is zero. Hence the contact force mapped at the joint

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space is given by the relation $J_i^T J_{fi}^T F_{ci} = J_{vi}^T F_{ci}$ where J_{vi} denotes the Jacobian for the translational velocity of the finger's rigid tip.

In the absence of gravity the robot finger dynamic model for $i=1,2$ is given by:

$$M_i(q_i)\ddot{q}_i + \left(\frac{1}{2}\dot{M}_i(q_i) + S(q_i,\dot{q}_i) + C_i\right)\dot{q}_i + J_i^T J_{fi}^T F_{ci} = u_i \tag{8}$$

where q_i is the joint position vector, $M_i(q_i)$ is the robot finger inertia matrix which is symmetric and positive definite, C_i denotes the viscous friction matrix, u_i denotes the vector of input torques and $S(q_i,\dot{q}_i)$ is a skew symmetric matrix.

Let m_o be the object's mass and I_o the object's inertia; then the object dynamics under the resulting force at the center of mass due to the contact forces is given by:

$$M_o\ddot{z} = J_{o1}^T F_{c1} + J_{o2}^T F_{c2}$$

where $M_o = \begin{bmatrix} m_o I_2 & 0 \\ 0 & I_o \end{bmatrix}$ or compactly

$$M_o\ddot{z} = J_o^T F_c. \tag{9}$$

Equations (8) and (9) describe the system dynamics. The state space $(q, z, \dot{q}, \dot{z})$ is of dimension 18 for two 3 d.o.f fingers. However, state variables are not independent; they are subject to the two contact area constraints between the soft fingertips and the rigid object surface. The contact area position and velocity constraints are (as found in the Appendix):

$$\Psi_i = Y_{ci} - \{k_i + (-1)^i \int_0^t (r - \Delta x_i(\tau))\dot{\phi}_i(\tau)\, d\tau\} = 0$$
$$\Phi_i = \dot{Y}_{ci} - (-1)^i [r - \Delta x_i(t)]\dot{\phi}_i(t) = 0.$$

Using the time derivative of equation (A.5), we can further write the contact area velocity constraints in the form:

$$\begin{aligned} \Phi_1 &= A_{11}^T \dot{q}_1 + A_{13}^T \dot{z} = 0 \\ \Phi_2 &= A_{22}^T \dot{q}_2 + A_{23}^T \dot{z} = 0 \end{aligned} \tag{10}$$

where $A_{ij} \in \Re^{3\times 1}$ are given below:

$$A_{ii} = J_{vi}^T \begin{bmatrix} -s_\theta \\ c_\theta \end{bmatrix} + (-1)^{i+1}(r - \Delta x_i)\, e_{ni},$$

$$A_{13} = \begin{bmatrix} \sin\theta \\ -\cos\theta \\ \ell/2 \end{bmatrix}, \quad A_{23} = \begin{bmatrix} \sin\theta \\ -\cos\theta \\ -\ell/2 \end{bmatrix}.$$

Note that positional constraints are not algebraic and hence the system is nonholonomic. However, velocity constraints (10) are in the form of linear relations with respect to the differentials of the generalised coordinates; this is a case for which the variational principle can be extended as described in reference [12]. Then, the complete set of Langrange's equation for this nonholonomic system requires the incorporation of the contact area velocity constraints through the use of Langrange multipliers which together with the velocity constraints now taken as first order differential equations fully describe the system. That is, we associate with each velocity constraint a corresponding Langrange multiplier λ_1 and λ_2 and describe the system dynamics as:

$$M_i(q_i)\ddot{q}_i + \left(\frac{1}{2}\dot{M}_i(q_i) + S(q_i, \dot{q}_i) + C_i\right)\dot{q}_i + J_i^T J_{fi}^T F_{ci} = u_i + \lambda_i A_{ii} \tag{11}$$

$$M_o\ddot{z} = J_o^T F_c + \lambda_1 A_{13} + \lambda_2 A_{23} \tag{12}$$

which together with equation (10) give a complete system description. A compact representation of the two fingers dynamics is:

$$M(q)\ddot{q} + H\dot{q} + J^T J_f^T F_c = u + \Lambda \tag{13}$$

where M and H are block diagonals of the matrices involved in the first and second term above and $F_c = \begin{bmatrix} F_{c1} \\ F_{c2} \end{bmatrix}$ and $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $\Lambda = \begin{bmatrix} \lambda_1 A_{11} \\ \lambda_2 A_{22} \end{bmatrix}$.

It is easily proved that the joint velocities and joint torque inputs form a conjugate (passive) pair. Taking the inner product of $\dot{q}$ with equation (13) we find using equation (5) that:

$$\dot{q}^T u = \frac{d}{dt}\left\{\frac{1}{2}\dot{q}^T M(q)\dot{q}\right\} + \dot{q}^T C\dot{q} + (J_o\dot{z} + V_r)^T F_c - \lambda_1 \dot{q}_1^T A_{11} - \lambda_2 \dot{q}_2^T A_{22}. \tag{14}$$

Using equation (12),

$$(J_o\dot{z})^T F_c = \frac{d}{dt}\left\{\frac{1}{2}\dot{z}^T M_o \dot{z}\right\} - \lambda_1 \dot{z}^T A_{13} - \lambda_2 \dot{z}^T A_{23}. \tag{15}$$

Using equation (7),

$$V_r^T F_c = \Delta\dot{x}_1 f_1(\Delta x_1) + \Delta\dot{x}_2 f_2(\Delta x_2) \text{ or } V_r^T F_c = \frac{dP}{dt} \tag{16}$$

where $P = \int_0^{\Delta x_1} f_1(\xi)\, d\xi + \int_0^{\Delta x_2} f_2(\xi)\, d\xi$ is the system potential energy which is positive since contact forces monotonically increase with Δx_i. Let K be the sum of the kinetic energies of the two fingers and the object which is positive.

$$K = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\dot{z}^T M_o \dot{z}. \tag{17}$$

Also, let the total system energy be

$$V_o = K + P. \tag{18}$$

Then, substituting equations (15) and (16) into equation (14) and using equation (10) we find that the inner product of $\dot{q}$ with u becomes:

$$\begin{aligned} \dot{q}^T u &= \frac{dV_o}{dt} + \dot{q}^T C\dot{q} - \lambda_1\{\dot{q}_1^T A_{11} + \dot{z}^T A_{13}\} - \lambda_2\{\dot{q}_2^T A_{22} + \dot{z}^T A_{23}\} \\ &= \frac{dV_o}{dt} + \dot{q}^T C\dot{q}. \end{aligned} \tag{19}$$

Since, V_o is a non-increasing function of t and $\dot{q}^T C\dot{q}$ is positive the integral of $\dot{q}^T u$ over (0, t) cannot become beyond the initial value of V_o for $t=0$ which demonstrates the passivity of the pair.

4. A FEEDBACK CONTROLLER FOR OBJECT MANIPULATION

The control which can achieve the desired object manipulation can be expressed as a superposition of four separate control inputs each one achieving stable grasping, rotation regulation and regulation of the x and y object coordinates:

$$u_i = u_{gi} + u_{ri} + u_{xi} + u_{yi}\,. \tag{20}$$

For a given internal force f_d the following control input can achieve dynamic stable grasping:

$$u_{gi} = J_i^T J_{fi}^T (F_{di} - u_{mi}) + u_{ei} - K_{vi}\dot{q}_i \tag{21}$$

where K_{vi} is a positive diagonal matrix and

$$F_{di} = (-1)^{i+1}\begin{bmatrix} c_\theta \\ s_\theta \end{bmatrix} f_d \tag{22}$$

$$u_{mi} = (-1)^{i+1}\begin{bmatrix} -s_\theta \\ c_\theta \end{bmatrix} f_d \frac{(\Delta x_1 + \Delta x_2)}{2r}\,\frac{Y_{c1} - Y_{c2}}{\ell} \tag{23}$$

$$u_{ei} = \frac{r - \Delta x_i}{2r}\, f_d (Y_{c1} - Y_{c2})\, e_{ni} \tag{24}$$

where $e_{ni} = [1\ \ 1\ \ 1]^T$ for a 3 d.o.f finger.

Note that the proposed control input for stable grasping consists of a signal that realizes the desired internal force (equation (22)), a feedback signal that balances a pair of moments to stop object rotation (equation (24)) and a small additional control signal (equation (23)) needed to ensure the negativeness of the Lyapunov function derivative as shown in the next section. Hence, the motion of the contact points has not been predefined, a demanding task of the planning stage found in most of the published works in this field, but the final values of Y_{c1}, Y_{c2} are determined by the dynamically stable grasping control signal.

For a given desired rotation θ_d we consider the following control input:

$$u_{ri} = J_i^T J_{fi}^T (-1)^{i+1}\begin{bmatrix} -s_\theta \\ c_\theta \end{bmatrix} u_\theta \tag{25}$$

where

$$u_\theta = \alpha\dot{\theta} + \beta\Delta\theta,\ \Delta\theta = \theta - \theta_d,\ \alpha,\ \beta > 0. \tag{26}$$

For a given desired position of the object's mass center on the x and y axis x_d and y_d correspondingly we consider the following control inputs:

$$u_{xi} = \gamma(x' - x_d)\Big\{-J_{vi}^T\begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1)^{i+1}(r - \Delta x_i)s_{\theta_d}e_{ni}$$
$$- J_i^T J_{fi}^T (-1)^{i+1}\frac{s_{\theta_d}}{\ell}\begin{bmatrix} -s_\theta \\ c_\theta \end{bmatrix}(\Delta x_1 - \Delta x_2)\Big\} \tag{27}$$

$$u_{yi} = \delta(y' - y_d)\Big\{-J_{vi}^T\begin{bmatrix} 0 \\ 1 \end{bmatrix} + (-1)^{i}(r - \Delta x_i)c_{\theta_d}e_{ni}$$
$$- J_i^T J_{fi}^T (-1)^{i+1}\frac{c_{\theta_d}}{\ell}\begin{bmatrix} -s_\theta \\ c_\theta \end{bmatrix}(\Delta x_1 - \Delta x_2)\Big\} \tag{28}$$

where

$$x' = \frac{1}{2}(x_{01} + x_{02}) + \frac{1}{2}(Y_{c1} + Y_{c2})s_{\theta_d} \tag{29}$$

$$y' = \frac{1}{2}(y_{01} + y_{02}) - \frac{1}{2}(Y_{c1} + Y_{c2})c_{\theta_d} \tag{30}$$

where control constants γ, $\delta > 0$.

Note that force sensing is not required by the above controllers. Measurements of the maximum displacement of deformation Δx_1, Δx_2 and the object rotation angle can be achieved by two optical sensors located closely to both sides of the soft fingertip as described in reference [13] where preliminary experimental results are also given.

5. STABILITY ANALYSIS

Theorem: Any solution of the closed loop system starting from an initial state in a subset of a bounded open set of the system state (q, z, $\dot{q}$, $\dot{z}$) which contains at least one equilibrium satisfying $\dot{q}=0$, $\dot{z}=0$, $\Delta x_1 = \Delta x_2 = \Delta x_d$ (or $f_1 = f_2 = f_d$), $Y_{c1} = Y_{c2}$ and $\theta = \theta_d$, $x = x_d$, $y = y_d$ tends asymptotically to it.

Proof: First note, that there are six positional requirements expressed by the above relations which do not uniquely determine the system configuration since there is no requirement on the specific value of $Y_{c1}(=Y_{c2})$. Thus, for a system with two 3 d.o.f fingers which posesses 7 d.o.f, the set of states satisfying the above relations is a one-dimensional manifold of the system's state space parameterized by the value of $Y_{c1}(=Y_{c2})$. Such a system is therefore able to achieve stable grasping as well as desired object position and orientation.

The proof is presented here by considering in turn each control input which constitutes the total control input (equation (20)). Let first the control input u_{gi} given by equations (21–24) be applied to the system while the rest of the control inputs are set to zero. Then the fingers closed loop dynamics are:

$$M(q)\ddot{q} + H\dot{q} + K_v\dot{q} + J^T J_f^T(\Delta F_c + u_m) - u_e - \Lambda = 0 \tag{31}$$

where K_v is block diagonal of K_{vi}, $\Delta F_c = F_c - F_d$, and u_m, u_d, u_e are the corresponding controls stacked in one column vector. Using equations (5) (12) and (10) the inner product of equation (31) with $\dot{q}$ yields:

$$\frac{dK}{dt} + \dot{q}^T(K_v + C)\dot{q} + V_r^T\{\Delta F_c + u_m\} + \dot{z}^T J_o^T\{u_m - F_d\}$$
$$- \frac{1}{2r}(Y_{c1} - Y_{c2})f_d\{(r - \Delta x_1)\dot{\gamma}_1 + (r - \Delta x_2)\dot{\gamma}_2\} = 0.$$

Using equation (7) we can find that $V_r^T u_m = 0$ and

$$V_r^T \Delta F_c = \Delta\dot{x}_1 \Delta f_1 + \Delta\dot{x}_2 \Delta f_2 = \delta\dot{x}_1 \Delta f_1 + \delta\dot{x}_2 \Delta f_2 = \frac{d\Delta P}{dt}$$

where $\delta x_i = \Delta x_i - \Delta x_d$ and

$$\Delta P = \int_0^{\delta x_1} \{f_1(\xi + \Delta x_d) - f_d\} d\xi + \int_0^{\delta x_2} \{f_2(\xi + \Delta x_d) - f_d\} d\xi$$

which is positive. We can also find that:

$$\dot{z}^T J_o^T \{u_m - F_d\} = -\dot{\theta}\ell(Y_{c1} - Y_{c2}) \frac{f_d}{\ell} \frac{\Delta x_1 + \Delta x_2}{2r} + \dot{\theta}(Y_{c1} - Y_{c2}) f_d$$

$$= \dot{\theta}(Y_{c1} - Y_{c2}) f_d \frac{(r - \Delta x_1) + (r - \Delta x_2)}{2r}$$

which when combined with the last term of the inner product can be written as $\frac{dS}{dt}$ where

$$S = \frac{f_d}{4r}(Y_{c1} - Y_{c2})^2.$$

Finally, we can write the inner product as: $\frac{dV_1}{dt} + W_1 = 0$

where $V_1 = K + \Delta P + S$, $W_1 = \dot{q}^T(K_v + C)\dot{q}$.

It is important to note that the scalar function V_1 cannot be a Lyapunov function for the system although its time derivative $\dot{V}_1 = -W_1$ is non-positive definite because V_1 is not positive definite in the manifold $M = \{(q, z, \dot{q}, \dot{z})$ under the contact constraints$\}$. However, if we assume that at time $t = 0$ the magnitude of V_1 is small enough to satisfy

$$V_1(0) < f_d \Delta x_d \int_0^{\Delta x_d} f_i(\xi) d\xi \text{ for } i = 1,2$$

which implies that both fingers maintain their contact with the object, it is possible to prove[14] that $\dot{q} \to 0$, $\ddot{q} \to 0$ as $t \to \infty$. On the other hand, if we differentiate $Y_{c1} - Y_{c2}$ taking from equation (A.5) and using equation (A.3) we can prove that $\ell\dot{\theta} = 0$ when $\dot{\gamma}_i$, $\dot{r}_{ti} \to 0$, and therefore prove that $\dot{\theta} \to 0$ as $t \to \infty$. This means that $\ddot{\theta} \to 0$ because $\ddot{\theta}$ is uniformly continous. Then, from equation (A.3) it follows that $\dot{Y}_{ci} \to 0$.

Now note that the object's motion equation (12) can be written using the analytic expressions for A_{13} and A_{23} as follows:

$$m\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} c_\theta \\ s_\theta \end{bmatrix}(\Delta f_1 - \Delta f_2) - \begin{bmatrix} -s_\theta \\ c_\theta \end{bmatrix}(\lambda_1 + \lambda_2) \qquad (32)$$

$$I_o\ddot{\theta} + Y_{c1}\Delta f_1 - Y_{c2}\Delta f_2 + (Y_{c1} - Y_{c2}) f_d = \frac{\ell}{2}(\lambda_1 - \lambda_2). \qquad (33)$$

If we project equation (32) in the direction tangent to the object surface we can find a relation connecting the Langrange multipliers and the object accelerations, i.e.:

$$m[-s_\theta \;\; c_\theta]\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = -(\lambda_1 + \lambda_2). \qquad (34)$$

If we now differentiate any one of equations (A.5) we can find that

$$[-s_\theta \;\; c_\theta]\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \to 0$$

and subsequently

$$[-s_\theta \;\; c_\theta]\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} \to 0$$

and therefore the left hand side of equation (34) goes to zero. Hence $\lambda_1 + \lambda_2 \to 0$ as $t \to \infty$.

The remaining terms of the closed loop system equation can be expressed as $Ah \to 0$ as $t \to \infty$ where:

$$A = \begin{bmatrix} J_{v1}^T \begin{bmatrix} -s_\theta \\ c_\theta \end{bmatrix} & (r - \Delta x_1)\, e_{n1} & J_1^T J_{f1}^T \begin{bmatrix} c_\theta \\ s_\theta \end{bmatrix} & 0_{n1\times 1} \\ -J_{v2}^T \begin{bmatrix} -s_\theta \\ c_\theta \end{bmatrix} & (r - \Delta x_2)\, e_{n2} & 0_{n2\times 1} & -J_2^T J_{f2}^T \begin{bmatrix} c_\theta \\ s_\theta \end{bmatrix} \end{bmatrix} \qquad (35)$$

and $h = [h_1\; h_2\; h_3\; h_4]^T$ with elements:

$$h_1 = (Y_{c1} - Y_{c2}) \frac{f_d}{\ell} \frac{\Delta x_1 + \Delta x_2}{2r} - \lambda_1$$

$$h_2 = h_1 - \frac{1}{2r} f_d (Y_{c1} - Y_{c2})$$

$$h_3 = \Delta f_1 \text{ and } h_4 = \Delta f_2.$$

Note that matrix A is 6×4 for two 3 d.o.f fingers and it is of full rank. Hence, $h \to 0$ and thus from the last two elements of h it follows that $\Delta f_i \to 0$. Then by subtracting h_1 from h_2 we get that $(Y_{c1} - Y_{c2}) \to 0$ as $t \to \infty$. Subsequently, from h_1 it follows that $\lambda_1 \to 0$.

Now, let the control input $u_r = [u_{r1}^T \;\; u_{r2}^T]^T$ be added to the system input. Then, the inner product of u_r with $\dot{q}$ is:

$$-\dot{q}^T u_r = \frac{dV_2}{dt} + W_2$$

where

$$V_2 = \frac{\ell}{2} \beta \Delta\theta^2 \geq 0 \text{ and } W_2 = \ell\alpha\dot{\theta}^2 \geq 0.$$

Hence from the positiveness of $V_1 + V_2$ and the non-positiveness of its derivative using the same arguments as above we can conclude the convergence of $\dot{q}$ and $\dot{\theta}$ and $\dot{Y}_{ci}$ to zero. From the closed loop equation and the convergence

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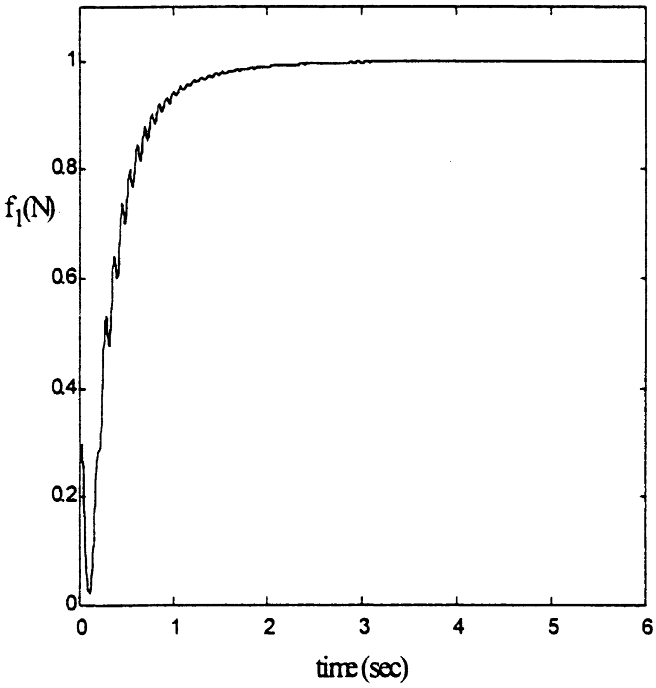

Fig. 3. Internal force at the left finger.

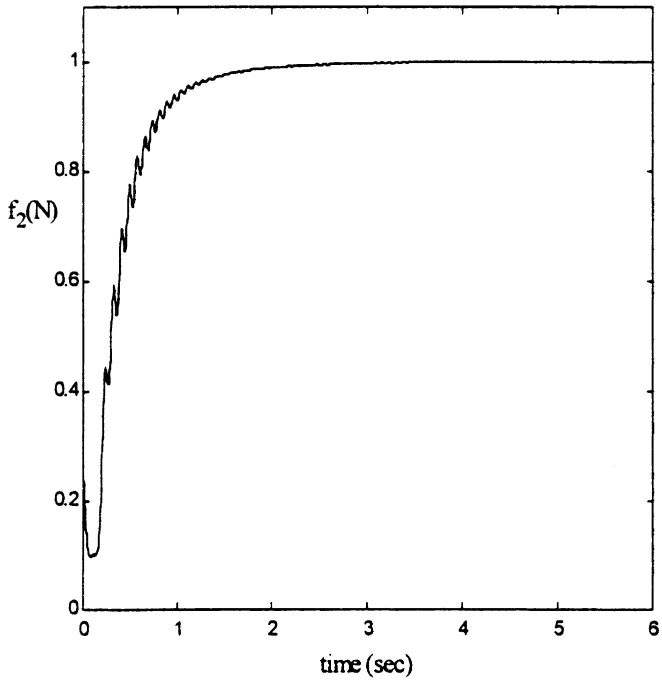

Fig. 4. Internal force at the right finger.

of the remaining terms of the closed loop system equation can be expressed by $Ah' \to 0$ as $t \to \infty$ where A is given by equation (35) and $h_1' = h_1 - \beta \Delta\theta$, $h_2' = h_1' - \frac{f_d}{2r}(Y_{c1} - Y_{c2})$ while the rest of the elements are the same. As before, $h' \to 0$, hence $\Delta f_i \to 0$ and $(Y_{c1} - Y_{c2}) \to 0$. Using equation (33) we can therefore find that $\lambda_1 - \lambda_2 \to 0$ which taken together with $\lambda_1 + \lambda_2 \to 0$ implies that $\lambda_1 \to 0$. Subsequently, from h_1' it follows that $\Delta\theta \to 0$. Note that stable grasping and rotation regulation could have been achieved with two 2 d.o.f fingers.

Now, let the control input $u_x = [u_{x1}^T \; u_{x2}^T]^T$ be added to the system input. Then:

$$
\begin{aligned}
-\dot{q}^T u_x &= \gamma(\dot{x}_{o1} + \dot{x}_{o2})(x' - x_d) + \gamma[-(r - \Delta x_1)\dot{\gamma}_1 \\
&\quad + (r - \Delta x_2)\dot{\gamma}_2] s_{\theta_d}(x' - x_d) - \gamma(\Delta x_1 - \Delta x_2)\dot{\theta} s_{\theta_d}(x' - x_d)
\end{aligned}
$$

which can be written as $\frac{dV_3}{dt}$ where $V_3 = \gamma(x' - x_d)^2 \geq 0$. When finally the control input u_{yi} is added to the system, the inner product of u_y with $\dot{q}$ can be written as $\frac{dV_4}{dt}$ where $V_4 = \delta(y' - y_d)^2 \geq 0$. Hence $V_1 + V_2 + V_3 + V_4$ is now a positive scalar function and its derivative is non-positive. As before, we can show the convergence of the system velocities to zero while the remaining terms of the closed loop system equation can be expressed as

$$
[A \;\; A_p] \begin{bmatrix} h'' \\ h_p \end{bmatrix} \to 0 \text{ as } t \to \infty
$$

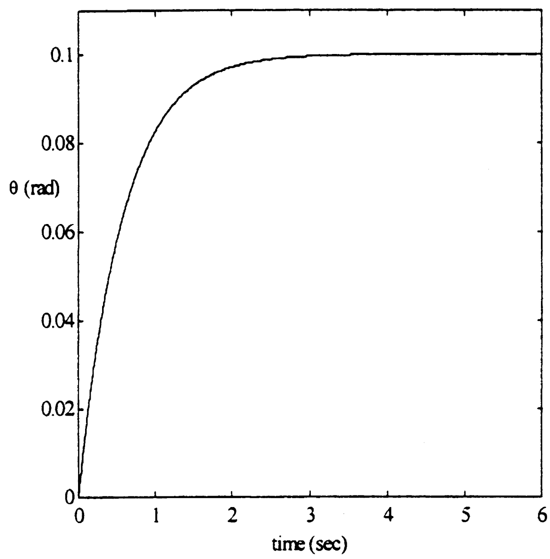

Fig. 5. Response of the object's angular position.

where

$$
A_p = \begin{bmatrix} J_{v1}^T \begin{bmatrix} 1 \\ 0 \end{bmatrix} - s_{\theta_d}(r - \Delta x_1)\, e_{n_1} & J_{v1}^T \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_{\theta_d}(r - \Delta x_1)\, e_{n_1} \\ J_{v2}^T \begin{bmatrix} 1 \\ 0 \end{bmatrix} + s_{\theta_d}(r - \Delta x_2)\, e_{n_2} & J_{v2}^T \begin{bmatrix} 0 \\ 1 \end{bmatrix} - c_{\theta_d}(r - \Delta x_2)\, e_{n_2} \end{bmatrix}
$$

$$
h_p = [\gamma(x' - x_d) \;\; \delta(y' - y_d)]^T.
$$

And the vector h'' has components:

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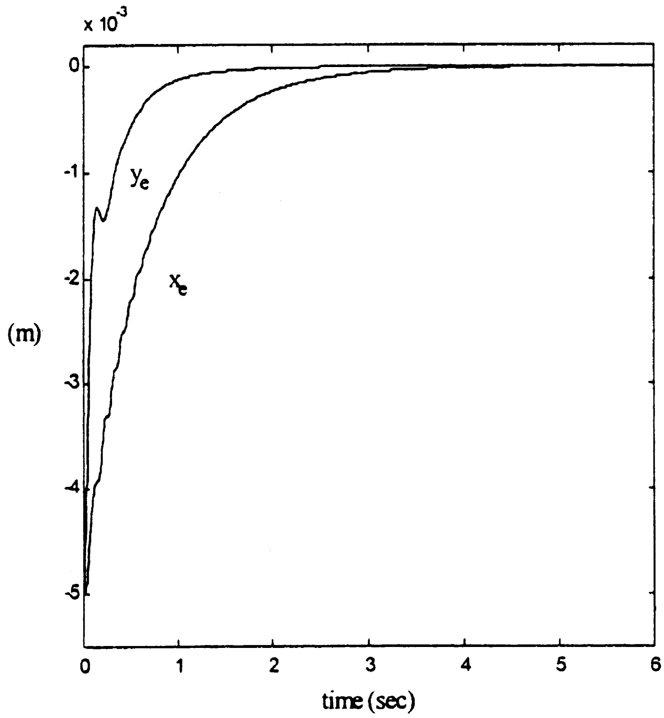

Fig. 6. Error responses in the x and y direction.

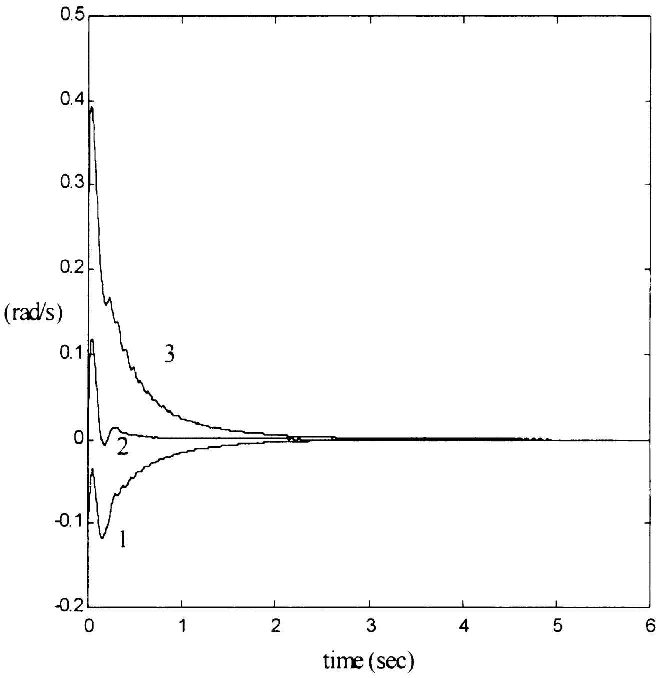

Fig. 7. Joint velocities of the left finger.

$$h_1'' = h_1' + (\Delta x_1 - \Delta x_2)\left\{\frac{s_{\theta_d}}{\ell}\gamma(x' - x_d) + \frac{c_{\theta_d}}{\ell}\delta(y' - y_d)\right\}$$

$$h_2'' = h_1'' - kf_d(Y_{c1} - Y_{c2})$$

while the rest of the elements are the same. As matrix $[A\ \ A_p]$ is now a 6×6 non-singular matrix, the convergence proof follows from $\begin{bmatrix} h'' \\ h_p \end{bmatrix} \to 0$ as before. Last, we can show

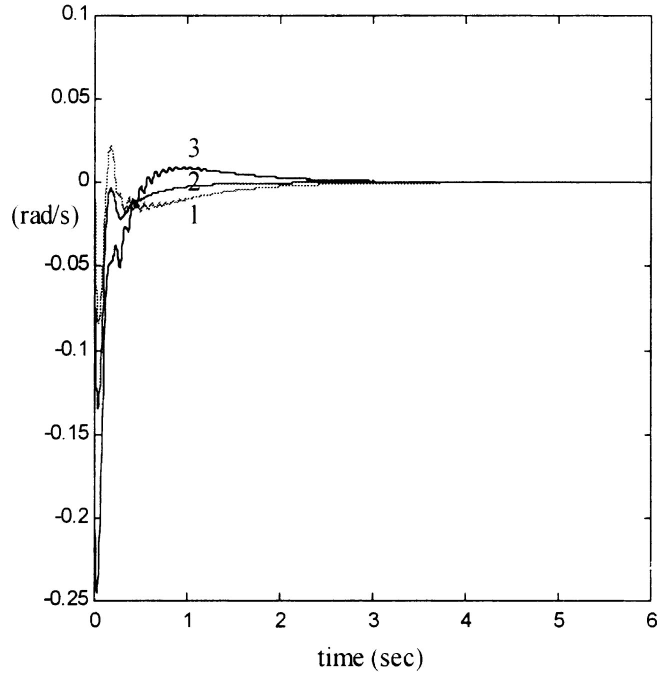

Fig. 8. Joint velocities of the right finger.

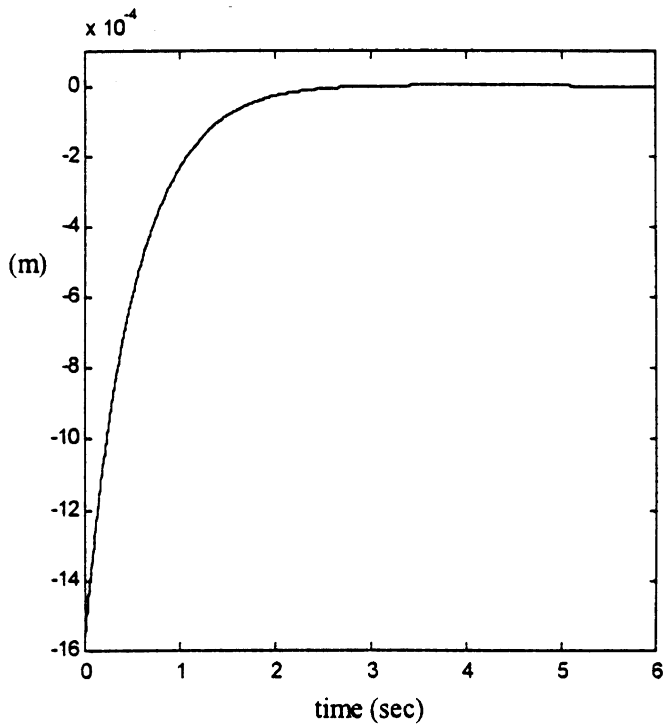

Fig. 9. Response of $Y_{c1} - Y_{c2}$.

that the convergence of x' and y' to the desired values implies the convergence of x and y to x_d and y_d. In fact, by adding relations $r_{oi} = r_{ti} + r_{fi} - r_o$ we can express the object's position on the x and y-axis as follows:

$$x = \frac{x_{o1} + x_{o2}}{2} + \frac{-\Delta x_1 + \Delta x_2}{2} c_\theta + \frac{Y_{c1} + Y_{c2}}{2} s_\theta$$

$$y = \frac{y_{o1} + y_{o2}}{2} + \frac{-\Delta x_1 + \Delta x_2}{2} s_\theta - \frac{Y_{c1} + Y_{c2}}{2} c_\theta .$$

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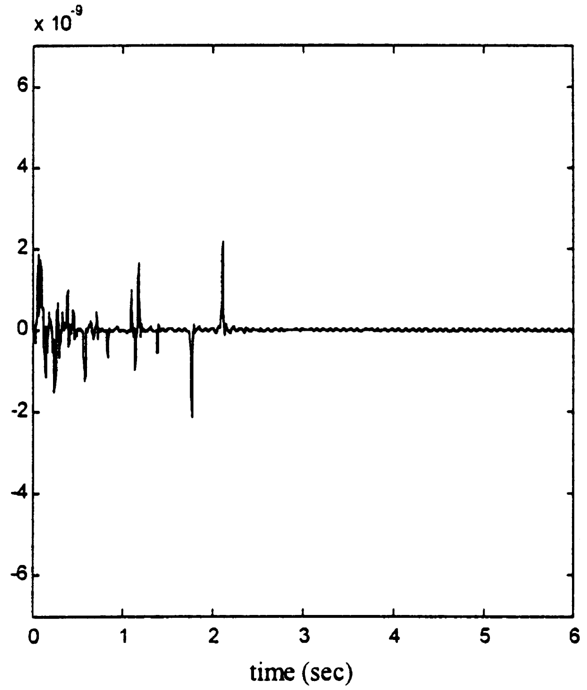

Fig. 10. Response of contact area position constraint Ψ_1.

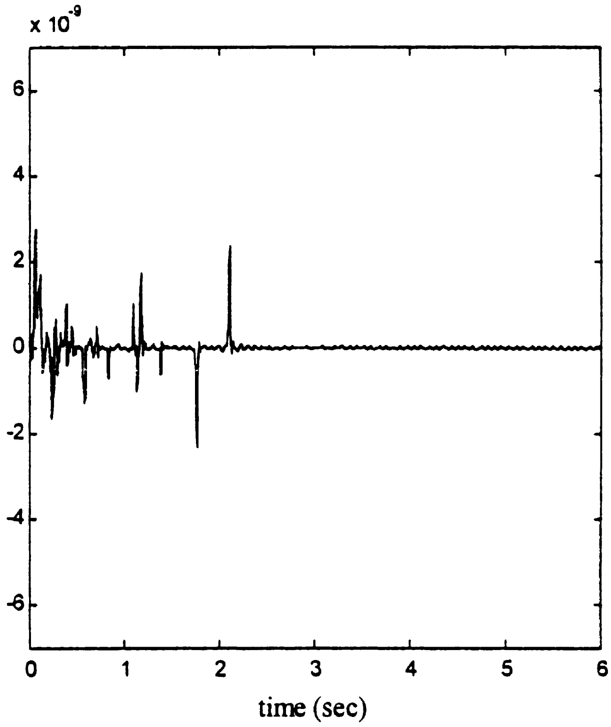

Fig. 11. Response of contact area position constraint Ψ_2.

It is now easy to see that if $\Delta x_1 = \Delta x_2 = \Delta x_d$ and $\theta = \theta_d$, then $x' \equiv x$ and $y' \equiv y$.

6. SYSTEM SIMULATION

In order to incorporate the contact area constraints into the simulation we differentiate the velocity constraints (equation (10)) to obtain:

$$\begin{aligned} A_{11}^T \ddot{q}_1 + A_{13}^T \ddot{z} &= -\dot{A}_{11}^T \dot{q}_1 - \dot{A}_{13}^T \dot{z} \\ A_{22}^T \ddot{q}_2 + A_{23}^T \ddot{z} &= -\dot{A}_{22}^T \dot{q}_2 - \dot{A}_{23}^T \dot{z} \end{aligned} \tag{36}$$

The system described by equations (11)–(12) and (36) can now be simulated as a set of 11 second order differential equations. However, the inclusion of the constraints in the form of Equation (36) does not guarantee the convergence of the contact area velocity and position constraints to zero. This problem has been successfully addressed by the constraint stabilization method in the solution of differential-algebraic equations.[15] According to this method the constraints are asymptotically stabilized using linear control theory as follows:

$$\dot{\Phi}_i + \xi \Phi_i + \omega \Psi_i = 0$$

where ξ, ω are appropriately chosen gains which ensure the fast convergence of both the contact position and velocity constrain to zero. Thus, instead of equation (36) we use:

$$\begin{aligned} A_{11}^T \ddot{q}_1 + A_{13}^T \ddot{z} &= -(\dot{A}_{11}^T + \xi A_{11}^T)\dot{q}_1 - (\dot{A}_{13}^T + \xi A_{13}^T)\dot{z} - \omega \Psi_1 \\ A_{22}^T \ddot{q}_2 + A_{23}^T \ddot{z} &= -(\dot{A}_{22}^T + \xi A_{22}^T)\dot{q}_2 - (\dot{A}_{23}^T + \xi A_{23}^T)\dot{z} - \omega \Psi_2 . \end{aligned} \tag{37}$$

A simulation for two identical planar revolute robotic fingers with 3 d.o.f. handling an object has been performed under the control law (equation (20)). Soft fingertips are spherical with 0.01 m radius. The reproducing forces are simulated by the non-linear functions $f_i = k_f \Delta x_i^2$ with $k_f = 250\,000$. The robotic fingers are assumed to be initially in contact with the rigid surface exerting an initial internal force. Our objective is to move the object from its initial position z_o to a desired position z_d with the soft fingertips exerting a desired internal force f_d.

Dynamic and kinematic parameters of the finger links and the object are shown in Table I. Table II shows the control gains used in the simulation. Coefficients ξ and ω in equation (37) are chosen as $\xi = 1200$ and $\omega = 360\,000$. Our objective is to move the object by 5 mm in x and y direction and rotate it by $+0.1$ rad. The desired internal forces is set to 1N.

Results on the response of the system state are plotted in Figures 3 to 11. The internal forces f_i, the object's angular

Table I. Parameters of the object and the finger links.

	Mass [kg]	Length [m]	Inertia [kgm^2]
link l_{1i}	0.3	0.08	0.0001625
link l_{2i}	0.28	0.07	0.00011667
link l_{3i}	0.28	0.07	0.00011667
Object	0.3	0.058	0.00025

Table II. Parameters of control inputs.

k_{v1i}	k_{v2i}	k_{v3i}	α	β	γ	δ
0.8	0.5	0.2	30	50	150	200

position and the position errors $x_e = x - x_d$, $y_e = y - y_d$ are shown in Figures 3, 4, 5 and 6. Figures 7 and 8 show the response of the joint velocities of the left and right finger respectively. Figure 9, shows the response of the contact relative position as expressed by the difference of $Y_{c1} - Y_{c2}$. Figures 10 and 11 show the response of the contact area position constraints. Results demonstrate the asymptotic stability of the system's equilibrium. All system states converge to the desired values within five seconds. The small oscillations that can be mainly observed on the response of f_i may come from the soft and flexible property of the fingertip material and the lack of damping in the object's motion equation but it is quite small in amplitude and disappears when the responses converge to the desired values. It is also worth noting that contact area constraints (Figures 10, 11) are practically zero (less than 10^{-8}) during the simulation.

7. CONCLUSION

This paper presents a feedback controller for the manipulation of a rectangular object by a pair of robotic fingers with spherical fingertips made of soft compressible material. It is assumed that the pressure distribution in the deformed area of each fingertip may be represented by a concentrated force at the center point of the area perpendicular to the object surface through a generally unknown or uncertain function with a magnitude dependent on the maximum displacement. The contact between the fingertips and the rigid object surface lead to nonholonomic constraints even for the planar case; however, the variational principle can be applied and the equation of motion is derived as a set of nonlinear differential equations with extra terms of Langrange multipliers incorporating the constraints. As the finger's last links are allowed to incline against the object surface, a contact motion is produced which is shown to be due to the rolling of the contacting bodies. The proposed feedback control law is a linear combination of simple control signals each achieving a specified subtask of the dexterous object pinching motion. Control signals use sensory feedback of a number of physical quantities regarding the object's angular position, the material deformation and the contact position. It is proved and demonstrated by simulation that the suggested controller drives the system asymptotically to the desired state at a stable grasping configuration. The controller's simplicity and effectiveness in producing a skillful object pinching motion suggests its possible usefulness in both robotic and prosthetic hands.

References

1. A.M. Okamura, N. Smaby and M.R. Cutkosky, "An overview of dexterous manipulation", *Proc. IEEE Int. Conf. Robotics and Automation*, San Francisco CA (2000) pp. 255–262.
2. Y. Yokokohji, M. Sakamoto and T. Yoshikawa, "Vision Aided Object Manipulation by a Multifingered Hand with Soft Fingertips", *Proc. IEEE Int. Conf. Robotics and Automation*, Detroit, Michigan (1999) pp. 3201–3208.
3. K.B. Shimoga, "Robot Grasp Synthesis Algorithms: A survey", *Int J. Robotics Research* **15** No. 3, 230–266 (1996).
4. T. Iberall, G. Sukhatme, D. Beattie and G. Bekey, "On the Development of EMG Control of a Prosthesis Using a Robotic Hand", *Proc. IEEE Int. Conf. Robotics and Automation*, San Diego, CA (1994) pp. 1753–1758.
5. P.J. Kyberd and P.H. Chappell, "The Southampton Hand: An Intelligent Myoelectric Prosthesis", *J. Rehabilitation Research and Development* **31** No. 4, 326–334 (1994).
6. W. Craelius, R.L. Abboudi and N.A. Newby, "Control of a Multi-Finger Prosthetic Hand", *Proc. Int. Conf. on Rehabilitation Robotics*, ICORR '99, Stanford, CA (1999) pp. 255–260.
7. L.R. Lin and H.P. Huang, "Mechanism Design of a New Multifingered Robot Hand", *Proc. IEEE Int. Conf. Robotics and Automation*, Minneapolis, Minnesota (1996) pp. 1471–1476.
8. S. Arimoto, K. Tahara, M. Yamaguchi, P.T.A. Nguyen and H-Y. Han, "Principle of superposition for controlling pinch motions by means of robot fingers with soft tips", *Robotica* **19** No. 1, 21–28 (2001).
9. S. Arimoto, K. Tahara, M. Yamaguchi, P.T.A. Nguyen and H-Y. Han, "Principle of superposition for controlling pinch motions by means of robot fingers with soft tips", *Proc. of the IFAC Symposium on Robot Control*, Wien, Austria (2000) pp. 421–426.
10. S Arimoto, P.T.A. Nguyen, H-Y. Han and Z. Doulgeri, "Dynamics and control of a set of dual fingers with soft tips", *Robotica* **18**, 71–80, (2000).
11. Z. Doulgeri and S. Arimoto, "Dynamic stable grasping by a pair of soft fingers", *Proc. of the 6th Intl. Conf. on Control Automation Robotics and Vision, ICARCV*, Singapore (2000) pp. 5–8.
12. H. Goldstein, *Classical Mechanics* (Addison-Wesley Publishing Company, 1980).
13. H-Y. Han, S. Arimoto, K. Tahara, M. Yamaguchi and P.T.A. Nguyen, "Robotic Pinching by means of a pair of soft fingers with sensory feedback", *Proc. 2001 IEEE Int. Conf. Robotics and Automation*, Seoul, Korea (2001) pp. 97–102.
14. S. Arimoto, "Reduction of complexity in learning dexterous multi-fingered motions: A theoretical exploration into a future problem C.E. Shannon raised", *Communications in Information and Control* **1** No. 1, 1–14 (2001).
15. J. Baumgarte, "Stabilization of constraints and integrals of motion in dynamical systems", *Compt. Meth. Appl. Mech. Eng.* **1**, 1–16 (1972).

APPENDIX – CONTACT AREA CONSTRAINTS

Let us assume initially that the finger's last link is normal to the surface and that the finger has just established contact. After time t the finger is inclined by an angle ϕ_i and is pressed against the surface thus deforrned by Δx_i and therefore producing the pressing force $f_i(\Delta x_i)$ at the centre point of the deformed area perpendicular to the object surface. Then, the distance traversed by the contact point (centre point) depends on the type of the soft material. A soft compressible material deforms in a way that both its shape and volume changes when pressed against a rigid surface, i.e. there is no mass spreading ahead and behind of the centerline of the contact. Hence, the contact point c_i in such material moves along the periphery of a circle of changing radius:

$$r(t) = r - \Delta x_i(t). \qquad (A.1)$$

The position of c_i can therefore be described by the following parametric equations: $x(t) = r(t)\cos[\phi_i(t)]$,

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$y(t) = r(t)\sin[\phi_i(t)]$ The distance traversed by the contact point on the rigid surface is equal to the arclength traversed at time t which can be found by:

$$s(t) = \int_0^t \sqrt{\dot{x}^2 + \dot{y}^2}\, d\tau = \int_0^t \sqrt{\dot{r}^2(\tau) + r^2(\tau)\dot{\phi}_i^2(\tau)}\, d\tau.$$

Using equation (A.1) and its derivative the above equation becomes:

$$s(t) = \int_0^t \sqrt{\Delta\dot{x}_i^2 + (r - \Delta x_i)^2\dot{\phi}_i^2(\tau)}\, d\tau.$$

If we now assume that $\Delta\dot{x}_i$ is small then $\Delta\dot{x}_i^2$ can be considered negligible. Hence we can write:

$$s(t) = \int_0^t (r - \Delta x_i(\tau))\,|\,\dot{\phi}_i(\tau)\,|\, d\tau, \quad \dot{s}(t) = (r - \Delta x_i)\,|\,\dot{\phi}_i(t)\,|\,.$$

Then, the contact point position and velocity for the rectangular object can be written in this case as:

$$Y_{ci} = k_i + (-1)^i \int_0^t (r - \Delta x_i(\tau))\dot{\phi}_i(\tau)\, d\tau \tag{A.2}$$

where $k_i = Y_{ci}(t = 0)$ and

$$\dot{Y}_{ci} = (-1)^i [r - \Delta x_i(t)]\dot{\phi}_i(t). \tag{A.3}$$

However, using the contact constraints, $r_{oi} = r_{ti} + r_{fi} - r_o$ expressed with respect to {o} we may find the following relations for the x and y coordinates:

$$\Delta x_1 = \ell/2 + r + [c_\theta \ \ s_\theta]r_{t1} - [c_\theta \ \ s_\theta]\begin{bmatrix} x \\ y \end{bmatrix}$$
$$\Delta x_2 = \ell/2 + r - [c_\theta \ \ s_\theta]r_{t2} + [c_\theta \ \ s_\theta]\begin{bmatrix} x \\ y \end{bmatrix} \tag{A.4}$$

$$Y_{c1} = [-s_\theta \ \ c_\theta]r_{t1} - [-s_\theta \ \ c_\theta]\begin{bmatrix} x \\ y \end{bmatrix}$$
$$Y_{c2} = [-s_\theta \ \ c_\theta]r_{t2} - [-s_\theta \ \ c_\theta]\begin{bmatrix} x \\ y \end{bmatrix}. \tag{A.5}$$

Equation (A.5) gives an expression of Y_{ci} as a function of the systems position and velocity variables (and we can also find such an expression for its derivative $\dot{Y}_{ci}$). Therefore, we can finally describe the contact area position and velocity constraints as follows:

$$\Psi_i = Y_{ci} - \{k_i + (-1)^i \int_0^t (r - \Delta x_i(\tau))\dot{\phi}_i(\tau)\, d\tau\} = 0$$
$$\Phi_i = \dot{Y}_{ci} - (-1)^i [r - \Delta x_i(t)]\dot{\phi}_i(t) = 0.$$

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