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COMMENTS ON: ORDERING PROPERTIES OF ORDER STATISTICS FROM HETEROGENEOUS POPULATIONS: A REVIEW WITH AN EMPHASIS ON SOME RECENT DEVELOPMENTS BY N. BALAKRISHNAN AND P. ZHAO

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I thank the authors for providing another excellent review on stochastic comparisons of order statistics after the review paper written by Kochar and Xu (2007).

Let $X_{\lambda_1}, \ldots, X_{\lambda_n}$ be a set of random variables with X_{λ_i} having distribution function F_{λ_i}, $i = 1, \ldots, n$. The behavior of various characteristics of statistic $\phi(X_{\lambda_1}, \ldots, X_{\lambda_n})$ with respect to $(\lambda_1, \ldots, \lambda_n)$ is interesting. In particular, attention has been given to the cases when either $\phi(X_{\lambda_1}, \ldots, X_{\lambda_n}) = X_{i:n}$ or $\phi(X_{\lambda_1}, \ldots, X_{\lambda_n}) = \sum_{i=1}^{n} X_{\lambda_i}$; and X_{λ_i} is exponential random variable with hazard rate λ_i. The notion of majorization and related topic introduced by Marshall, Arnold, and Olkin (2011) is one of the main tools and techniques to study such a behavior and derive the interesting results; as have been done by the authors and the other colleagues refereed in the text. On the other hand, various notions of stochastic orderings like location orderings, dispersion orderings and skewness orderings come to the picture and also play a very important role to explain the change of characteristics of the statistics like mean, survival function, hazard rate function, dispersion measures, and skewness measures when we switch the vector of λ_i's to another vector λ_i^*'s according to some pre-orderings like majorization, p-larger and reciprocal majorization.

It is surprising that the comparisons of order statistics from two different heterogeneous samples according to some notions stronger than the usual stochastic order like the hazard rate order and the likelihood ratio order hold only for the 2nd order statistic of a sample of size 2 (cf. Sect. 2.1). Let $X_{\lambda_1}, \ldots, X_{\lambda_n}$ be independent exponential random variables with hazard rate λ_i, $i = 1, \ldots, n$. Then, the hazard rate function of $X_{k:n}$, $k = 1, \ldots, n$ may not be a Schur concave (cf. Boland et al. 1994). Navarrov [5] further considered this problem and obtained among some other results that there exists $a > 0$, such that the hazard rate of $X_{k:n}$, say $h(t; \lambda_1, \ldots, \lambda_n)$ is Schur concave for $t > a$. It will be good if the authors consider Navarrov's paper and its references as related articles to this review and incorporate the results obtained in Navarrov [5] and the results in other related papers.

The results of Section 2.2, which compare the skewness and dispersion of $X_{k:n}$'s from a set of independent exponential random variables following a multiple-outlier model hold for all $k \geq 1$ and $n \geq 1$ which make the results more applicable than those which are true for only $n = k = 2$.

In Section 2.2, the number of λ_1's and λ_2's are the same for the both samples of X's and Y's. Let us assume that the number of λ_1's and λ_2's corresponding to Y's sample are respectively p' and q', $p' + q' = n'$. Now, under what restrictions on p, q, p', q', n, and n' we can get similar results to those in Section 2.2.

Open problem 4 is interesting, since the inequality given in the statement of Theorem 2.17 depends on α which is an unusual condition.

Comparisons of order statistics from heterogeneous sample with those from homogeneous sample make more sense when the common parameter of homogenous sample can be expressed as one of the usual means, arithmetic mean, geometric mean, and harmonic mean (we may have some idea about the values of these means without having the exact values of λ_i's). For example, λ_{mrl} in (2.20) cannot be computed if we do not know the exact values of λ_i's.

There is one more review paper on this topic by Boland et al. [2] that the authors may refer to it in Page 1.

I thank Professor Sheldon Ross for giving me this opportunity to be one of the discussant of this paper.

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