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PROSPECTIVE LONGEVITY RISK ANALYSIS

By G. Woo, C. J. Martin, C. Hornsby and A. W. Coburn

ABSTRACT

Mortality improvement has traditionally been analysed using an array of statistical methods, and extrapolated to make actuarial projections. This paper presents a forward-looking approach to longevity risk analysis which is based on stochastic modelling of the underlying causes of mortality improvement, due to changes in lifestyle, health environment, and advances in medical science. The rationale for this approach is similar to that adopted for modelling other types of dynamic insurance risk, e.g. natural catastrophes, where risk analysts construct a stochastic ensemble of events that might happen in the future, rather than rely on a retrospective analysis of the non-stationary and comparatively brief historical record.

Another feature of prospective longevity risk analysis, which is shared with catastrophe risk modelling, is the objective of capturing vulnerability data at a high resolution, to maximise the benefit of detailed modelling capability down to individual risk factor level. Already, the use by insurers of postcode data for U.K. flood risk assessment has carried over to U.K. mortality assessment. Powered by fast numerical computation and parameterised with high quality geographical data, hydrological models of flood risk have superseded the traditional statistical insurance loss models. A decade later, medically-motivated computational models of mortality risk can be expected to gain increasing prominence in longevity risk management.

KEYWORDS

Longevity Risk; Mortality Improvement; Medicine; Geroscience; Catastrophe Modelling

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1. Introduction

1.1 *Prospective View of Risk*

1.1.1 Intensive actuarial research into longevity risk has evolved increasingly elaborate and sophisticated statistical projections of future mortality (see e.g. Pitacco *et al.*, 2009). However, projection credibility is limited by the use of past mortality data, which may not capture dynamic trends in future mortality improvement. Indeed, it is widely recognised (e.g. CMI, 2009) that, in the long-term, mortality change may be driven by very different forces from the recent past. Furthermore, in the absence of a probabilistic framework for modelling future mortality change, there may be little recourse but to rely on actuarial opinion on long-term improvement rates.

1.1.2 Plotted as a fan chart, the spread of confidence bounds around a central statistical longevity projection provides an indication of the

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progressive increase in uncertainty over time. Review of the indifferent record of 20th century projections suggests a minimum level of uncertainty, but very high levels of uncertainty may border on medical if not statistical implausibility. Whatever the merits of individual projections, a common feature of statistical projections is a lack of transparency in understanding the key risk drivers at high confidence levels. Detailed insight into the causal pathways towards longevity is lost.

1.1.3 Drawing an analogy with the GDP fan charts produced quarterly by the Bank of England, in February 2007 the 90% confidence fan charts did not encompass the recession zone of negative GDP growth. Had the fan charts been constructed prospectively from a stochastic ensemble of future economic scenarios, including a catastrophic housing market down-turn, the additional insight gained might have mitigated the decision to raise interest rates successively in May and July 2007. Undue reliance on historical time series may be profoundly misleading. Events may occur without historical precedent. Rating of U.S. mortgage-backed securities did not allow for the correlated decline in house prices across the entire country in 2008; a downturn unprecedented in U.S. history, giving rise to the expression 'subprime meltdown'.

1.1.4 New vocabulary is a portent of dynamic risk. An innovation in longevity risk is the coinage of a new word 'geroscience' for the study of the interface between ageing and age-related diseases. Ageing is the most important risk factor for disease in the western world. In 2007, the U.S. National Institutes of Health granted $25 million to the Buck Institute for Age Research to create the new discipline of geroscience. This is an innovative interdisciplinary initiative, bringing together molecular biologists, neuroscientists, cell biologists, geneticists, endocrinologists, pharmacologists and mathematicians, to understand better the processes of ageing, and to elucidate their role in the mechanics of disease. Ageing and disease are now perceived to have similar mechanisms, as endorsed by the award of the 2009 Nobel Prize for Medicine to the discoverers of telomerase, an enzyme important both for cell replication and cancer growth. With the expectation of sustained progress, biologists of ageing have advised that we should be prepared to adjust for significant increases in lifespan during this century.

1.2 *A Review of Catastrophe Insurance Risk Modelling*

1.2.1 Over the past several decades, probabilistic risk assessment of extreme insurance risks has been radically transformed from an academic pursuit to a major international business sector. Career specialisation within the actuarial profession makes it helpful to review, for the benefit of life and pensions actuaries unfamiliar with property insurance, some key facts about catastrophe insurance modelling. Until the mid-1980s, the insurance risk management of natural hazards, such as floods, windstorms and earthquakes, was very simplistic, limited often to a crude deterministic estimation of

Probable Maximum Loss. Information on a U.S. property portfolio might not even have exposure data at state level, let alone city or address level, and there might be total ignorance as to building hazard vulnerability. Insurers who requested further information would lose market advantage to those unruffled by the exposure uncertainty. This dearth of information started to change with the advent of the desktop computer, when it became economically practical for better exposure information to be captured, and for the portfolio risk to be quantified probabilistically using the first generation of catastrophe insurance models run on personal computers.

1.2.2 A fundamental tenet of catastrophe modelling is that no historical time series of loss experience is adequate for catastrophe risk management. This has been an expensive recurrent lesson for the insurance industry. Both Hurricane Andrew, which swept through Florida in 1992, and the Northridge earthquake, which struck southern California in 1994, greatly exceeded prior Probable Maximum Loss estimates, and exposed the shortcomings of using finite historical time series of loss data for statistical projection of future loss. In the 1970s and 1980s, the annual California earthquake loss ratio was very small in most years, and peaked at about 130% in 1989. However, the loss ratio in 1994, the year of the Northridge earthquake, was 2,272.7% (Embrechts *et al*., 1997).

1.2.3 Catastrophe risk analysts approach risk assessment in a different methodological direction from statisticians, following the classic scientific paradigm in developing structural causal models of the underlying physical phenomena (Woo, 1999). In sequence, the underlying threat source is modelled, then the vulnerability to the threat is assessed, and the loss to an insurance portfolio is evaluated. Through construction of event-trees, risk is disaggregated into its component parts, which then become more amenable to parameterisation through objective technical analysis and informed expert judgement.

1.2.4 Rather than characterising the historical time series by a single deterministic projection, an ensemble of potential future events is constructed which collectively spans the range of future possibility. This involves teams of seismologists, meteorologists and hydrologists in developing stochastic models of earthquake, windstorm and flood occurrence, and mapping the damaging hazard footprints at a fine spatial scale. To complement the detailed modelling of the geographical footprints of future scenarios, emphasis is placed on the acquisition of high resolution data on exposure geography and hazard vulnerability. Earthquake risk, for example, is sensitive to the proximity of a building to an active fault, and its standard of seismic design. Effort to capture spatial and engineering design information is necessary in reducing the epistemic uncertainty associated with portfolio risk, and is now standard industry practice.

1.2.5 It is commonplace for natural catastrophe metaphors to be used to describe all manner of extreme risks where historical information may have

limited value for risk projection. For example, the hazard terms, 'seismic shock', 'tsunami' and 'hurricane', have all been applied to crises on Wall Street (see e.g. Kansas, 2009). The conceptual framework and computational methodology for modelling natural hazards has broad application to financial risks and has already been applied to man-made dangers to life, such as terrorism, and has been extended to cover pandemic disease (RMS, 2010). Indeed, excess mortality risk to a life insurance portfolio can now be quantified using detailed structural models of the principal drivers of excess mortality: natural hazards, terrorist attacks, and pandemics. This methodological advance beyond actuarial extrapolation of historical excess mortality experience has motivated its further development in general mortality modelling.

2. Individual Mortality Risk Modelling

2.1 *System for Prognostic Modelling*

2.1.1 Just as ever faster computer hardware speed has facilitated the development of natural hazard insurance modelling at an individual property level, so modern computer technology allows mortality modelling to be undertaken on an individual basis. At the time when catastrophe insurance modelling developed in the 1980s, actuarial interest in modelling mortality improvement was vitiated by prevailing high interest rates. But outside the insurance industry, clinical interest in mortality risk assessment was growing rapidly through findings of the Framingham Heart Study and other longitudinal health studies. Amongst the Framingham research milestones of the 1980s, high levels of high density lipoprotein (HDL) cholesterol were found to reduce the risk of death (Levy & Brink, 2005).

2.1.2 The clinical opportunities for applying medical modelling of mortality have encouraged the development of software tools to facilitate risk-informed decision-making. One initiative is the Cardiovascular Health Improvement Model (CHIME), outlined in Martin *et al*. (2008). Healthcare providers need to model future patterns of need for health services, and to identify the cost effectiveness of different intervention strategies. In clinical medicine, cardiovascular risk is a key criterion for identifying those who will most benefit from interventions designed to prevent cardiovascular disease and death. Utilising a variety of methods, including regression equations, scoring systems, Bayesian belief networks and neural networks, mortality and morbidity risk models have been increasingly encroaching on standard clinical practice (Puddu *et al*., 2009; JBS 2, 2005).

2.1.3 In extending the application domain to life insurance and pensions, this modelling tool has been expanded to cover a full range of disease risks. A key part of the design of the system is the ability to enable a choice of models to best fit the context of the simulation and the data

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available. This computer-intensive software simulation tool (PrognoSys), produces individual mortality risk curves allowing for personal medical and lifestyle risk factors, e.g. blood pressure, cholesterol, diabetes, smoking, alcohol consumption, body-mass-index, etc. According to an individual's lifestyle traits (e.g. smoker or not), and health status (e.g. diabetic or not), statistical survival curve projections can be made, informed and validated by extensive longitudinal medical studies of the principal life-threatening diseases. A schematic flow-chart of the mortality simulation model is reproduced in Figure 1.

2.1.4 Disease prevalence is markedly dependent on socio-economic class, and data might be mined down to the geographical level of a general practitioner's surgery. In the future, the acquisition of health factor data on pensioners may become no more surprising than data on property vulnerability. In the meantime, for very large pensions, there may be benefits in investing extra effort to enrich the individual data to increase the resolution on important risk factors that would refine the analysis, such as height and weight data, smoking and other lifestyle information.

2.2 *Individual Mortality Profiles*

2.2.1 A pension portfolio can be characterised in terms of the relative frequency of individualised population segments with specific risk factor characteristics, for example smokers and non-smokers, etc. The procedure for imputing risk factor distributions makes use of whatever information is made available, e.g. socio-economic class, postcode data, size of pension, as well as age and gender.

2.2.2 Exploratory longevity analysis can be conducted to assess the risk management implications of shifts in the mix of the portfolio, by generating a revised set of survival curves that would result from a new mix. To illustrate the potential knowledge gain from discriminating survival probabilities for different pensioner groups, Figure 2 compares, for a 50 year-old man, the probability of age of death by cause in an average smoker and non-smoker.

3. Prospective Mortality Risk Modelling

3.1 *Geroscience Advancement Stochastic Process*

3.1.1 The probability of death by age profiles in Figure 2 pertain to the current status of medical science and healthcare. To encompass the spectrum of potential mortality change developments, the mortality modelling tool must be coupled with a future projection which recognises that geroscience advancements are themselves intrinsically stochastic processes, marked by a considerable degree of randomness. In this section we breakdown possible advances into five generic categories for forecasting: lifestyle, health

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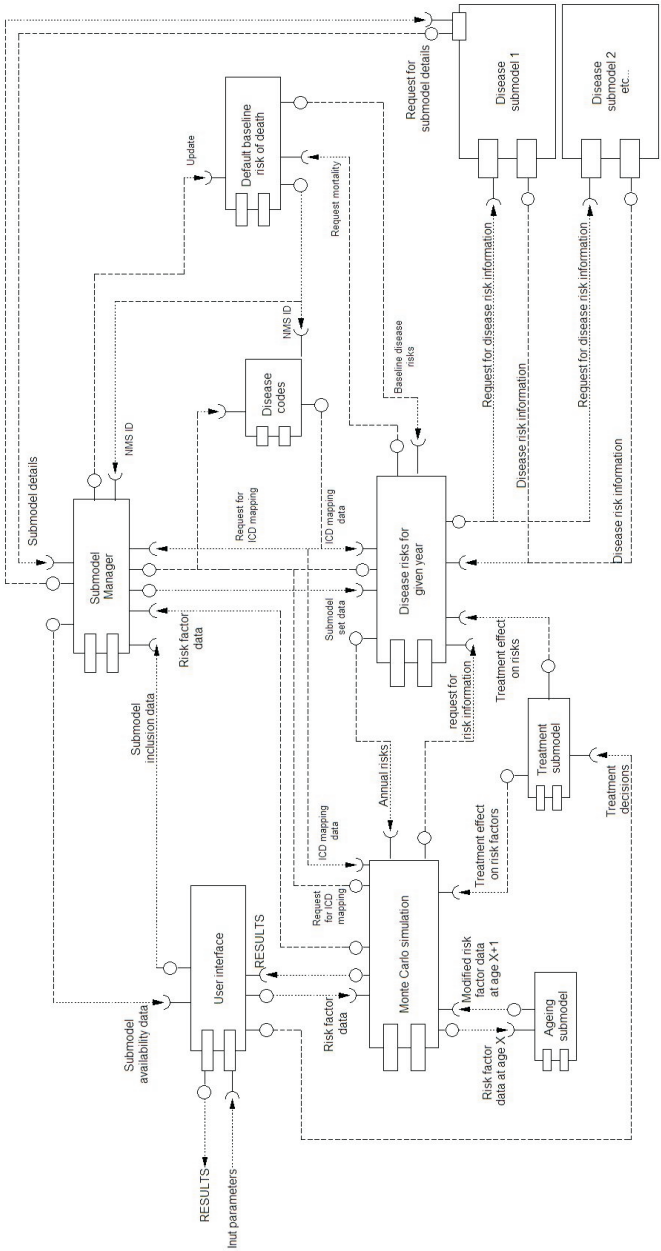

Figure 1. Schematic flow-chart of mortality model simulation

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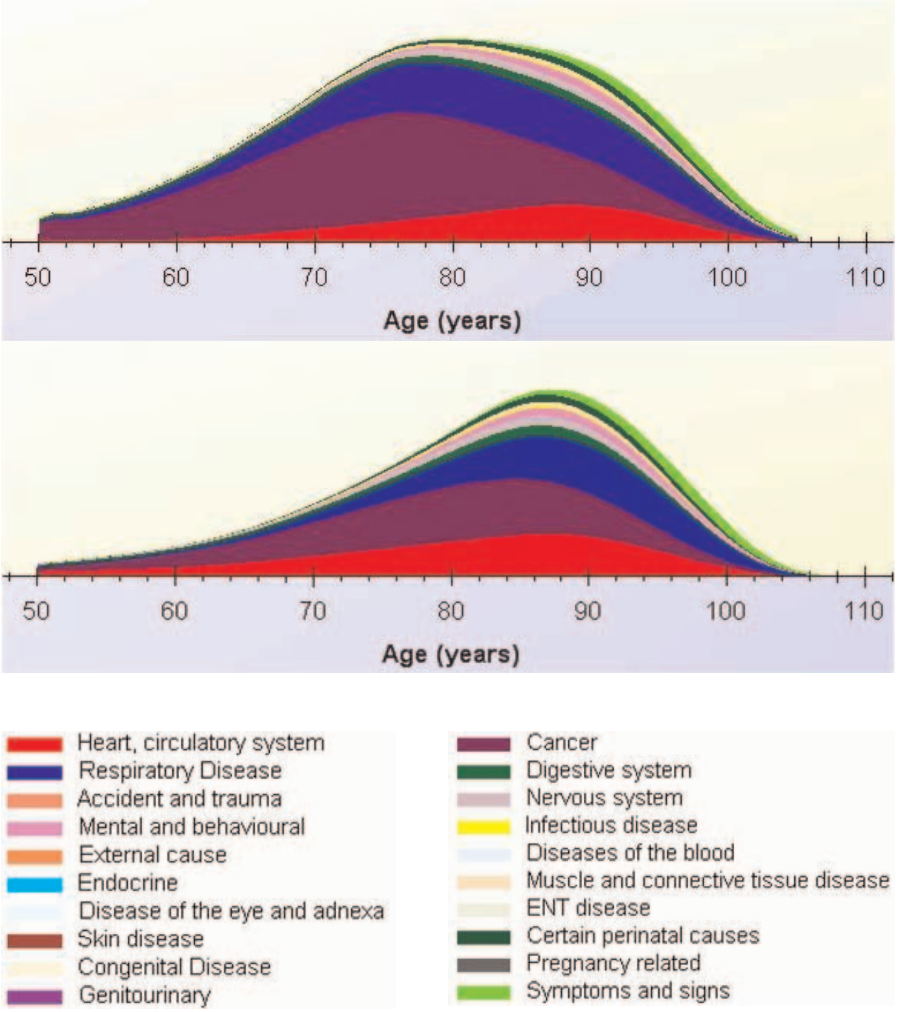

Figure 2. Comparison of probability of age of death by cause in an average smoking (upper graph), and non-smoking (lower graph), 50 year-old man

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environment, disease reduction, regenerative medicine and the biology of ageing.

3.2 *Personal Lifestyle Category*

3.2.1 The behavioural sciences of social psychology, consumer marketing, and health economics provide guidance for alternative future projections of lifestyle. Government intervention in promoting healthier lifestyles is a significant external factor. Extensive research into smoking trends (Future Foundation, 2005) and obesity trends out to 2050 (McPherson *et al*., 2005) provide the basis for stochastic modelling of U.K. smoking and obesity. Regarding smoking prevalence, this research suggests that as the proportion of smokers in the population falls, public attitudes to government intervention may become more positive, providing a popular mandate for more draconian restrictions.

3.3 *Healthcare Environment Category*

3.3.1 After the contributions of lifestyle change and medical advancement have been explicitly accounted for, the residual mortality change is associated broadly with the healthcare environment. This covers hygiene, sanitation, knowledge of health issues, availability of healthcare education, and attitude towards healthcare provision such as visiting the doctor.

3.4 *Drug Discovery Category*

3.4.1 This covers the medical discovery of new chemical entities, including medication and vaccines. The pace of discovery is hastened by the broadening international dimension of scientific collaborative research, escalating computational power for drug molecule design, and advanced laboratory technology. However, progress is subject to the economic law of diminishing returns.

3.5 *Regenerative Medicine Category*

3.5.1 This covers the emerging discipline of regenerative medicine, encompassing nanomedicine and stem cell therapy, potentially providing the means to replace cells damaged by trauma or devastating diseases such as Alzheimer's, Parkinson's, heart disease, cancer and diabetes.

3.6 *Biology of Ageing Category*

3.6.1 This covers the molecular genetics and systems biology of senescence, which is fundamentally linked with age-related diseases. The evolving understanding of the ageing process holds the promise of indicating strategies for maintaining homeostasis and slowing the rate of ageing, with the consequent prospect of human lifespan increases in the 21st century.

4. The Path of Medical Advancement

4.1 *Serendipity in Medical Discovery*

4.1.1 Since the Renaissance, every century has introduced medical innovation and advanced progress in medical science. With advanced medical research conducted collaboratively on a global scale, the degree of innovation in prospect for the 21st century holds particularly great promise. For mortality change to be projected into an uncertain medical future, the underlying stochastic process of medical advancement needs to be modelled. This requires construction of a meta-model of medical progress.

4.1.2 The unpredictability of medical research is a frustration for those whose lives depend on medical discovery. Yet, it is precisely this characteristic unpredictability which facilitates the development of a stochastic model. Serendipity plays some part in all technological progress, no more so than in medical progress. Serendipity has been defined as the art of finding what we are not looking for by looking for what we are not finding (Quéau, 1986). From penicillin to tamoxifen, the history of medical breakthroughs (e.g. Le Fanu, 1999) is replete with examples of accidental unforeseen discovery. Indeed, an entire book (Meyers, 2007) has been written on the role of serendipity in modern medical breakthroughs. Research to find a new drug to treat angina led to Viagra. As an example of complementary discovery, breast cancer research has led to new insights into brain development and vice versa. Making an important discovery whilst trying to discover the very opposite is a research student's lottery jackpot. This was the fortunate experience of Jay Mclean, who discovered the anti-coagulant heparin, whilst endeavouring to find a blood clotting agent.

4.1.3 Many of the most important breakthroughs in medicine have come from unexpected sources in seemingly unrelated fields, and have depended on luck, accident and error. But human sagacity is required to recognise an opportunity opened by serendipity. As Louis Pasteur said, chance favours only the prepared mind. A crucial factor in medical progress is continued international funding for creative research. Investigations that seemed totally irrelevant to any practical objective have yielded most of the major discoveries of medicine. Arthur Kornberg, Nobel Laureate in Medicine for his DNA discoveries, has graphically described the way that progress is actually made, '*Medical research is still more a game of pool than billiards. You score points regardless of which pocket the ball goes into*' (Kornberg, 2005). A scatter-gun metaphor for medical progress is thus more apt than the linear model of steady advancement that is often the public perception. Progress does not follow a straight path from point A to point B. Rather, point X is reached in the course of looking to reach point Y.

4.1.4 The serendipitous random walk of medical science is the modelling paradigm for the latter medical progress in categories 3, 4 and 5, that is, drug discovery, regenerative medicine, and the biology of ageing. For each of

these three categories, a metric can be defined which gauges progress relative to the maximum mortality reduction potentially achievable.

4.1.5 Let D be the number of different disease chapters. For each specific disease j, the maximum potentially achievable mortality reduction associated with the kth category ($k = 3, 4, 5$) is designated as $V_k(j)$. At future time t, progress in mortality reduction can be gauged relative to $V_k(j)$. Then the stochastic process for the kth category $G_k(t)$ can be defined as the quotient:

$$G_k(t) = \frac{\sum_{j=1}^{D} C_j(t) V_k(j)}{\sum_{j=1}^{D} V_k(j)} \tag{1}$$

where the coefficients $C_j(t)$ which chart relative progress in tackling disease j at time t, vary from 0 to 1. $G_k(t)$ asymptotes to 1 for large times t, as the $C_j(t)$ approach 1 and the maximum achievable mortality reduction is achieved. The practical time period for longevity forecasting is up to fifty years, which is sufficiently short for this asymptote not to be attained for any category.

4.1.6 The formal analysis of the scientific discovery process (Cheng, 2001) suggests that most computational models of discovery can be conceptualised as performing a recursive search of a space of possible states. This is particularly true of modern medical research, where laboratory experiments address the complexity of systems biology and face the daunting combinatorial problem of testing for a wide variety of factors, which may be clinically effective either singly or in specific combinations.

4.1.7 Accordingly, over and above the smooth evolution of progress, a random walk component is added to reflect the serendipitous heuristic search aspect of scientific discovery; the haphazard aspects of the clinical trials and regulatory process; the possibility of latent side-effects and litigation; medical research malpractice; and socio-ethical objections to the new dimensions in medical treatment. The procedure for simulating a trajectory for advances might be a stochastic process $G_k(t)$ based on the generic dynamical equation below

$$\frac{dG_k(t)}{dt} = f(t, G_k(t)) + \sigma(t, G_k(t)) \cdot W(t). \tag{2}$$

4.1.8 This includes a noise factor $W(t)$ which reflects the inherent randomness in each stochastic process, and makes deterministic forecasting of medical advances impossible. As exemplified by gene therapy, negative as well as positive developments happen. Stochastic modelling at the meta-level of medical progress, $G_k(t)$ contrasts with the construction of top-down stochastic models for mortality $\mu(t)$ itself (e.g. Bauer *et al.*, 2010), which do not make explicit the specific causal drivers of mortality change:

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$$d\mu(t) = a(t) \cdot dt + \sigma(t) \cdot dW(t). \quad (3)$$

The equation for $G_k(t)$ is discretised with time step Δ as:

$$G_k(t + \Delta) - G_k(t) = f(t, G_k(t)) \cdot \Delta + \sigma(t, G_k(t)) \cdot W(t) \cdot \Delta. \quad (4)$$

4.1.9 An appropriate and practical discretisation time interval for longevity risk simulation is several years. Given diligent regulatory healthcare oversight, and the imperative for new discoveries to do no harm, this is the shortest practical time frame in which notable developments in mortality improvement are recognised to take place.

4.1.10 In his history of the rise and fall of modern medicine, Le Fanu (1999) has argued that the law of diminishing returns applies to medical research. This can be introduced by following the health econometric model of Lichtenberg (2005), where mortality improvement due to drug discovery is represented as a logarithmic function of medical progress, specifically the cumulative number of New Chemical Entities (NCE). This incorporates the law of diminishing returns in a transparent and tractable manner.

4.1.11 Lichtenberg's statistical analysis of the impact of new drug launches on longevity provides an empirical rationale for a simple adaptation of a linear trend model, to include just one additional parameter b which modulates the trend β_k over time. The larger this modulation parameter, the greater is the deviation from linearity at large times. Defining the trend uncertainty ratio as $u_k = \sigma_k/\beta_k$ and R as a random number deviate drawn from the standard Normal Distribution:

$$G_k(t + \Delta) - G_k(t) = \Delta \cdot \beta_k \frac{1}{[1 + bt]}(1 + u_k R). \quad (5)$$

4.1.12 The parameter b is varied within the simulation ensemble, and is different for the three categories: drug discovery, regenerative medicine, and the biology of ageing. The more mature the research domain, the larger b should be, reflecting the increasing economic price of innovation.

4.1.13 This model encodes a representation of the substantial fortuitous component to medical progress, and generates a cone of uncertainty around future projections: the more distant the time horizon, the greater is the possible range of progress. The cone is bounded by estimates of the least and greatest plausible progress trend rate. The uppermost plausible trend rate can be estimated from assessing the optimal system logistics of the complete research and development cycle from animal laboratory to human trials, to regulatory approval and clinical usage. The relative likelihood of alternative trend rates may be estimated by extrapolating recent experience, exploring event-trees of scientific progress paths, and crowd-sourcing the spread of expert medical science opinion. Simulation of $G_k(t)$ trajectories yields a

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swathe of alternative possible realisable scenarios, which collectively span the space of mortality improvement from geroscience advancement.

5. Mortality Improvement Risk Metrics

5.1 *Longevity Exceedance Probability*

5.1.1 The stochastic modelling methodology outlined in Section 4 lends itself to the computation of probabilistic risk metrics, insightful for longevity risk management. Where risk criteria or guidelines are expressed in terms of an extreme return period, e.g. 200 years, this methodology provides a coherent quantitative framework for prudent portfolio-specific risk management. In the absence of this type of probabilistic risk framework, the adequacy of a risk management strategy can be difficult to assess systematically. In particular, when the extreme stress values are selected rather arbitrarily, deterministic scenario analysis may be no more satisfactory in respect of mortality improvement in the 21st century than it has been for earthquake magnitude or hurricane landfall intensity in the last century.

5.1.2 The Solvency Capital Requirement (SCR) under Solvency II is set at 99.5% VaR over one year. The 99.5% criterion is equivalent to a 1-in-200 probability. Whereas various mortality risk models have been adapted for SCR analysis (e.g. Börger, 2009), the medically-based approach proposed here is well suited not just to facilitate this computation in accordance with state-of-the-art medical knowledge, but to identify the key longevity factors which govern this tail risk. Furthermore, it is capable of relating back in risk terms the stipulated longevity shock of a 25% reduction of mortality.

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