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2018 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM 2018

Udine, Italy

July 23–28, 2018

Logic Colloquium 2018, the annual European Summer Meeting of the Association of Symbolic Logic, was hosted by the University of Udine. The meeting took place from July 23 to July 28, 2018. It was organised by members of the Department of Mathematics, Computer Science, and Physics of the University of Udine and was held in the Department of Economics and Statistics.

Funding for the conference was provided by Association for Symbolic Logic (ASL); the US National Science Foundation; Università degli Studi di Udine; Dipartimento di Scienze Matematiche, Informatiche e Fisiche, Università degli Studi di Udine; Istituto Nazionale di Alta Matematica—GNSAGA; Associazione Italiana di Logica e sue Applicazioni; Società Italiana di Logica e Filosofia delle Scienze; Italian Chapter of the European Association for Theoretical Computer Science; and the sponsor AMGA Gruppo Energia & Servizi.

The success of the meeting owes a great deal to the enthusiasm and hard work of the Local Organizing Committee under the leadership of its cochairs, Giovanna D'Agostino and Angelo Montanari (University of Udine). The other members were Vincenzo Dimonte, Guido Gherardi, Alberto Marcone, Franco Parlamento, Carla Piazza, Dario Delle Monica, Marta Fiori Carones, Emanuele Frittaion, Nicola Gigante, Alberto Molinari, and Manlio Valenti.

The Program Committee consisted of Dugald Macpherson (University of Leeds, Chair), Stéphane Demri (CNRS), Alexander S. Kechris (California Institute of Technology), Chris Laskowski (University of Maryland), Alberto Marcone (Università degli Studi di Udine), Antonio Montalban (UC Berkeley), Pavel Pudlák (Czech Academy of Sciences), Gila Sher (UC San Diego), and Dima Sinapova (University of Illinois at Chicago).

The program included two tutorial courses, twelve invited lectures, including the Twenty-ninth Annual Gödel Lecture, twenty-four invited lectures in six special sessions (on computability theory, descriptive set theory and dynamical systems, model theory, philosophy of logic and mathematics, proof theory and constructivism, and temporal and multivalued logics), and 87 contributed talks. There were about 190 participants, and ASL travel grants were awarded to 33 students and recent Ph.D's.

The following tutorial courses were given:

Ulrike Sattler (University of Manchester), *Description logics, ontologies, and automated reasoning*: *An introduction*.

Katrin Tent (WWU Münster), *Model-theoretic ampleness*.

The following invited plenary lectures were presented:

Rod Downey (Victoria University, Wellington), (the Gödel Lecture), *Algorithmic randomness*.

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Marianna Antonutti Marfori (LMU München), *On the significance of mathematical hierarchies*.

Albert Atserias (Universitat Politècnica de Catalunya), *What can not be solved by the ellipsoid method*.

Vasco Brattka (Universität der Bundeswehr München), *On the computational content of theorems*.

Agata Ciabattoni (TU Wien), *Analytic calculi for substructural logics: Theory and applications*.

Paola D'Aquino (Università degli Studi della Campania), *Complex exponential field*.

Paolo Oliva (Queen Mary University of London), *Relational proof interpretations*.

Ludovic Patey (Institut Camille Jordan, Lyon), *Ramsey's Theorem from a computable perspective*.

Anush Tserunyan (University of Illinois at Urbana-Champaign), *Ergodic theorems and descriptive combinatorics*.

Spencer Unger (Tel Aviv University), *Stationary reflection*.

Matteo Viale (Università degli Studi di Torino), *Forcing as a tool to prove theorems*.

Dag Westerstahl (Stockholm University), *Logical constants and logical consequence*.

More information about the meeting can be found at the conference website, `https://lc18.uniud.it/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
DUGALD MACPHERSON

Abstracts of Invited Tutorials

▶ ULI SATTLER, *Description logics, ontologies, and automated reasoning: An introduction*.
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Description logics (DL) [1, 2] form the logical basis of state-of-the-art ontology languages, in particular the Semantic Web Ontology language OWL [3]. They have been first developed as the formalisation of semantic networks and frames, and are "coincidental" cousins of modal logic and the guarded fragment, and hence decidable fragments of first-order logic.

In the last three decades, we have seen a wide range of contributions and applications, due to mutually beneficial interactions between the following areas of activity:

- variants, extensions, and combinations of description logics being investigated with respect to their decidability, computational complexity, model theory, and other relevant properties;
- automated reasoners being developed, constantly optimised to cater for ever more demanding application scenarios, extended to cater for a wide range of reasoning tasks, and supported by other tools;
- tools such as editors, integrated development environments, and programmatic APIs being developed and constantly improved that integrate well with reasoners and support domain experts in modelling;
- applications—in particular from bio-health applications but also from other knowledge-heavy domains—that benefit from the "semantic lense" that description logic theories provide and the reasoning services offered via DL reasoners, and that require novel, nonstandard reasoning services such as module extraction or entailment explanations.

This development was helped by the standardisation of the syntax and extensions to this syntax to annotate/comment logical theories and axioms with relevant book-keeping information.

In this tutorial, I will give an introduction to description logics, their relationship to modal and first-order logics, and the four areas highlighted above. This introduction is aimed at anybody with a general background in logic and an interest in learning more about the field of description logics, knowledge representation, and ontology engineering.

[1] F. Baader, D. Calvanese, D. McGuinness, D. Nardi, and P. F. Patel-Schneider, ***The Description Logic Handbook*: *Theory, Implementation, and Applications***, Cambridge University Press, 2003.

[2] F. Baader, I. Horrocks, C. Lutz, and U. Sattler, ***Introduction to Description Logic***, Cambridge University Press, 2017.

[3] B. C. Grau, I. Horrocks, B. Motik, B. Parsia, P. F. Patel-Schneider, and U. Sattler, *OWL 2*: *The next step for OWL*. ***Journal of Web Semantics***, vol. 6 (2008), no. 4, pp. 309–322.

► KATRIN TENT, *Model theoretic ampleness*.
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The notion of ampleness originates in algebraic geometry and characterizes an embeddability property into projective spaces.

This notion turned out to be crucial in the characterization of Zariski geometrie due to Hrushovski and Zilber.

Pillay subsequently introduced a model theoretic version of ampleness. His definition can be seen as an attempt to characterize projective spaces just using model theoretic independence. It remained an open question whether ampleness might be sufficient to characterize a strongly minimal structure as being an algebraic curve.

In this series of lectures, I will give an introduction to model theoretic independence and explain the definition of ampleness. I will then go on to explain a number of different examples of ample structures, starting with projective spaces and other kinds of buildings, sketching ampleness in the free group. Finally, I will explain recent constructions of ample strongly minimal structures not arising as an algebraic curve.

Abstract of the invited Gödel Lecture

► ROD DOWNEY, *Algorithmic randomness*.
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In spite of the fact that elementary probability theory tells us that all sequences of n tosses of a fair coin are equally likely, our intuition tells us that some sequences are more random than others. Is there a reasonable mathematical theory of randomness of individual objects rather than one of expected behavior of distributions? In this talk I will discuss work in the area of mathematics devoted to interpreting randomness through computation. I will begin with Borel, von Mises, and Turing and finish discussing some things we have learned in recent years. The lecture should be accessible to graduate students.

Abstracts of invited Plenary talks

► MARIANNA ANTONUTTI-MARFORI, *On the significance of mathematical hierarchies*.
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The study of hierarchies in mathematics has been a very active field of research since the 1940s. Hierarchies are usually taken to classify objects (or collections thereof) according to a measure of complexity along some dimension, such that the level at which a given object appears in the hierarchy measures the degree of difficulty of constructing the object in question, or equivalently, the difficulty of verifying its existence. Theories considered mathematically "natural" often occupy special places in these hierarchies and can sometimes be put into interesting correspondences with foundational or philosophical approaches to mathematics, according to the strength of their existential assumptions.

In this talk, I will propose a different, complementary view of the lessons we can draw from the study of mathematical hierarchies. According to this view, hierarchies measure the relative distance from the axiomatic assumptions that we make on the basis of our pretheoretical understanding of a certain domain of mathematical objects, by means of countable or uncountable iterations of inference patterns that we recognise as correct. In outlining this view, I will consider how the study of hierarchies developed from the new formal analyses of concepts such as computation, consistency, interpretation, and model, that emerged in the early 20th century, and I will suggest that a deeper understanding of the historical development of mathematical hierarchies can help illuminate their significance.

► ALBERT ATSERIAS, *What can not be solved by the ellipsoid method*?
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The ellipsoid method, developed in the 1960s for nonlinear programming, and rigorously analyzed by Khachiyan in the 1970s for linear programming, is a powerful algorithm for solving linear optimization problems over convex sets that are given by a separation oracle. One of the important features of the method is that it gives a polynomial-time algorithm for solving not only explicitly given linear programs but also certain implicitly given exponentially big linear or semidefinite programs that arise in combinatorial contexts. The massive flexibility of this method turns the following question into a challenge: what are the limits of the ellipsoid method? In other words, what are the combinatorial problems that the ellipsoid method can provably *not* solve in polynomial time? In this talk, we will show how the methods of mathematical logic, concretely, the methods of descriptive complexity and finite model theory, provide good answers to some of these questions.

► VASCO BRATTKA, *On the computational content of theorems*.
Department of Computer Science, Universität der Bundeswehr München, 85577 Neubiberg, Germany and Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7700, South Africa.
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To analyze the computational content of theorems is a research topic at least since Turing's seminal work on computable numbers in which he started the investigation of computable versions of theorems in analysis. In the sequel, this topic was taken up by many other researchers such as Specker, Lacombe, Shore and Nerode, Pour-El and Richards [2], and Weihrauch [4]. A related but formally different approach has been started by Friedman and Simpson [3], who have characterized axioms that are sufficient and often necessary to prove certain theorems in second-order arithmetic. This approach is best known under the name reverse mathematics. In recent years, the interaction between these two research trends has been intensified and overlaps in what is called Weihrauch complexity. Weihrauch complexity is a computability theoretic approach to the classification of the computational content of theorems that yields results that can be seen as a uniform and resource-sensitive version of reverse mathematics. The benefit of this theory is that it yields fine grained computational results that answer typical questions from the computable analysis perspective, while being compatible with reverse mathematics. Sometimes results can be imported from reverse mathematics and computable analysis, but often completely new methods and techniques are required. We will present a survey on this approach that is based on a recent survey article [1] on this topic.

[1] V. Brattka, G. Gherardi, and A. Pauly, *Weihrauch complexity in computable analysis*, 2017, arXiv:1707.03202.

[2] M. B. Pour-El and J. I. Richards, ***Computability in Analysis and Physics***, Springer, Berlin, 1989.

[3] S. G. Simpson, ***Subsystems of Second Order Arithmetic***, second ed., Cambridge University Press, Poughkeepsie, 2009.

[4] K. Weihrauch, ***Computable Analysis***, Springer, Berlin, 2000.

▸ AGATA CIABATTONI, *Analytic calculi for substructural logics: Theory and applications*.
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Substructural logics are axiomatic extensions of full Lambek calculus. They encompass, among many others, classical, intuitionistic, intermediate, fuzzy, and relevant logics. In my talk, I will outline some results towards a uniform and systematic introduction of cut-free sequent and hypersequent calculi for substructural logics.

The calculi are defined by integrating proof theoretic and algebraic techniques, starting from Hilbert systems [4, 5, 6].

The hypersequent calculi are used to provide a concurrent computational interpretation for many intermediate logics, classical logic included. This solves Avron's conjecture [3]. We use indeed the Curry–Howard correspondence to obtain new typed concurrent λ-calculi, each of which features a specific communication mechanism and implements forms of code mobility [1, 2].

[1] F. Aschieri, A. Ciabattoni, and F. A. Genco, *Gödel logic: From natural deduction to parallel computation*, ***Proceedings of Logic in Computer Science*** (LICS 2017), 2017, pp. 1–12.

[2] ———, *Classical proofs and parallel programs*, submitted, 2018.

[3] A. Avron, *Hypersequents, logical consequence and intermediate logics for concurrency*. ***Annals of Mathematics and Artificial Intelligence***, vol. 4 (1991), pp. 225–248.

[4] A. Ciabattoni, N. Galatos, and K. Terui, *Algebraic proof theory: Hypersequents and hypercompletions*. ***Annals of Pure and Applied Logic***, vol. 168 (2017), no. 3, pp. 693–737.

[5] ———, *Algebraic proof theory for substructural logics: Cut-elimination and completions*. ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 3, pp. 266–290.

[6] ———, *From axioms to analytic rules in nonclassical logics*, ***Proceedings of Logic in Computer Science*** (LICS 2008), 2008, pp. 229–240.

▸ PAOLA D'AQUINO, *Complex exponential field*.
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In his article [2], Zilber identifies a new class of exponential fields (pseudo-exponential fields), proves a categoricity result in every uncountable cardinality, and puts forward the dramatic conjecture that the classical complex exponential field is the unique model of power continuum. The huge importance of this conjecture for the classical case is that Zilber has, unconditionally, established, for the pseudo-exponential fields, geometrically natural criteria for solvability of systems of exponential equations, whereas in the classical case, only a very few such criteria have been established by using hard complex analysis, for example, Nevanlinna Theory. In the last 15 years, much attention has been given to extend classical results for the complex exponential field to the pseudo-exponential fields, and vice versa much effort has been put in proving for $\mathbb{C}$ properties of solutions of exponential polynomials which follow from the axioms of Zilber. The analytic methods have been substituted by algebraic and diophantine-geometrical arguments. I will review some of the first results on this and I will present some more recent achievements obtained in collaboration with A. Fornasiero and G. Terzo.

[1] P. D'Aquino, A. Fornasiero, and G. Terzo, *Generic solutions of equations with iterated exponentials*. ***Transactions of the American Mathematical Society***, vol. 370 (2018), no. 2, pp. 1393–1407.

[2] B. Zilber, *Pseudo-exponentiation on algebraically closed fields of characteristic zero*. ***Annals of Pure and Applied Logic***, vol. 132 (2004), no. 1, pp. 67–95.

▸ YAIR HAYUT AND SPENCER UNGER, *Stationary reflection*.
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Stationary reflection is an important notion in the study of compactness principles in set theory. The existence of a nonreflecting stationary set is enough to construct many objects which witness the noncompactness of various properties, examples include noncompactness of the chromatic number of graphs and the extent of freeness of abelian groups. In the other direction, stationary reflection (the assertion that every stationary set reflects) is consistent relative to the existence of large cardinals.

At the successor of a singular cardinal, the previous known upperbound was infinitely many supercompact cardinals. In particular, Magidor showed that stationary reflection at $\aleph_{\omega+1}$ is consistent from this hypothesis. We improve this upperbound by showing that stationary reflection at $\aleph_{\omega+1}$ is consistent relative to the existence of a cardinal κ which is κ^+-Π^1_1-subcompact. Under GCH this large cardinal assumption follows from κ^+-supercompactness.

► PAULO OLIVA, *Relational proof interpretations*.
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Functional interpretations come in two flavours: one either interprets formulas as *sets* of realisers or as *relations* between potential realisers and counter-realisers. In the first group, we have the various realizability interpretations, such as Kleene's original numerical realizability, or Kreisel's modified realizability. In the second class, we have Dialectica-like interpretations, such as Gödel's functional interpretation or its Diller-Nahm variant. In this talk, we review the observation of [2] that a relational version of modified realizability also exists, and that its set-based definition can be derived from the relational one. We stress the advantages of this (more general) relational approach: it allows for a unification of realizability and Dialectica interpretations, enabling a modified realizability of linear logic [3], and a hybrid interpretation that combines various interpretations [4]. Surprisingly, this even includes truth-variants of these interpretations [1] and explains these in terms of the linear logic exponentials. We conclude by discussing recent work on a similar (relational) generalisation and unification for type-free (partial) interpretations such as Kleene's original realizability.

[1] J. Gaspar and P. Oliva, *Proof interpretations with truth*. ***Mathematical Logic Quarterly***, vol. 56 (2010), no. 6, pp. 591–610.

[2] P. Oliva, *Unifying functional interpretations*. ***Notre Dame Journal of Formal Logic***, vol. 47 (2006), no. 2, pp. 263–290.

[3] ———, *Modified realizability interpretation of classical linear logic*, ***Proceedings of the Twenty Second Annual IEEE Symposium on Logic in Computer Science LICS'07***, IEEE Press,Wroclaw, Poland, 2007, pp. 431–442.

[4] ———, *Hybrid functional interpretations of linear and intuitionistic logic*. ***Journal of Logic and Computation***, vol. 22 (2012), no. 2, pp. 305–328.

► LUDOVIC PATEY, *Ramsey's theorem from a computable perspective*.
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Ramsey's theorem for n-tuples and k colors (RT^n_k) asserts that given a k-coloring of $[\mathbb{N}]^n$, there exists an infinite set H such that $[H]^n$ is monochromatic. This theorem is not computably true, in that there is a computable such coloring with no infinite computable monochromatic set. Ramsey's theorem can be seen as a mathematical problem, in terms of *instances* and *solutions*. An instance is a k-coloring f of $[\mathbb{N}]^n$, and a solution to f is an infinite set H such that $[H]^n$ is monochromatic. A natural question to ask is "how hard is it to compute a solution given an instance of Ramsey's theorem?" The study of the computational strength of Ramsey's theorem is a long journey, started with Carl Jockusch, with high feats, such as Seetapun's theorem and Liu's theorem. This talk aims to be a gentle introduction to the modern analysis of Ramsey's theorem from the viewpoint of proof-theory, computability theory, and Weihrauch degrees. These approaches happened to be a very fruitful line of

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research, leading to the development of new techniques in various areas. Some important questions remain however open, and we shall stress the remaining challenges.

▶ ANUSH TSERUNYAN, *Descriptive combinatorics and ergodic theorems*.
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"Gee Professor Kechris, descriptive set theory sure is powerful, and beautiful too!" was my friend's suggestion on how to phrase my email to my future Ph.D. advisor asking for a reading course. I still believe this, of course, and I'll try to convince you as well by exhibiting an example of how modern descriptive set theoretic thinking combined with combinatorial and measure theoretic arguments yields a pointwise ergodic theorem for quasi-probability-measure-preserving locally countable graphs. This can be viewed as a general analogue of pointwise ergodic theorems for group actions, as a group action naturally induces a graph, its Cayley–Schreier graph. The theorem states that ergodicity (measure kinetic indecomposability) of a graph amounts to locally approximating global averages of L^1-functions via increasing subgraphs with finite connected components.

▶ MATTEO VIALE, *Forcing as a tool to prove theorems*.
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Forcing is a fundamental working tool of set theorists, and is the standard method to obtain independence results. The aim of this talk was to outline that forcing can also be used to find the correct solution for certain types of mathematical problems. Specifically, we will present two examples of metamatemathical arguments (based on generic absoluteness results and forcing) which can be used to answer two rather concrete questions, one related to Schanuel's conjecture on the transcendence properties of the complex exponential and the other related to the classification problem for countable abelian groups [1, 3]. We will also give a general picture of the generic absoluteness results mainly by Woodin (for second-order arithmetic) and myself (for fragments of third-order arithmetic in combination with forcing axioms) [2] and their connections with large cardinals, forcing axioms, and with other nonconstructive principles such as the axiom of choice and the Baire category theorem.

[1] F. Calderoni and S. Thomas, *The bi-embeddability relation for countable abelian groups*. ***Transactions of the AMS***, DOI: https://doi.org/10.1090/tran/7513.

[2] M. Viale, *Category forcings,* MM^{+++}*, and generic absoluteness for the theory of strong forcing axioms*. ***Journal of the American Mathematical Society***, vol. 29 (2016), no. 3, pp. 675–728.

[3] ———, *Forcing the truth of a weak form of Schanuel's conjecture*. ***Confluentes Mathematicae***, vol. 8 (2016), no. 2, pp. 59–83.

▶ DAG WESTERSTÅHL, *Logical constants and logical consequence*.
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Logical consequence is a (*the*?) central notion in logic, and consequence relations in general and their properties is a familiar topic in the discipline. The dual notion of a logical constant has received less attention. Yet consequence depends on constants: as Bolzano and (100 years later) Tarski made precise, every choice of a set X of constants yields a corresponding consequence relation $\Rightarrow_X$, defined in terms of truth preservation under replacement/reinterpretation of symbols *outside* X. How to choose X? Both Bolzano and Tarski emphasized the importance of this question, and admitted they had no answer (at the time; Tarski later suggested an answer in terms of permutation invariance). However, our intuitions about "what follows from what" are stronger than those about what is constant, and it makes sense to try to see how constants depend on consequence, by "extracting" the constants from a given consequence relation. A natural way to make this precise yields a

Galois connection establishing the duality between sets of constants and Bolzano/Tarskian consequence relations.

Logical consequence relations are also defined syntactically, via rules. If such a relation coincides with the Bolzano/Tarskian consequence relation given by the standard logical symbols with their standard interpretation, we have (soundness and) *completeness*: syntax matches the semantics. But the other side of the coin, that semantics matches syntax in the sense that the meaning of the logical symbols is (at least to a large extent) determined by the relation of logical consequence, seems fundamental too. Such *categoricity* results were the main focus of Carnap's 1943 book *The Formalization of Logic*, but the topic has been rather neglected since.

I will report on ongoing work together with Denis Bonnay on these issues, first presenting the abstract relations between constants and consequence, and then some concrete categoricity results (work begun in [1, 2]). While the picture for first-order logic and its extensions is fairly clear, new conceptual issues, and some open questions arise in the case of other logics such as modal logic.

[1] D. Bonnay and D. Westerståhl, *Consequence mining. Constants versus consequence relations*. ***Journal of Philosophical Logic***, vol. 41 (2012), no. 4, pp. 671–709.

[2] ———, *Compositionality solves Carnap's problem*. ***Erkenntnis***, vol. 81 (2016), no. 4, pp. 721–739.

Abstracts of invited talks in the Special Session on Computability Theory

▸ JOHANNA N. Y. FRANKLIN AND TIMOTHY H. MCNICHOLL, *Lowness and computable metric spaces*.
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In the past five years, lowness, a familiar notion in other areas of computability theory, has been introduced into computable structure theory; specific types of structures studied include equivalence structures and various types of linear orders. Recently, we have extended this work to computable metric spaces. Here, we present some results on lowness for computably presented metric spaces and then, more specifically, Banach spaces.

▸ TAKAYUKI KIHARA, *On the structure of the Wadge degrees of BQO-valued Borel functions*.
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In this talk, we give a full description of the Wadge degrees of Borel functions from ω^ω to a better-quasi-ordering $\mathcal{Q}$. More precisely, for any countable ordinal ξ, we show that the Wadge degrees of $\utilde{\mathbf{\Delta}}^0_{1+\xi}$-measurable functions $\omega^\omega \to \mathcal{Q}$ can be represented by countable joins of the ξ-th transfinite nests of $\mathcal{Q}$-labeled well-founded trees [2]. This result generalizes and unifies former works by Wadge, Duparc, Selivanov, and others. This also has a consequence in computability theory, since it is shown that there is a natural isomorphism between the structure of the Wadge degrees of $\mathcal{Q}$-valued functions and that of the "natural" many-one degrees of $\mathcal{Q}$-valued problems [1].

Our main theorem is completely described in the descriptive-set-theoretic language; nevertheless, our proof requires various computability-theoretic tools such as Marcone–Montalbán's *Turing jump operator via true stages*, and (the uniform version of) the *Friedberg jump inversion theorem*. Thus, our work provides a new application of computability theory to descriptive set theory.

This is joint work with Antonio Montalbán.

[1] T. Kihara and A. Montalbán, *The uniform Martin's conjecture for many-one degrees*. ***Transactions of the American Mathematical Society***, to appear.

[2] ———, *On the structure of the Wadge degrees of BQO-valued Borel functions*, submitted.

▸ TAKAYUKI KIHARA AND LINDA BROWN WESTRICK, *Topology and parallelized Weihrauch reducibility*.
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The parallelized Weihrauch reducibility on functions $f, g : 2^\omega \to \mathbb{R}$ is defined as follows: $f \leq_W \hat{g}$ if and only if f is Weihrauch reducible to ω-many copies of g. We give some alternate characterizations of this and related notions, relating them to the choice of topology on 2^ω and on the space of functions $f, g : 2^\omega \to \mathbb{R}$.

▸ MICHAEL MCINERNEY AND KENG MENG NG, *A randomness-free characterization of strong jump traceability*.
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The strongly jump traceable degrees have been shown to be very useful and have been used to illustrate the many connections between algorithmic randomness and classical computability theory. It is a very robust class, with many different characterizations. Unfortunately, all of the characterizations obtained so far have used notions of randomness (either directly or indirectly). Given that the class of strongly jump traceable degrees exhibit strong connections with randomness and computability, it is therefore natural to ask if a degree-theoretic characterization can be obtained for strong jump traceability that does not utilize randomness.

We prove that the strongly jump traceable degrees are exactly those Δ_2^0 degrees that are jump traceable preserving, i.e., a degree $\boldsymbol{a}$ is JT-preserving if $\boldsymbol{a} \cup \boldsymbol{x}$ is jump traceable whenever $\boldsymbol{x}$ is jump traceable. We also give several other degree-theoretic characterizations of strong jump traceability.

Abstracts of invited talks in the Special Session on Descriptive Set Theory and Dynamical Systems

▸ CLINTON CONLEY, *Realizing abstract systems of congruences*.
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An abstract system of congruences is an equivalence relation on the power set of a finite set meant to encode various possibilities for equidecomposability for some ostensible starting partition. For example, the assertion $\{0\}E\{0, 1, 2\}$ may be read to mean part 0 is congruent with the union of parts 0, 1, and2. If now some group acts on a space X, we say a partition of X realizes a given abstract system of congruences if the congruence equivalence relation coincides with the induced notion of equidecomposability for the action. Wagon has characterized which such abstract systems of congruences can be realized in the action of rotations of the 2-sphere. We develop an analogous characterization of systems of congruences realized by partitions with the property of Baire, and also investigate some other natural actions.

This is joint work with Andrew Marks and Spencer Unger.

▸ JULIEN MELLERAY, *Orbit equivalence of Toeplitz flows is Borel complete*.
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A homeomorphism of the Cantor space is *minimal* if all its orbits are dense. Given two minimal homeomorphisms g, h of the Cantor space, one says that g and h are orbit equivalent if there exists a homeomorphism which maps each g-orbit onto an h-orbit.

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I will explain why the relation of orbit equivalence of minimal homeomorphisms is Borel complete; more precisely, the relation of orbit equivalence between Toeplitz subshifts is Borel complete, which stands in contrast with the fact that the relation of isomorphism on subshifts is essentially countable.

Perhaps surprisingly, notions from model theory (ultrahomogeneity and Fraïssé limits) play a part in the proof, and I will try to explain why.

▸ TODOR TSANKOV, *Universal minimal flows relative to a URS*.
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A *uniformly recurrent subgroup* (*URS*) of a locally compact group G is a minimal, conjugation invariant, closed subset of the space of closed subgroups of G. To every minimal action of G, one can naturally associate its stabilizer URS and thus understanding the URSs of a given group gives important information about its nonfree, minimal actions. URSs were introduced by Glasner and Weiss and they left open the following basic question: does every URS arise as the stabilizer URS of a minimal action? We answer this question in the affirmative by a universal construction.

This is joint work with Nicolás Matte Bon.

▸ ROBIN TUCKER-DROB, *Measure equivalence, superrigidity, and weak Pinsker entropy*.
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We show that the class B, of discrete groups, which satisfy the conclusion of Popa's Cocycle Superrigidity Theorem for Bernoulli actions, is invariant under measure equivalence. We generalize this to the setting of discrete p.m.p. groupoids, and as a consequence, we deduce that any nonamenable lattice in a product of two noncompact, locally compact second countable groups, must belong to B. We also introduce a measure-conjugacy invariant called weak Pinsker entropy and show that, if G is a group in the class B, then weak Pinsker entropy is an orbit-equivalence invariant of every essentially free p.m.p. action of G.

Abstracts of invited talks in the Special Session on Model Theory

▸ GABRIEL CONANT, *Pseudofinite groups and tame arithmetic regularity*.
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Arithmetic regularity, developed by Green in 2005, uses discrete Fourier analysis to produce regular decompositions of subsets of finite abelian groups, giving an arithmetic analogue of Szemerédi regularity for finite graphs. More recently, several authors have obtained various "strong arithmetic regularity" results for subsets of finite groups satisfying restricted assumptions. On the combinatorial side, this includes quantitative results assuming stability (Terry–Wolf) or bounded VC-dimension (Alon–Fox–Zhao; Sisask) in finite abelian groups. On the model-theoretic side, in joint work with Pillay and Terry, we give qualitative arithmetic regularity results for finite groups under the same tameness assumptions, by applying local stability and NIP theory to pseudofinite groups. The common ground for these two approaches lies in the study of approximate subgroups, especially the work of Hrushovski and of Breuillard–Green–Tao. In this talk, I will present several of these results, with a focus on the underlying connections between model theory and additive combinatorics.

▸ ADRIEN DELORO, *Linearisation in model theory*.
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Model theory deals with algebraic structures in its own way, but the presence of a field is always a good thing. Typically fields come from the very basic Schur lemma in representation theory; of course in model theory one wants definable versions. The most famous result in this vein is Zilber's classical observation that definable fields emerge in many abstract groups of finite Morley rank. But it is not the only such result, nor is finite Morley rank the only model-theoretic framework.

We try to provide the ultimate version of linearisation theorems, in a single statement generalising every Schur–Zilber result known so far and extending them to the natural context of "finite-dimensional theories" (which will be discussed).

This reports on joint work with Frank Wagner.

▸ NADJA HEMPEL AND DANIEL PALACIN, *Division rings with ranks.*
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In this talk, we analyze division rings which admit a well-behaved ordinal valued rank function on definable sets that behaves like a rudimentary notion of dimension. These are called superrosy division rings. Examples are the quaternions, any superstable division ring (which are known to be algebraically closed fields by theorems of Macintyre [3]/Cherlin–Shelah [1]), and more generally supersimple division rings (which are commutative by a result of Pillay, Scanlon, and Wagner [4]). We show that any superrosy division ring has finite dimension over its center, generalizing the aforementioned results [2]. If time permits, we will also present some results on division rings of finite burden and weight one [2].

[1] G. Cherlin and S. Shelah, *Superstable fields and groups*. ***Annals of Mathematical Logic***, vol. 18 (1980), pp. 227–270.

[2] N. Hempel and D. Palacin, *Division rings with ranks*. ***Proceedings of the American Mathematical Society***, vol. 146 (2018), no. 2, pp. 803–817.

[3] A. Macintyre, *ω_1-categorical fields*. ***Fundamenta Mathematicae***, vol. 70 (1971), no. 3, pp. 253–270.

[4] A. Pillay, T. Scanlon, and F. Wagner, *Supersimple fields and division rings*. ***Mathematical Research Letters***, vol. 5 (1998), pp. 473–483.

Abstracts of invited talks in the Special Session on Philosophy of Logics and Mathematics

▸ BENEDICT EASTAUGH, *Does nonclassical truth impair mathematical reasoning?*
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Weakening the principles of classical logic in order to retain desirable properties of intensional notions such as truth has been widely embraced as a response to the intensional paradoxes. Advocates of classical logic who resist logical revision have argued that our standard for reasoning with intensional notions should not be different from that employed in our best scientific and mathematical theories. A specific version of this argument, due to Halbach [1], uses a proof-theoretic analysis of two classical and nonclassical theories of Kripkean truth (known as KF and PKF) to show that when we give up classical logic, we must in the process give up important nonsemantic patterns of reasoning, in particular in mathematics. The nonclassical logician has a natural response to Halbach's argument: they can bite the bullet and argue that even if one must give up the universality of these patterns of reasoning, one does not thereby lose too much in the way of genuine mathematics. However, despite first appearances, one does give up substantial mathematics by accepting nonclassical logic in

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this context. Drawing on work in reverse mathematics by Montalbán [2] and Neeman [3], we show that an ordinary mathematical theorem concerning indecomposable linear orderings is proof-theoretically reducible to the classical theory of Kripkean truth KF, but not to the weaker nonclassical theory PKF.

This is joint work with Carlo Nicolai.

[1] V. Halbach, ***Axiomatic Theories of Truth***, Cambridge University Press, 2011.

[2] A. Montalbán, *Indecomposable linear orderings and hyperarithmetical analysis*. ***Journal of Mathematical Logic***, vol. 6 (2006), pp. 89–120.

[3] I. Neeman, *The strength of Jullien's indecomposability theorem*. ***Journal of Mathematical Logic***, vol. 8 (2008), no. 1, pp. 93–119.

▸ BRICE HALIMI, *Kreisel's problem*.
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Is any logical consequence of ZFC ensured to be true? (Q) Kreisel and Boolos both proposed an answer to (Q), taking "truth" to mean truth in the background set-theoretic universe. My talk will advocate another answer, which lies at the level of the models of ZFC, so that "truth" remains the usual semantic notion. This other, model-scaled answer relies on the fact that any model of ZFC can be shown to contain "internal models," and thus can be compared to a particular background universe itself.

After defining logical consequence w.r.t. any given model of ZFC, I will set out the answer to the original question (Q) prompted by this definition, and present further results bearing on internal models. Finally, the semantics interpreting internal models as accessible possible worlds leads to an "internal modal logic" in which internal reflection is shown to correspond to modal reflexivity, and resplendency to the modal axiom **4**.

▸ SIMON HEWITT, *Some arguments for* Ex Contradictione Quodlibet.
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The principle that any proposition whatsoever follows from a contradiction, *Ex Contradictione Quodlibet* (ECN), is enshrined in classical, intuitionistic, and related logics. Although it is frequently encountered as counterintuitive, and has been questioned by relevance and paraconsistent logicians, little has been done to supply philosophical motivation for ECN.

To the extent that there has been philosophical debate concerning the principle, this has been against the background of an alternative on which some propositions (but not necessarily everthing) follow from a contradiction, as with Priest's **LP**. In contrast to this, I explore the prospects for a debate between the proponent of ECN and the historically important view that *no proposition* follows from a contradiction. The hope is that this will allow us to get clearer about both ECN and the potential of this alternative (which I call *Ex Contradictione Nihil*).

I lay out four arguments for *ECQ*: C. I. Lewis' "proof" of the principle (a version of which was discovered by William of Soissons in the 12th century); an argument from Tarski's analysis of consequence; an argument from the necessary truth-preservation, and an argument from the requirements of proof-theoretic harmony. In each case, I diagnose a circularity in the case for ECQ: there is a point at which someone not antecedently disposed to accept ECQ could, and should, object to the argument. This sheds light on the form a logical theory embodying should take, with respect to structural rules and model-theoretic definitions of consequence, and suggests that if debate is to move beyond *impasse*, then it will have to take place on a terrain that is somehow more fundamental. I end by suggesting that the theory of meaning is a candidate for such a terrain.

▸ GIL SAGI, *Logicality and semantic theory*.
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In this talk, we address the question of whether there is a logical consequence relation in natural language, combining our previous work on semantic constraints and recent work by Michael Glanzberg.

In previous work [3], we proposed a model-theoretic framework for logical consequence where there is no strict division of the vocabulary into the logical and the nonlogical terms. The class of models is set by *semantic constraints*: statements in the metalanguage that restrict the interpretation of terms in the language, not necessarily fixing them completely. The framework is a generalization of standard first-order logic, and the standard semantic clauses can be reformulated as semantic constraints. In recent work, we consider criteria for logicality of semantic constraints. We generalize the criterion of invariance under isomorphisms for logical terms to apply to semantic constraints. The correct generalization, we claim there, is to the requirement that the class of models satisfying a constraint be closed under isomorphisms.

We apply these results to the question of logic in natural language. In a recent article [1], Michael Glanzberg claims that natural language does not have a logical consequence relation. He argues for this thesis by observing contemporary natural language semantics, and claiming that it cannot distinguish logical consequence from other sorts of entailment. In the first part of the talk, we criticize Glanzberg's arguments, and claim that they do indeed leave room for a logical consequence relation in natural language.

We then discuss another recent article by Glanzberg [2], where he proposes a thesis of partiality in the explanatory force of linguistic theory. He observes that semantic clauses such as

1. $[\![\text{Ann}]\!] = \text{Ann}$
2. $[\![\text{smokes}]\!] = \lambda x \in D_e.\ x \text{ smokes}$

appear to have only weak explanatory force. Some lexical items, for example, predicates, are provided a type by the semantic theory, and while they are also provided with an extension as their semantic value—what their extension is not explained by the theory—but is rather taken for granted. According to Glanzberg, "semantics, narrowly construed as part of our linguistic competence, is only a partial determinant of content." We apply our framework of semantic constraints to isolate the explanatory parts of semantic clauses as those above. We claim that the semantic constraints obtained in this way will be invariant under isomorphisms, in the sense defined before. We thus conclude that semantic theory proper (as delineated by Glanzberg) is a theory of logical consequence in natural language, by the lights of a widely accepted philosophical account of logicality.

[1] M. Glanzberg, *Logical consequence and natural lanaguage*, ***Foundations of Logical Consequence*** (C. Caret and O. Hjortland, editors), Oxford University Press, Oxford, 2015, pp. 71–120.

[2] ———, *Explanation and partiality in semantic theory*, ***Metasemantics: New Essays on the Foundations of Meaning*** (A. Burgess and B. Sherman, editors), Oxford University Press, Oxford, 2014, pp. 259–292.

[3] G. Sagi, *Formality in logic: From logical terms to semantic constraints*. ***Logique et Analyse***, vol. 227 (2014), pp. 259–276.

► NICOLE WYATT, *Logics and explanations*.
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Logics, it seems, explain things. To take a classic example, Russell deploys first-order logic to explain the behaviour of "The present king of France is bald." The debate over the effectiveness of his explanation takes for granted that it is at least possible to use a logic to explain natural language phenomena. But it is a bit of puzzle as to how this works. This talk offers a anti-exceptionalist account of explanation in logic, and sketches some consequences for logical pluralists.

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Abstracts of invited talks in the Special Session on Proof Theory and Constructivism

▸ RYOTA AKIYOSHI, *"Proofs as programs" revisited.*
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Schwichtenberg has developed the program called "Proofs as Programs" by measuring the "complexity as programs" of proofs in an arithmetical system (with recursion) in [7]. Technically, he used Arai's observation in [4] to use a slow growing hierarchy $G_a(\cdot)$ in order to "climb down" tree ordinals due to Buchholz [5]. We remark that this is going back to Girard's hierarchy comparison theorem [6].

In this talk, we sketch another approach to this program by focusing on parameter-free subsystems of Girard's System F. There are at least two advantages in this approach. (i) It is simpler and smoother than the original one. (ii) It is relatively easy and uniform to extend our results to stronger fragments (corresponding to theories of iterated inductive definitions based on intuitionistic logic).

For explaining basic ideas, we focus on the weakest parameter-free fragment of System F. In this fragment called $\mathbb{F}_0$, a second-order type $\forall\alpha.\ A$ is permitted only if A is $\forall$-free and contains no other variable than X.

The upperbound of the complexity of terms in $\mathbb{F}_0$ is computed as follows. For a finite term M in $\mathbb{F}_0$, we define the relation $\vdash^{\alpha}_{m} M : A$ saying that (i) the upperbound of the complexity of M is measured by a tree ordinal α, (ii) the "cut-rule" of the rank $<m$ is sufficient. In particular, we introduce a miniaturized version of the Ω-rule due to Buchholz [5]. Next, we prove theorems for the relation $\vdash^{\alpha}_{m} M : A$ corresponding to the predicative cut-elimination and impredicative cut-elimination theorems in proof-theory.

Now, the upperbound theorem is stated as follows:

THEOREM 1. *Let f be a representable function in $\mathbb{F}_0$ with $M : \mathbf{N} \Rightarrow \mathbf{N}$. Then, $\models_0^{D_0(d\times(n+1))}$ $\mathcal{D}_0\mathcal{D}_1^m(M\mathsf{S}^n0) : \mathbf{N}$ with $d = D_1^m(\Omega \times m)$ for some $m \geq 4$. Therefore, there is m such that for all $n \geq m \geq 4$*

$$|\mathcal{D}_0\mathcal{D}_1^m(M\mathsf{S}^n0)| < G_{D_0D_1^{m+2}0}(n).$$

(Here, D_1 is the standard exponential function with the base ω, and D_0 is the collapsing function from Ω_2 into Ω_1.)

The lowerbound is proved by using Schwichtenberg's result [7] and Aehlig's one [1]:

THEOREM 2. *The function expressed by the formula $\forall x \exists y (D_0D_1^m0)[x]^y = 0$ in arithmetic is representable in $\mathbb{F}_0$.*

If time is permitting, we sketch how to extend our approach into stronger parameter-free fragments of System F by climbing up the hierarchy defined by Aehlig [1].

Acknowledgment. This work is partially supported by JSPS KAKENHI 16K16690 and 17H02265.

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[2] R. AKIYOSHI, *An ordinal-free Proof of the complete cut-elimination theorem for Π^1_1-CA + BI with the ω-rule*. ***The Mints' Memorial Issue of the IfCoLog Journal of Logics and their Applications***, vol. 4 (2017), no. 4, pp. 867–884.

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[4] T. ARAI, *A slow growing analogue to Buchholz' proof*. ***Annals of Pure and Applied Logic***, vol. 54 (1991), no. 2, pp. 101–120.

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▶ MARTÍN HÖTZEL ESCARDÓ, *Univalent mathematics at work*.
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I will illustrate how to work in Voevodsky's univalent mathematics in practice, both on article, informally, and in (cubical) Agda, formally, with examples from my recent work on constructive mathematics.

▶ ALESSANDRA PALMIGIANO, *Constructive canonicity of inductive inequalities*.
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This talk, based on [8], discusses the canonicity of inductive inequalities in a constructive meta-theory, for classes of logics algebraically captured by varieties of normal and regular lattice expansions. These results are obtained using the tools of *unified correspondence theory* [6, 10, 7, 9, 3, 2, 13] and contribute to develop a theoretical environment in which different proof techniques for canonicity and correspondence can be systematically compared and connected with each other (cf. [12, 16, 11, 17]). These canonicity results contribute to not only applications of semantic results to proof-theoretic issues [14, 1, 15] but also of logic to social sciences [5, 4].

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[5] ———, *Categories*: *How I learned to stop worrying and love two sorts*, ***Workshop on Logic, Language, Information, and Computation*: *WoLLIC 2017, Proceedings***, Springer, 2016, pp. 145–164, preprint, arXiv:1604.00777.

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[8] ———, *Constructive canonicity of inductive inequalities*, submitted, preprint, `arXiv:1603.08341`.

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► CHUANGJIE XU, *Unifying functional interpretations for nonstandard/uniform arithmetic*.
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We extend Oliva's method [3] to develop a parameterised functional interpretation for nonstandard arithmetic. By instantiating the parameterised relations, we obtain the Herbrand functional interpretations introduced in [2, 4] for nonstandard arithmetic as well as the usual, well-known ones such as modified realisability for Berger's uniform Heyting arithmetic [1]. We implement it in the Agda proof assistant by making use of Agda's parameterised modules. This allows us to extract computer programs from proofs in the nonstandard/uniform arithmetic via different instantiations of our parameterised functional interpretation.

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[2] F. Ferreira and J. Gaspar, *Nonstandardness and the bounded functional interpretation*. ***Annals of Pure and Applied Logic***, vol. 166 (2015), no. 6, pp. 701–712.

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[4] B. van den Berg, E. Briseid, and P. Safari, *A functional interpretation for nonstandard arithmetic*. ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 12, pp. 1962–1994.

Abstracts of invited talks in the Special Session on Temporal and Multivalued Logics

► AGI KURUCZ, *Horn fragments of the Halpern–Shoham interval temporal logic* $\mathcal{HS}$.
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The elegance and expressive power of $\mathcal{HS}$ have attracted the attention of the temporal and modal logic communities, as well as many other areas of computer science, AI, philosophy and linguistics. As $\mathcal{HS}$-satisfiability over various timelines is undecidable [2], the quest for "tame" fragments began in the 2000s, and have resulted in a substantial body of literature that identified a number of ways of reducing its expressive power. We discuss recent results on the computational complexity of Horn and core fragments of $\mathcal{HS}$ [1].

[1] D. Bresolin, A. Kurucz, E. Muñoz-Velasco, V. Ryzhikov, G. Sciavicco, and M. Zakharyaschev, *Horn fragments of the Halpern–Shoham interval temporal logic*. ***ACM Transactions on Computational Logic***, vol. 18 (2017), no. 3, pp. 22:1–22:39.

[2] J. Halpern and Y. Shoham, *A propositional modal logic of time intervals*. ***Journal of the ACM***, vol. 38 (1991), no. 4, pp. 935–962.

► DANIELE MUNDICI, *Łukasiewicz logic*: *98 years*.
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After almost a century since its origination, Łukasiewicz logic is still a vibrant research topic. A recent development is the differential semantics of Łukasiewicz infinite-valued propositional logic L_∞, according to which a formula θ follows from a set Θ of premises if, and only if, the following two conditions are satisfied:

1. Every model m of Θ is a model of θ (as per the Bolzano–Tarski paradigm), and
2. Any infinitesimal perturbation $m + \mathrm{d}m$ which is a model of Θ is a model of θ.

Condition 2 makes sense because the space of models is endowed with the topological-differential structure of a Tychonoff cube. This notion of semantic consequence turns out to be equivalent to the classical notion of syntactic consequence—stating that θ is derivable by finitely many applications of modus ponens to a finite subset of ($\Theta \cup$ tautologies of L_∞). This is the completeness theorem for L_∞. If Θ is finite, Condition 2 automatically follows from Condition 1 (Hay's theorem). If Θ is infinite this is no longer true in general (Wójcicki's theorem).

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► PARITOSH K. PANDYA, *Expressive-completeness and deciability of metric temporal logics: Recent progress*.
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The celebrated Kamp theorem established expressive completeness of linear temporal logic (LTL) with respect to $FO[<]$, the first-order theory of words. The equally celebrated Buchi theorem established (effective) expressive completeness of monadic second-order logic $MSO[<]$ with respect to deterministic finite state automata (DFA). The latter also provides a method for deciding satisfiability and model checking of $MSO[<]$ formulae using algorithms over DFA.

Generalizing this approach to real-time logics has been problematic. In this talk, we will survey some recent progress in formulating expressively complete real-time logics. Recent results of Hunter, Ouaknine, and Worrell [4, 3] have led to metric temporal logics which are expressively complete for $FO[<, +1]$, the first-order theory of timed words with metric distance predicate $+1$. However, these logics have undecidable satisfiability.

Towards the quest for expressive and decidable real-time logics, we will survey past work on logics [1, 8, 2], which explored expressive completeness with nondeterministic timed automata (NTA). We will then look at recent attempts at extending metric temporal logic to get expressive completeness with 1-clock alternating timed automata ($1ATA$). A monadic second-order logic with guarded metric quantifiers, $QkMSO$, has decidable satisfiability and model checking. This logic will be presented and its expressive power will be characterised using a subclass of $1ATA$ as well as a metric temporal logic $RegMTL$. The logic $RegMTL$ (see [5]) extends the well-known $MTL[U_I]$ [7] with a regular expression modality.

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► AMANDA VIDAL, *Many-valued modal logics: Axiomatizability issues*.
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Modal logic is one of the most developed and studied nonclassical logics, yielding a beautiful equilibrium between complexity and expressibility. Generalizations of the concepts of necessity and possibility offer a rich setting to model and study notions from many different areas, including proof-theory, temporal and epistemic concepts, and workflow in software applications. On the other hand, substructural logics provide a formal framework to manage vague and resource-sensitive information in a very general (and so, adaptable) fashion. *Many-valued modal logics* is a field in ongoing development. While the first publications on the topic can be traced back to the 90s [5, 6], it has been only in the latter years when a more systematic work has been developed, addressing the axiomatizability question in general, characterization and study of the model-theoretic notions analogous to the ones from the classical case, decidability and applicability issues, etc (see e.g., [7], [3, 4], [1], [9], [8], [2],...).

In this talk, we present some recent results for these logics, focused on their decidability and axiomatizability. In particular, we exhibit a large family of many-valued modal logics, including several very natural cases (including modal expansions of Łukasiewicz and Product logics, two of the three main BL extensions) that even if they enjoy the finite model property, are undecidable. Thus, no r.e. axiomatization can exist for them, in contrast to what happens with their propositional counterparts. We show that simple properties in the underlying class of algebras (noncontractivity of the strong conjunction operation, and a technical but easy notion related to the existence and behavior of the infimum of certain values) allow the construction of a set of formulas capturing the Post correspondence Problem in a large family of modal logics. In the particular case of Łukasiewicz logic, this allow us, together with completeness with respect to the so-called witnessed models (where, in contrast to other many-valued modal logics, each modal formula is evaluated as that formula in some successor world) to also show that the usually called global Łukasiewic modal logic cannot be axiomatized.

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Abstracts of contributed talks

▸ PIOTR BŁASZCZYK, *Trends in the history of infinity*.
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Cantor established two kinds of infinity: cardinal and ordinal numbers, each with its own arithmetic and its own relation *greater than*. In modern developments, ordinal numbers are special sets, cardinal numbers are specific ordinal numbers. In both cases, the set of natural numbers $\mathbb{N}$ makes the yardstick of infinity—be it the cardinal number $\aleph_0$ or the ordinal ω. However, while Cantor infinities try to extend the arithmetic of finite numbers, the addition and multiplication of ordinal numbers are not commutative. Moreover, while there are many possible well-orderings on the set $\mathbb{N}$, Cantor considered the *natural* one; in [2] he also considered the *natural* order of the real numbers. Cantor could never explain what does mean *natural* order in mathematical terms.

In [4], Euler introduced numbers that exceed any finite number. Still, while in his development finite numbers form an ordered field, Euler infinite numbers also belong to the ordered field. Consequently, when N is infinite, so is $N - 1$ and $N/2$.

Thus, Cantor's and Euler's investigations exemplify competing trends in the history of infinity founded on set theory or algebra.

We will argue that the theory of surreal numbers developed in [2] provides a uniform perspective that allows one to compare these two trends. We will argue that the perspective of ordered fields provides a more general and consistent account of infinity. (a) The theory of real closed fields developed in [3] provides mathematical reasons to treat a total order as a *natural* one. Namely, in some fields, e.g., in the field of surreal numbers, there is only one total order compatible with addition and multiplication. (b) Surreal numbers include ordinal numbers. (c) The addition and multiplication of ordinal numbers is commutative when they are taken as surreals. (d) There exist negative and fractional ordinal numbers, when ordinal numbers are considered as elements of the filed of surreals.

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▸ GIULIA BATTILOTTI, MILOS BOROZAN, AND ROSAPIA LAURO GROTTO, *Reading Bi-logic in first-order language*.
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We see how one can formalise the main aspects of Matte Blanco's Bi-logic [3] in first-order language [1, 2, 4]. In particular, the formalisation allows easily to read also Freud's original way to distinguish between "word presentation" and "thing presentation," introduced in [5], that is at the root of his theory, in the distinction between closed terms and variables, that had been introduced short before in formal logic. Moreover, we suggest how the psychoanalytical problems so involved could be discussed in terms of the modal logic S4 and in terms of linear logic, both of which are aimed to overcome the limitations of first-order logic, due to the formal notion of term. Then, we think that logic should consider Freud's view of the problem, and, in general, that the particular clustering of the logical notions induced by the reading of the psychoanalytical notions, and conversely, can offer a new opportunity for both subjects.

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▸ NIKOLAY BAZHENOV, EKATERINA FOKINA, DINO ROSSEGGER, AND LUCA SAN MAURO, *Computable bi-embeddable categoricity*.
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The study of morphisms between computable structures is a main theme in computable structure theory since Fröhlich and Shepherdson, and Mal'cev exhibited computable structures that are not computably isomorphic.

We study the complexity of embeddings between bi-embeddable computable structures. To this end, we define and investigate the notions (*relative*) *b.e. categoricity* and *degree of b.e. categoricity*. These notions are analogues to the well studied notions for isomorphisms between computable structures. As general results we obtain that every Turing degree $\mathbf{d}$ that is d-c.e. above $\mathbf{0}^{(\alpha)}$ for α a computable successor ordinal is the degree of b.e. categoricity of a structure and that $\mathbf{0}'$ b.e. categoricity is Π^1_1 complete.

We furthermore prove that every computable equivalence structure has degree of categoricity $\mathbf{0}$, $\mathbf{0}'$, or $\mathbf{0}''$ and characterize the computable b.e. categorical structures in several natural classes of structures.

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▸ NIKOLAY BAZHENOV, MANAT MUSTAFA, AND MARS YAMALEEV, *Elementary theories and hereditary undecidability for semilattices of numberings*.
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The main motive in the study of degree structures of all kinds has been the question of the decidability or undecidability of their first-order theories. This is a fundamental question that is an important goal in the analysing of these structures. A decision procedure implies and requires a full understanding and control of the first-order properties of a structure [4]. In this talk, we will show undecidability for theories of upper semilattices that arises from the theory of numberings [1]. We use the following approach: given a level of complexity, say Σ^0_α, we consider the upper semilattice $R_{\Sigma^0_\alpha}$ of all Σ^0_α-computable numberings of all Σ^0_α-computable families of subsets of $\mathbb{N}$. We prove that the theory of the semilattice of all computable numberings is m-equivalent to the first-order arithmetic. We show that the theory of the semilattice of all numberings is m-equivalent to the second-order arithmetic. We also obtain a lower bound for the m-degree of the theory of the semilattice of all Σ^0_α-computable numberings, where $\alpha \geq 2$ is a computable successor ordinal. Furthermore, it is shown that for any of the theories T mentioned above, the Π_5-fragment of T is hereditarily undecidable. Similar results are obtained for the commutative monoid of all computably enumerable equivalence relations on $\mathbb{N}$, under composition.

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► DAVID BÉLANGER, CHITAT CHONG, WEI WANG, TIN LOK WONG, AND YUE YANG, *Some variants of weak pigeonholes principle and WWKL*.

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Recently Avigad et al. [1] introduced a variant of WWKL, $2-\mathsf{WWKL}$, which says that if a Δ^0_2 binary tree T and a positive ration satisfy $\forall n(|T \cap 2^n| > \delta 2^n)$ then T has an infinite path. They proved that $2-\mathsf{WWKL}$ implies $B\Sigma^0_2$ over RCA_0. Combining this with a theorem of Conidis and Slaman [2], we know that the first-order theory of $2-\mathsf{WWKL}$ is axiomatized by $B\Sigma^0_2$. Obviously, $2-\mathsf{WWKL}$ implies $2-\mathsf{RAN}$, the existence of a 2-random. Slaman proved that $\mathsf{RCA}_0 + 2 - \mathsf{RAN} \nvdash B\Sigma_2$. But the first-order theory of $\mathsf{RCA}_0 + 2 - \mathsf{RAN}$ remains unknown.

Here, we introduce an interpolation between $2-\mathsf{WWKL}$ and $2-\mathsf{RAN}$, denoted by $2-\mathsf{WWKL}(\geq 2^{-1})$, which says that if a Δ^0_2 binary tree T satisfies $\forall n(|T \cap 2^n| \geq 2^{n-1})$ then T has an infinite path. It turns out that the first-order theory of $\mathsf{RCA}_0 + 2 - \mathsf{WWKL}(\geq 2^{-1})$ has interesting connections with $\Sigma^0_2 - \mathsf{WPHP}$ (there exists no Σ^0_2 injection from any $2a$ to a) where WPHP stands for Weak Pigeonholes Principle, and the cardinal scheme (there exists no first-order definable injection from $\mathbb{N}$ to any a). For example, the first-order theory of

$2-\mathsf{WWKL}(\geq 2^{-1})$ is axiomatized by $\Sigma^0_2-\mathsf{WPHP}$ which lies strictly between $B\Sigma^0_2$ and the cardinal scheme for Σ^0_2 injections.

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▶ S. BONZIO, T. MORASCHINI, AND M. PRA BALDI, *Logics of variable inclusion and Płonka sums of matrices*.

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It is always possible to associate with an arbitrary propositional logic $\vdash$, two new substitution-invariant consequence relations $\vdash^l$ and $\vdash^r$, which satisfies, respectively, a *left* and a *right variable inclusion principle*, as follows:

$$\Gamma \vdash^l \varphi \iff \text{there is } \Delta \subseteq \Gamma \text{ s.t. } \mathsf{Var}(\Delta) \subseteq \mathsf{Var}(\varphi) \text{ and } \Delta \vdash \varphi,$$

and

$$\Gamma \vdash^r \varphi \iff \begin{cases} \Gamma \vdash \varphi \text{ and } \mathsf{Var}(\varphi) \subseteq \mathsf{Var}(\Gamma) & \text{or} \\ \Sigma \subseteq \Gamma, \end{cases}$$

where Σ is a set of inconsistency terms for $\vdash$. Accordingly, we say that the logics $\vdash^l$ and $\vdash^r$ are, respectively, the *left* and the *right variable inclusion companions* of $\vdash$.

Prototypical examples of variable inclusion companions are found in the realm of three-valued logics. For instance, the left and the right variable inclusion companions of classical (propositional) logic are, respectively, *paraconsistent weak Kleene logic* (PWK for short) [4], and *Bochvar logic* [1].

Recent work [2] linked PWK to the algebraic theory of regular varieties, i.e., equational classes axiomatized by equations $\varphi \approx \psi$ such that $\mathsf{Var}(\varphi) = \mathsf{Var}(\psi)$. The representation theory iof regular varieties is largely due to the pioneering work of Płonka [5], and is tightly related to a special class-operator $\mathcal{P}\text{ł}(\cdot)$ nowadays called *Płonka sums*. This observations led us to investigate the relations between left and right variable inclusion companions and Płonka sums in full generality. Our study is carried on in the conceptual framework of abstract algebraic logic [3, 6].

At first, we define an appropriate notion of *direct system* of logical matrices, and we lift the construction of Płonka sums from algebras to logical matrices. This new technique allows to provide a completeness theorem for arbitrary logics of variable inclusion $(\vdash^l, \vdash^r)$ by performing Płonka sums over direct systems of models of $\vdash$. Then, we present a general method to transform every Hilbert-style calculus for a finitary logic $\vdash$ with a *partition function* into complete Hilbert-style calculi for $\vdash^l$ and $\vdash^r$ respectively. Moreover, we describe the structure of the matrix semantics $\mathsf{Mod}^{\mathsf{Su}}(\vdash^l)$, $\mathsf{Mod}^{\mathsf{Su}}(\vdash^r)$, given by the so-called Suszko reduced models of $\vdash^l, \vdash^r$. We close our investigation by determining the location of logics of variable inclusion in the Leibniz hierarchy.

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▶ OLIVIER BOURNEZ AND SABRINA OUAZZANI, *Computability and cheap nonstandard analysis*.
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Nonstandard Analysis (NSA) is an area of mathematics dealing with notions of infinitesimal and infinitely large numbers, in which many statements from classical analysis can be expressed very naturally. Cheap NSA introduced by Terence Tao in 2012 [2] is based on the idea that considering that a property holds eventually is sufficient to give the essence of many of its statements. This provides constructibility but at some (acceptable) price.

We consider computability in cheap NSA. We prove that many concepts from computable analysis as well as several concepts from computability can be very elegantly and alternatively presented in this framework. For example, a standard real x is proved to be computable in the classical sense (of computable analysis) iff there exists some cheap nonstandard computable rational $\frac{p}{q}$, such that $\left|x - \frac{p}{q}\right| \leq \epsilon$ for some effective infinitesimal ϵ. As another example, a function $f\colon [0, 1] \to \mathbb{R}$ is computable in the classical sense (of computable analysis) iff it satisfies some discretization property (there exists some computable δ such that if $|x - y| \leq \delta$ then $|f(x) - f(y)| \leq \epsilon$.) and it has some uniform approximation function over the rationals.

We illustrate on various statements from analysis and computable analysis that cheap NSA provides a dual view and elegant dual proofs to several statements already known in these fields. Further applications of the framework to ordinal time computations are hence made possible.

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▶ FILIPPO CALDERONI, *The bi-embeddability relation for countable abelian groups*.
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We analyze the Borel complexity of the bi-embeddability relation for different classes of countable abelian groups. Most notably, we use the Ulm theory to prove that bi-embeddability is incomparable with isomorphism in the case of torsion groups, and p-groups for any fixed prime number p. As I will explain, our result contrasts the arguable thesis that bi-embeddability for countable abelian p-groups has strictly simpler complete invariants than isomorphism.

This is joint work with Simon Thomas.

▶ DOMENICO CANTONE AND ALBERTO POLICRITI, *Encoding sets as real numbers*.
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In 1937, W. Ackermann proposed the following encoding of hereditarily finite sets by natural numbers:

$$\mathbb{N}_A(x) := \sum_{y \in x} 2^{\mathbb{N}_A(y)}. \tag{1}$$

The encoding $\mathbb{N}_A$ is simple, elegant, and highly expressive for a number of reasons. On the one hand, it builds a strong bridge between two foundational mathematical structures: (hereditarily finite) sets and (natural) numbers. On the other hand, it enables the representation of the *characteristic function* of hereditarily finite sets in terms of the usual notation for natural numbers by sequences of binary digits. That is: y belongs to x if and only if the $\mathbb{N}_A(y)$-th digit in the binary expansion of $\mathbb{N}_A(x)$ is equal to 1. As one would expect, the string of 0s and 1s representing $\mathbb{N}_A(x)$ is nothing but (a representation of) the characteristic function of x.

We study a very simple variation of the encoding $\mathbb{N}_A$, applicable to a larger collection of sets. The proposed variation is obtained by simply putting a minus sign before each exponent in (1), resulting in the expression:

$$\mathbb{R}_A(x) := \sum_{y \in x} 2^{-\mathbb{R}_A(y)}. \tag{2}$$

The range of $\mathbb{R}_A$ is now a collection of *real* numbers and its domain can be enlarged to include *non–well-founded* hereditarily finite sets, that is, sets defined by (finite) systems of equations of the following form

$$\begin{cases} x_1 & = & \{x_{1,1}, \dots, x_{1,m_1}\} \\ x_2 & = & \{x_{2,1}, \dots, x_{2,m_2}\} \\ \vdots & & \vdots \\ x_n & = & \{x_{n,1}, \dots, x_{n,m_n}\}, \end{cases} \tag{3}$$

with *bisimilarity* as equality criterion (see [2] and [3], where the term *hyperset* is also used). For instance, the special case of the single equation $x_1 = \{x_1\}$, resulting in the equation $x = 2^{-x}$, provides the code of the unique (under bisimilarity) hyperset $\Omega = \{\Omega\}$.[1]

As a matter of fact, $\mathbb{N}_A$ is defined inductively and this is perfectly in line with our intuition on the very basic properties of the collection of natural numbers $\mathbb{N}$ and the collection of hereditarily finite sets HF—called HF^0 in [3]. The definition of $\mathbb{R}_A$, instead, is not inductive and it requires a more careful analysis, as it must be *proved* to injectively associate (real) numbers to sets.

The injectivity of $\mathbb{R}_A$ on the collection of non–well-founded sets—henceforth, to be referred to as $\mathsf{HF}^{1/2}$, see [3]—was conjectured in [6] and is still an open problem. Here, we prove that, given any finite collection $\hbar_1, \dots, \hbar_n$ of elements of $\mathsf{HF}^{1/2}$ satisfying a minimal system of set-theoretic equations of the form (3) in the variables $x_1, \dots, x_n$, we can univocally determine real numbers $\mathbb{R}_A(\hbar_1), \dots, \mathbb{R}_A(\hbar_n)$ satisfying the following system of equations:

$$\begin{cases} \mathbb{R}_A(\hbar_1) = \sum_{k=1}^{m_1} 2^{-\mathbb{R}_A(\hbar_{1,k})} \\ \mathbb{R}_A(\hbar_2) = \sum_{k=1}^{m_2} 2^{-\mathbb{R}_A(\hbar_{2,k})} \\ \quad \vdots \qquad\qquad \vdots \\ \mathbb{R}_A(\hbar_n) = \sum_{k=1}^{m_n} 2^{-\mathbb{R}_A(\hbar_{n,k})}. \end{cases}$$

This preliminary result shows that the definition of $\mathbb{R}_A$ is well given, as it associates a unique (real) number to evey non–well-founded hereditarily finite set. This extends to $\mathsf{HF}^{1/2}$ the first of the properties that the encoding $\mathbb{N}_A$ enjoys with respect to HF. Should $\mathbb{R}_A$ also enjoy the injectivity property, our proposed adaptation of $\mathbb{N}_A$ would be completely satisfiying, and $\mathbb{R}_A$ could be coherently dubbed an *encoding* for non–well-founded sets.

In the course of our proof, we shall also introduce a procedure that drives us to the real numbers $\mathbb{R}_A(\hbar_1), \dots, \mathbb{R}_A(\hbar_n)$ mentioned above by way of successive approximations. In the well-founded case, our procedure will converge in a finite number of steps, whereas in the non–well-founded case, infinitely many steps will be required for convergence.

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[1]Notice that the solution to the equation $x = e^{-x}$ is the so-called *omega* constant, introduced by Lambert in [5] and studied also by Euler in [4].

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▸ DAVIDE CATTA, ALDA MARI, MICHEL PARIGOT, AND CHRISTIAN RETORÉ, *Natural language variants of universal quantification in first-order modal logic*.
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Natural languages offer a variety of universal quantifiers, both intra and cross-linguistically. French has three wordings of universal quantification: *chaque* (singular; $\sim$ *each*), *tout* (singular; $\not\sim$ *every*)—and a third one, *tous les* (plural; $\sim$ *all*), left out from this study because it involves second–order objects.

Based on linguistic data, and granted parametrization of the lexical functions to both sets of individuals and worlds, we show that *chaque* and *tout* differ in the modal properties of their domains of quantification: with *chaque* the world is fixed (as the actual one), with *tout* the set of worlds varies with the individuals. We treat exceptions as pertaining to individuals for *chaque* and to worlds for *tout*, by forcing conditions on accessibility paths.

We will finally discuss the semantics of *tout* in comparison to the silent quantifier GEN that linguists have described as a generic unselective quantifier binding both individuals and worlds variables, with worlds restricted to "normal" ones. [1]

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▸ ANAHIT CHUBARYAN AND ARTUR KHAMISYAN, *On some universal proof system for all versions of many-valued logics*.
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The current research refers to the problem of constructing some universal Gentzen-like proof system for all versions of propositional many-valued logic (MVL) such that for every variant of MVL any propositional proof system can be presented in described form. Some generalization of Kalmar's proof of deducibility for two-valued tautologies in the classical propositional logic gives us a possibility to suggest some simple method for proving the completeness for described systems.

Let E_k be the set $\left\{0, \frac{1}{k-1}, \ldots, \frac{k-2}{k-1}, 1\right\}$. We use the well-known notions of propositional many-valued formula, which defined as usual from propositional variables with values from E_k, (may be also propositional constants), parentheses (,), and logical connectives &, $\vee$, $\supset$, $\neg$, every of which can be defined by different well-known modes. For propositional variable

p and $\boldsymbol{\delta} = \frac{i}{k-1}$ $(0 \leqslant i \leqslant k-1)$ we define additionally "exponent" functions:

$\boldsymbol{p^\delta}$ as $(p \supset \delta)\&(\delta \supset p)$ with Lukasiewicz's implication (1) exponent,
$\boldsymbol{p^\delta}$ as p with $(k-1)^i$ cyclically permuting negation (2) exponent,

and introduce the additional notion of formula: for every formulas A and B the expression $\mathbf{A^B}$ (for both modes) is formula also. For every propositional variable p in k-valued logic $p^0, p^{1/(k-1)}, \ldots, p^{(k-2)/(k-1)}$ and p^1 in sense of both exponent modes are the *literals*. In every MVL either only **1** or every of values $\frac{1}{2} \leqslant \frac{i}{\mathbf{k-1}} \leqslant 1$ can be fixed as *designated values*. A formula φ with variables $p_1, p_2, \ldots p_n$ is called ***k**-tautology* if for every $\tilde{\delta} = (\delta_1, \delta_2, \ldots, \delta_n) \in E_k^n$ assigning δ_j $(1 \leqslant j \leqslant n)$ to each p_j gives the value 1 (or some value $\frac{1}{2} \leqslant \frac{i}{k-1} \leqslant 1$) of φ.

Universal system (US) *for all versions of MVL is defined as follow.* For sequent we use the denotation $\boldsymbol{\Gamma} \vdash \Delta$, where $\boldsymbol{\Gamma}$ and Δ are finite (may be empty) sequences (or sets) of propositional formulas. For every literal $\boldsymbol{C}$ and for any set of literals $\boldsymbol{\Gamma}$ the axiom sxeme of propositional system *US is* $\boldsymbol{\Gamma, C \rightarrow C}$. For every formulas $\boldsymbol{A}, \boldsymbol{B}$, for any set of literals $\boldsymbol{\Gamma}$, for each $\boldsymbol{\sigma_1}, \boldsymbol{\sigma_2}, \boldsymbol{\sigma}$ from the set E_k and for $* \in \{\&, \vee, \supset\}$ the logical rules of US are:

$$\vdash * \frac{\Gamma \vdash A^{\sigma_1} \text{ and } \Gamma \vdash B^{\sigma_2}}{\Gamma \vdash (A * B)^{\varphi_*(A,B,\sigma_1,\sigma_2)}}, \quad \vdash \exp \frac{\Gamma \vdash A^{\sigma_1} \text{ and } \Gamma \vdash B^{\sigma_2}}{\Gamma \vdash (A^B)^{\varphi_{\exp}(A,B,\sigma_1,\sigma_2)}}, \quad \vdash \neg \frac{\Gamma \vdash A^{\sigma}}{\Gamma \vdash (\neg A)^{\varphi_\neg(A,\sigma)}},$$

$$\text{literals elimination } \vdash \frac{\Gamma, p^0 \vdash A, \Gamma, p^{\frac{1}{k-1}} \vdash A, \ldots, \Gamma, p^{\frac{k-2}{k-1}} \vdash A, \Gamma, p^1 \vdash A}{\Gamma \vdash A},$$

where many-valued functions $\varphi_*(A, B, \sigma_1, \sigma_2)$, $\varphi_{\exp}(A, B, \sigma_1, \sigma_2)$ and $\varphi_\neg(A, \sigma)$, must be defined individually for each version of MVL such, that

1. formulas $A^{\sigma_1} \supset (B^{\sigma_2} \supset (A * B)^{\varphi_*(A,B,\sigma_1,\sigma_2)})$, $A^{\sigma_1} \supset (B^{\sigma_2} \supset (A^B)^{\varphi_{\exp}(A,B,\sigma_1,\sigma_2)})$ and $A^\sigma \supset (\neg A)^{\varphi_\neg(A,\sigma)}$ must be k-tautology in this version,

2. if for some $\sigma_1, \sigma_2, \sigma$ the value of $\sigma_1 * \sigma_2$ $(\sigma_1^{\sigma_2}, \neg\sigma)$ is one of *designed values* in this version of MVL, then $(\sigma_1 * \sigma_2)^{\varphi_*(\sigma_1,\sigma_2,\sigma_1,\sigma_2)}) = \sigma_1 * \sigma_2((\sigma_1^{\sigma_2})^{\varphi_{\exp}(\sigma_1,\sigma_2,\sigma_1,\sigma_2)} = \sigma_1^{\sigma_2}, (\neg\sigma)^{\varphi_\neg(\sigma,\sigma)} = \neg\sigma)$.

THEOREM. *Any sequent* $\vdash \boldsymbol{A}$ *is derived in* ***US*** *iff formula* $\boldsymbol{A}$ *is* k*-tautology.*

► ANAHIT CHUBARYAN AND GARIK PETROSYAN, *On the relations between the proof complexity measures of strongly equal k-tautologies in some proof systems.*
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In the mean time many interesting applications of many-valued logic (MVL) were found in such fields as Logic, Mathematics, Formal Verification, Artificial Intelligence, Operations Research, Computational Biology, Cryptography, Data Mining, Machine Learning, and Hardware Design; therefore, the investigations of proof complexity for different systems of MVL are very important.

The traditional assumption that all tautologies as Boolean functions are equal to each other is not fine-grained enough to support a sharp distinction among tautologies. The authors of [1] have provided a different picture of equality for classical tautologies. They have introduced the notion of strong equality of 2-valued tautologies on the basis of determinative conjunct notion. The idea to revise the notion of equivalence between tautologies in such way that is takes into account an appropriate measure of their "complexity." It was proved in [2, 3] that in "weak" proof systems the strongly equal 2-valued tautologies have the same proof complexities, while in the "strong" proof systems the measures of proof complexities for strongly equal tautologies can essentially differ from each other. Here, we generalize the notions of determinative conjunct and strongly equal tautologies for MVL and compare the proof complexity measures of *strongly equal many-valued tautologies* in some proof systems of MVL. We prove the following statement.

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THEOREM. *There are the sequences of strongly equal k-valued $(k \geqslant 3)$ tautologies A_n and B_n such, that (a) the lengths and sizes of A_n and B_n proofs in some "weak" systems of MVL are the same, (b) if $\mathbf{\Phi}$ is one of "strong" systems of MVL, then*

$$\boldsymbol{t}^{\Psi}_{A_n} = \boldsymbol{O}(\mathbf{1}), \quad \boldsymbol{l}^{\Psi}_{A_n} = \boldsymbol{O}(\boldsymbol{n}),$$
$$\boldsymbol{t}^{\Psi}_{B_n} = \mathbf{\Omega}(\boldsymbol{n}), \quad \boldsymbol{l}^{\Psi}_{B_n} = \mathbf{\Omega}(\boldsymbol{n}^2),$$

where $\boldsymbol{t}^{\Psi}_{A_n}$ $(\boldsymbol{t}^{\Psi}_{B_n})$ and $\boldsymbol{l}^{\Psi}_{A_n}$ $(\boldsymbol{l}^{\Psi}_{B_n})$ are the minimal measures of $\mathbf{\Phi}$-proofs lengths and sizes for formulas A_n (B_n).

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▸ ALBERTO CIAFFAGLIONE, PIETRO DI GIANANTONIO, FURIO HONSELL, MARINA LENISA, AND IVAN SCAGNETTO, *Reversible computation and principal types in $\lambda^!$-calculus*.
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In [1], S. Abramsky discusses *reversible computation* in a game-theoretic setting using partial *involutions*, *i.e.*, functions such that $f(u) = v \Leftrightarrow f(v) = u$. The construction is a special case of a general categorical paradigm [2, 3], which amounts to defining a *combinatory algebra* starting from a *Geometry of Interaction* (GoI) *Situation* in a traced symmetric monoidal category. Involutions amount to history-free strategies and apply according to *GoI symmetric feedback*/Girard's *Execution Formula*.

We highlight a *duality* between the GoI interpretation of a λ-term as an involution and its *principal type* w.r.t. an *intersection types discipline* for a refinement of λ-calculus inspired by Linear Logic, the $\lambda^!$-calculus.

The grammar of types is: $\mu ::= \alpha \mid \mu \to \mu \mid !_v\mu \mid \mu \wedge \mu$.

The grammar of $\lambda^!$-terms is: $M ::= x \mid MN \mid \lambda x.M \mid \lambda !x.M \mid !M$, where λ-abstractions can be taken only if x occurs at most once and is not in the scope of a!. Reduction rules are extended with a *!-pattern* β-reduction.

We define inductively the judgements: "$\Vdash M : \sigma$", "the term M has principal type scheme σ", and "$\mathcal{T}(\alpha, \sigma) = u \leftrightarrow v$", "the type-variable α in the principal type σ generates the component $u \leftrightarrow v$ of an involution." We have

THEOREM. *Given $M, N \in \Lambda^!$ such that*

$\Vdash M : \sigma_1 \to \sigma_2$, $\Vdash N : \tau$,

$\cdot\ f_N = \{u \leftrightarrow v \mid \exists \alpha \in \tau.\ \mathcal{T}(\alpha, \tau) = u \leftrightarrow v\}$

$\cdot\ f_M \bullet_{GoI} f_N = \{u \leftrightarrow v \mid S = MGU(\sigma_1, \tau) \wedge \exists \alpha \in S(\sigma_2).\ (\mathcal{T}(\alpha, S(\sigma_2)) = u \leftrightarrow v)\}$,

where f_N denotes the interpretation of N in GoI, $\bullet_{GoI}$ denotes application in GoI, and MGU denotes the "most general unifier."

The above theorem unveils three conceptually independent, but ultimately equivalent, accounts of *application* in the λ-calculus: *β-reduction*, GoI application of involutions, and *unification* of principal types. Furthermore, we prove that involutions are denotations of combinators iff they generate the principal type of a λ-term, thus answering an open question raised in [1].

The present work extends [4], where the purely affine fragment of the GoI combinatory algebra of involutions and purely affine λ-calculus have been investigated.

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► CEZARY CIEŚLIŃSKI, *Axiomatic theories of truth based on weak and strong Kleene logic*.
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One of the main research topics in the area of axiomatic theories of truth has been that of assessing their strength. A subtle measure of strength has been proposed by Fujimoto in [2]. Namely, denoting by L_T the result of adding a new predicate "$T(x)$" to the language of arithmetic, we say that the truth theory Th_1 is relatively truth-definable in Th_2 iff there is a formula $\theta(x) \in L_T$ such that for every $\psi \in L_T$, if $Th_1 \vdash \psi$, then $Th_2 \vdash \psi(\theta(x)/T(x))$.

If Th_2 defines the truth predicate of Th_1, then Th_2 is not *conceptually* weaker than Th_1, as Th_2 contains the resources permitting to reproduce the concept of truth of Th_1.

We will compare the conceptual strength of two axiomatic theories of truth: *KF* and *WKF*. The first one has been designed to capture Kripke's fixed-point construction based on Strong Kleene logic. The second one is based on the Weak Kleene evaluation schema.

In [2] Fujimoto proved that *WKF* is relatively truth-definable in *KF*. However, it has been an open question whether *KF* is relatively truth-definable in *WKF*.

We will provide the negative answer to this question, one that does not depend on the choice of language and coding. We consider this remarkable, because various important properties of Weak Kleene fixed-point construction are not absolute in this sense (see [1]).

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► NUNZIA COSMO, *Notes on the semantic refractions of the intuitionistic logic*.
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My research proposal concerns the evaluation of a possible semantic for the dual-intuitionistic logic. In particular, the purpose of this research is to validate the universalisation of Kripke's semantics for the modal and intuitionistic logic and also for the dual-intuitionistic logic; the possibility of the exstention of the kripkeian's logic is investigated through the examination of semantic of the Da Costa's paraconsistent logic [2, 3], which in Priest is a part of Rauszer's Brouwer–Heyting logic [11]. This research project is conducted through an analysis of the history of semantic for the logical truths and laws of intuitionistic logic, from Brouwer theory [1] and from Heyting formalism [4]—with their theoretical developments (G. Kreisel, J. Myhill, and A. S. Troelstra)—to Kripke's semantic [6] and to Rauszer and Hiroshis' theories [5, 12, 13].

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► STAMATIS DIMOPOULOS, *Strong compactness and the continuum function*.
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Strong compactness may be one of the established large cardinal notions in set theory, but it is not robust when it comes to forcing. In particular, it is still open whether it is possible to control the continuum function and at the same time preserve strong compactness, without relying on stronger properties such as supercompactness. In an ongoing work with A. Apter, we look at special cases where the preservation of strong compactness is possible, while having some control over the continuum function.

Initially, we show that assuming only a partial degree of supercompactness, it is possible to violate GCH at a nonsupercompact strongly compact cardinal, while preserving the full extent of its strong compactness. Also, we show that certain Easton functions can be realised while preserving the strong compactness of the least measurable limit of supercompact cardinals. Finally, we show how to force a violation of GCH at all strongly compact cardinals, in models where strong compactness coincides with supercompactness.

► DAVID FERNÁNDEZ DUQUE AND CHRISTIAN RETORÉ, *Logic and topology: Some connections*.
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One of the oldest connections between logic and topology is McKinsey and Tarski's seminal result stating that S4 is sound and complete for its topological interpretation over the real line and other topological spaces, a result that has since been extended and sharpened and continues to be studied to this day.

This basic construction naturally yields semantics for intuitionistic logic via the Gödel–Tarski translation, which in turn gives rise to models of first-order intuitionistic logics. The (pre)sheaves over a topological space (or a pretopology) yield models for which the intuitioinistic predicate calculus is complete. They can be seen as Kripke models with additional properties and enjoy a structure that is more familiar in common mathematics.

These connections between logic and topology have led to a wealth of applications that have become increasingly relevant with developments in computing. Topological semantics for modal logic make them an efficient tool for spatial reasoning, and topological semantics of intuitionistic logic have led to models of computation including denotational semantics of lambda calculi and more recently the development of Homotopy Type Theory.

In this talk, we will give brief overview of these connections between logic and topology. We will discuss both historical and technical aspects of the field.

► MIRKO ENGLER, *Pathological well-orderings and proof-theoretic ordinals*.
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We consider proof-theoretic ordinals for theories of first- and of second-order languages. The former may be defined as the supremum of order types of p.r. well-orderings for which a theory T proves the corresponding schema of transfinite induction. The latter (also called Π^1_1-ordinal) may be defined as the supremum of order types of p.r. well-orderings whose well-foundedness is provable in T. While it is known [2], that for any $\alpha < \omega_1^{CK}$ there are order-relations $\prec$ (not defining a well-ordering) of order type α s.t. $PA \vdash TI(\prec)$, the Π^1_1-ordinal is often claimed to provide a stable and convenient measure of proof-theoretic strength of theories. Moreover, it is claimed to be not dependent on any special concepts of natural well-orderings.

In this contribution, we extend a result from [1] and first show that $PA \vdash TI(\prec)$ for any $\alpha < \omega_1^{CK}$, where $\prec$ is of order type α and actually represents a well-ordering. Furthermore, we adapt the construction to prove a similar result for 2nd order and discuss the resulting problems for defining Π^1_1-ordinals.

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▸ MARTA FIORI CARONES, *The strength of a theorem about subgraphs with nice properties*.
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In 1979, Ivan Rival and Bill Sands [1] proved that each infinite graph G has an infinite subgraph H such that each vertex of G is adjacent to none or to one or to infinitely many vertices of H. This statement, showing the existence of a substructure with some property in every infinite graphs, resembles Ramsey's theorem, which assures that each infinite graph has a complete or a totally disconnected subgraph. Actually, the authors presented the statement as a variation of Ramsey's theorem because, while renouncing the complete information about H itself, it gives some information about the adjacency structure of H with respect to G. Despite the superficial similarity with Ramsey's theorem, this principle is not what in reverse mathematics is called a "Ramsey's type principle"; in fact only if the graph is locally finite each subgraph of a solution is still a solution.

We investigated this statement (restricted to countable graphs) from the viewpoint of reverse mathematics, establishing that it is equivalent to ACA_0. Hence, the coding power of this statement is at least $0'$, but we suspect that it is higher. Moreover, if there is a computable bound to the degree of the vertices of G, then the theorem is computably true.

We will also discuss the reverse mathematics of a related theorem of Rival and Sands proved in the same article: each countably infinite partial order P of finite width has an infinite chain C such that every element of P is comparable with either none or cofinitely many elements of C.

This is joint work with Paul Shafer and Giovanni Soldà.

[1] I. Rival and B. Sands, *On the adjacency of vertices to the vertices of an infinite subgraph*. ***Journal of the London Mathematical Society***, vol. 2 (1980), no. 3, pp. 393–400.

▸ EKATERINA FOKINA, TIMO KÖTZING, AND LUCA SAN MAURO, *Learning equivalence structures*.
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Algorithmic learning theory is a vast research program, initiated by Gold [1] in the 1960s, that comprises different models of learning in the limit. It deals with the question of how a

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learner, provided with more and more data about some environment, is eventually able to achieve systematic knowledge about it.

In this article, we investigate computable structures through the lens of algorithmic learning theory (see, e.g., [2] for another study in this direction). We introduce the following framework. Let $\mathbb{K}$ be a class of structures. Suppose we have an effective numbering $\mathcal{A}_0, \mathcal{A}_1, \ldots, \mathcal{A}_i, \ldots$ of computable structures from $\mathbb{K}$. Let M be a partial computable function which takes for its inputs finite substructures of a structure $\mathcal{A}$ from the class $\mathbb{K}$ (where $\mathcal{A}^0 \subseteq \mathcal{A}^1 \subseteq \cdots \subseteq \mathcal{A}^i \subseteq \cdots$ and $\mathcal{A} = \bigcup_i \mathcal{A}^i$) and which either goes undefined or returns a number of program. For a finite initial substructure $\mathcal{A}^i$, if $M(\mathcal{A}^i)\downarrow = n$, then n represents M's conjecture or hypothesis as to an index for $\mathcal{A}$ in the abovementioned numbering. The learner M $\mathbf{EX}_{\cong}$*-learns* $\mathcal{A}$ if there exists a number n such that $\mathcal{A} \cong \mathcal{A}_n$ and $M(\mathcal{A}^i)\downarrow = n$, for all but finitely many i. A family of structures $\mathfrak{A}$ is $\mathbf{EX}_{\cong}$*-learnable* if there is M that learns all $\mathcal{A} \in \mathfrak{A}$.

We focus on equivalence structures and study which families of equivalence structures with domain ω are $\mathbf{EX}_{\cong}$*-learnable*. We also unveil a natural hierarchy of different notions of learnability by replacing isomorphism with other relations expressing structural similarity, such as bi-embeddability ad bi-homomorphism.

[1] M. E. Gold, *Language identification in the limit*. ***Information and Control***, vol. 10 (1967), no. 5, pp. 447–474.

[2] V. Harizanov and F. Stephan, *On the learnability of vector spaces*. ***Journal of Computer and System Sciences***, vol. 73 (2007), no. 1, pp. 109–122.

► BALTHASAR GRABMAYR, *A step towards a coordinate free version of Gödel's Second Theorem*.
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[2] locates three sources of indeterminacy in the formalization of a consistency statement for a theory T: (I) the choice of a proof system, (II) the choice of a coding system and (III) the choice of a specific formula representing the axiom set of T. According to [2], "Feferman's solution [1] to deal with the indeterminacy is to employ a fixed choice for (I) and (II) and to make (III) part of the individuation of the theory" (p. 544). [2]'s own approach rests on fixed choices for (II) and (III) but is independent of (I).

The primary result of this talk is to eliminate the dependency on (II), by proving the invariance of Gödel's Second Theorem with regard to acceptable numberings. This involves two steps. Firstly, I discuss the notion of acceptability of a numbering and argue that the computability of a numbering is a necessary condition for its acceptability. A precise notion of computability then allows the formerly vague invariance claim to be restated as a (meta-)mathematical theorem, whose proof will be outlined in the second part of the talk.

[1] S. Feferman, *Arithmetization of metamathematics in a general setting*. ***Fundamenta Mathematicae***, vol. 49 (1960), pp. 35–92.

[2] A. Visser, *Can we make the second incompleteness theorem coordinate free*? ***Journal of Logic and Computation***, vol. 21 (2011), no. 4, pp. 543–560.

► HAROLD T. HODES, *Modal logic out of moodal logic*.
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Let *Fml* be the set of formulas generated from a set of propositional variables using the logical constants $\perp$, $\supset$, $\Box$ (and &, $\vee$, $\Diamond$ for the intuitionistic case). Form the set *MFml* of marked formulas by prefixing a formula with a moodal markers: **0** and **1** (and $\mathbf{1}^+$ for the intuitionistic case). Note: moodal markers are not operators—they cannot be iterated. Heuristic: **0** indicates acceptance as true at the actual world; **1** indicates acceptance as true at an arbitrary accessible world; $\mathbf{1}^+$ indicates acceptance as true at an arbitrary accessible$^+$ world. Classical and intuitionistic normal modal logics turn out to be **0**-level fragments of conceptually deeper moodal logics.

Using marked formulas, we can formalize many normal modal logics so as to characterize $\Box$ (and $\Diamond$ in the intuitionistic case) by introduction and elimination rules: the introduction rules for $\Box$ (and $\Diamond$) "freezes" a formula φ marked by $\mathbf{1}$ (or perhaps $\mathbf{1}^+$ in the intuitionistic case) into $\Box\varphi$ (and $\Diamond\varphi$) marked by $\mathbf{0}$; the corresponding elimination rules "unfreezes" $\mathbf{0}\Box\varphi$ (and $\mathbf{0}\Diamond\varphi$).

We will introduce the modal moodal logics Intuitionistic K [Classical K] by model-theoretically defining IK-consequence [CK-consequence] on marked formulas. Our model-theory for IK builds on Plotkin and Sterling's semantics; for CK it builds on Kripke's semantics.

This talk will focus on Natural Deduction formalizations for these logics. Necessitation and the standard axioms are not proof-theoretically "rock-bottom": necessitation is a derived rule; the familar K-axioms, and in the intuitionistic case the other axioms offered by Plotkin and Sterling, are all provable. Time permitting, I will discuss some other familiar normal modal logics.

Note added May 1 2018: I have figured out how to avoid using $\mathbf{1}^+$ in the intuitionistic case.

▸ RAMON JANSANA AND TOMMASO MORASCHINI, *Relational semantics, ordered algebras, and quantifiers for deductive systems*.

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Relational semantics has proved to be a fundamental tool in the mathematical and philosophical understanding of many nonclassical logics, including intuitionistic, modal, and substructural logics. Nevertheless, the evolution of the *general* theory of relational semantics is far behind that of algebraic semantics. In this talk [4], we present a first abstract approach to relational semantics, in the spirit of Abstract Algebraic Logic [2], which turns out to be connected with the theory of completions of ordered algebras [6, 1, 3].

Even though, for the sake of simplicity, we will confine our discussion to the *local* aspects [5] of relational semantics, our approach can be extended harmlessly to *arbitrary* propositional logics and, in particular, to global consequences of normal modal logics and to arbitrary substructural logics. More in detail, we will consider two basic questions which need to be addressed by any truly general theory of relational semantics:

- Can we make precise the idea that a logic has a local relational semantics?
- In case a logic has a local relational semantics, what are its distinguished relational models?

As a matter of fact, our approach encompasses that of canonical extensions of arbitrary lattices, but diverges from the known approaches when applied to non–lattice-based logics. This is due to the fact that our approach is logic-based, and produces completions and relational semantics which reflect the behaviour of the logic under consideration. This makes it especially fruitful in the study of purely intensional fragments (which are not lattice based). Interestingly enough, as a by-product of our approach, we are able to associate to every local consequence a class of distinguished *ordered algebras*, thus justifying on general grounds the empiric observation that most algebras of logics are intrinsically ordered.

If time allows, we will consider the following problems as well:

- Can we use the distinguished relational semantics of a propositional logic $\vdash$ to introduce a semantically defined first-order extension $\vdash$?
- Can we axiomatize this first-order extension relative to a given axiomatization of the propositional logic $\vdash$?

[1] M. J. Dunn, M. Gehrke, and A. Palmigiano, *Canonical extensions and relational completeness of some substructural logics*. ***The Journal of Symbolic Logic***, vol. 70 (2005), no. 3, pp. 713–740.

[2] J. M. Font, ***Abstract Algebraic Logic—An Introductory Textbook***, Studies in Logic: Mathematical Logic and Foundations, vol. 60, College Publications, London, 2016.

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[3] M. Gehrke, R. Jansana, and A. Palmigiano, *Δ_1-completions of a poset*. ***Order***, vol. 30 (2013), no. 1, pp. 39–64.

[4] R. Jansana and T. Moraschini, *Relational semantics, ordered algebras, and quantifiers for deductive systems*, manuscript, 2017.

[5] M. Kracht, *Modal consequence relations*, ***Handbook of Modal Logic***, vol. 3. Elsevier Science Inc., New York, 2006.

[6] H. M. MacNeille, *Partially ordered sets*. ***Transactions of the American Mathematical Society***, vol. 42 (1937), no. 3, pp. 416–460.

▸ SÁNDOR JENEI, *Group representation and Hahn-type embedding for a class of involutive residuated chains with an application in substructural fuzzy logic*.
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Hahn's celebrated embedding theorem asserts that linearly ordered Abelian groups embed in the lexicographic product of real groups. Conrad, Harvey, and Holland generalized Hahn's theorem for lattice-ordered Abelian groups [1]. We prove a representation theorem for a class of involutive residuated semigroups, namely, for group-like FL_e-chains which possess only finitely many idempotents.

An FL_e-algebra [2] is a structure $\mathbf{X} = (X, \wedge, \vee, \circledast, \to_\circledast, t, f)$ such that $(X, \wedge, \vee)$ is a lattice, $(X, \leq, \circledast, t)$ is a commutative, monoid, and f is an arbitrary constant. One defines $x' = x \to_\circledast f$ and calls $\mathbf{X}$ involutive if $(x')' = x$ holds. Call $\mathbf{X}$ group-like if it is involutive and $t = f$. Since for involutive FL_e-chains $t' = f$ holds, one extremal situation is the integral case (when t is the top element of the universe and hence f is its bottom one) and the other extremal situation is the group-like case (when the two constants coincide). Prominent classes of group-like FL_e-chains are totally ordered Abelian groups and odd Sugihara chains, the latter constitute an algebraic semantics of a logic at the intersection of relevance logic and fuzzy logic. These two classes are extremal in the sense that lattice-ordered Abelian groups have a single idempotent element, whereas all elements of any odd Sugihara algebra are idempotent.

The representation uses only linearly ordered Abelian groups and a newly introduced construction, called partial lexicographic product. As a corollary, we extend Hahn's theorem to this class of semigroups by showing that any such algebra embeds in some partial-lexicographic product of linearly ordered Abelian groups. As an application for this embedding, we show the finite strong standard completeness of an axiomatic extension of the Involutive Uninorm Logic **IUL** [4] by $\boldsymbol{t} \Leftrightarrow \boldsymbol{f}$.

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[1] P. F. Conrad, J. Harvey, and W. C. Holland, *The Hahn embedding theorem for lattice-ordered groups*. ***Transactions of the American Mathematical Society***, vol. 108 (1963), pp. 143–169.

[2] N. Galatos, P. Jipsen, T. Kowalski, and H. Ono, ***Residuated Lattices: An Algebraic Glimpse at Substructural Logics***, Studies in Logic and the Foundations of Mathematics, Elsevier, 2007.

[3] H. Hahn, *Über die nichtarchimedischen Grössensysteme*. ***Sitzungsberichte der Akademie der Wissenschaften, Wien, Mathematik-Naturwissenschaften Klasse***, vol. 116 (1907), no. IIa, pp. 601–655.

[4] G. Metcalfe and F. Montagna, *Substructural fuzzy logics*. ***The Journal of Symbolic Logic***, vol. 72 (2007), no. 3, pp. 834–864.

▸ ALEXANDER JONES, *An axiomatic theory of truth and paradox*.
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In this article, I present a new axiomatic theory of truth, state key theorems about this theory, and discuss its treatment of the semantic paradoxes.

The theory follows the spirit of Tarskian and contextual approaches to truth: truth is treated as a typed notion (sentences are not true absolutely, but true relative to a particular

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level of the language) and this allows a broadly classical treatment of truth. Where my theory differs to previous theories is that truth is treated as a binary, rather than unary, predicate, and this allows for quantification over the levels of the language. This results in a more expressive theory of truth that allows internal proofs of some natural statements that must ordinarily be proven outside of the theory. I state some key theorems about this theory, and why it might be viewed as a relatively attractive alternative to other typed theories of truth, before moving on to a discussion of its relation to the semantic paradoxes.

There are a family of Liar-like and truth-teller-like sentences which the theory has to deal with. Some of these are provable, some of their negations are provable, but all are provably untrue. The theory carries with it an internal definition of "pathological" sentences. This definition allows Liar-like sentences to be provably untrue, without falling into inconsistency. I provide a philosophical interpretation of this definition as sentences which are not "truth-apt" and consider some other examples of pathological sentences, in particular those which quantify absolutely generally over all levels of the truth predicate. I propose that this is an interesting new axiomatic theory of truth with intriguing formal features that add new depth to contextualist approaches to the semantic paradoxes.

▶ JERZY KRÓL, TORSTEN ASSELMEYER-MALUGA, KRZYSZTOF BIELAS, AND PAWEŁ KLIMASARA, *Set-theoretic forcing in low-dimensional differential topology and cosmology*.
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We propose a cosmological model as a mathematical tool for relating smoothness structures on $\mathbb{R}^4$ with the Cohen and random forcings. It refines the model where the initial quantum state defines the lattice of projections $\mathbb{L}(\mathcal{H})$ of a Hilbert space $\mathcal{H}$ [3]. The maximal Boolean subalgebras of $\mathbb{L}(\mathcal{H})$ are typically atomless measure algebras B supporting random forcing. Internal real numbers of the Boolean-valued models V^B parametrize the spacetime points, the smooth regions of the spacetime gain exotic smoothness of $\mathbb{R}^4$. Additionally, it predicts the value of the cosmological constant [1].

Cohen forcing distinguishes exotic smoothness structures of $\mathbb{R}^4$. One turns to Calkin algebra $\mathcal{C}(\mathcal{H}) = B(\mathcal{H})/\mathcal{K}$ and $\mathrm{Aut}(\mathbb{L}(\mathcal{H}))$, $\mathrm{Aut}(\mathcal{C}(\mathcal{H}))$. Classically $\mathcal{C}(\mathcal{H})$ is $P(\mathbb{N})/fin$ representing the hyperfinite extensions of Casson handles in nonstandard models of $\mathbb{N}$. The trivial automorphisms of $P(\mathbb{N})/fin$ define operations on handlebodies and diffeomorphisms of exotic R^4's [2]. Additionally, if an atlas of $\mathbb{R}^4$ contains transition functions from the Cohen extensions $\mathbf{L}[a]$ of Gödel universe $\mathbf{L}$ it defines exotic R^4.

[1] T. Asselmeyer-Maluga and J. Król, *How to obtain a cosmological constant from small exotic* $\mathbb{R}^4$. ***Physics of the Dark Universe***, vol. 19 (2018), pp. 66–77.

[2] J. Król, *Model and set-theoretic aspects of exotic smoothness structures on* $\mathbb{R}^4$, ***At the Frontier of Spacetime*** (T. Asselmeyer-Maluga, editor), Fundamental Theories of Physics, vol. 183, Springer, Switzerland, 2016, pp. 217–240.

[3] J. Król, T. Asselmeyer-Maluga, K. Bielas, and P. Klimasara, *From quantum to cosmological regime. The role of forcing and exotic 4-smoothness*. ***Universe***, vol. 3 (2017), p. 31.

▶ QUENTIN LAMBOTTE AND FRANÇOISE POINT, *On expansions of* $(\mathbf{Z}, +, 0)$.
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Call a (strictly increasing) sequence $R = (r_n)$ of natural numbers *regular* if it satisfies the following two conditions:

1. R is eventually periodic modulo m for all $m \geq 2$;
2. $\lim_{n\to\infty} r_{n+1}/r_n = \theta \in \mathbf{R}^{>1} \cup \{\infty\}$ and, if θ is algebraic, then R satisfies a recurrence relation whose characteristic polynomial is the minimal polynomial of θ.

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THEOREM 1 ([2]). *Let R be a regular sequence. Then $\mathcal{Z}_R = (\mathbf{Z}, +, 0, R)$ is superstable of U-rank ω.*

The proof follows the strategy used by D. Palacin and R. Sklinos [3] to show that, when $q > 1$, $\mathcal{Z}_{(q^n)}$ and $\mathcal{Z}_{(n!)}$ are superstable, with U-rank ω (the first result was also shown, using different methods, by B. Poizat in [5]). Independently of our work, the results of [3] were generalized, using the same strategy, by Gabriel Conant in [1]. The results in [1] and Theorem 1 have a nontrivial overlap: the case $\theta = \infty$ is completely treated by Conant without the periodicity assumption while our result concerning recurrence relations is more general.

A function $\mathtt{A}\colon \mathbf{N} \to \mathbf{Z}$ of the form $\mathtt{A}(n) = a_0 r_n + a_1 r_{n+1} + \cdots + a_d r_{n+d}$, $a_0, \ldots, a_d \in \mathbf{Z}$, is called an *operator* on R. Let $\mathcal{L}$ be the language

$$\mathcal{L} = \{+, -, 0, 1, D_n | n > 1\} \cup \{R, S, S^{-1}\} \cup \{\Sigma_{\bar{\mathtt{A}}} | \bar{\mathtt{A}} \text{ is a tuple of operators}\},$$

where D_n is a predicate for $n\mathbf{Z}$, S is the successor function on R $(S(r_n) = r_{n+1})$, S^{-1} its inverse and $\Sigma_{\bar{\mathtt{A}}}$ is a predicate for the image of the function $\mathtt{A}_1 + \cdots + \mathtt{A}_k$.

THEOREM 2. *Let R be a regular sequence. Then $Th(\mathcal{Z}_R)$ has quantifier elimination in $\mathcal{L}$.*

Theorem 2 corresponds to known results in Presburger arithmetic (see [6] and [4]), and allows us to prove directly that $\mathcal{Z}_R$ is superstable.

[1] G. CONANT, *Stability and sparsity in sets of natural numbers*, preprint, arXiv:1701.01387.

[2] Q. LAMBOTTE and F. POINT, *On Expansions of* $(\mathbf{Z}, +, 0)$, preprint, arXiv:1702.04795.

[3] D. PALACIN and R. SKLINOS, *On superstable expansions of free abelian groups*. ***Notre Dame Journal of Formal Logic***, vol. 59 (2018), no. 2, pp. 157–169.

[4] F. POINT, *On decidable extensions of Presburger arithmetic*: *From A. Bertrand numeration systems to Pisot numbers*. ***The Journal of Symbolic Logic***, vol. 65 (2000), no. 3, pp. 1347–1374.

[5] B. POIZAT, *Supergénérix*. ***Journal of Algebra***, vol. 404 (2014), pp. 240–270.

[6] A. L. SEMENOV, *On certain extension of the arithmetic of addition of natural numbers*. ***Mathematics of the USSR-Izvestia***, vol. 22 (1992), no. 2, pp. 401–418.

▸ JUNGUK LEE, *Valued hyperfields, truncated discrete valuation rings, and valued fields*.
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In [2], M. Krasner introduced a notion of valued hyperfield analogous to a valued field with a multivalued addition operation, and used it to do a theory of limits of local fields. In [1], P. Deligne did the theory of limits of local fields in a different way by defining a notion of triple, which consists of truncated discrete valuation rings and some additional data. Typical examples of a valued hyperfield and a truncated discrete valuation ring are the n-th valued hyper field, which is quotient of a valued field by a multiplicative subgroup of the form $1 + \mathfrak{m}^n$, where $\mathfrak{m}$ is the maximal ideal of a valuation ring, and the n-th residue ring, which is a quotient of a valuation ring by the n-th power of the maximal ideal.

J. Tolliver in [4] showed that discrete valued hyperfields and triples are essentially same, stated by P. Deligne in [1] without a proof. In [3] W. Lee and the author showed that given complete discrete valued fields of mixed characteristic with perfect residue fields, any homomorphism between the n-th residue rings of the valued fields is lifted to a homomorphism between the valued fields for large enough n. This lifting process is functorial.

Motivated by above results in [4] and [3], we show that given complete discrete valued fields of mixed characteristic with perfect residue fields, any homomorphism between the n-th valued hyperfields of the valued fields can be lifted to a homomorphism between the valued fields for large enough n, which is functorial. We also compute an upper bound of such a minimal n effectively depending only on the ramification index. Most of all, any homomorphism between the first valued hyperfields of valued fields is uniquely lifted to a homomorphism between the valued fields in the case of tamely ramified valued fields. From this lifting result, we prove a relative completeness AKE-theorem via valued hyperfields for finitely ramified valued fields with perfect residue fields.

[1] P. Deligne, *Les corps locaux de caractéristique p, limits de corps locaux de caractéristique* 0, ***Representations des groupes reductifs sur un corps local*** (J.-N. Bernstein, P. Deligne, D. Kazhdan, and M.-F. Vigneras, editors), Travaux en cours, Hermann, Paris, 1984, pp. 119–157.

[2] M. Krasner, *Approximation des corps valués complets de caractéristique* $p \neq 0$ *par ceux caractéristique* 0, ***1957 Colloque d'algèbre supérieure, tenu á Bruxelles du 19 au 22 décembre***.

[3] J. Lee and W. Lee, *On the structure of certain valued fields*, preprint.

[4] J. Tolliver, *An equivalence between two approaches to limits of local fields*. ***Journal of Number Theory***, 2016, pp. 473–492.

▶ JUDIT X. MADARÁSZ, MIKE STANNETT, AND GERGELY SZÉKELY, *Frames and coordinate systems in the formalization of Einstein's special principle of relativity*.

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In the literature, there are several informal treatments and discussions of Einstein's special principle of relativity (SPR) and its consequences. The main problem with informal approaches in physics (and science in general) is that they often lead to confusions and misunderstandings.

Of course, formalization in itself does not save us from bumping into apparent contradictions. For example Rindler, in his book [3, p. 40] referring to Dixon, claims that "the principle of relativity is equivalent to the isotropy (of space) and the homogeneity (of space and time)." Contrary to this claim, the construction proving Theorem 2 in article [1] gives an anisotropic extension of the standard model of special relativity which still satisfies SPR These two claims appear to be a direct contradictions of each other. Of course, the contradiction is only apparent since something else is meant by "the principle of relativity" in [1] and [3]. Even the mathematical frameworks of the two formulations are different. So there are inequivalent formulations of SPR in the literature. Therefore, it seems so natural to ask: which formulation is the "true one"?

The "original" SPR is just an informal idea that goes back at least to Galileo's famous Dialogo. So it is not surprising that it has several different formalizations since an idea can clearly be formulated in several different ways based on the many choices one has to make when turning an idea into a formal statement. Therefore, the right question is not which formulation is the "true one," but how the different formulations are related to one another logically.

In [2], we have already compared three different formalizations of SPR within a mathematical logic based axiomatic framework developed by the Andréka–Németi group and investigated various auxiliary assumptions that make these three formalizations equivalent. Now, we are going to use the same framework and Rindler's distinctions between inertial frames and inertial coordinate systems to investigate the logical connection between the two versions of the principle from [1] and [3]. We will see that SPR in [3] is understood for coordinate systems and the construction in [1] satisfies SPR understood for reference frames.

Based on Galileo's ship argument, we will also argue that the original intuition behind SPR is in some sense better reflected if we formulate it for reference frames only and hence does not imply isotropy.

[1] H. Andréka, J. X. Madarász, I. Németi, M. Stannett, and G. Székely, *Faster than light motion does not imply time travel*. ***Classical and Quantum Gravity***, vol. 31 (2014), no. 9, paper 095005, 11 pp.

[2] M. Stannett, J. X. Madarász, and G. Székely, *Three different formalisations of Einstein's relativity principle*. ***The Review of Symbolic Logic***, vol. 10 (2017), no. 3, pp. 530–548.

[3] W. Rindler, ***Relativity: Special, General, and Cosmological***, Oxford University Press, Oxford, 2001.

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► JOSÉ M. MÉNDEZ, GEMMA ROBLES, AND FRANCISCO SALTO, *Basic quasi-Boolean extensions of relevant logics*.
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Let L be a negationless relevant logic. A Boolean negation (B-negation) can be introduced in L by adding to it the following axioms: (a1) $(A \wedge \sim A) \rightarrow B$, (a2) $B \rightarrow (A \vee \sim A)$. This way of introducing B-negation in relevant logics suggests the definition of two families of quasi-Boolean negation (QB-negation) extensions of relevant logics. One of them, intuitionistic in character, has a1 but not a2; the other one, dual intuitionistic in nature, has a2 but lacks a1. The aim of this article is to begin the investigation of both families of QB-negation extensions of relevant logics. B-negation extensions of relevant logics are of both logical and philosophical interest (cf. [1, pp. 376, ff]). It is to be expected that QB-negation extensions of the same logics (not considered in the literature, as far as we know) will have a similar logical interest.

Acknowledgment. Work supported by research project FFI2017-82878-P, financed by the Spanish Ministry of Economy, Industry, and Competitiveness.

[1] R. Routley, R. K. Meyer, V. Plumwood, and R. T. Brady, ***Relevance Logic and Their Rivals***, vol. 1, Ridgeview, Atascadero, CA, 1982.

► RUSSELL MILLER, *Hilbert's tenth problem as a pseudojump operator*.
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When considering subrings of $\mathbb{Q}$, it is natural to define the *HTP-operator* to be the pseudojump operator sending each subset W of the set $\mathbb{P}$ of prime numbers to the set

$$HTP(R_W) = \{f \in \mathbb{Z}[X_1, X_2, \dots] \colon f = 0 \text{ has a solution in } \mathbb{Z}[W^{-1}]\},$$

known as *Hilbert's Tenth Problem* for the subring $R_W = \mathbb{Z}[W^{-1}]$ of $\mathbb{Q}$. We show that every Σ^0_2 Turing degree above $\mathbf{0}'$ is the degree of $HTP(R_W)$ for some Π^0_1 set W. Moreover, this operator does not respect Turing equivalence. Indeed, using the technique of high permitting, we can give an example where the HTP operator reverses the (strict) Turing reductions: $V <_T W$, yet $HTP(R_W) <_T HTP(R_V)$. (The set V here is Π^0_1, while W is Σ^0_1.)

These results use joint work with Ken Kramer.

► RYSZARD MIREK, *Linear perspective in Renaissance*.
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Renaissance mathematicians and geometers like Piero della Francesca and Luca Pacioli refers directly or indirectly to Euclidean geometry. But what is new in the Renaissance concerns linear perspective. Piero della Francesca in Proposition 1.12 shows how to draw in perspective a surface of undefined shape, which is located in profile as a straight line. According to Proposition 1.13, which is known as the first new European theorem in geometry after Fibonacci, one can add a square that represents the object to be drawn in reality in a horizontal plane. Then, we draw from the position of the eye of a hypothetical observer visual rays to the corners of the square. At the same time, this proposition can be used to interpret Renaissance paintings of Piero della Francesca's, applying the strict rules of geometry and perspective. The proposition is directly applicable in the masterpieces painted by Piero della Francesca, namely, in his The *Flagellation of Christ* and *The Baptism of Christ*. In turn, in Proposition I.8, he shows for the first time that the perspective images of orthogonals converge to a *centric point*. In *De Pictura* Alberti merely assumed the truth of this result.

My goal here is to provide a detailed analysis of the methods of inference that are employed in the Renaissance treatises with particular emphasis on a method of natural deduction that takes into account the importance of diagrams within formal proofs.

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Hilbert's ε-calculus is based on an extension of the language of predicate logic by a term-forming operator ε [1]. Two fundamental results about the ε-calculus, the first and second epsilon theorem, play a role similar to that which the cut-elimination theorem plays in sequent calculus. In particular, Herbrand's Theorem is a consequence of the epsilon theorems. Moser and Zach study the epsilon theorems and the complexity of the elimination procedure underlying their proof, as well as the length of Herbrand disjunctions of existential theorems obtained by this elimination procedure [2]. We extend their results to ε-calculus with equality predicate.

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[2] G. Moser and R. Zach, *The epsilon calculus and Herbrand complexity*. ***Studia Logica***, vol. 82 (2006), no. 1, pp. 133–155.

▸ GIANLUCA PAOLINI, *Model theory of free projective planes*.
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We prove that the theory of open projective planes is complete and strictly stable. As a corollary, we prove that Marshall Hall's free projective planes $(\pi^n : 4 \leq n \leq \omega)$ are all elementary equivalent and that their common theory is strictly stable and decidable (being in fact the theory of open projective planes).

This work is joint with Tapani Hyttinen.

▸ FRANCO PARLAMENTO AND FLAVIO PREVIALE, *Absorbing the structural rules in the sequent calculus with additional atomic rules*.
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A multisuccedent sequent calculus for intuitionistic logic free of structural rules was presented in [1] and a detailed proof of their admissibility, based on [2], appeared in [5]. A single succedent version of that calculus was adopted in [6]. In all cases, the proof of the admissibility of the structural rules relies, as for the classical **G3** system, on the hight-preserving admissibility of the contraction rule, that, when atomic rules are added to the calculus, may fail. Letting **G3[mic]** be the slight variants described in [3] for minimal, intuitionistic and classical logic of the calculi in [5], and $\mathcal{R}$ any set of atomic rules of the following form:

$$\frac{\vec{Q}_1, \Gamma_1 \Rightarrow \Delta_1, \vec{Q}'_1 \ \ldots \ \vec{Q}_n, \Gamma_n \Rightarrow \Delta_n, \vec{Q}'_n}{\vec{P}, \Gamma_1, \ldots, \Gamma_n \Rightarrow \Delta_1, \ldots, \Delta_n, \vec{P'}} \quad ,$$

where $\vec{Q}_1, \vec{Q}'_1, \ldots, \vec{Q}_n, \vec{Q}'_n, \vec{P}, \vec{P'}$ are sequences (possibly empty) of atomic formulae and $\Gamma_1, \ldots, \Gamma_n, \Delta_1, \ldots \Delta_n$ are finite sequences (possibly empty) of formulae that are not active in the rule, and letting $\mathbf{G3[mic]}^{\mathcal{R}}$ denote the calculi obtained by adding to **G3[mic]** the rules in $\mathcal{R}$ and Cut_{cs} the contex-sharing cut rule, we have the following:

Theorem 1. *For any set of atomic rules $\mathcal{R}$, any derivation in $\mathbf{G3[mic]}^{\mathcal{R}} + Cut_{cs}$ can be transformed into a derivation in the same system in which the rules in $\mathcal{R}$ and the Cut_{cs} rule are applied before any logical rule.*

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Corollary 2. *If the structural rules are admissible in the calculus that contains only the initial sequents and the rules in* $\mathcal{R}$, *then they are admissible in* **G3[mic]**$^{\mathcal{R}}$ *as well.*

Two applications are the following. (1) For $\mathcal{R} = \emptyset$ we have a proof of the admissibility of the *Cut* rule in **G3[mic]** independent from the height-preserving admissibility of the contraction rule. (2) Let Ref and Repl be the following rules for equality, introduced in [4] and adopted in the second edition [7] of [6]

$$\frac{t = t, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\ \text{Ref} \qquad \frac{s = r, P[x/s], P[x/r], \Gamma \Rightarrow \Delta}{s = r, P[x/s], \Gamma \Rightarrow \Delta}\ \text{Repl}$$

The contraction rule is not height preserving admissible in $\mathcal{R} = \{\text{Ref}, \text{Repl}\}$, yet we obtain that the structural rules are admissible in **G3[mic]**$^{\mathcal{R}}$, thus extending the result proved in [4] in the case t, r, and s are restricted to be constants.

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[7] ———, ***Basic Proof Theory***, second ed., Cambridge University Press, Cambridge, 2000.

► CHRISTIAN RETORÉ AND LÉO ZARADZKI, *Individuals, equivalences and quotients in type theoretical semantics*.

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In natural language semantics, individuation and the nature of individuals is highly debated. If one says *I carried all the books from the shelf to the cellar because I read them all* and if there were five books, two of them being *Dubliners*, five books were carried, whereas four were read. This raises the question whether individuals are the same or not, and type theoretical semantics with equality types offer new perspectives on this question.

Mathematically, sameness corresponds to equivalence: equivalent objects enjoy some common properties, their equivalence classes can be proved to be equal. Equivalence can be coarser or finer grained: a book may be defined as a novel, as an edition of this novel or as a particular copy of some particular edition.

We show how linguistically motivated quotient constructions can be integrated in type theoretical semantics, with insights from some recent work on quotients in proof assistants—indeed both cases require to only encode computable quotients.

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[2] C. Cohen, *Pragmatic quotient types in Coq*, ***ITP2013 International Conference on Interactive Theorem Proving, vol. 7998*** (S. Blazy, C. Paulin-Mohring, and D. Pichardie, editors), Springer, Rennes, 2013, pp. 213–228.

[3] C. Retoré, *The Montagovian generative lexicon* ΛTy_n: *A type theoretical framework for natural language semantics*, ***19th International Conference on Types for Proofs and Programs (TYPES 2013)*** (R. Matthes and A. Schubert, editors), Leibniz International Proceedings in Informatics (LIPIcs), vol. 26, Toulouse, 2014, pp. 202–229.

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► IBRAHIM SENTURK AND TAHSIN ONER, *Regarding Aristotelian logic as a Sheffer stroke basic algebra*.
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In this work, our aim is to tackle with the Aristotelian logic by means of Carroll's diagrammatic method. For this purpose, we firstly define a formal system which gives us a formal approach to logical reasoning with diagrams for representations of the fundamental propositions in Aristotelian logic. In the sequel, we show that syllogisms are closed under the syllogistic criterion of inference which is the deletion of middle term. Therefore, it allows for the formalism which comprise synchronically bilateral and trilateral diagrammatical appearance and a naive algorithmic nature. And also, there is no specific knowledge or exclusive ability needed in order to understand and use it.

In another perspective, we give a morphism from categorical syllogistic system to a Sheffer stroke basic algebra. To this end, we give a reduction of basic algebras only by using the Sheffer stroke operation. Thereupon, we explain the quantitative relationship between two terms by means of bilateral diagrams. So, we obtain possible conclusion values of bilateral diagrams of premises. Finally, by using these sets, we construct a complete bridge between Sheffer stroke basic algebra and categorical logical system.

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► PAUL SHAFER AND ANDREA SORBI, *Comparing the degrees of enumerability and the closed Medvedev degrees*.
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For sets $\mathcal{A}, \mathcal{B} \subseteq \omega^\omega$, recall that $\mathcal{A}$ *Medvedev reduces* to $\mathcal{B}$ if there is a Turing functional Φ such that $\Phi(f) \in \mathcal{A}$ whenever $f \in \mathcal{B}$. The degree structure induced by Medvedev reducibility is called the *Medvedev degrees*.

Both the Turing degrees and the enumeration degrees embed into the Medvedev degrees: map the Turing degree of $f \in \omega^\omega$ to the Medvedev degree of $\{f\}$, and map the enumeration degree of (a nonempty) $A \in 2^\omega$ to the Medvedev degree of $\{g : \operatorname{ran}(g) = A\}$. The embedding of the Turing degrees into the Medvedev degrees is particularly nice. Every Turing degree is mapped to the Medvedev degree of a closed set (in particular, a singleton), and the range of the embedding is definable (a theorem of Dyment and Medvedev). On the other hand, little is known about the embedding of the enumeration degrees in the Medvedev degrees. For example, whether or not the range of the embedding is definable is a longstanding open question of Rogers.

Call a Medvedev degree *closed* if it is the degree of a closed subset of ω^ω, and call a Medvedev degree a *degree of enumerability* if it is in the range of the embedding of the enumeration degrees into the Medvedev degrees. We explore the distribution of the degrees

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of enumerability with respect to the closed degrees and find that many situations occur. There are nonzero closed degrees that do not bound nonzero degrees of enumerability, there are nonzero degrees of enumerability that do not bound nonzero closed degrees, and there are degrees that are nontrivially both degrees of enumerability and closed degrees. We also show that the compact degrees of enumerability exactly correspond to the cototal enumeration degrees.

► ASSAF SHANI, *Borel equivalence relations and symmetric models*.
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We develop a correspondence between the study of Borel equivalence relations induced by closed subgroups of S_∞, and the study of symmetric models of set theory without choice, and apply it to solve questions of [1].

In [1], the possible values of *potential complexity* of Borel equivalence relations which are induced by actions of closed subgroups of S_∞ are completely classified. To that end, they refine the Friedman–Stanley hierarchy $\cong_\alpha$, $\alpha < \omega_1$, by defining equivalence relations $\cong^*_{\lambda+1}$, $2 \leq \lambda < \omega_1$. E.g., $\cong^*_n$ has potential complexity $D(\mathbf{\Pi}^0_n)$.

Moreover, they define equivalence relations $\cong^*_{\lambda+1,0} \leq_B \cong^*_{\lambda+1}$, and show that they correspond to actions by "well-behaved" closed subgroups of S_∞. That is, if $E \leq_B \cong^*_{\lambda+1}$ is induced by a Borel G-action of a closed subgroup G of S_∞ *which admits an invariant compatible metric*, then $E \leq_B \cong^*_{\lambda+1,0}$. Furthermore, they prove that for any countable ordinal α, $\cong^*_{\alpha+3,0} <_B \cong^*_{\alpha+3}$.

They ask whether the remaining reductions are also strict, and conjecture that they are. We confirm this, and focus on the minimal open cases:

THEOREM 1. $\cong^*_{\omega+1,0} <_B \cong^*_{\omega+1}$ *and* $\cong^*_{\omega+2,0} <_B \cong^*_{\omega+2}$.

The proof goes through studying symmetric models generated by generic invariants of these equivalence relations. To make the connection with Borel reducibility we use tools developed by Zapletal.

We use models constructed in [3], separating the "*generalized Kinna–Wagner principles*," KWP^n, which state that every set can be embedded into the n'th-power set of an ordinal. Towards proving Theorem 1, we show the consistency of $\mathrm{KWP}^{\omega+1} \wedge \neg\mathrm{KWP}^{\omega}$, answering a question of [2].

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[3] G. P. MONRO, *Models of ZF with the same sets of sets of ordinals*. ***Fundamenta Mathematicae***, vol. 80 (1973), no. 2, pp. 105–110.

► ANDREI SIPOŞ, *Quantitative results on the method of averaged projections*.
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A recent direction of research in convex optimization has been the extension of classical results that hold in normed spaces to various nonlinear analogues, e.g., the class of CAT(0) spaces. One of such extensions refers to the class of *firmly nonexpansive* mappings (that play a central role in convex optimization, as they encompass a large number of concrete and useful cases such as proximal mappings or resolvents), whose nonlinear generalization was introduced in [1]. A slightly larger class of mappings (which coincides with the former in the context of Hilbert spaces) consists of those that satisfy *property* (P_2), introduced in [2], where the fact that the composition $T := T_2 \circ T_1$ of two such mappings with $Fix(T) \neq \emptyset$ is asymptotically regular is proven.

The primary application of this kind of algorithm is the *alternating projections method*, where the two mappings are the projection operators on two closed, convex, nonempty subsets

A and B of the space X. The goal is, then, to find *best approximation pairs* corresponding to those sets, i.e., pairs $(a, b) \in A \times B$ such that $d(a, b) = d(A, B)$, or, at least, points that approximate such pairs to any prior degree.

A related method, the *method of averaged projections*, replaces the composition of projections from above with a weighted average of them, i.e., a convex combination. A classical way of proving the efficacy of this method in the usual Euclidean space (and recently also in Hilbert spaces [3, Section 5]) is by reducing it to the first one through the replacing of the combined mapping with a concatenated transformation in a product space with the averaged metric, followed by a projection on the diagonal. Our goal is to extend that line of argument to self-mappings satisfying property (P_2)—in particular, to firmly nonexpansive self-mappings—in the setting of CAT(0) spaces.

In addition, we will also establish effective versions of such asymptotic results, in the sense of proof mining. Proof mining is an applied subfield of mathematical logic, developed primarily by U. Kohlenbach and his collaborators ([4] is the standard introduction, while more recent surveys are [5, 6]), that aims to provide quantitative information (witnesses and bounds) for numerical entities which are shown to exist by proofs which cannot be necessarily said to be fully constructive. In nonlinear analysis, which has been a primary focus for such work, the relevant information is usually found within convergence statements, where the problem is to find an explicit formula for the N_ε such that for any $\varepsilon > 0$, the elements of the sequence of index greater than N_ε are ε-close to the limit. The result above is an instance of this, as asymptotic regularity is clearly a statement of convergence, and it has indeed been analyzed from this viewpoint in [7].

The results presented in this talk may be found in [8].

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► WILLIAM STAFFORD, *The untyped λ-calculus as a foundation for natural language semantics: The problem of computational intractability*.
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Natural language appears to contain hyperintensional settings, but it has proven difficult to formalise hyperintensionality using Montague's approach of treating intensions as functions from worlds to assignments. Fox and Lappin (2005) have proposed Property Theory with Curry Types (PTCT) as an alternative foundation for natural language. They claim that PTCT formalises hyperintensionality and polysemy but is a computationally weaker system than the traditional Montague grammars. In this article, we dispute their claim to have generated a computationally weaker system. Firstly, their argument is based on the claim that Montague grammars are second-order systems and so stronger than PTCT, which is

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a first-order system. Second-order systems are only stronger than first-order systems if we use the full semantics. This means that Montague grammars are of equivalent strength to PTCT if they use a weaker semantics. Secondly, PTCT uses the untyped λ-calculus, whereas Montague grammars uses the typed λ-calculi. The untyped calculus is Turing complete, whereas the typed systems are not. From this point of view, we see that PTCT might in fact be a stronger formal system than Montague grammars.

[1] C. Fox and S. Lappin, ***Foundations of Intensional Semantics***, Blackwell, 2005.

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We show that if $(M, \in_1, \in_2)$ satisfies the first-order Zermelo–Fraenkel axioms of set theory when the membership-relation is $\in_1$ and also when the membership-relation is $\in_2$, and in both cases the formulas are allowed to contain both $\in_1$ and $\in_2$, then $(M, \in_1) \cong (M, \in_2)$, and the isomorphism is definable in $(M, \in_1, \in_2)$. This extends a theorem of Zermelo [2] from 1930. A similar result holds for first-order Peano arithmetic, extending the categoricity result of Dedekind [1] of second-order Peano arithmetic. The proof of this is similar, but somewhat easier.

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▸ CATERINA VIOLA, *Piecewise linear valued constraint satisfaction problems*.
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Valued constraint satisfaction problems (VCSPs) are a class of combinatorial optimisation problems. Recently, the computational complexity of all VCSPs for finite sets of cost functions over finite domains has been classified completely. Many natural optimisation problems, however, cannot be formulated as VCSPs over a finite domain. One example is the famous linear programming problem, where the task is to optimise a linear function subject to linear inequalities. This can be modelled as a VCSP over $\mathbb{Q}$, the set of rational numbers, by allowing unary linear cost functions and cost functions of higher arity to express the crisp (i.e., hard) linear inequalities. I will present our work that initiates the systematic investigation of infinite-domain VCSPs by studying the complexity of VCSPs for classes of piecewise linear cost functions, i.e., cost functions admitting a first-order definition on $(\mathbb{Q}; +, <, 1)$.

This is joint work with Manuel Bodirsky, Technische Universität Dresden, and Marcello Mamino, Università di Pisa.

▸ FARN WANG AND WEN-HONG CHIANG, *Statistical model-checking uncertain responses written in LTL with confidence*.
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Model-checking [2] has been an important technology that aims to bridge the logic community and the industry. However, the assumption of rigid and complete models of the model-checking technology has been a major incompatibility block to the industry projects. In practice, there is usually no complete and precise model at all. Moreover, most specification properties cannot be simplistically evaluated as "true" (for satisfaction) or "false" (for failure). This is particularly true in the case of evaluating system responses. For example, if we try to download a music file from a server via the internet. Usually the users can tolerate a few downloading failures. To avoid frustration of the users, the developers may also want to lift the download success ratio to $p = 95\%$ if the download request immediately follows an unfulfilled request. This can be specified with the following *Linear Temporal Logic* (*LTL*) property [3]:

$$\Box((\mathit{downloadFail} \wedge \bigcirc(\mathit{downloadOrCancel} \wedge \bigcirc \mathit{download})) \rightarrow \bigcirc\bigcirc\bigcirc \mathit{downloadSucc}).$$

But clearly the model-checking technology cannot aid in answering how many consecutive failures can be compiled to a confident test verdict of a server defect.

In this work, we adopt the LTL statistical model-checking [4] which extends the LTL model-checking problem to the following.

Definition (Model Set Evaluation Problem with Chernoff Confidence (MSEPCC)). Given a set S of traces (for a system with uncertain responses due to environment interference) and an LTL formula ϕ, what is the Chernoff Bound of probability that S is an evidence that the system implements ϕ?

We have implemented a tool, *System Response Tester with Confidence* (*SRTC*), that utilizes the classic concept of Chernoff bounds [1] for Bernoulli variables and sample sizes to calculate a bound on probability that a set of randomized traces is an evidence of system failure to achieve a response success ratio. There are the following technical issues in collecting trace sets for evaluating Chernoff bounds.

- **Efficiency:** The generation of traces cannot be too random for our purpose to trigger the precondition for the target response property.
- **Randomness:** The Chernoff bounds are based on the assumption that the samples are randomly collected. We propose to randomize the lengths of the test cases. Intuitively, the lengths of test sessions should constitute a Poisson distribution.

SRTC can efficiently generate trigger sequences for a given response property with lengths of the sequences roughly constitute a Poisson distribution. We experimented with several Android apps from Google play. Experiment data show that SRTC could be handy in evaluating issues related to response properties expressed in LTL with the confidence values derived from a Chernoff bound.

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▸ AIBAT YESHKEYEV, *Hereditary β-cosemantic Jonsson theories*.
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It is well known that the perfect Jonsson theory can be studied using of the first-order properties of the center of this theory and its semantic model, since the center of the Jonsson theory is the model companion of it. Imperfect Jonsson theories at the moment have not been studied. For example, a bright example of this fact is the theory of all groups. We know that this theory is Jonsson, but does not have a model companion, and the structure of its semantic model is unknown to us. In this thesis, we will consider a special subclass of Jonsson theories, namely, hereditary β-cosemantic Jonsson theories. The need to introduce such a class was due to the following questions: (1) to find a reasonable approach to describing imperfect Jonsson theories; (2) finding the conditions for preserving the theory's consistency with some enrichment of the language; and (3) finding conditions for preserving the definability of a type with the appropriate stability obtained as a result of enriching the language. The proposed method for solving the first problem is to use the central type and forcing companion in some permissible enrichment of the language of the Jonsson theory under consideration. The second and third tasks appear automatically after the proposed method to solve the first problem. All three problems are independent in themselves and they have connections with known questions for complete theories. For example, in problem 2, there is a question about the existence of an amalgam and joint embedding property. In connection with Problem 3, well known are the results of T. G. Mustafin [2] and E. A. Palyutin [3] about new types of stability for complete theories. In the works of B. Poizat [4], E. Bouscaren [1] considered the problem of completeness of elementary pairs. It is clear that the concepts of heredity and β-cosemanticity will be a refinement of these problems in the framework of studying, generally speaking, incomplete Jonsson theories.

We introduce the following definitions necessary for the above purposes.

DEFINITION 1. An enrichment $\overline{T}$ of the Jonsson theory T is said to be permissible if any type in this enrichment is definable in the framework of $\overline{T}$-stability.

DEFINITION 2. The Jonsson theory is said to be hereditary, if in any of its permissible enrichment any expansion of it in this enrichment will be Jonsson theory.

DEFINITION 3. The Jonsson theory T is called (α, β_i)-cosemantic, where $i \in \overline{1, \alpha}$, if the quotient of the Jonsson spectrum [5] of any model of the theory T by cosemanticness has α equivalence classes, each of which has the power β_i, respectively.

If $\alpha = 1$, then we omit the first index and say that the Jonsson theory T β-cosemantic.

[1] E. BOUSCAREN, *Elementary pairs of models*. ***Annals of Pure and Applied Logic***, vol. 45 (1989), pp. 129–137.

[2] T. G. MUSTAFIN, *New concepts of the stability of theories*, ***Proceedings of the Soviet–French Colloquium on Model Theory***, Karaganda, 1990, pp. 112–125.

[3] E. A. PALYUTIN, *E^*-stable theories*. ***Algebra and Logic***, vol. 41 (2003), no. 2, pp. 194–210.

[4] B. POIZAT, *Paires de structures stables*. ***The Journal of Symbolic Logic***, vol. 48 (1983), no. 2, pp. 239–249.

[5] A. R. YESHKEYEV and O. I. ULBRIKHT, *JSp-cosemanticness and JSB property of Abelian groups*. ***Siberian Electronic Mathematical Reports***, vol. 13 (2016), pp. 861–874.

Abstracts of articles submitted by title

► A. ALIBEK, B. S. BAIZHANOV, B. SH. KULPESHOV, AND T. S. ZAMBARNAYA, *The countable spectrum of weakly o-minimal theories of convexity rank 1*.
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Here, we discuss the Vaught's problem for weakly o-minimal theories of convexity rank 1. Convexity rank has been introduced in [2]. In particular, a theory has convexity rank 1 if there is no parametrically definable equivalence relation with an infinite number of infinite convex classes. Obviously, any o-minimal theory has convexity rank 1.

As it is known, in [4] the Vaught's conjecture for o-minimal theories was solved. Recently in [3], the Vaught's conjecture for quite o-minimal theories was solved. From the above works, it follows that these theories have the same spectrum, namely such a theory has either continuum of countable models, or exactly $6^a 3^b$ countable models for nonnegative integers a and b.

In [1] B. S. Baizhanov and A. Alibek have constructed for every ordinal κ with $4 \le \kappa \le \omega$ examples of weakly o-minimal theories having exactly κ countable models. All these examples have convexity rank 1. The following theorem describes the countable spectrum of weakly o-minimal theories of convexity rank 1 (which already differs from the countable spectrum of o-minimal theories):

THEOREM 1. *Let T be a weakly o-minimal theory of convexity rank* 1 *in a countable language. Then exactly one of the following possibilities holds*:

(1) *T is $\aleph_0$-categorical*

(2) *T has k countable models, where $3 \leq k < \omega$*
(3) *T has ω countable models*
(4) *T has 2^{ω} countable models.*

[1] A. Alibek and B. S. Baizhanov, *Examples of countable models of a weakly o-minimal theory*. ***International Journal of Mathematics and Physics***, vol. 3 (2012), no. 2, pp. 1–8.

[2] B. S. Kulpeshov, *Weakly o-minimal structures and some of their properties*. ***The Journal of Symbolic Logic***, vol. 63 (1998), no. 4, pp. 1511–1528.

[3] B. S. Kulpeshov and S. V. Sudoplatov, *Vaught's conjecture for quite o-minimal theories*. ***Annals of Pure and Applied Logic***, vol. 168 (2017), no. 1, pp. 129–149.

[4] L. L. Mayer, *Vaught's conjecture for o-minimal theories*. ***The Journal of Symbolic Logic***, vol. 53 (1988), pp. 146–159.

► S. S. BAIZHANOV AND B. SH. KULPESHOV, *On expansions of $\aleph_0$-categorical weakly o-minimal structures by binary predicates*.
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The present talk is concerned with the notion of *weak o-minimality* originally deeply studied in [2]. A subset A of a linearly ordered structure M is *convex* if for any $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \dots \rangle$ such that any definable (with parameters) subset of the structure M is a finite union of convex sets in M.

In [1] expansions of 1-indiscernible $\aleph_0$-categorical weakly o-minimal structures by an equivalence relation were studied. Here, we study the question of preserving properties at expanding models of weakly o-minimal theories by an arbitrary binary predicate.

Consider the following example:

Example 1. Let $M := \langle \mathbb{Q} \times \mathbb{Q}, < \rangle$ be a linearly ordered structure on the set $\mathbb{Q} \times \mathbb{Q}$, ordered lexicographically. Obviously, M is an $\aleph_0$-categorical o-minimal structure. Introduce the following binary formulas $E(x, y)$ and $R_1(x, y)$ on the set $\mathbb{Q} \times \mathbb{Q}$: for any $a = (m_1, n_1), b = (m_2, n_2) \in \mathbb{Q} \times \mathbb{Q}$

$$E(a, b) \Leftrightarrow m_1 = m_2$$

$$R_1(a, b) \Leftrightarrow m_1 = m_2 \wedge n_1 \leq n_2 < n_1 + \sqrt{2}$$

Let $R(x, y) := y \leq x \wedge E(x, y) \wedge \neg R_1(x, y)$ and $M' := \langle \mathbb{Q} \times \mathbb{Q}, <, R^2 \rangle$ be an expansion of M by binary predicate $R(x, y)$. Obviously, $R(M', a)$ is convex and $a < R(M', a)$ for any $a \in M'$.

It can be established that M' is an 1-indiscernible weakly o-minimal structure, but $Th(M')$ is not $\aleph_0$-categorical.

Here, we discuss necessary and sufficient conditions when an expansion of an 1-indiscernible $\aleph_0$-categorical weakly o-minimal theory by a binary predicate is both weakly o-minimal and $\aleph_0$-categorical.

[1] S. S. Baizhanov and B. S. Kulpeshov, *Expanding 1-indsicernible countably categorical weakly o-minimal theories by equivalence relations*. ***Siberian Electronic Mathematical Reports***, vol. 15 (2018), pp. 106–114.

[2] H. D. Macpherson, D. Marker, and C. Steinhorn, *Weakly o-minimal structures and real closed fields*. ***Transactions of the American Mathematical Society***, vol. 352 (2000), pp. 5435–5483.

► ALEXANDR BESSONOV, *Gödel's first incompleteness theorem cannot be used as an argument against Hilbert's program*.
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An argument against realizability of Hilbert's program of finitary grounding mathematics based on the Gödel's second incompleteness theorem is generally built as follows. Let the formal Dedekind–Peano arithmetic (PA) be consistent. Suppose that there is an informal finitary consistency proof of PA. By von Neumann's thesis (every finitary informal proof is formalizable in PA), such a proof would be Gödel-style formalizable in PA. As a result, a formula, say

$$\forall y \neg \mathrm{Prov}(\ulcorner(\boldsymbol{0} = \boldsymbol{1})\urcorner, y)$$

expressing the unprovability of $\boldsymbol{0} = \boldsymbol{1}$ and, hence, the consistency of PA, would turn out to be provable, which would contradict the second incompleteness theorem; see, e.g., [2]. (Here $\mathrm{Prov}(x, y)$ numeralwise expresses a provability predicate that satisfies Hilbert–Bernays–Löb conditions; $\ulcorner A \urcorner$ denotes the Gödel number of A.)

Obviously, the von Neumann reasoning above applies if, instead of $\boldsymbol{0} = \boldsymbol{1}$, we take any other formula, in particular G or $\neg G$, where G is Gödel's unsolvable formula. Thus, neither $\forall y \neg \mathrm{Prov}(\ulcorner G \urcorner, y)$ nor $\forall y \neg \mathrm{Prov}(\ulcorner \neg G \urcorner, y)$ can be proved in PA, for otherwise we will be led to a contradiction with the second incompleteness theorem. This implies that an informal finitary proof of Gödel's first incompleteness theorem does not exist.

The impossibility of an informal finitary proof of the first incompleteness theorem implies the nonlegitimacy of any argument against realizability of Hilbert's program based on this theorem: from the standpoint of consistent finitism, any informal proof of the first theorem is doubtful, since it is not finitary.

In [1] it was shown that argumentation against realizability of Hilbert's program based on the second Gödel's incompleteness theorem is incorrect from the outset. Now we can conclude that the textbook proposition, according to which both Gödel's incompleteness theorems serve as decisive arguments against feasibility of Hilbert's finitistic program is wrong.

[1] A. Bessonov, *Gödel's second incompleteness theorem cannot be used as an argument against Hilbert's program*, this Bulletin, vol. 24 (2018), no. 2, pp. 235–236.

[2] R. Zach, *Hilbert's program then and now*, ***Handbook of the Philosophy of Science*** (D. Jacquette, editor), Philosophy of Logic, vol. 5, Elsevier BV, Amsterdam, 2006, pp. 431–432.

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We present a system $\mathbf{LKprob}(\varepsilon)$ (see [1], [7], and [2]) making it possible to work with expressions of the form $\Gamma \vdash^n \Delta$, a generalization of Gentzen's sequents $\Gamma \vdash \Delta$ of classical propositional logic **LK**, with the intended meaning that 'the probability of the sequent $\Gamma \vdash \Delta$ is greater than or equal to $1 - n\varepsilon$', for a given small real $\varepsilon > 0$ and any natural number n (see [4] and [5]). For instance, $\mathbf{LKprob}(\varepsilon)$ is based on rules of the following form:

$$\frac{\Gamma A B \vdash^n \Delta}{\Gamma A \wedge B \vdash^n \Delta}(\wedge\vdash) \quad \frac{\Gamma \vdash^n A\Delta \quad \Gamma \vdash^m B\Delta}{\Gamma \vdash^{m+n} A \wedge B\Delta}(\vdash\wedge).$$

A model for $\mathbf{LKprob}(\varepsilon)$ is a mapping $p\colon \mathrm{Seq} \to I \cap [0,1]$, where $I = \{1 - n\varepsilon | n \in \mathbf{N}\}$, satisfying the following conditions: (*i*) $p(A \vdash A) = 1$, for any formula A; (*ii*) if $p(AB \vdash) = 1$, then $p(\vdash AB) = p(\vdash A) + p(\vdash B)$, for any formulae A and B; (*iii*) if sequents $\Gamma \vdash \Delta$ and $\Pi \vdash \Lambda$ are equivalent in **LK**, in sense that there are proofs for both sequents $\bigwedge \Gamma \to \bigvee \Delta \vdash \bigwedge \Pi \to \bigvee \Lambda$ and $\bigwedge \Pi \to \bigvee \Lambda \vdash \bigwedge \Gamma \to \bigvee \Delta$ in **LK**, then $p(\Gamma \vdash \Delta) = p(\Pi \vdash \Lambda)$.

A theory $\mathbf{LKprob}(\varepsilon)(\sigma_1, \dots, \sigma_n)$, an extension of $\mathbf{LKprob}(\varepsilon)$ by the list of new axioms $\sigma_1, \dots, \sigma_n$, is said to be consistent iff there exists a sequent $\Gamma \vdash^0 \Delta$ which is unprovable in $\mathbf{LKprob}(\varepsilon)(\sigma_1, \dots, \sigma_n)$. We prove that each consistent theory can be extended to a maximal consistent theory, and as a consequence, soundness and completeness is also proved.

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The system **LKprob**(ε) can be considered a particular case of our general approach to probabilization of sequent calculus (see [1], [6], and [3]).

[1] M. Boričić, *Hypothetical syllogism rule probabilized*, this Bulletin, vol. 20 (2014), no. 3, pp. 401–402, abstract in ***Logic Colloquium 2012***.

[2] ———, *Models for the probabilistic sequent calculus*, this Bulletin, vol. 21 (2015), no. 1, p. 60, abstract in ***Logic Colloquium 2014***.

[3] ———, *Suppes-style rules for probability logic*, this Bulletin, vol. 22 (2016), no. 3, p. 431, abstract in ***Logic Colloquium 2015***.

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[5] ———, *Sequent calculus for classical logic probabilized*. ***Archive for Mathematical Logic***, to appear.

[6] P. Suppes, *Probabilistic inference and the concept of total evidence*, ***Aspects of Inductive Inference*** (J. Hintikka and P. Suppes, editors), North-Holland, Amsterdam, 1966, pp. 49–55.

[7] C. G. Wagner, *Modus tollens probabilized*. ***British Journal for the Philosophy of Science***, vol. 54 (2004), no. 4, pp. 747–753.

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Logics—distinguishing the semantic consequence relation from the syntactic deducibility relation—are three-part systems composed of a language, a semantic system of interpretations, and a syntactic system of deductions [2]. In contrast, theories—distinguishing truths from theorems—were regarded as certain systems of interpreted sentences—systems closed under logical deducibility: every sentence of a theory's language deducible from the theory's theorems using the theory's underlying logic is one of its theorems [5].

Although attention focuses on a theory's truths and theorems, reflection on what Church [1] called a theory's underlying logic reveals that a theory can be regarded as five-part system: the three-parts of its underlying logic, its intended interpretation, and its intended theorems—the intended interpretation being a member of its underlying logic's semantics and its intended theorems being members of its underlying logic's language.

Until Łukasiewicz started arguing otherwise in the 1920s [4, pp. 15, 73, passim], people generally regarded Aristotle's syllogistic as a logic and not as a theory. In the 1970s, Austin, Corcoran, Smiley, and others began finding difficulties reconciling the Łukasiewicz view with Aristotle's text. Corcoran and Smiley, working independently, proposed logics that fit parts of the text that Łukasiewicz couldn't treat [3]—construing Aristotle's syllogistic as a logic and not as a theory.

In retrospect, we see that the semantics of a Corcoran–Smiley underlying logic leaves room for, and perhaps requires, in the metalanguage a Łukasiewicz-style theory of terms [4, Chapter III]. This article explores the extent to which the antithetical Łukasiewicz and Corcoran–Smiley approaches can be synthesized into a more adequate account of Aristotle's ***Prior Analytics***.

[1] A. Church, ***Introduction to Mathematical Logic***, Princeton University Press, 1956.

[2] J. Corcoran, *Gaps between logical theory and mathematical practice*, ***Methodological Unity of Science***, Kluwer, 1973.

[3] ———, *Aristotle's demonstrative logic*. ***History and Philosophy of Logic***, vol. 30 (2009), pp. 1–20.

[4] J. Łukasiewicz, ***Aristotle's Syllogistic***, Oxford University Press, 1951.

[5] A. Tarski, ***Introduction to Logic***, Dover, 1995.

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Church [1], Tarski [3], and other mathematically oriented logicians use the adjectives *concrete* and *abstract* as in traditional subject-object epistemology where *concrete* thinking *subjects* make *abstract* judgments about *concrete* objects. In some correspondence theories of truth, abstract propositions are true or false in virtue of *concrete* facts [2]. Teaching geometry requires distinguishing between *concrete triangles* and their *abstract shapes*. Besides absolute, or attributive, uses of *concrete* and *abstract*, we also find relative uses. Every sentence of the form $(p \vee q)$ is *more concrete* than its form. Conversely, the form $(p \vee q)$ is *more abstract* than any sentence having it.

We survey uses and conspicuous nonuses in logic of *concrete* and *abstract* including cognates: *to abstract*, *abstraction*, etc. We also consider distinctions often confused or identified with concrete-abstract distinctions, e.g., physical/nonphysical and individual/universal.

We identify many occurrences of *concrete* as fillers or rhetorical expletives: deleting them leaves the sentences's propositional meanings intact. In this use, every example is a concrete example and every concrete example is an example. Equally empty substitutes come readily to mind: *particular*, *specific*, *individual*, and so on.

Remarkably, Church [1] uses *abstract* frequently while completely avoiding *concrete*. However, in Tarski [3] the exact opposite holds: there *concrete* occurs frequently while *abstract* is largely absent. Logicians considered include Aristotle, al-Farabi, Ockham, Boole, al-Ahsa'i, De Morgan, Peirce, Frege, Russell, Carnap, Gödel, Quine, and Lawvere.

In primary senses, *concrete* and *abstract* are correlative adjectives like *old* and *young*. It is difficult to determine what is being conveyed by calling something *concrete* or *abstract* unless writers give examples where they would apply one and not the other and they present some indication of their criteria for applying each word.

[1] A. Church, ***Introduction to Mathematical Logic***, Princeton University Press, 1956.

[2] J. Corcoran, *Sentence, proposition, judgment, statement, and fact*, ***Many Sides of Logic***, College Publications, 2009.

[3] A. Tarski, ***Logic, Semantics, Metamathematics***, Hackett, 1983.

▸ JOHN CORCORAN AND JOSÉ MIGUEL SAGÜILLO, *Argument validity, form omnivalidity, and schema panvalidity*.
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Consider a standard one-sorted first-order language L. Instances of argument-schemata [3] are concrete [premise-conclusion] arguments using L's sentences. Model-theoretically, an argument is *valid* if its conclusion is true in every model of its premise-set; *invalid* if its conclusion is false in some model of its premise-set. Every argument is either valid or invalid; no argument is both.

Using [logical] form as in [1, 2], every concrete argument has exactly one abstract form. Every two arguments having the same form are both valid or both invalid.

Validity and invalidity apply to concrete arguments and not to abstract forms. An argument form is defined to be *omnivalid* if all arguments having it are valid; *nullovalid* if all arguments having it are invalid [4]. Every argument form is either omnivalid or nullovalid, and not both.

Notice that some argument-schemata have valid instances and invalid instances [3]. Consider the schema of all one-premise arguments:

$$\frac{P}{Q}.$$

Putting $2 = 1$ for P and for Q yields a valid instance; putting $2 = 1$ for P and $3 = 2$ for Q yields an invalid instance.

An argument-schema is *panvalid* if every instance is valid, *paninvalid* if every instance is invalid, and *neutrovalid* if some but not every instance is valid [5].

We discuss how these three concepts can be used to give fresh understanding of classical results, to suggest new avenues of research, and to correct confusions and errors in the literature.

[1] J. Corcoran, *Argumentations and logic*. ***Argumentation***, vol. 3 (1989), pp. 17–43.
[2] ———, *First-order logical form*, this Bulletin, vol. 10 (2004), p. 445.
[3] ———, *Schemata*, this Bulletin, vol. 12 (2006), pp. 219–240.
[4] ———, *Logic teaching in the 21st century*. ***Quadripartita Ratio: Revista de Argumentación y Retórica***, vol. 1 (2016), pp. 1–34.
[5] ———, *Panvalid four-connective argument schemata*, this Bulletin, vol. 23 (2017), pp. 209–210.

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Formal rigour seems to be more important to philosophical logic than to mathematics. However, worth is made up by substance, and it is not clear what the content of modern modal logic precisely is.

The subject was conceived to avoid the so-called paradoxes of "material" implication like $\bot \to \phi$ and $\phi \to \top$, but "strict" implication shows them likewise, namely, $\bot \Rightarrow \alpha$ $\big(\text{eq. } \Box(\bot \to \alpha)\big)$, $\alpha \Rightarrow \top$.

Possible worlds are already given on the nonmodal level. For instance, structures (models) are the possible worlds of predicate logic.

The notion of world seems to have been misleading. Nonequivalent possible worlds are incompatible: the union of their theories is inconsistent. So they cannot exist simultaneously. Isn't the notion of accessibility therefore dubious?

On the basis of a given logic, say PL, it is clear what the logical modalities are: ϕ is *logically necessary* iff $\models \phi$. Accordingly, ϕ is *logically possible* iff the negation of ϕ is not logically necessary, viz. $\not\models \neg\phi$. Thus any atomic proposition p is logically possible—naturally. However, $\Diamond p$ is *not* a theorem of any of the systems of modern modal logic. The deficiency is caused by uniform substitution, which must not be generalised to the modal level. Proper implementation uniquely leads to the semantic system C (like Carnap) and the evident deductive system S of ours. Cf. The Bulletin of Symbolic Logic 20, pp. 238, 264–265.

The logical modalities in the narrow sense can be generalised to the Σ-*modalities*: σ is Σ-*necessary* iff Σ seq σ, σ is Σ-*nonnecessary* iff σ is not Σ-necessary, ... , where *seq* $:= \models = \vdash$. We proved that the Σ-modalities exhaust the *objective modalities*, where a necessity in whatever sense is *objective* if it is closed under seq. (BSL 21, 239-0; BSL 22, 585)

The implementation of our mathematized conception of adequately formalising the Σ-modalities leads to one and only one consequence relation, namely Σ $seq^{\Box}$ α, which can be constructed as well semantically ($\mathcal{M}, V \Vdash_{\Sigma} \Box\alpha$:iff for all $\mathcal{N} \models \Sigma$ and all $W\colon \mathcal{N}, W \Vdash_{\Sigma} \alpha$, $\Sigma \Vdash \alpha$) as deductively ($\Sigma \Vdash\!\!- \alpha$:iff $\Sigma \cup \{\neg\Box\sigma : \Sigma \not\models \sigma\} \vdash_{\mathrm{QNI}} \alpha$).

The transition to this *metalogical extension* forks validity and *general consequence*: $\Vdash \alpha$ does not imply $/\!/\!= \alpha$, viz. $\Sigma \Vdash \alpha$ for all Σ, whence a separate deductive characterization of the general consequences is required, which coincide with the theorems of (a suitable quantificational version of) S5 probably. (BSL 23, 211-2)

It's not that laws like (NR), (K), (T), (4), (5) were false: wrong conception, unjustified pluralism, persisting in uniform substitution and monotony, misguided semantics, and severe incompleteness have lead modern modal logic away from science.

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A Brief on Direct Product Languages and Models Languages $L_1, \dots, L_w$ with signatures S_i's, ... , $S_i < S_{i+1}$. Applying inclusion ordering on the signatures $S*$ we have morphic preorders on the free trees on the signature TS_i.

Proposition 1. *There is a small complete category on the infinitary language fragments definable with the S_i's based on the direct product on TS_i.*

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Consider the fragment topology defined on the author defined on Kiesler L^1_w, K fragments. Let us have a glimpse on n-types and positive local realizability (Nourani 2007–2015). The set of all complete n-types over T is denoted $n(T)$.

THEOREM 1 (Nourani 2014). *There is a generic functor on the category the omitting n-types realizing a direct product model.*

THEOREM 2. *The category based on L^1_w, K fragments and preorder morphisms is a convergent space category.*

[1] C. F. NOURANI and P. EKLUND, *Term functors and product models*: *A brief*, ***Joint MM AMS-MAM, Atlanta, Georgia***, abstract 1125-VJ-2308, 2017.

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This work continues Beklemishev's work [2], in which derived topologies and corresponding generalizations of stationary sets were introduced and investigated. The starting point of these studies lies rather in proof theory than in set theory. Esakia's pioneer work [4] connected scattered spaces to Gödel–Löb's logic GL. This put beginning of studies of topological completeness of its extension, Japaridze's provability logic GLP. Solving the problem, Beklemishev introduced [2] the following construction: given a topology τ on a set X, the *derived* (or *next*) topology τ^+ on X is generated by τ and the sets $d(A)$ for all $A \subseteq X$, where $d(A)$ is the Cantor derivative of A given by τ, i.e., the set of τ-limit points of A. He considered ω-sequences of topologies defined by letting $\tau_0 = \tau$ and $\tau_{n+1} = \tau_n^+$. If τ_0 is the left (or Alexandroff) topology on an ordinal θ, then τ_1 is the standard interval topology of θ, τ_2 is the so-called club topology whose limit points are ordinals $\alpha < \theta$ of uncountable cofinality and $d_2(A) = \{\alpha < \theta \colon A \cap \alpha \text{ is stationary in } \alpha\}$, and τ_3 is called the Mahlo topology. It was shown [2] that τ_{n+2} is included into the topology $\tau_{\Pi^1_n}$ whose limit points are Π^1_n-indescribable ordinals, so a weakly compact suffices to have τ_3 nondiscrete, and later [1] that in L (the constructible universe), τ_{n+2} and $\tau_{\Pi^1_n}$ coincide. An overview with some additional information can be found in [3].

We consider sequences of derived topologies of arbitrary length by letting $\tau_0 = \tau$, $\tau_{\alpha+1} = \tau_\alpha^+$, and $\tau_\alpha =$ the topology generated by all τ_β, $\beta < \alpha$, if α is a limit ordinal. We show that, for $\alpha \geq \omega$, under an appropriate definition of Π^1_α-formulas of second-order infinitary languages, τ_α is included into $\tau_{\Pi^1_\alpha}$. Evaluating size of the least ordinals indescribable in higher order infinitary languages, we show, e.g., that if κ is the least Π^1_n-indescribable in $\mathcal{L}^2_{\alpha,\alpha}$ (the second-order language with connectives and quantifiers of arity $<\alpha$), then it is so already in $\mathcal{L}^2_{\omega,\omega}$ (the second-order finitary language). Furthermore, if θ is the ω-Erdős cardinal, then the set $\{\alpha < \theta \colon \alpha \text{ is } \mathcal{L}^\alpha_{\alpha,\alpha}\text{-indescribable}\}$ is stationary in θ, and if θ is a measurable cardinal and U a normal ultrafilter on θ, then this set is in U, therefore, almost all $\alpha < \theta$ are τ_α-limit points. The hierarchy of derived topologies on θ extends up to θ^+ by diagonalizing: for $\xi < \theta^+$ define $d_\xi(A) = \{\alpha < \theta \colon \alpha \in d_{\xi_\alpha}(A)\}$ if $\operatorname{cf} \xi = \theta$ with $\lim_{\alpha\to\theta} \xi_\alpha = \xi$, and as before otherwise. If θ is measurable, then all the topologies τ_ξ also are nondiscrete.

[1] J. BAGARIA, M. MAGIDOR, and H. SAKAI, *Reflection and indescribability in the constructible universe*. ***Israel Journal of Mathematics***, vol. 208 (2015), no. 1, pp. 1–11.

[2] L. D. BEKLEMISHEV, *On GLP spaces*, circulated manuscript, 2009.

[3] L. D. BEKLEMISHEV and D. GABELAIA, *Topological interpretations of provability logic*, ***Leo Esakia on Duality in Modal and Intuitionistic Logics*** (G. Bezhanishvili, editor), Outstanding Contributions to Logic, vol. 4, 2014, pp. 257–290.

[4] L. ESAKIA, *Diagonal constructions, Löb formula and Cantor's scattered space*, Studies in Logic and Semantics, Metsniereba, Tbilisi, 1981, pp. 128–143 (in Russian).