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REAL WAGE RIGIDITY, HETEROGENEITY, AND THE BUSINESS CYCLES

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This paper investigates the quantitative implications of real wage rigidities and heterogeneity for two long-lasting puzzles in the business cycle literature: the low correlation between total hours worked and labor productivity and the large volatility of the labor wedge, defined as a gap between the marginal rate of substitution of aggregate leisure for aggregate consumption and the marginal product of aggregate labor. I shed light on these issues by extending a heterogeneous-agent model with an indivisible labor supply choice to real wage rigidities. I find that a small amount of real wage stickiness would be sufficient to resolve both anomalies when long-term wage contracts and heterogeneity are taken into account.

Keywords: Real Wage Rigidity, Heterogeneity, Labor Wedge, Hours–Productivity Correlation

1. INTRODUCTION

Understanding labor market dynamics is important in the business cycle literature since labor income is a main source of total income; hence, fluctuations in variables related to labor market may directly affect economic welfare. Although the equilibrium business cycle models based on the representative agent (RA) have had a lot of success in accounting for most of the stylized facts of business cycles, they cannot account for some important issues regarding labor market dynamics. For example, the high volatility of the labor wedge, defined as a gap between the marginal rate of substitution (MRS) between consumption and leisure and the aggregate productivity (or marginal product of labor, MPL), is a well-known puzzle in the business cycle literature. In addition, the low correlation between aggregate hours of work and aggregate labor productivity is still an open question. These two puzzles are central issues in the literature on the

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real business cycle (RBC) theory as they are closely related to welfare costs of the business fluctuations [Gali et al. (2007) and Chang and Kim (2007)].[1] This study addresses both anomalies simultaneously by introducing heterogeneity[2] and real wage rigidities, which have often been abstracted in the neoclassical frameworks.[3] Once heterogeneity is taken into account, aggregate variables are not determined by the representative household's optimality condition, and they are nothing but the sum of individual variables. In this sense, heterogeneity across economic agents connotes aggregation bias, and analyzing behaviors of agents at the individual level is crucial to understand the dynamics of the aggregate variables. Chang and Kim (2007), for instance, find that aggregation errors induced by heterogeneity of individual households endogenously generate the labor wedge. Besides heterogeneity, according to Gali et al. (2007) and Shimer (2009), labor market frictions including real wage rigidities are the key factors for labor market dynamics. However, most of the existing studies on wage stickiness are based on the RA [Cho and Cooley (1995), Cho et al. (1997), Gali et al. (2007), Uhlig (2007), and Abbritti and Weber (2010)], and there have been surprisingly few attempts to analyze the implications of real wage rigidities for the dynamics of labor markets in the context of HA economies.[4] Therefore, this study explores the quantitative relevance of wage rigidities and heterogeneity for labor market dynamics focusing on the two enduring anomalies.

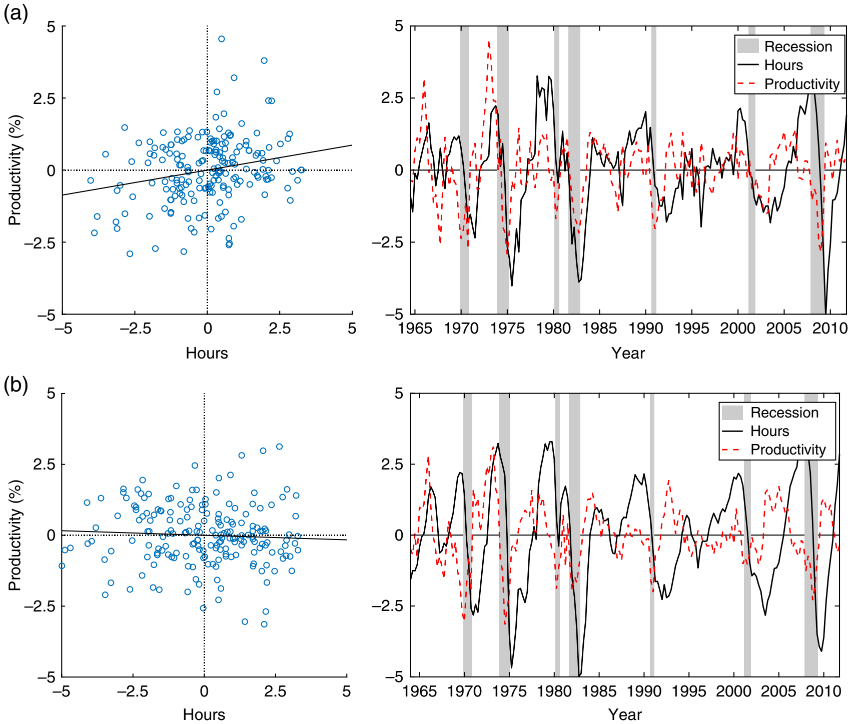
To this end, I extend a HA model with an indivisible labor supply choice to real wage rigidities. There are three main features in the model economy. First, a household is assumed to not fully insure against idiosyncratic productivity shocks that she faces: asset markets are incomplete as in Huggett (1993) and Aiyagari (1994). This feature generates rich heterogeneity across households' individual characteristics, such as employment, wealth, income, and consumption. Second, it is assumed that a labor supply decision for a household is indivisible, following Hansen (1985) and Chang and Kim (2007). It is well known that extensive margins of hours worked are important to explain the variation in total hours of work. Third, the model economy allows for the presence of real wage rigidity following Gali et al. (2007) and Shimer (2009), who suggest that labor market frictions including wage rigidities are important explaining the labor wedge. I assume that wage rigidities arise from wage contracts agreed to by households and firms as in the work of Cho and Cooley (1995) and Cho et al. (1997).

The main findings of this study are summarized as follows. First, I find that the correlation coefficient between hours and productivity decreases when the index of wage stickiness or the length of wage contracts increases. This is because hours and productivity no longer move along the labor supply curve due to the wage contract. Heterogeneity also helps explain the low correlation between two macro variables, since the labor supply curve fluctuates in response to aggregate shocks [Chang and Kim (2007)]. Second, I also find that the volatility of labor wedge increases with the index of wage rigidity or the wage contract length. More volatile hours worked and the lower correlation between aggregate hours and productivity, which are induced by wage rigidity, produce the larger cyclical movement in the labor wedge.[5] Heterogeneity also plays a role in accounting for the large volatile wedge since the wedge is endogenously generated when heterogeneity is taken into account. From these findings, I conclude that a small amount of real wage rigidity would be enough to resolve both puzzles when long-term wage contracts and heterogeneity are considered.

A large body of the literature has provided various explanations for the low correlation between hours and productivity and the large volatile wedge. Key contributions are Benhabib et al. (1991), Cho and Cooley (1995), Hall (1997), Gali et al. (2007), and Shimer (2009). Benhabib et al. (1991) and Hall (1997) consider exogenous shocks to the labor supply schedule for an explanation for the puzzles. Benhabib et al. (1991) incorporate home production technology into a standard RBC framework, while Hall (1997) analyzes preference shifts as an exogenous shock to the labor supply curve. Cho and Cooley (1995) incorporate nominal wage contracts and monetary shocks into a RBC model, and their model can generate the low correlation between hours and productivity. Other important works on the labor wedge are Gali et al. (2007) and Shimer (2009). Gali et al. (2007) study the efficiency gap (the labor wedge) as a measure of the welfare costs in the labor market based on a New Keynesian model economy and find that labor market frictions are the key factor for the gap. Furthermore, Shimer (2009) finds that search frictions combined with real wage stickiness endogenously produce the labor wedge. All these papers are based on the RA economy. I contribute to this literature by developing a model with HAs to account for the two anomalies as in Chang and Kim (2007) and Takahashi (2014).

This study is also closely related and complementary to a chain of quantitative papers based on a HA model for the two puzzles. Chang and Kim (2007) provide one of the first frameworks linking heterogeneity to the labor wedge. They develop a model economy of incomplete capital markets with discrete labor supply and idiosyncratic labor productivity shocks and show that the measured labor wedge arises endogenously due to rich heterogeneity across agents and incomplete markets.[6] Takahashi (2015) builds a HA dynamic stochastic general equilibrium (DSGE) model in the presence of time-varying wage risks. He finds that fluctuations in idiosyncratic wage risks resolve the hours–productivity puzzle and the large cyclical movement in the wedge. Differentiating from Chang and Kim (2007) and Takahashi (2014), the main contribution of this article is that I explicitly consider real wage rigidities, including long-term wage contracts, in a HA DSGE model.

The remainder of this paper is organized as follows. Section 2 summarizes the two puzzles using US aggregate data. In Section 3, I will build an incomplete asset market model with real wage rigidities and HAs. The parameter values are determined in Section 4. Section 5 summarizes the findings of the model economies. In Section 6, I examine the role of heterogeneity in the two anomalies. Section 7 concludes.



Note. All variables are logged and detrended by the Hodrick–Prescott filter with a smoothing parameter of 1600.

FIGURE 1. Hours and productivity. (a) Household survey and (b) establishment survey.

2. EMPIRICAL FACTS: TWO PUZZLES

In this section, I summarize empirical evidence regarding the two puzzles using US aggregate data spanning from 1964:I to 2011:IV. Other than total hours of work, US macroeconomic data used in this paper are standard as in the business cycle literature.[7] Since I believe that choosing a measure for hours worked is most important to compute reliable aggregate productivity and the labor wedge, I use two types of datasets for hours of work: the household survey (HS) and the establishment survey (ES).[8] The ES is taken from the Current Employment Statistics (CES), which is conducted by the Bureau of Labor Statistics. The source for the household-level survey data is Cociuba et al. (2009). They use the Consumer Population Survey to compute total hours worked.

2.1. Puzzle 1: Low Correlation between Productivity and Hours

The relation between the aggregate labor productivity and aggregate hours is summarized in Figure 1. The aggregate labor productivity is computed as Y/H, where

Y is the real gross domestic production (GDP) in the private business sector, and H is the aggregate hours of work. The upper panel of Figure 1 exhibits the correlation between productivity and hours, which are computed with the HS. As observed in the scatter plot in the top-left panel, productivity does not seem to be correlated with hours. The correlation coefficient between the two macro variables is very small (0.23). The consistent pattern is also seen in the upper right panel, which shows the cyclical components of productivity and hours over long periods of time. Hours often fluctuate in the opposite direction to productivity, which may cause the low correlation between the two time series. A similar relationship is also found in the bottom panel of Figure 1, where productivity and hours are computed using the ES. The correlation coefficient between the two variables with the ES is −0.06.

2.2. Puzzle 2: Large Volatile Labor Wedge

Following Chang and Kim (2007) and others, the labor wedge is defined as the gap between the MRS of aggregate leisure for aggregate consumption (*MRS*) and the marginal product of aggregate labor (*MPL*), that is,

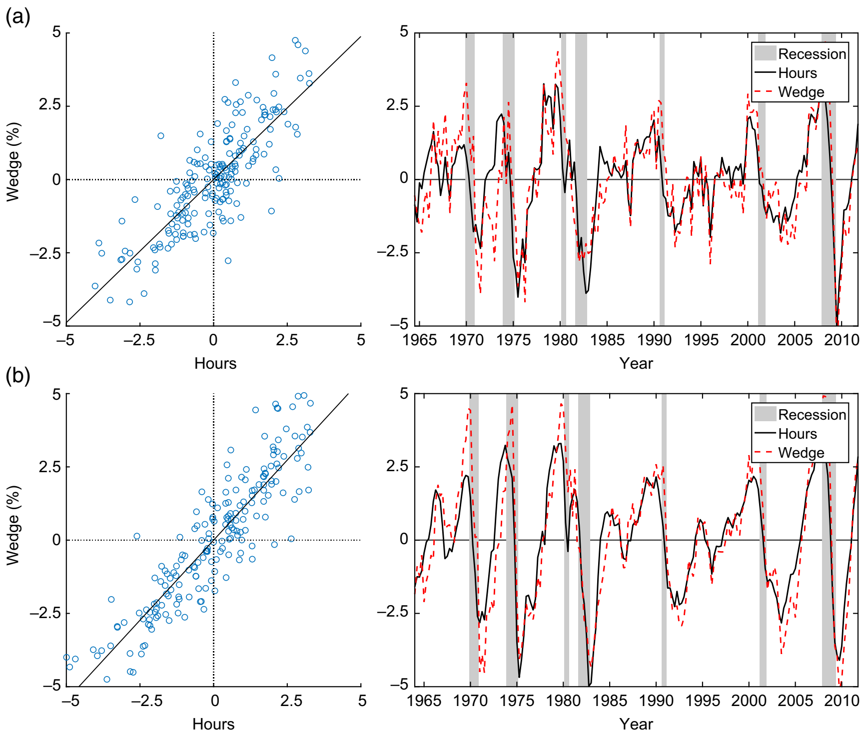
$$\log(wedge) = \log(CH^{\frac{1}{\chi}}) - \log(Y/H), \tag{1}$$

where C is the aggregate expenditures on nondurable goods and services, and χ is the aggregate labor supply elasticity.[9] $MRS(= CH^{\frac{1}{\chi}})$ is derived from the functional form of household's utility which will be discussed in Section 3. Figure 2 exhibits the relation between the aggregate hours and the labor wedge. Regardless of what survey data I use, the correlation coefficient of the wedge with hours of work is high: 0.82 (0.90) in the HS (in the ES). More importantly, the labor wedge is highly volatile. The relative volatility of the wedge to Y is 0.86 (1.08) in the HS (in the ES), which is larger than that of aggregate hours of work, 0.72 (0.89). These findings are consistent with previous work, such as that of Chang and Kim (2007), Takahashi (2014), and Karabarbounis (2014).

To sum up, the correlation between aggregate productivity and hours worked is low and the labor wedge is large volatile.[10] As discussed in Chang and Kim (2007), these two facts are closely related. According to equation (1), the labor wedge arises since hours worked and productivity show a fairly low correlation. In the next section, I present a DSGE model economy with heterogeneous households and real wage rigidities to account for the two puzzles.

3. THE MODEL

I build a simple DSGE model with a large population of heterogeneous households in labor productivity and with real wage rigidities. It is assumed that households cannot fully insure against individual productivity shocks, which follow a stochastic process. That is, capital markets are incomplete following



Note. The labor wedge is computed from equation (1). All variables are logged and detrended by the Hodrick–Prescott filter with a smoothing parameter of 1600.

FIGURE 2. Hours and labor wedge. (a) Household survey and (b) establishment survey.

Huggett (1993) and Aiyagari (1994). A labor supply decision for each household is assumed to be indivisible as in Hansen (1985) and Chang and Kim (2007). In particular, the model economy allows for the presence of real wage rigidities following Gali et al. (2007) and Shimer (2009), who argue that labor market frictions, including real wage stickiness, are key factors for the labor wedge. As in Cho and Cooley (1995) and Cho et al. (1997), real wage rigidities arise from wage contracts agreed to by households and firms. In these senses, the model economy in this paper is an extended version of Chang and Kim (2007), who feature extensive margins of labor supply in a HA model. It is also in line with Cho and Cooley (1995), Cho et al. (1997), Gali et al. (2007), and Shimer (2009) in that wage rigidities are explicitly considered in the model economy.

3.1. Real Wage Contracts

In this subsection, I discuss the wage contract rule. Related papers are Cho and Cooley (1995), Cho et al. (1997), Janko (2007), and Janko (2008). It is assumed that real wage rigidities arise from wage contracts agreed to by households and firms. Following Hall (2005) among others, I use a natural formulation of the contract wages: the contract wage is a weighted average of the expected wage and the desired wage. Consider a k-period wage contact. As in Janko (2007), wage contracts are partially set in a synchronized manner: the expected wages for period $t-k+1$ to t are determined at the end of period $t-k$, whereas the desired wage is determined based on the information available in each period. To make the wage contract operational in an internally consistent rational-expectations model of the economy, we need more explicit assumptions about how households and firms determine the contract wages. One of typical approaches in the literature is that wages are determined in such a way that *markets are expected to clear* during the period where the wage applies [Gray (1976), Blinde and Mankiw (1984), and Cho and Cooley (1995)]. I follow this *expected-market-clearing approach* and assume that the expectation formation for wages in the economy is based on the forecasting function for wage rates from the economy with no wage rigidities. Specifically, the forecasting function for the market wage rate, w, is assumed to take log-linear functions of aggregate capital K and aggregate productivity Z[11]:

$$\log w_t = b_0 + b_1 \log K_t + b_2 \log Z_t. \tag{2}$$

The coefficients for the forecasting function can be obtained from the market-clearing economy, where there are no wage rigidities. Suppose that the firms and the households are in the period of $t-k+j$, where j is defined as follows:

$$j = \begin{cases} q & \text{for } q > 0, \\ k & \text{for } q = 0, \end{cases} \tag{3}$$

where q is the remainder after division of t by k.[12] Now I formalize how the expected and desire wages are determined. The expected wages for k successive periods are set simultaneously at the end of period $t-k$. w^e_{t-k+j} denotes the expected wage rate in period $t-k+j$ conditional on the information available in period $t-k$. Following Cho and Cooley (1995), w^e_{t-k+j} is computed as an expected value of equation (2) such that:

$$\log w^e_{t-k+j} = \mathbb{E}\left[b_0 + b_1 \log K_{t-k+j} + b_2 \log Z_{t-k+j} \mid \Omega_{t-k}\right], \tag{4}$$

where $\Omega_{t-k} \equiv (K_{t-k}, Z_{t-k})$.[13] Similarly, the desired wage rate, w^o_{t-k+j}, can be defined as:

$$\log w^o_{t-k+j} = b_0 + b_1 \log K_{t-k+j} + b_2 \log Z_{t-k+j}. \tag{5}$$

That is, the desired wages are also set using equation (4) but based on the information available in period $t-k+j$. This implies that the desired wage is set according to the wage rate rule from the market-clearing economy [Cho and Cooley (1995)]. Now I define real wage rigidity by employing a notion of a wage contract rule following Hall (2005). I assume that the real contract wage, w^c_{t-k+j}, is a weighted average of the expected wage and the desired wage:

$$w^c_{t-k+j} = \lambda w^e_{t-k+j} + (1-\lambda) w^o_{t-k+j}, \tag{6}$$

where $0 \leq \lambda \leq 1$ is the index of real wage rigidity.[14] Equation (6) implies that if there is no wage rigidity in the model economy ($\lambda = 0$), the economy collapse into the frictionless one: the desired and the contract wages are the same as the market equilibrium wage. According to equations (4) and (5), w^e_{t-k+j} is expressed by the functions of K_{t-k} and Z_{t-k}, and w^o_{t-k+j} is determined by the current state variables (K_{t-k+j} and Z_{t-k+j}). Thus, the contract wage rate, w^c_{t-k+j}, is a function of the set of the past information (K_{t-k}, Z_{t-k}) and the set of the current information (K_{t-k+j}, Z_{t-k+j}). Intuitively, the contract wage depends on two parameters: the degree of wage rigidity, λ, and the length of wage contracts, k.

3.2. Representative Firm

The production technology for the representative firm is represented by the constant returns-to-scale Cobb–Douglas function:

$$F(K, L, Z) \equiv Z K^{\theta} L^{1-\theta},$$

where K, L, and θ denote the aggregate capital, aggregate effective labor, and capital income share, respectively. The aggregate productivity, Z, follows an autoregressive (AR(1)) process in logs:

$$\log Z_{t+1} = \rho_Z \log Z_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma_Z^2).$$

Following Cho and Cooley (1995) and Cho et al. (1997), given the wage contract rule, it is assumed that the representative firm decides how much aggregate effective labor it demands. In other words, the households cede the right to make decisions about aggregate labor to the firm. Hence, given the contract wage rate, the aggregate effective labor L is determined by firm's profit maximization condition. Of course, given w^c and the real rental price for capital r, the demand for capital is also determined by the first-order conditions. That is,

$$w^c = (1-\theta) Z (K/L)^{\theta},$$

and

$$r + \delta = \theta Z (K/L)^{\theta - 1},$$

where δ is the capital depreciation rate.

3.3. Heterogeneous Households

Each household maximizes the expected lifetime utility:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\log c_t - \Psi \frac{h_t^{1+1/\eta}}{1+1/\eta}\right)\right],$$

where $0 < \beta < 1$ is the time discount factor, c_t is the consumption, h_t is the hours of work, $\Psi > 0$ is a parameter for disutility from working, and η is the micro

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Frisch elasticity of labor supply. A labor choice by each household is assumed to be indivisible following Hansen (1985), Rogerson (1988), and Chang and Kim (2007). In other words, a household makes a decision to work for a fixed amount of hours ($h = \overline{h}$) or none ($h = 0$). Accordingly, there are two occupational choices in the model economy: an employed worker and a nonemployed worker.

It is assumed that households face idiosyncratic labor productivity shocks, x, which evolves with transition probabilities $P_x(x'|x) = Pr(x_{t+1} = x'|x_t = x)$. Asset markets are incomplete following Huggett (1993) and Aiyagari (1994): households cannot issue any assets contingent on their future idiosyncratic risks, x. Borrowing is allowed for a household, but the borrowing limit, $\overline{a}$, is fixed. I assume that the aggregate shocks and the individual shocks are independent of each other. In addition, the idiosyncratic risks are assumed to follow a log-AR(1) process:

$$\log x' = \rho_x \log x + \varepsilon_x, \quad \varepsilon_x \sim N(0, {\sigma_x}^2).$$

Suppose that a household is in period $t - k + j$, where j is defined in equation (3). For each household, the individual state is the vector $\gamma \equiv (a, x)$, where a denotes assets that a household holds. As mentioned above, since w^c_{t-k+j} is a function of μ_{t-k}, Z_{t-k}, μ_{t-k+j}, and Z_{t-k+j}, the aggregate state is the vector $\Gamma \equiv (\mu, Z, \mu_{-k}, Z_{-k}, j)$, where μ is a joint distribution of assets and idiosyncratic labor productivity across households, and $Z_{-k}(\mu_{-k})$ is the aggregate productivity shock (the joint distribution) in the period of $t - k$. Notice that the future aggregate state is the vector $\Gamma' \equiv (\mu', Z', \mu_{-k}, Z_{-k}, j + 1)$ when $j < k$ since the household's expectation is still based on information in the period of $t - k$, while it is the vector $\Gamma' \equiv (\mu', Z', \mu, Z, 1)$ when $j = k$ since, in period $t + 1$, the base period for the expectation will change from the period of $t - k$ to the period of t.

Under the wage contract, households should provide the efficiency units of labor that the firm demands since they agree to cede the right to decide the aggregate effective labor to the representative firm. In the RA model, it is easy to fulfill the wage contract for the representative household since all identical households are employed and supply the same amount of labor. In the HA model economy, however, the "employment allocation rule"—how different households allocate their hours—matters as reservation wages are different across households. Since the employment status of a household affects her consumption and saving decisions, a reasonable assumption of labor supply decisions for households is a critical issue. To this end, I introduce two sequential steps for households' problems. The first step is the employment allocation rule: given the wage contract (or the aggregate effective labor), households make a decision on their employment. The second step is the standard consumption-saving decisions: given the wage contract and the employment decisions, households decide how much they consume and save.

3.3.1. Step 1: Employment decision (employment allocation rule). Since households agree to cede the right to decide an aggregate efficient labor to the representative firm, they should provide effective labor that the firm demands, denoted by L^c, hereafter. If the economy is based on the RA assumption, all identical households should supply the same amount of labor, L^c. However, in the HA model economy, since each household has a different reservation wage, it is needed to have an additional assumption how different households allocate their hours differently to fulfill the wage contract. Hence, I assume that households are employed in ascending order by their reservation wage rates until they provide L^c. Specifically, suppose that $\widetilde{w}$ is the wage rate at which households endogenously provide effective labor of L^c. In other words, a household, whose reservation wage rate is less than or equal to $\widetilde{w}$, should work. This implies that households make labor supply decisions under $\widetilde{w}$. Formally, to fulfill the wage contract, a household, whose reservation wage rate, $w^R(\gamma, \Gamma)$, is less than or equal to $\widetilde{w}$, should work:

$$h(\gamma, \Gamma) = \begin{cases} \bar{h} & \text{if } \widetilde{w} \geq w^R(\gamma, \Gamma), \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Thus, even if employment decisions are made endogenously under $\widetilde{w}$, some are voluntarily (non-)employed while others are involuntarily (non-)employed because of the wage contracts. Of course, the size of the involuntary labor choices depends on the degree of wage rigidity or the length of wage contracts. Let us consider a recursive equilibrium to characterize $\widetilde{w}$ and labor supply choices for households, $h(\gamma, \Gamma)$. The value function for an employed worker $\widetilde{V}^W(\gamma, \Gamma)$ is

$$\widetilde{V}^W(\gamma, \Gamma) = \max_{c,a'} \left\{ \log c - \Psi \frac{\bar{h}^{1+1/\eta}}{1+1/\eta} + \beta \mathbb{E}\left[\widetilde{V}(\gamma', \Gamma')\right] \right\}$$

subject to

$$c = \widetilde{w} x \bar{h} + (1+r)a - a', \ c \geq 0, a' \geq \bar{a},$$

and

$$\mu' = \Theta(\Gamma),$$

where Θ denotes a transition operator for μ, and $\bar{a}$ is a borrowing constraint that limits the fixed amount of debt.

The value function for a nonemployed worker, denoted by $\widetilde{V}^N(\gamma, \Gamma)$, is

$$\widetilde{V}^N(\gamma, \Gamma) = \max_{c,a'} \left\{ \log c + \beta \mathbb{E}\left[\widetilde{V}(\gamma', \Gamma')\right] \right\}$$

subject to

$$c = (1+r)a - a', \ c \geq 0, a' \geq \bar{a} \text{ and } \mu' = \Theta(\Gamma).$$

The household's employment decision $h(\gamma,\Gamma)\in\left\{0,\bar{h}\right\}$ is defined as:

$$\widetilde{V}(\gamma,\Gamma)=\max_{h\in\left\{0,\bar{h}\right\}}\left\{\widetilde{V}^{W}(\gamma,\Gamma),\widetilde{V}^{N}(\gamma,\Gamma)\right\}. \quad (8)$$

Of course, the employment decision determined by equation (8) is consistent with equation (7).

3.3.2. Step 2: Consumption-saving decision. The next step of a decision for a household is a consumption-investment decision. Given employment status $h(\gamma,\Gamma)$, decisions for consumption and savings are made under the contract wage w^c such that:

$$V(\gamma,\Gamma)=\max_{c,a'}\left\{\log c-\Psi\frac{h(\gamma,\Gamma)^{1+1/\eta}}{1+1/\eta}+\beta\mathbb{E}\left[V(\gamma',\Gamma')\right]\right\}$$

subject to

$$c=w^c xh(\gamma,\Gamma)+(1+r)a,\ c\geq 0, a'\geq\bar{a}\text{ and }\mu'=\Theta(\Gamma).$$

To sum up, employment decisions are made under $\widetilde{w}$ to supply the labor the firm demands, while consumption-investment decisions are under w^c. Notice that the real return to capital, r, is the same for the two decisions.

3.4. Definition of Equilibrium

A recursive competitive equilibrium is a set of inputs $\{K(\Gamma),L^c(\Gamma)\}$, a set of factor pricing functions $\{w^c(\Gamma),r(\Gamma),\widetilde{w}(\Gamma)\}$, a set of value functions $\{\widetilde{V}^W(\gamma,\Gamma),\widetilde{V}^N(\gamma,\Gamma),\ \widetilde{V}(\gamma,\Gamma),\ V(\gamma,\Gamma)\}$, a set of policy functions $\{h(\gamma,\Gamma),c(\gamma,\Gamma),a'(\gamma,\Gamma)\}$, and a forecasting function $\Theta(\Gamma)$ such that:

1. Real wage contracts.
   - Contract wage setting rule:

   $$w^c(\Gamma)=\lambda w^e(\Gamma)+(1-\lambda)w^o(\Gamma).$$

   - Employment decisions: given $\widetilde{w}(\Gamma)$ and $r(\Gamma)$, the optimal employment decision rule $h(\gamma,\Gamma)$ solves the value functions $\widetilde{V}^W(\gamma,\Gamma)$, $\widetilde{V}^N(\gamma,\Gamma)$, and $\widetilde{V}(\gamma,\Gamma)$. That is, a household whose reservation wage rate $w^R(\gamma,\Gamma)$ is less than or equal to $\widetilde{w}$ should work

   $$h(\gamma,\Gamma)=\begin{cases}\bar{h} & \text{if } \widetilde{w}\geq w^R(\gamma,\Gamma),\\ 0 & \text{otherwise.}\end{cases} \quad (9)$$

   - Households supply the effective labor that the firm demands:

   $$L^c(\Gamma)=\int xh(\gamma,\Gamma)d\mu. \quad (10)$$

2. Consumption-saving decisions: Given $h(\gamma,\Gamma)$, $w^c(\Gamma)$ and $r(\Gamma)$, the optimal decision rules $c(\gamma,\Gamma)$ and $a'(\gamma,\Gamma)$ solve the value function $V(\gamma,\Gamma)$.

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TABLE 1. Parameters of the model economy

Parameter	Value	Description
β	0.983125	Time discount factor
$\overline{h}$	1/3	Extensive margin for hours worked
η	0.4	Labor supply elasticity
Ψ	166.2	Parameter for disutility from working
ρ_x	0.929	Persistence of productivity shocks
σ_x	0.227	Standard deviation of productivity shocks
$\overline{a}$	−2.0	Borrowing constraint
θ	0.36	Capital income share
δ	0.025	Capital depreciation rate
ρ_Z	0.95	Persistence of aggregate productivity shock
σ_Z	0.007	Standard deviation of innovation to aggregate productivity

3. The firms optimize: for all Γ, $F_L(K(\Gamma), L^c(\Gamma), Z) = w^c(\Gamma)$ and $F_K(K(\Gamma), L^c(\Gamma, Z) = r(\Gamma) + \delta$.
4. Capital market clears: for all Γ, $K(\Gamma) = \int a d\mu$.
5. Goods market clears: for all Γ, $F(K(\Gamma), L^c(\Gamma), Z) + (1-\delta)(\Gamma) = C(\Gamma) + K'(\Gamma)$, where $C(\Gamma) = \int c(\gamma, \Gamma) d\mu$.
6. Aggregate behaviors are consistent with individual ones: $\Theta(\Gamma)$ is consistent with the actual law of motion implied by the optimal policy function $a'(\gamma, \Gamma)$.

Notice that even though wage rigidity is introduced, the standard capital and goods market-clearing conditions hold in the economy. However, the wage contract prevents households from optimally making their labor supply decisions, so the labor market does not clear. Since households agree to cede the right to decide the aggregate effective labor to the representative firm, equation (10) must hold with the fulfillment of the wage contract. As in equation (9), the employment allocation rule is associated with $\widetilde{w}$ under which households endogenously decide their employment until they provide $L^c(\Gamma)$.

4. CALIBRATION

Table 1 summarizes parameter values used in the model economies. The parameter η, which corresponds to the micro elasticity of labor supply, is set to 0.4 based on the findings that estimates of the microelasticity of labor supply are around 0-0.5. However, since a labor supply decision is discrete, choosing any values of η does not affect simulation results. An extensive margin for the labor supply, $\overline{h}$, is chosen to be $1/3$. I choose the time discount factor, β, and the disutility parameter of working, Ψ, to target a 1% quarterly return to capital and a 60% employment rate for employed workers.[15] The borrowing constraint, $\overline{a}$, is set to -2.0, following Chang and Kim (2007).[16]

For the individual labor productivity shock process, I choose $\rho_x = 0.929$ and $\sigma_x = 0.227$ following Chang and Kim (2007), which is estimated with the AR(1) wage process using the PSID for 1979–1992.[17] For aggregate productivity shocks,

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TABLE 2. Cyclicality of aggregate variables

Variable	Data (HS)	Data (ES)	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$
$\rho(Y, C)$	0.81	0.81	0.91	0.87	0.83	0.79
$\rho(Y, I)$	0.95	0.95	0.99	0.99	0.99	0.99
$\rho(Y, H)$	0.84	0.86	0.96	0.89	0.87	0.87
$\rho(H, Y/H)$	0.23	−0.06	0.81	0.20	−0.28	−0.55
$\rho(H, wedge)$	0.82	0.90	0.94	0.94	0.97	0.98
$\rho(Y, gini)$	−0.34	−0.34	−0.96	−0.92	−0.91	−0.91

Note: The labor wedge is computed from equation (1). All variables are logged and detrended by the Hodrick–Prescott filter. $\rho(A, B)$ denotes the correlation between variables A and B. HS and ES denote the household survey and the establishment survey, respectively. *gini* denotes the income Gini coefficient.

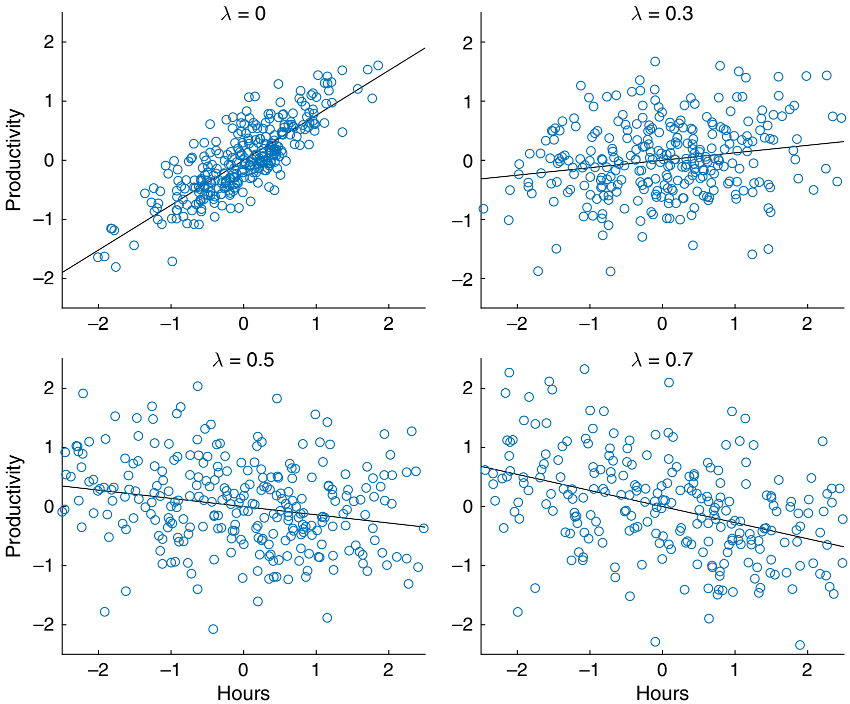
I simply choose $\rho_z = 0.95$ and $\sigma_z = 0.007$ following Kydland and Prescott (1982). The capital income share, θ, is 0.36, and the quarterly depreciation rate, δ, is 2.5%. Unfortunately, there is little empirical evidence on the index of real wage rigidity, given the wage contract rule. Hence, I choose a set of values for λ, which are 0, 0.3, 0.5, and 0.7. As far as the length of wage contracts, k, is concerned, I consider one-, four-, and eight-period contracts, that is, $k = 1$, 4, and 8.

5. FINDINGS

5.1. One-Period Wage Contract Case

As a benchmark case, I first summarize the key findings of the model economy with one-period wage contract ($k = 1$) to study the role of wage rigidity indexes, λ.

5.1.1. Wage rigidity and hours–productivity correlation. The cyclical properties of the aggregate variables for model economies are summarized in Table 2. Comovements between output and other key variables, such as consumption and investment, are well replicated in all the model economies as in the standard RBC models. The main focus in this paper is on variables related to labor markets. Table 2 and Figure 3 show that the correlation coefficient of hours and productivity decreases as the index of wage stickiness increases. The correlation coefficients of the two macro series in the model economies when $\lambda = 0.3$ and $\lambda = 0.7$ are 0.20 and −0.55, respectively, while it is 0.81 in the market-clearing economy ($\lambda = 0$). Thus, the model economies with small amounts of wage stickiness reproduce reasonably well the fact that hours of work are not strongly correlated with productivity. The intuition is simple. During expansions, the aggregate labor demand curve shifts to the right, but the contract wage and hours are not determined at an intersection between the labor supply and demand curves due to the wage contract. Hence, wage rigidities prevent the wage and hours from moving together along the labor supply curve with aggregate shocks. Regarding



Note. Variables are logged and detrended by the Hodrick–Prescott filter with a smoothing parameter of 1600 for each λ.

FIGURE 3. Correlation between productivity and hours.

the cyclicality of the income distribution, the model economy replicates reasonably the counter-cyclical the income Gini coefficient, but the cyclicality of the income Gini does not change significantly with the index of wage stickiness.[18]

5.1.2. Wage rigidity and volatility of labor wedge. Table 3 reports the volatilities of the aggregate variables for the US economy and the corresponding variables simulated by model economies. The model economy without wage rigidities (the market- clearing economy) shows an output volatility of 1.26, explaining around 60% of cyclical variation of output in the US data. The model economies with wage rigidities exhibit larger output variation: the volatility of output is 1.40 when $\lambda = 0.3$ and 1.67 when $\lambda = 0.7$. Sample statistics of other variables are similar to those in the standard RBC models: consumption and investment is around 30–40% and three times as volatile as output, respectively.

A distinguishing feature of the model economy is the labor wedge. The wage stickiness plays an important role in generating the large volatile wedge. According to Table 3 and Figure 3, the volatility of the wedge increases with the degree of wage stickiness: the volatilities of the wedge relative to output in

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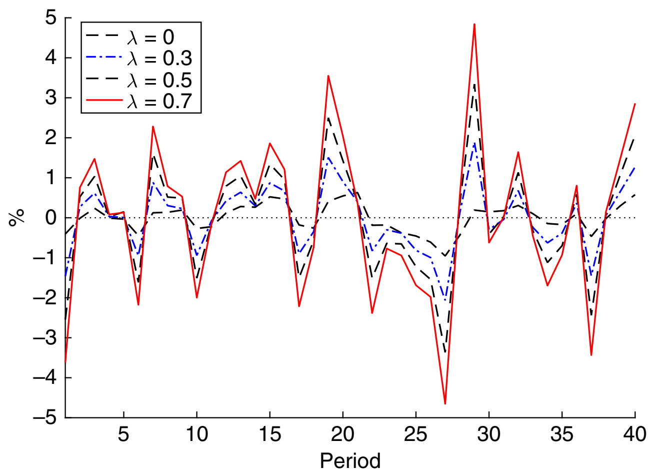
TABLE 3. Volatilities of aggregate variables

Variable	Data (HS)	Data (ES)	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$
σ_Y	2.11	2.11	1.26	1.40	1.53	1.67
σ_C/σ_Y	0.43	0.43	0.35	0.33	0.31	0.29
σ_I/σ_Y	2.67	2.67	3.00	3.11	3.20	3.28
σ_H/σ_Y	0.72	0.89	0.54	0.79	1.01	1.20
$\sigma_{Y/H}/\sigma_Y$	0.55	0.51	0.51	0.47	0.52	0.59
σ_{wedge}/σ_Y	0.86	1.08	0.23	0.66	1.00	1.27
σ_{gini}/σ_Y	0.99	0.99	0.58	0.76	0.90	1.03

Note: The labor wedge is computed from equation (1). All variables are logged and detrended by the Hodrick–Prescott filter. σ_A denotes the standard deviation (multiplied by 100) of a variable A. HS and ES denote the household survey and the establishment survey, respectively. *gini* denotes the income Gini coefficient.



Note. The labor wedge is computed from equation (1). The series is logged and detrended by the Hodrick–Prescott filter with a smoothing parameter of 1600 for each λ.

FIGURE 4. Volatility of labor wedge.

the model economies when $\lambda = 0.3$ and $\lambda = 0.7$ are 0.66 and 1.27, respectively. However, the market-clearing model economy fails to replicate the cyclical variation of the labor wedge in the US economy: relative volatility of the wedge is 0.23 when $\lambda = 0$. According to equation (1), the volatility of wedge is positively related to the cyclical movement in hours worked (H) and the volatility of productivity (Y/H), and negatively related to the correlation between hours and productivity. Firstly, the relative volatility of aggregate hours worked gets larger with the degree of wage rigidity: the cyclical variation of hours of work relative to output is 0.54 and 1.20 with wage rigidity indexes of 0.3 and 0.7, respectively. Second, interestingly, there are small differences in the cyclical variation of productivity

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relative to output across model economies with different λ. Lastly, as found earlier in Table 2, the correlation between hours and productivity decreases as the index of wage rigidity increases. From the three findings above, I conclude that large volatile hours worked and the low correlation coefficient between hours and productivity produce the large cyclical movement in the wedge.[19]

Another interesting fact is that the relative volatility of the income Gini coefficient increases with the degree of wage rigidity: the volatilities of the Gini coefficient relative to output are 0.76 and 1.03 with wage rigidity stickiness of 0.3 and 0.7, respectively.[20] As found in Table 3, a higher degree of wage rigidity leads to a larger variation of employment. As discussed by Castaneda et al. (1998), the counter-cyclical Gini coefficient (see Table 2) implies that employment increases or decreases from the bottom of the income distribution over the business cycles. Therefore, the large cyclical movement in employment induced by wage rigidity results in the large variation in the Gini coefficient.

5.2. Multiperiod Wage Contract Case

In the benchmark case above, the length of wage contracts is a quarter. However, in reality, the contract periods are much longer: four or eight quarters. Hence, I examine how the cyclical behavior of the economy varies with the length of the wage contracts, focusing on the two puzzles. In fact, the long-term wage contracts play a significant role in accounting for the business cycle properties of the economy. For example, Cho and Cooley (1995) find that the length of wage contracts is important in the cyclical variation of aggregate variables. Cho et al. (1997) also quantitatively estimate the welfare cost of nominal wage contracting across the contract periods and find that the welfare costs can vary over the contract length.

Summary statistics of the aggregate variables simulated by model economies over contract periods are summarized in Table 4. Given the wage rigidity index ($\lambda = 0.3$), the longer wage contract periods generate the larger volatile labor wedge and the lower correlation coefficients between productivity and hours. The relative volatility of the labor wedge is 0.90 in the four-period wage contract and around 1 in the eight-period wage contract, while it is 0.66 in the short-term wage contract rule ($k = 1$). The correlation coefficients between productivity and hours in the long-term wage contracts are small negative numbers, while the correlation is 0.20 in the short-term wage contract. These results imply that the length of wage contracts plays a role in implicitly generating stickier wages.

Next, an interesting question is how large index of wage stickiness is required to solve the two puzzles when long-term wage contracts are considered. Table 5 reports the required degree of the wage stickiness to reproduce the correlation between hours and productivity and the relative volatility of the labor wedge of the US economy.[21] According to the upper panel of Table 5, the required index of wage rigidity is around 0.20–0.25 to obtain the zero correlation coefficient. As far as the volatility of the labor wedge is concerned, the second row of Table 5 suggests that around 0.30–0.35 of the wage rigidity index is needed to replicate

Table 4. Volatilities and cyclicality of aggregate variables: long-period wage contracts

Variable	Data (HS)	Data (ES)	$k=0$	$k=1$	$k=4$	$k=8$
σ_Y	2.11	2.11	1.26	1.40	1.56	1.69
σ_C/σ_Y	0.43	0.43	0.35	0.33	0.31	0.30
σ_I/σ_Y	2.67	2.67	3.00	3.11	3.20	3.27
σ_H/σ_Y	0.72	0.89	0.54	0.79	0.97	1.06
$\sigma_{Y/H}/\sigma_Y$	0.55	0.51	0.51	0.47	0.43	0.41
σ_{wedge}/σ_Y	0.86	1.08	0.23	0.66	0.90	1.01
$\rho(Y, C)$	0.81	0.81	0.91	0.87	0.83	0.80
$\rho(Y, I)$	0.95	0.95	0.99	0.99	0.99	0.98
$\rho(Y, H)$	0.84	0.86	0.96	0.89	0.91	0.92
$\rho(H, Y/H)$	0.23	−0.06	0.81	0.20	−0.15	−0.33
$\rho(H, wedge)$	0.82	0.90	0.94	0.94	0.98	0.98

Note: All variables are logged and detrended by the Hodrick–Prescott filter with a smoothing parameter of 1600. σ_A denotes the standard deviation (multiplied by 100) of a variable A and $\rho(A, B)$ denotes the correlation between variables A and B. HS and ES denote the household survey and the establishment survey, respectively. $\lambda = 0.3$ for all k other than $k = 0$. $k = 0$ denotes the absence of wage rigidities in the model economy.

Table 5. Required index of wage rigidity to replicate the US data

	Four-period contract	Eight-period contract
$\rho(H, Y/H)$	0.26	0.22
σ_{wedge}/σ_Y	0.35	0.29

Note: The labor wedge is computed from equation (1). All variables are logged and detrended by the Hodrick–Prescott filter with a smoothing parameter of 1600.

the average of the volatility of the wedge relative to output in the US economy, which is around one. Therefore, I argue that a small amount of real wage rigidity would be enough to reproduce the low correlation between productivity and hours and the large volatility of the labor wedge when the long-term wage contracts are considered in the HA model. These findings are consistent with empirical work on real wage rigidity supporting the evidence that the degree of real wage rigidity in the USA is small (Dickens et al. 2007; Holden and Wulfsberg, 2009; Deelen and Verbeek, 2015).[22]

6. ROLE OF HETEROGENEITY

In some previous related work based on the RA economies, the labor wedge arises when other features are introduced into the model economy [Benhabib et al. 1991, Hall, 1997, Gali et al. 2007, and Shimer, 2009]. Not only that, previous work in the context of the RA models with wage rigidity—Cho and Cooley (1995) for

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TABLE 6. Volatilities and cyclicality of aggregate variables: RA model

Variable	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$	$k = 4$	$k = 8$
σ_Y	1.27	1.39	1.51	1.64	1.54	1.65
σ_C/σ_Y	0.35	0.55	0.64	0.73	0.51	0.49
σ_I/σ_Y	3.00	2.41	2.21	2.02	2.65	2.75
σ_H/σ_Y	0.42	0.60	0.73	0.84	0.70	0.75
$\sigma_{Y/H}/\sigma_Y$	0.60	0.50	0.46	0.43	0.43	0.38
σ_{wedge}/σ_Y	0.00	0.66	0.98	1.25	0.70	0.74
$\rho(Y, C)$	0.92	0.96	0.95	0.94	0.90	0.87
$\rho(Y, I)$	0.99	0.98	0.96	0.94	0.97	0.97
$\rho(Y, H)$	0.95	0.92	0.91	0.91	0.93	0.95
$\rho(H, Y/H)$	0.95	0.65	0.40	0.17	0.55	0.52
$\rho(H, wedge)$	0.00	0.90	0.94	0.96	0.92	0.93

Note: All variables are logged and detrended by the Hodrick–Prescott filter with a smoothing parameter of 1600. σ_A denotes the standard deviation (multiplied by 100) of a variable A and $\rho(A, B)$ denotes the correlation between variables A and B. $k = 1$ for all λ other than $\lambda = 0$, and $\lambda = 0.3$ for all k.

example—has also successfully reproduced the fact that hours of work are not highly correlated with labor productivity.[23] Therefore, it seems natural to ask why heterogeneity is necessary and what the role of heterogeneity is in the two puzzles. To investigate the importance of heterogeneity for resolving the two anomalies, I compare the key business cycle properties of the RA and HA model economy, controlling for wage rigidities. The basic assumptions for the wage contacts in the RA model are the same as those in the HA model other than the employment decisions for households. Since households are identical in the RA model, they cannot decide who works or not as in the HA model. Hence, given the contract wage, the representative household is assumed to provide L^c, which is determined by firm's profit maximization condition.[24] For calibration in the RA model, I choose $\beta = 0.99$ to target a 1% quarterly return to capital, and I set the disutility parameter of working, Ψ, to match the aggregate hours of 0.2. Importantly, the labor supply elasticity, η, is chosen to be 1.5, which is the same as the aggregate labor supply elasticity generated by the HA model.[25] Other remaining parameters are the same as those in the HA model.

Summary statistics of the aggregate variables simulated by the RA model economies over the indexes of wage rigidity and the length of wage contracts are summarized in Table 6. As in the HA model economy, in the RA model, the correlation coefficient of hours and productivity decreases, and the volatility of the wedge increases as the degree of wage stickiness or the length of wage contracts increases. However, the effect of wage stickiness is much less in the RA model. For example, when $\lambda = 0.3$ with $k = 4$, in the RA model, the volatility of the wedge is 0.70, and the correlation of hours and productivity is 0.55, while they are 0.95 and -0.15, respectively, in the HA model economy. Not only that, according to Table 7, the required degree of the wage stickiness under the RA

TABLE 7. Required index of wage rigidity to replicate the US data: RA model

	Four-period contract	Eight-period contract
$\rho(H, Y/H)$	0.65	0.57
σ_{wedge}/σ_Y	0.50	0.46

Note: The labor wedge is computed from equation (1). All variables are logged and detrended by the Hodrick–Prescott filter with a smoothing parameter of 1600.

model with long-term wage contracts, which is able to reproduce the two key business cycle moments of the US economy, is around 0.5–0.7, whereas it is around 0.20–0.35 in the HA model as in Table 5. From these counterfactual analyses, I conclude that heterogeneity plays an important role in solving the two puzzles. First, heterogeneity allows the labor supply curve to evolve over time since reservation wage distribution or wealth distribution is time-varying in response to the aggregate shocks in the presence of heterogeneity [Chang and Kim (2007)]. As a result, heterogeneity also plays a role in breaking the positive linear relation between productivity and hours and, in turn, generates the low correlation between the two time series. Furthermore, as found in Chang and Kim (2007), the labor wedge arises endogenously in the presence of heterogeneity. Table 3 and Figure 3 present that the volatility of the wedge is positive in the HA model even if there is no wage rigidity in the labor market ($\lambda = 0$), while it is zero in the RA model when $\lambda = 0$.[26] As mentioned above, the empirical evidence, found by Dickens et al. (2007), Holden and Wulfsberg (2009), and Deelen and Verbeek (2015), suggests that the size of real wage rigidity in the US economy is small. Therefore, I argue that the HA model with wage rigidities is compatible with the empirical findings while the RA model may not.

7. CONCLUSION

This paper studies the quantitative implications of real wage rigidities and heterogeneity for the two long-standing puzzles in the business cycle literature, the weak comovement of hours worked with labor productivity, and the large cyclical movement in the labor wedge. I shed light on these issues by extending a HA model with an indivisible labor supply choice to real wage rigidities.

The main findings of this paper can be summarized as follows. I find that the correlation coefficient between hours and productivity decreases, and the volatility of the labor wedge increases when the index of wage stickiness or the length of wage contracts increases. Heterogeneity also plays a role in solving the two puzzles since heterogeneity allows the aggregate labor supply curve to move in response to aggregate productivity shocks and the wedge to be endogenously produced. From these results, I argue that a small amount of real wage stickiness would be sufficient to resolve both anomalies when long-term wage contracts and heterogeneity are taken into account.

NOTES

1. Gali et al. (2007) show that the wedge can be used as a measure of the lost surplus in the labor market, and Chang and Kim (2007) argue that the labor wedge may arise because of the low correlation between hours worked and productivity.

2. In this paper, a term "heterogeneity" involves asset market incompleteness.

3. Cho and Cooley (1995) incorporate nominal wage contracts into a RBC model, while Krusell and Smith (1998) among others build a heterogeneous-agent (HA) model with aggregate productivity shocks in the context of the neoclassical economies.

4. Benigno and Ricci (2011) investigate the macroeconomic implications of downward nominal wage rigidities using a model economy with aggregate and idiosyncratic shocks. Schulz (2015) builds a search and matching model with heterogeneity and sorting, which endogenously generates wage rigidity.

5. An indivisible labor decision plays a significant role in breaking the tight link between consumption and labor at both individual and aggregate levels, which also affects the volatility of the labor wedge [Chang and Kim (2007)].

6. Takahashi (2014) finds that there are errors in the computational method of Chang and Kim (2007), and their model economy actually generates the strong correlation between productivity and hours and the less volatile labor wedge.

7. See Subsection A1 in the appendix below for details of the data sources.

8. Similarly, Karabarbounis (2014) also uses both the household-level and the establishment-level survey data for hours of work when he computes the labor wedge.

9. It should be noted that microlabor supply elasticity is calibrated, but the macroelasticity of labor supply is endogenously generated due to heterogeneity and indivisibility of labor supply. I choose $\chi = 1.5$, which is computed from steady-state reservation wage distribution generated by the baseline model economy. The value is in the range of the estimates for the aggregate labor supply elasticity in the macrolabor literature.

10. Some studies [e.g., Gali and Gambetti (2009)] point out a change in the US business cycles around the mid-1980s. I find that these empirical findings are robust to the sample periods starting from 1985: the relative volatility of the wedge is around one, and the correlation between aggregate productivity and hours worked is around zero.

11. For an equilibrium with aggregate dynamics, I follow the procedure suggested Krusell and Smith (1998): a very high precision can be obtained by approximating the distribution across characteristics of households, μ, using the first moment of it. Hence, the mean asset, K, is used in forecasting the law of motion for μ.

12. Hence, $1 \leq j \leq k$.

13. $\log w^e_{t-k+j}$ can be computed recursively using the law of motions for the capital and the aggregate productivity [Cho and Cooley (1995)].

14. Similarly, Hall (2005) also assumes that the wage is a weighted average between the previous wage (the predetermined wage) and the Nash-bargain wage (the desired wage)

15. Recent US data such as the Panel Study of Income Dynamics (PSID) and the Survey of Consumer Finances consistently report that the employment rates are around 60%.

16. $-\bar{a}(=2)$ is around doubled quarterly average income of the model.

17. Chang and Kim (2007) estimate the AR(1) process of the logged wage rate using the maximum-likelihood estimation method of Heckman (1979) to fix the selection bias problem, where the wages of households who did not work are not observable in the data.

18. The statistics for $\rho(Y, gini)$ in the data is from Oh (2014).

19. The correlation coefficient of the wedge with aggregate hours worked is also reasonably replicated: it is large in the model economies as in the US data (see Table 2).

20. The statistics for σ_{gini}/σ_Y in the data is from Oh (2014).

21. I use the cubic spline interpolation method to approximate the relative volatility of the labor wedge and the correlation between hours and productivity with five grid points for λ. Then I compute the required λ which replicates the US data.

22. These results are also in line with work of Cho and Cooley (1995), who show that output volatility measured in the US data is replicated with a very small amount of wage rigidity when a long-term wage contract is considered.

23. Cho and Cooley (1995) yield a very small correlation between productivity and hours using a model with nominal wage contracts and monetary shocks.

24. The RA model in this paper is similar to the model of Cho and Cooley (1995), but Cho and Cooley (1995) consider nominal wage rigidities with monetary policies while this work introduces real wage rigidities without nominal shocks.

25. In a HA model economy with indivisible labor, macrolabor supply elasticity can be computed from steady-state reservation wage distribution. See Chang and Kim (2006) for details.

26. Of course, the indivisibility of a labor decision also helps to solve the puzzles. See Chang and Kim (2007) for the role of the indivisible labor.

27. All nominal variables are deflated by the GDP deflator (Fred ID: GDPDEF).

28. Given the contract wage w_j^c, the real interest rate r_j can be computed from the firm's profit maximization: $r_j = \theta Z^{1/\theta}[\frac{(1-\theta)}{w_j^c}]^{(1-\theta)/\theta} - \delta$.

29. The transition probabilities for x and Z are approximated using Tauchen (1986).

30. The return to capital r can be computed by: $r = \theta Z^{1/\theta}[\frac{(1-\theta)}{w^c}]^{(1-\theta)/\theta} - \delta$.

31. I drop the first 500 periods to eliminate the impact of the arbitrary choice of initial aggregate state variables.

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APPENDIX A: DATA AND COMPUTATIONAL ALGORITHM

A.1. US AGGREGATE DATA

This subsection explains the US aggregate data sources used for the moments in the tables and figures. All data except hours worked for the household survey are taken from St. Louis Fred economic database. Data frequency is a quarter. The sample period of the data is from 1964:I to 2011:IV.[27]

- Private Gross Output (Y): Real GDP (Fred ID: GDPC1) − Real Government Consumption Expenditures & Gross Investment (GCEC1).
- Consumption (C): Personal Consumption Expenditures: Nondurable Goods + Personal Consumption Expenditures: Services.
- Investment (I): Real Gross Private Domestic Investment (GPDIC1) + Personal Consumption Expenditures: Durable Goods.
- Hours worked (H).
  - Household Survey: Data constructed by Cociuba et al. (2009).
  - Establishment Survey: Average Weekly Hours of Production and Nonsupervisory Employees: Total private (AWHNONAG)× All Employees: Total Private Industries (USPRIV).

A.2. THE COMPUTATIONAL ALGORITHM

I summarize the computational algorithm used for the model economy with the four-period wage contract ($k = 4$). I provide the computational methods only for an equilibrium with aggregate fluctuations since the computational algorithm for the steady-state economy is the same as in Chang and Kim (2007).

Step 1. I construct grids for aggregate state variables such as the aggregate capital and the aggregate productivity, and individual state variables such as the individual labor productivity and asset holdings. For the aggregate capital, K, I construct five grid points in the range of $[0.9K_s, 1.1K_s]$, where K_s is the steady-state level of aggregate capital. For the logged aggregate productivity, $\widehat{Z} \equiv \log Z$, I construct five grid points in the range of $[-3\sigma_{\widehat{Z}}, 3\sigma_{\widehat{Z}}]$, where $\sigma_{\widehat{Z}} = \sigma_Z/\sqrt{1-\rho_Z^2}$. Individual logged labor productivity, $\widehat{x} \equiv \log x$, is spaced in the range of $[-3\sigma_{\widehat{x}}, 3\sigma_{\widehat{x}}]$ with 17 grid points, where $\sigma_{\widehat{x}} = \sigma_x/\sqrt{1-\rho_x^2}$. The grid points for K, $\widehat{Z}$, and $\widehat{x}$ are equally spaced. I use 75 grid points for the assets, a, in the range of $[-2, 160]$. The grid points for a are not equally spaced: more points are assigned on the lower asset range.

Step 2. I obtain the coefficients of equation (2) from the market-clearing economy. I obtain the contract wage w_j^c using equation (2), for each j defined in equation (3).

Step 3. For each j, I parameterize the forecasting function for the future aggregate capital and $\widetilde{w}_j$ for the employment decision such that:

$$\log K_j' = a_{0j} + a_{1j} \log K + a_{2j} \log Z + a_{3j} \log K_{-4} + a_{4j} \log Z_{-4}, \qquad \textbf{(A1)}$$

$$\log \widetilde{w}_j = b_{0j} + b_{1j} \log K + b_{2j} \log Z + b_{3j} \log K_{-4} + b_{4j} \log Z_{-4}. \qquad \textbf{(A2)}$$

Step 4. Given the forecasting functions of (A1) and (A2), and the contract wage, w_j^c, I solve the optimization problems for the individual households.[28] The steps are as follows:

(a) Employment decision: for each j, given the forecasting functions of (A1) and (A2), I solve the optimization problems for individual households and obtain the employment decision rule, h_j, and the value function $\widetilde{V}_j$.[29]

(b) Consumption-saving decision: for each j, given the forecasting functions of (A1) and (A2), the contract wage, w_j^c, and the employment decision rule,

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h_j, I solve the optimization problems for individual households and obtain the policy functions for consumption c_j and asset holdings a'_j, and the value function V_j.

Step 5. I generate simulated data for 3500 periods using the value functions for individuals obtained in Step 2. The details are as follows.

(a) I set the initial conditions for K, Z and $\mu(a,x)$.

(b) Given the aggregate state variables and the market-clearing forecasting function (2), I compute the contract wage w^c.[30]

(c) Employment decision: given the aggregate state variables, forecasting functions of (A1), the evaluated value function obtained in Step 4(a), $\widetilde{V}(a,x)$, and $\hat{\widetilde{w}}$ as a guess for $\widetilde{w}$, I obtain the employment decision rule, $h(a,x)$, for households.

(d) I check if the households provide the effective labor demanded the representative firm under $\hat{\widetilde{w}}$: $L^c \equiv ((1-\theta)Z/\hat{\widetilde{w}})^{1/\theta}K = \int h(a,x)x\mu$. If not, reset $\hat{\widetilde{w}}$ and go to Step 5(c).

(e) Consumption-saving decision: given the aggregate state variables, forecasting functions of (A1), the evaluated value function obtained in Step 4(a), $V(a,x)$, the employment decision rule $h(a,x)$ obtained in Step 5(c), and the contract wage rate w^c, I obtain the decision rules for consumption $c(a,x)$ and saving $a'(a,x)$ for households.

(f) I compute macro variables using μ: $C = \int c(a,x)\mu$, $L^c = \int h(a,x)x\mu$, $K' = \int a'(a,x)\mu$, $H = \int h(a,x)\mu$, $Y = ZK^\theta L^{c(1-\theta)}$, and $I = Y - C$.

(g) Obtain the next period measure $\mu'(a,x)$ using $a'(a,x)$ and transition probabilities for x.

Step 6. For each j, I obtain the new coefficients for the forecasting functions by the ordinary least squares estimation using the simulated time series.[31] If the new coefficients are close enough to the previous ones, the simulation is done. Otherwise, I update the coefficients, and go to Step 4.

Step 7. I check the goodness of fit for the forecasting functions using R^2. The high accuracy is obtained such that:

$j = 1$,

$$\log K' = 0.0993 + 0.9966 \log K + 0.1400 \log Z - 0.0404 \log K_{-4} - 0.0455 \log Z_{-4}, \qquad R^2 = 0.9999,$$

$$\log \widetilde{w} = -0.2081 + 2.1009 \log K + 1.3328 \log Z - 1.6276 \log K_{-4} - 0.5699 \log Z_{-4}, \qquad R^2 = 0.9981,$$

$j = 2$,

$$\log K' = 0.1001 + 0.9743 \log K + 0.1401 \log Z - 0.0186 \log K_{-4} - 0.0427 \log Z_{-4}, \qquad R^2 = 0.9999,$$

$$\log \widetilde{w} = -0.2707 + 1.5498 \log K + 1.3136 \log Z - 1.0488 \log K_{-4} - 0.5937 \log Z_{-4}, \qquad R^2 = 0.9985,$$

$j = 3$,

$$\log K' = 0.0973 + 0.9772 \log K + 0.1389 \log Z - 0.0201 \log K_{-4} - 0.0421 \log Z_{-4}, \qquad R^2 = 0.9999,$$

$$\log \widetilde{w} = -0.3440 + 1.3937 \log K + 1.2469 \log Z - 0.8603 \log K_{-4} - 0.5713 \log Z_{-4}, \qquad R^2 = 0.9988,$$

$j = 4$,

$$\log K' = 0.0954 + 0.9761 \log K + 0.1387 \log Z - 0.0182 \log K_{-4} - 0.0415 \log Z_{-4}, \qquad R^2 = 0.9999,$$

$$\log \widetilde{w} = -0.3809 + 1.2587 \log K + 1.2045 \log Z - 0.7092 \log K_{-4} - 0.5478 \log Z_{-4}, \qquad R^2 = 0.9984.$$

TABLE A.1. Wage rigidity and accuracy for forecasting functions

	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$
Capital equation	0.9999	0.9999	0.9999	0.9999
Wage equation	0.9998	0.9995	0.9988	0.9980

Note: Accuracy (R^2) of the forecasting functions across different wage rigidity indexes in the model with one-period contract case ($k = 1$).

A.3. WAGE RIGIDITY AND ACCURACY OF FORECASTING FUNCTIONS

Table A.1 shows the accuracy of the forecasting functions across different wage rigidity indexes (λ) in the model with a one-period contract case ($k = 1$). I use R^2 as a measure for the goodness of fit. As shown in Table A.1, the high accuracy is obtained for both capital and wage equations even if R^2 for the wage equation slightly decreases with larger λ.