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Parametric coupling of light waves and a surface polariton in a plasma bounded by free space

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Abstract. We investigate a three-wave decay interaction involving a surface wave in which a bulk electromagnetic plasma wave decays into another bulk electromagnetic plasma wave and a surface polariton. It is shown that a p-polarized light wave transmitted from vacuum into a plasma at a particular angle of incidence can propagate as a bulk electromagnetic plasma wave without attenuation. The nonlinear boundary conditions for the surface wave are formulated in terms of the surface charge and the volume current, taking full account of the rippling of the free boundary. The coupled-mode equations are derived and solved in the parametric approximation to obtain the threshold and the growth rate. We assess the relative strength of the kinematic and the dynamic nonlinearities in the parametric interaction.

1. Introduction

During the past few decades, extensive investigations have been made of nonlinear wave interactions and parametric instabilities in infinite plasmas (Davidson 1972; Simon and Thompson 1975; Weiland and Wilhelmsson 1977). However, there have been few such investigations for bounded plasmas. It is well known that surface waves can propagate along the interface between a plasma and a vacuum (or dielectric) (Boardman 1982; Halevi 1992). Surface waves can be used for plasma diagnostics (Shivarova et al. 1975) and for sustaining a plasma that can be used in plasma processing (Chaker et al. 1986). Furthermore, surface waves are relevant to laser fusion (Aliev and Brodin 1990) and to astrophysical problems in the magnetosphere and the solar corona (Buti 1985).

In view of the various applications and occurrences of surface waves, it would be interesting to consider wave–wave interactions involving surface waves. Atanassov et al. (1981) considered a nonlinear interaction of three high-frequency electrostatic surface waves that produce a low-frequency density perturbation. Aliev and Brodin (1990) investigated the excitation of a surface wave and a volume plasma wave in an inhomogeneous plasma by a p-polarized pump wave. Brodin and Lundberg (1991) considered the same problem, including thermal effects. Lindgren et al. (1982) developed a general theory of three-wave interaction in plasmas with sharp boundaries by using the diffuse charge distribution model.

In this paper, we deal with a specific problem, namely the decay of a light wave in a plasma bounded by a vacuum into another light wave and a surface

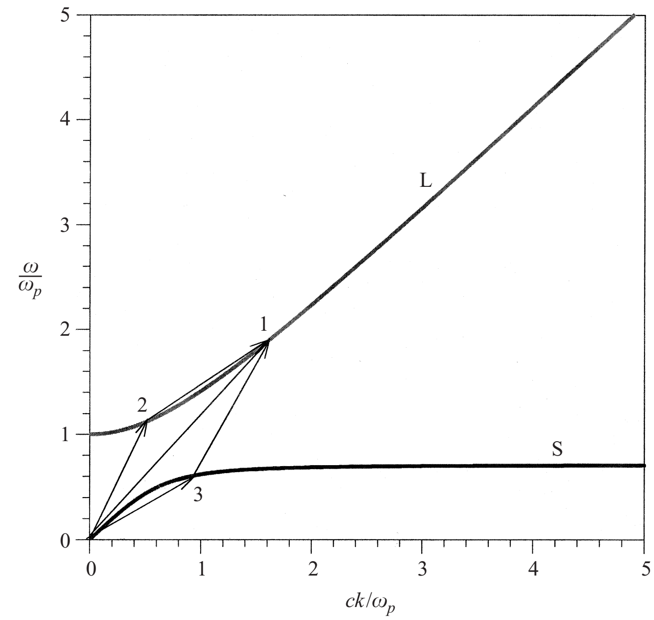

Figure 1. Dispersion curves for a light wave (L) and a surface polariton (S) in a plasma, and the plausibility of the three-wave resonant matching conditions.

polariton. A surface polariton is a low-frequency electron surface wave that is a mixed mode with both longitudinal and transverse components. The dispersion relation is

$$\omega^2 = c^2k^2 + \tfrac{1}{2}\omega_p^2 - \tfrac{1}{2}(\omega_p^4 + 4c^4k^4)^{1/2}, \tag{1}$$

where $\omega_p\,(= (4\pi Ne^2/m)^{1/2})$ is the electron plasma frequency and the remaining notation is standard. The light wave in the plasma is a bulk electromagnetic wave with dispersion relation

$$\omega^2 = \omega_p^2 + c^2k^2. \tag{2}$$

The decay of the high-frequency bulk electromagnetic wave (with frequency ω_1 and wavenumber k_1) into another high-frequency bulk electromagnetic wave (with ω_2 and k_2) and a surface polariton (with ω_3 and k_3) whose dispersion relation is given by (1) requires the resonance matching conditions

$$\omega_1 = \omega_2 + \omega_3, \tag{3a}$$

$$k_1 = k_2 + k_3. \tag{3b}$$

The decay interaction involving the surface polariton appears to be very similar to Brillouin scattering, and the plausibility of the resonance conditions (3) is depicted in Fig. 1.

In this paper, we assume that the waves propagate in the z direction and that the unperturbed interface is the plane $x = 0$, separating the plasma ($x > 0$) and vacuum ($x < 0$). In deriving the coupled-mode equations, we retain all the nonlinearities that can be responsible for the resonant interactions. In particular, nonlinear boundary conditions are used for the surface wave and special attention is paid to the rippling effect of the deformable free boundary. When the boundary is fixed, the important nonlinearity is a dynamic nonlinearity ($\mathbf{v}\cdot\nabla\mathbf{v}$ and $\mathbf{v}\times\mathbf{B}$ force). If the boundary is deformable, the kinematic nonlinearity plays a role. We assess the relative importance of these two nonlinearities on the parametric decay instability.

This paper is organized as follows. In Sec. 2, the high-frequency light wave that is transmitted to the plasma from the vacuum is described, and its coupling with the low-frequency surface wave is investigated to derive the coupled-mode equation. In Sec. 3, a nonlinear wave equation for the low-frequency surface wave is formulated. In Sec. 4, nonlinear boundary conditions are set up, allowing for the rippling effect of the moving boundary. In Sec. 5, a perturbation analysis is carried out for the nonlinear equation, and a coupled-mode equation for the low-frequency wave is derived. In Sec. 6, the coupled-mode equations are solved in the parametric approximation to find the thresholds and growth rates. A discussion is given in Sec. 7.

2. Equation for high-frequency light waves

We assume that a pump wave of frequency ω_1, polarized parallel to the plane of incidence ((x,z) plane) is incident upon the interface $x=0$ between the plasma ($x>0$) and vacuum ($x<0$) from the vacuum side with an angle of incidence θ_0. The electric field of the pump wave is then written as

$$\mathbf{E}_0 = E_0(\hat{\mathbf{x}}\sin\theta_0 - \hat{\mathbf{z}}\cos\theta_0)e^{i(\omega_1/c)(x\cos\theta_0+z\sin\theta_0)-i\omega_1 t}. \tag{4}$$

We choose the angle θ_0 so that $\sin\theta_0$ is equal to the refractive index of the plasma:

$$\sin\theta_0 = \left(1-\frac{\omega_p^2}{\omega_1^2}\right)^{1/2}. \tag{5}$$

Then internal reflection occurs (Jackson 1975), and the transmitted electric field takes the form

$$\mathbf{E}_1 = \hat{\mathbf{x}}\tilde{E}_x(\omega_1)e^{ik_1 z-i\omega_1 t}. \tag{6}$$

The reflected wave can be written as

$$\mathbf{E}_0' = E_0'(\hat{\mathbf{x}}\sin\theta_0 + \hat{\mathbf{z}}\cos\theta_0)e^{i(\omega_1/c)(-x\cos\theta_0+z\sin\theta_0)-i\omega_1 t}. \tag{7}$$

Since the continuity conditions for the tangential components of the electric and magnetic fields and the normal components of the electric displacement should be satisfied, we need first of all the condition

$$k_1 = \frac{\omega_1}{c}\sin\theta_0. \tag{8}$$

In view of the condition (5) that we impose, we see that the transmitted wave satisfies the dispersion relation (2).

The continuity conditions yield

$$\tilde{E}_x(\omega_1) = \frac{2E_0}{\sin\theta_0}, \tag{9a}$$

$$E_0' = E_0. \tag{9b}$$

Owing to the three-wave resonant interaction, the pump wave given by (6) can decay into a scattered high-frequency wave (frequency ω_2) and a low-frequency (frequency ω_3) surface wave, and the daughter waves can grow simultaneously. We can assume that the scattered wave is of the form

$$\mathbf{E}_2 = \hat{\mathbf{x}}\tilde{E}_x(z,t)e^{ikz-i\omega t} \equiv \hat{\mathbf{x}}E_x, \tag{10}$$

where the amplitude $\tilde{E}_x$ is slowly varying owing to the nonlinear interactions. The high-frequency field in (10) is governed by the following Maxwell wave equations and the cold-electron equation of motion:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2}\right)E_x = 4\pi e\frac{\partial}{\partial t}[(N+n)v_x] \tag{11}$$

$$\frac{\partial E_x}{\partial z} = -\frac{1}{c}\frac{\partial B_y}{\partial t} \tag{12}$$

$$\left(\frac{\partial}{\partial t} + \nu_H\right)v_x + \frac{e}{m}E_x = -v_x\frac{\partial v_x}{\partial x} - v_z\frac{\partial v_x}{\partial z} - \frac{e}{mc}(\mathbf{v}\times\mathbf{B})_x. \tag{13}$$

In the above, ν_H is the collision frequency for the high-frequency wave, the linear terms are the high-frequency scattered wave quantities, and the quadratic terms give rise to the high-frequency components by mixing the high-frequency pump wave of frequency ω_1 and the low-frequency surface wave of frequency ω_3.

In (11), N is the equilibrium electron number density and n is the electron density perturbation. Since no density perturbation is associated with either the light wave or the surface wave, it is sufficient to consider only the linear current in (11). The high-frequency quantities are the electromagnetic components consisting of (v_x, E_x, B_y), while the low-frequency variables are the transverse magnetic (TM) mode set $(v_x, v_z, E_x, E_z, B_y)$. All the other components can be put equal to zero. Thus the Lorentz force in (13) is $(\mathbf{v}\times\mathbf{B})_x = -v_z B_y$.

The evolution equation for a slowly varying amplitude $\tilde{E}_x(z,t)$ can be most easily obtained by solving (11)–(13) in terms of the multiple-scale series

$$E_x(z,t) = \epsilon E_x^{(1)}(z,t) + \epsilon^2 E_x^{(2)}(z,t) + \dots, \tag{14}$$

and similar expression for other variables. In (14), ϵ is ordering parameter scaling the time and the space coordinates as $t = t_0 + \epsilon t_1 + \dots$, $z = z_0 + \epsilon z_1 + \dots$. Then one can break down (11)–(13) order by order as follows:

order ϵ,

$$\left(\frac{\partial^2}{\partial t_0^2} - c^2\frac{\partial^2}{\partial z_0^2}\right)E_x^{(1)} = 4\pi eN\frac{\partial}{\partial t_0}v_x^{(1)}, \tag{15}$$

$$\frac{\partial E_x^{(1)}}{\partial z_0} = -\frac{1}{c}\frac{\partial B_y^{(1)}}{\partial t_0}, \tag{16}$$

$$\frac{\partial v_x^{(1)}}{\partial t_0} = -\frac{e}{m}E_x^{(1)}; \tag{17}$$

order ϵ^2,

$$\left(\frac{\partial^2}{\partial t_0^2} - c^2\frac{\partial^2}{\partial z_0^2}\right)E_x^{(2)} + 2\left(\frac{\partial^2}{\partial t_1\,\partial t_0} - c^2\frac{\partial}{\partial z_1}\frac{\partial}{\partial z_0}\right)E_x^{(1)} = 4\pi eN\left(\frac{\partial}{\partial t_1}v_x^{(1)} + \frac{\partial}{\partial t_0}v_x^{(2)}\right), \tag{18}$$

$$\frac{\partial}{\partial t_0}v_x^{(2)} + \left(\nu_H + \frac{\partial}{\partial t_1}\right)v_x^{(1)} + v_x^{(1)}\frac{\partial}{\partial x}v_x^{(1)} + v_z^{(1)}\frac{\partial}{\partial z_0}v_x^{(1)} = -\frac{e}{m}E_x^{(2)} + \frac{e}{mc}v_z^{(1)}B_y^{(1)}. \tag{19}$$

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The linear equations (15) and (17) are combined to give

$$\left(\frac{\partial^2}{\partial t_0^2}-c^2\frac{\partial^2}{\partial z_0^2}+\omega_p^2\right)E_x^{(1)}=0, \tag{20}$$

which is solved by

$$E_x^{(1)}=\tilde{E}_x^{(1)}(z_1,t_1)e^{ikz_0-i\omega t_0}+\text{c.c.}, \tag{21}$$

with the dispersion relation given by (2). The Fourier amplitudes of other quantities, denoted by tildes (˜), are

$$\tilde{v}_x^{(1)}=-\frac{ie}{\omega m}\tilde{E}_x^{(1)}, \tag{22}$$

$$\tilde{B}_y^{(1)}=\frac{ck}{\omega}\tilde{E}_x^{(1)}. \tag{23}$$

The second-order equations (18) and (19) can be arranged in the form

$$\left(\frac{\partial^2}{\partial t_0^2}-c^2\frac{\partial^2}{\partial t_0^2}+\omega_p^2\right)E_x^{(2)}=-2\left(\frac{\partial^2}{\partial t_1\,\partial t_0}-c^2\frac{\partial^2}{\partial z_1\,\partial z_0}\right)E_x^{(1)}+\frac{c}{\omega}\nu_H\,\omega_p^2\,E_x^{(1)}-4\pi eNP, \tag{24}$$

where

$$P=v_x^{(1)}\frac{\partial}{\partial x}v_x^{(1)}+v_z^{(1)}\frac{\partial}{\partial z_0}v_x^{(1)}-\frac{e}{mc}v_z^{(1)}B_y^{(1)}. \tag{25}$$

The right-hand side of (24) contains the secularity-causing terms, which vary as $e^{ikz_0-i\omega t_0}$ and should be removed by requiring that

$$\left(\frac{\partial}{\partial t_1}+\Gamma_H+\frac{k_2c^2}{\omega_2}\frac{\partial}{\partial z_1}\right)\tilde{E}_x^{(1)}(\omega_2)+\frac{i2\pi eN}{\omega_2}\tilde{P}(k_2,\omega_2)=0, \tag{26a}$$

where

$$\Gamma_H=\frac{\nu_H\,\omega_p^2}{2\omega_2^2} \tag{26b}$$

is the effective collisional damping rate for the high-frequency wave and $\tilde{P}(k_2,\omega_2)$ is the high-frequency component of the nonlinear force given by (25). In (26), we have explicitly introduced the frequency as ω_2 to show that the equation is the evolution equation for the ω_2 wave. To complete this section, we have to evaluate the nonlinear coupling term $\tilde{P}(k_2,\omega_2)$. Although we have not solved for the low-frequency surface wave, let us agree to use (71) (Sec. 5 below), which are the linear solutions for the surface wave variables. The low-frequency variables are x-dependent, but it is sufficient to evaluate $\tilde{P}$ at $x=0$. Then, in view of the resonance condition (3), $\tilde{P}(k_2,\omega_2)$ is obtained as

$$\begin{aligned}\tilde{P}(k_2,\omega_2)&=v_x(\omega_1)\frac{\partial}{\partial x}v_x^*(\omega_3)+v_z^*(\omega_3)\frac{\partial}{\partial z_0}v_x(\omega_1)-\frac{e}{mc}B_y(\omega_1)v_z^*(\omega_3)\\&=\frac{ie^2k_3}{m^2\omega_1\omega_3}\tilde{E}_x(\omega_1)\tilde{F}^*(\omega_3),\end{aligned} \tag{27}$$

where we have used $v_z(\omega_1)=0$ for the pump wave, and $\tilde{E}_x(\omega_1)$ is given by (9a). Substituting (27) into (26), we finally obtain the coupling equation

$$\left(\frac{\partial}{\partial t_1}+\Gamma_H+\frac{c^2k_2}{\omega_2}\frac{\partial}{\partial z_1}\right)\tilde{E}_x^{(1)}(\omega_2)=\frac{ek_3\,\omega_p^2}{2m\omega_1\,\omega_2\,\omega_3}\tilde{E}_x(\omega_1)\tilde{F}^*(\omega_3). \tag{28}$$

3. Low-frequency equation

To describe the nonlinear low-frequency surface wave, we have the following set of fluid and Maxwell equations:

$$\frac{\partial n}{\partial t}+\boldsymbol{\nabla}\cdot[(n+N)\mathbf{v}]=0, \tag{29}$$

$$\left(\frac{\partial}{\partial t}+\nu_L\right)\mathbf{v}+\mathbf{v}\cdot\boldsymbol{\nabla}\mathbf{v}=-\frac{e}{m}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right), \tag{30}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E}=-4\pi en, \tag{31}$$

$$\boldsymbol{\nabla}\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \tag{32}$$

$$\boldsymbol{\nabla}\times\mathbf{B}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}-\frac{4\pi e}{c}(N+n)\mathbf{v}. \tag{33}$$

In the above, linear terms represent the low-frequency (ω_3) surface-wave quantities, and the quadratic terms are beatings of two high-frequency waves to excite the ω_3 wave. Thus we can discard the nonlinear current $n\mathbf{v}$ in (29) and (33) because $n=0$ for the high-frequency light wave. Furthermore, the convective derivative term $\mathbf{v}\cdot\boldsymbol{\nabla}\mathbf{v}$ does not give rise to a low-frequency component because $v_z=0$ for a light wave and $\partial/\partial x=0$ when operating on a high-frequency quantity. As previously noted, our low-frequency variables are the TM mode set (E_x, E_z, v_x, v_z, B_y), and the rest can be put to zero. In (30), ν_L is the collision frequency for the low-frequency wave. Eliminating $\mathbf{B}$ between (32) and (33) and using (30), we obtain

$$\frac{\partial^2 E_z}{\partial x\partial z}+A^2E_x=0, \tag{34}$$

where

$$A=\left\{\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\frac{\omega_p^2}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}+\nu_L\right)^{-1}-\frac{\partial^2}{\partial z^2}\right\}^{1/2}, \tag{35}$$

$$\left\{\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\frac{\omega_p^2}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}+\nu_L\right)^{-1}\right\}\left(\frac{\partial^2}{\partial x^2}-A^2\right)E_z=\frac{\omega_p^2}{c^3}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}+\nu_L\right)^{-1}A^2(v_xB_y). \tag{36}$$

Some remarks are in order regarding the inverse operator $(\partial/\partial t+\nu_L)^{-1}$ and the operators A and A^{-1}. These operators act on the two-scale functions such as $f(z_1,t_1)e^{ikz_0-i\omega t_0}$, where z_0 and t_0 are fast variables and z_1 and t_1 are slow variables. The derivatives $\partial/\partial t$ and $\partial/\partial z$ are derivative-expanded in the fashion $\partial/\partial t=\partial/\partial t_0+\epsilon\,\partial/\partial t_1$ and $\partial/\partial z=\partial/\partial z_0+\epsilon\,\partial/\partial z_1$. Taking ν_L as a quantity of order ϵ, and treating $\partial/\partial t_0$ and $\partial/\partial t_1$ as algebraic quantities and using the binomial expansion, we can expand as

$$\left(\frac{\partial}{\partial t}+\nu_L\right)^{-1}=\left(\frac{\partial}{\partial t_0}\right)^{-1}-\epsilon\left(\frac{\partial}{\partial t_0}\right)^{-2}\left(\frac{\partial}{\partial t_1}+\nu_L\right), \tag{37}$$

which obviously satisfies the relation $(\partial/\partial t+\nu_L)(\partial/\partial t+\nu_L)^{-1}=1$. Since $\partial/\partial t_0=-i\omega$ and $\partial/\partial z_0=ik$, these operators and the inverses are in fact algebraic quantities, and the meaning of the inverse operators in the above is not ambiguous. An operator involving a fractional power such as A in (35) will also be derivative-expanded as in (37), and again its meaning is not ambiguous. Thus one obtains the following expansion for the operator A:

$$A = A_0 + \varepsilon A_1$$
$$= \alpha + i\varepsilon\left(\frac{\partial\alpha}{\partial\omega}\frac{\partial}{\partial t_1} - \frac{\partial\alpha}{\partial k}\frac{\partial}{\partial z_1}\right) + \varepsilon\nu_L\left.\frac{\partial A}{\partial\nu_L}\right|_0, \tag{38}$$

where

$$\left.\frac{\partial A}{\partial \nu_L}\right|_0 = \frac{-i\omega_p^2}{2c^2\omega\alpha}, \tag{39}$$

evaluating the derivative at $\nu_L = \partial/\partial t_1 = \partial/\partial z_1 = 0$, and

$$\alpha(k,\omega) = \left(k^2 + \frac{\omega_p^2 - \omega^2}{c^2}\right)^{1/2}. \tag{40}$$

Likewise, we have

$$A^{-1} = (A^{-1})_0 + \varepsilon(A^{-1})_1$$
$$= \alpha^{-1} + i\varepsilon\left(\frac{\partial\alpha^{-1}}{\partial\omega}\frac{\partial}{\partial t_1} - \frac{\partial\alpha^{-1}}{\partial k}\frac{\partial}{\partial z_1}\right) + \varepsilon\nu_L\left.\frac{\partial A^{-1}}{\partial\nu L}\right|_0. \tag{41}$$

In the vacuum, the low-frequency electric field is determined by

$$\left(\frac{\partial^2}{\partial x^2} - \Lambda^2\right)E_z = 0, \tag{42}$$

$$\Lambda^2 E_x + \frac{\partial^2 E_z}{\partial x \partial z} = 0, \tag{43}$$

where Λ is an operator defined by

$$\Lambda\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial z}\right) = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)^{1/2}. \tag{44}$$

The perturbation expansion of the operator Λ parallels that of A: putting $\omega_p = \nu_L = 0$ in (38) and (41) gives the corresponding formulae for Λ, with the replacement $\alpha \to \lambda$, where

$$\lambda = \left(k^2 - \frac{\omega^2}{c^2}\right)^{1/2}. \tag{45}$$

We first discuss the vacuum solutions. The x dependence of the function E_z in (42) is usually sought in the form of a power series, as is often done for water waves or Rayleigh–Taylor instability problems (Dodd et al. 1982; Infeld 1989). In fact, the derivation of the power-series solution is facilitated by regarding the operators $\partial/\partial t$ and $\partial/\partial z$ as algebraic quantities. Then (42) is solved by

$$E_z(x,z,t) = e^{x\Lambda(\partial/\partial t,\, \partial/\partial z)}\, W(z,t), \tag{46}$$

where the exponential function should be read as a power series and the operator Λ acts on the unknown function $W(z,t)$, which is determined from the appropriate boundary conditions. Then (43) yields

$$E_x(x,z,t) = -e^{x\Lambda}\,\Lambda^{-1}\frac{\partial W}{\partial z}. \tag{47}$$

Turning to the plasma equations (34) and (36), we note that the solution E_z as well as E_x consists of two parts – the homogeneous and the particular solutions:

$$E_z = E_{zH} + E_{zS}, \qquad E_x = E_{xH} + E_{xS}. \tag{48}$$

The homogeneous solution E_{zH} solves (36) with the right-hand side put equal to zero, and thus can be written as

$$E_{zH} = e^{-xA} F(z,t), \tag{49}$$

where $F(z,t)$ is an unknown function yet to be determined by the boundary conditions. The homogeneous solution of E_x is determined from (34):

$$E_{xH} = e^{-xA} A^{-1} \frac{\partial F}{\partial z}. \tag{50}$$

The particular solutions E_{zS} and E_{xS} of (34) and (36) are determined with the right-hand side of (36) constructed by an iterative procedure, i.e. by using the lower-order solutions for the quadratic terms. In our perturbation scheme, the lowest-order solutions are extracted from the homogeneous solutions.

4. Boundary conditions

The solutions obtained for the plasma and the vacuum regions in the preceding sections should be matched on the interface

$$S(x,y,z) = x - \xi(z,t) = 0 \tag{51}$$

through appropriate boundary conditions. First, u, the normal velocity of the moving boundary $S(x,z,t)$, is expressed by

$$u = \frac{-\partial S/\partial t}{|\nabla S|} = \frac{\partial \xi/\partial t}{\{1+(\partial \xi/\partial z)^2\}^{1/2}}, \tag{52}$$

and the unit vector normal to the surface $S = 0$ is

$$\hat{\mathbf{n}} = \frac{\hat{\mathbf{x}} - \hat{\mathbf{z}}\partial \xi/\partial z}{\{1+(\partial \xi/\partial z)^2\}^{1/2}}. \tag{53}$$

The kinematic boundary condition $u = \hat{\mathbf{n}}\cdot\mathbf{v}$ (where $\mathbf{v}$ is evaluated at $x = \xi(z,t)$) gives the relation

$$v_x = \frac{d\xi}{dt} = \frac{\partial \xi}{\partial t} + v_z \frac{\partial \xi}{\partial z}, \tag{54}$$

as expected. In the following, it should be borne in mind that the surface elevation ξ has all three components of high and low frequencies.

The boundary conditions on the moving interface were obtained by Kruskal and Schwarzschild (1954), and we have

$$[\hat{\mathbf{n}} \times \mathbf{E}] = \frac{u}{c}[\mathbf{B}], \tag{55}$$

$$[\hat{\mathbf{n}}\cdot\mathbf{E}] = 4\pi\sigma, \tag{56}$$

where the bracket $[\mathbf{B}] \equiv \mathbf{B}(\text{plasma}) - \mathbf{B}(\text{vacuum})$ evaluated at $x = \xi$. Other bracketed quantities have similar meanings. In (56), the surface charge density σ is as given by Sedov (1972):

$$\frac{\partial \sigma}{\partial t} + \hat{\mathbf{n}} \cdot \mathbf{J} + \nabla \cdot \mathbf{J}^*_{\parallel} = 0, \tag{57}$$

where $\mathbf{J}^*_{\parallel}$ is the surface current density flowing on the interface and will be assumed to be zero. It would be instructive to derive (57) in an alternative way. By definition, we have

$$\sigma = -e \int_{-\delta}^{\delta} n \, dx,$$

where the surface charge layer is denoted by the range $(-\delta, \delta)$, and we assume that the interface $x = 0$ is fixed for simplicity. Using (29) gives

$$\frac{\partial \sigma}{\partial t} = e \int_{-\delta}^{\delta} \left(\frac{\partial}{\partial x} \{(n+N)v_x\} + \frac{\partial}{\partial z} \{(n+N)v_z\} \right) dx \tag{57$'$}$$

The first integral is $e[(n+N)v_x]_{0^+} = J_x$. Equation (57) is a generalization of (57$'$). The second integral in (57$'$) can give rise to higher-order surface currents (Vladimirov et al. 1994; Lee and Hong 1998), but we neglect those terms here for simplicity. Using $u = \hat{\mathbf{n}} \cdot \mathbf{v}$ in (57) yields

$$\frac{\partial \sigma}{\partial t} = e(n+N) \frac{\partial \xi / \partial t}{\{1 + (\partial \xi / \partial z)^2\}^{1/2}}. \tag{58}$$

The y component of (55) reads

$$-[E_z] = \frac{\partial \xi}{\partial z}[E_x] + \frac{1}{c} \frac{\partial \xi}{\partial t}[B_y]. \tag{59}$$

Equations (56) and (58) yield

$$[E_x] - 4\pi e N \xi = [E_z] \frac{\partial \xi}{\partial z}. \tag{60}$$

In writing (60), we have neglected terms of order ϵ^3. Equations (59) and (60) are general jump conditions valid for both high- and low-frequency waves. The last term of (59) can be omitted, since $[B_y] = 0$ in the linear approximation for both light and surface waves. If (59) is used in (60) for $[E_z]$, the last term of (60) turns out to be of order ε^3, and can be neglected. Thus we have

$$[E_z] = -\frac{\partial \xi}{\partial z}[E_x], \tag{59$'$}$$

$$[E_x] = 4\pi e N \xi. \tag{60$'$}$$

The linear equation (60$'$) is nothing other than the continuity of the normal component of the electric displacement vector (Lee 1995). Equation (59$'$) is the nonlinear version of the jump of the tangential component of the electric field.

Equation (59$'$) takes the form

$$F - W = \xi(AF + \Lambda W) - E_{zS}(\xi) - 4\pi e N \xi \frac{\partial \xi}{\partial z} \equiv N_B(\xi), \tag{61}$$

where we have used (46) and (49), and have expanded the exponential operators. In (61), F and W are the low-frequency variables, and the quadratic terms are the low-frequency beats of two high-frequency waves. The first term on the right-hand side of (61) can be omitted, since it does not generate any low-frequency component.

It seems more advantageous to have the homogeneous solutions appear explicitly in (60′).

Differentiating (60′) with respect to t gives

$$\frac{\partial}{\partial t}\left\{e^{-\xi A}A^{-1}\frac{\partial F}{\partial z}+E_{xS}(\xi)+e^{\xi\Lambda}\Lambda^{-1}\frac{\partial W}{\partial z}\right\}=4\pi eN\left\{v_x(\xi)-v_z(\xi)\frac{\partial \xi}{\partial z}\right\} \tag{62}$$

where we have used (47), (50) and (54). Differentiating (62) with respect to t again and using (30), we obtain

$$\begin{aligned}\frac{\partial^2}{\partial t^2}\left(e^{-\xi A}A^{-1}\frac{\partial F}{\partial z}+e^{\xi\Lambda}\Lambda^{-1}\frac{\partial W}{\partial z}\right)&+\omega_p^2\,e^{-\xi A}A^{-1}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}+\nu_L\right)^{-1}\frac{\partial F}{\partial z}\\ &=4\pi eN\frac{\partial}{\partial t}v_{xS}(\xi)-4\pi eN\frac{\partial}{\partial t}\left\{v_z(\xi)\frac{\partial\xi}{\partial z}\right\}\\ &\quad-\frac{\partial^2}{\partial t^2}E_{xS}(\xi)+\omega_p^2\frac{\partial \xi}{\partial t}e^{-\xi A}\left(\frac{\partial}{\partial t}+\nu_L\right)^{-1}\frac{\partial F}{\partial z}\\ &\equiv N_A(\xi),\end{aligned} \tag{63}$$

where

$$v_{xS}(\xi)=-\frac{e}{m}\left[\left(\frac{\partial}{\partial t}+\nu_L\right)^{-1}E_{xS}(x,z,t)\right]_{x=\xi}. \tag{64}$$

Equations (61) and (63) are the two equations that we analyse in the multiple-scale perturbation scheme in the next section. Since F and W are low-frequency quantities and the nonlinear quadratic terms are the beating terms of two high-frequency waves, it can be seen that we can put $\xi=0$ on the left-hand side of (63), and $v_z(\xi)=0$ and the $(\partial\xi/\partial t)\partial F/\partial z$ term can be discarded in $N_A(\xi)$.

5. Perturbation analysis

We expand the various quantities in (61) and (63) in perturbation series as

$$F(z,t)=\varepsilon F^{(1)}(z_0,t_0,z_1,t_1)+\varepsilon^2F^{(2)}+\dots \quad . \tag{65}$$

The quantities W, ξ, v_x, B_y, E_{xH} and E_{zH} are expanded in this fashion. However, the particular solutions E_{xS} and E_{zS} are expanded as $E_{xS}=\varepsilon^2E_{xS}^{(2)}+\varepsilon^3E_{xS}^{(3)}+\dots$. The operators have already been expanded in Sec. 3.

By means of the above perturbation scheme, we can break down (61) and (63) order by order. At order ϵ, we have the relations

$$F^{(1)}=W^{(1)}, \tag{66}$$

$$\mathscr{L}_0F^{(1)}=0, \tag{67}$$

with

$$\mathscr{L}_0 = \left(\frac{\partial^2}{\partial t_0^2}+\omega_p^2\right)(A^{-1})_0 \frac{\partial}{\partial z_0}+\frac{\partial^2}{\partial t_0^2}(\Lambda^{-1})_0 \frac{\partial}{\partial z_0}. \tag{68}$$

Equation (67) admits a solution in the form

$$F^{(1)}(z,t) = \tilde{F}(z_1,t_1)e^{i\theta}, \tag{69}$$

with $\theta = kz_0-\omega t_0$ and the linear dispersion relation (Zhelyazkov 1987; Lee 1995)

$$D(k,\omega) = \frac{1}{\lambda}+\frac{1}{\alpha}\left(1-\frac{\omega_p^2}{\omega^2}\right) = 0, \tag{70}$$

where α and λ have been introduced in (40) and (45). Corresponding to $F^{(1)}$, we have the following linear solutions in the plasma:

$$v_x^{(1)} = \frac{ek}{m\omega\alpha}e^{-x\alpha}\tilde{F}e^{i\theta}, \tag{71a}$$

$$v_z^{(1)} = -\frac{ie}{m\omega}e^{-x\alpha}\tilde{F}e^{i\theta}, \tag{71b}$$

$$n^{(1)} = 0, \tag{71c}$$

$$B_y^{(1)} = -i\frac{c}{\omega}\left(\alpha-\frac{k^2}{\alpha}\right)e^{-x\alpha}\tilde{F}e^{i\theta}, \tag{71d}$$

$$\xi^{(1)} = \frac{iek}{m\omega^2\alpha}\tilde{F}e^{i\theta}. \tag{71e}$$

Likewise the order-ε vacuum solutions for the surface wave are

$$E_z^{(1)} = e^{x\lambda}\tilde{F}e^{i\theta}, \tag{71f}$$

$$E_x^{(1)} = -\frac{ik}{\lambda}e^{x\lambda}\tilde{F}e^{i\theta}, \tag{71g}$$

$$B_y^{(1)} = -\frac{i\omega}{c\lambda}e^{x\lambda}\tilde{F}e^{i\theta}. \tag{71h}$$

It can be seen that $B_y^{(1)}$ and $E_z^{(1)}$ are continuous across $x=0$ in the linear approximation. When we evaluate $N_A(\xi)$ and $N_B(\xi)$ in (61) and (63), we need the quantities at the boundary $x=\xi$. The exponential factor $e^{-\xi\alpha} = 1-\xi\alpha+\frac{1}{2}\xi^2\alpha^2+\dots$ generates all the higher-order terms. Since we need only the quadratic terms, it is sufficient here to evaluate the boundary values at $x=0$ in (71).

Next we move on to the order-ε^2 equations. At order ε^2, (61) and (63) yield, taking ν_L as a quantity of order ϵ,

$$W^{(2)} = F^{(2)}-N_B^{(2)}, \tag{72}$$

$$\mathscr{L}_0 F^{(2)}+\mathscr{L}_1 F^{(1)} = \nu_L\,\omega_p^2 A_0^{-1}\left(\frac{\partial}{\partial t_0}\right)^{-1}\frac{\partial F^{(1)}}{\partial z_0}+\frac{\partial^2}{\partial t_0^2}\Lambda_0^{-1}\frac{\partial}{\partial z_0}N_B^{(2)}(\xi)+N_A^{(2)}(\xi), \tag{73}$$

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where

$$\mathscr{L}_1 = \left(\frac{\partial^2}{\partial t_0^2}+\omega_p^2\right)\left\{(A^{-1})_1\frac{\partial}{\partial z_0}+A_0^{-1}\frac{\partial}{\partial z_1}\right\}$$

$$+2\frac{\partial^2}{\partial t_0\,\partial t_1}(A_0^{-1}+\Lambda_0^{-1})\frac{\partial}{\partial z_0}+\frac{\partial^2}{\partial t_0^2}\left\{(\Lambda^{-1})_1\frac{\partial}{\partial z_0}+\Lambda_0^{-1}\frac{\partial}{\partial z_1}\right\}, \quad (74)$$

$$N_A^{(2)} = 4\pi eN\frac{\partial}{\partial t_0}v_{xS}^{(2)}(\xi=0)-\frac{\partial^2}{\partial t_0^2}E_{xS}^{(2)}(\xi=0), \quad (75)$$

$$N_B^{(2)}(\xi) = \xi^{(1)}(A_0F^{(1)}+\Lambda_0W^{(1)})-4\pi eN\xi^{(1)}\frac{\partial\xi^{(1)}}{\partial z_0}-E_{zS}^{(2)}(\xi=0). \quad (76)$$

The $\mathscr{L}_1F^{(1)}$ term in (61) can be shown to reduce to

$$k\omega^2\left(\frac{\partial D}{\partial\omega}\frac{\partial F^{(1)}}{\partial t_1}-\frac{\partial D}{\partial k}\frac{\partial F^{(1)}}{\partial z_1}\right) \quad (77)$$

upon using the dispersion relation (70). If (73) is read as the low-frequency (ω_3 frequency) equation, the term (77) and the ω_3 frequency terms on the right-hand side generated by the beats satisfying the resonance matching conditions (3) cause the secularity when one attempts to solve (73) for $F^{(2)}$. All these secularity-causing terms should be removed by requiring that

$$k\omega^2\left(\frac{\partial D}{\partial\omega}\frac{\partial}{\partial t_1}-\frac{\partial D}{\partial k}\frac{\partial}{\partial z_1}\right)F^{(1)}+\nu_L\left\{\left(\frac{\partial^2}{\partial t_0^2}+\omega_p^2\right)\frac{i\omega_p^2}{2c^2\omega\alpha^3}-\omega_p^2A_0^{-1}\left(\frac{\partial}{\partial t_0}\right)^{-1}\right\}\frac{\partial F^{(1)}}{\partial z_0}$$

$$= N_A^{(2)}(\omega)-\frac{ik}{\lambda}\omega^2N_B^{(2)}(\omega), \quad (78)$$

where ω stands for $\omega_3 = \omega_1-\omega_2$, the low frequency. Equation (78) is the desired coupled-mode equation, and it remains for us to evaluate the coupling terms on the right-hand side. In writing (78), we have tacitly assumed that the amplitude of the light wave is of order ε (weak pump). When one evaluates the coupling terms, one needs the elevation of the perturbed interface oscillating with frequency ω_1 or ω_2, which is obtained from the linear equation $\partial\xi^{(1)}(\omega_1)/\partial t = v_x^{(1)}(\omega_1)$ as, with the aid of (22),

$$\xi^{(1)}(\omega_1) = \frac{e\tilde{E}_x(\omega_1)}{m\omega_1^2}e^{i(k_1z_0-\omega_1t_0)}, \quad (79a)$$

$$\xi^{(2)}(\omega_2) = \frac{e\tilde{E}_x(\omega_2)}{m\omega_2^2}e^{i(k_2z_0-\omega_2t_0)}. \quad (79b)$$

Omitting the details, we only present essential intermediate steps in the following. Equation (36) gives the particular solution

$$E_{zS}^{(2)}(\omega_3) = \frac{iek_3\omega_p^2}{m\omega_1\omega_2(\omega_3^2-\omega_p^2)}\tilde{E}_x(\omega_1)\tilde{E}_x^*(\omega_2)e^{i(k_3z_0-\omega_3t_0)}. \quad (80)$$

Since $\partial E_{zS}/\partial x = 0$, we have, from (34) and (64),

$$E_{xS}^{(2)}(\omega_3) = v_{xS}^{(2)} = 0, \quad (81)$$

and we have $N_A^{(2)} = 0$. Equation (76) yields

$$N_B^{(2)}(\omega_3) = \frac{ie\omega_p^2 k_3}{m\omega_1\omega_2}\left(\frac{1}{\omega_p^2-\omega_3^2}-\frac{1}{\omega_1\omega_2}\right)\tilde{E}_x(\omega_1)\tilde{E}_x^*(\omega_2). \tag{82}$$

Then the coupled-mode equation (78) takes the form

$$\left(\frac{\partial}{\partial t_1}+\frac{\partial\omega_3}{\partial k_3}\frac{\partial}{\partial z_1}+\Gamma_L\right)\tilde{F}(\omega_3) = \eta_3\,\tilde{E}_x(\omega_1)\tilde{E}_x^*(\omega_2), \tag{83}$$

where

$$\Gamma_L = \frac{\nu_L(1+k_3^2/\alpha_3^2)\omega_p^2}{2\alpha_3\,\omega_3^3\,\partial D_3/\partial\omega_3}, \tag{84}$$

$$\eta_3 = \frac{e\omega_p^2\,k_3}{m\omega_1\omega_2\lambda_3\,\partial D_3/\partial\omega_3}\left(\frac{1}{\omega_p^2-\omega_3^2}-\frac{1}{\omega_1\omega_2}\right), \tag{85}$$

with $D_3 = D(k_3, \omega_3)$. If we assume that the interface is fixed and unmoving, the second term ($1/\omega_1\omega_2$ term) of the coupling coefficient η_3 disappears. Equations (28) and (83) are the desired two coupled-mode equations.

6. Parametric instability

In this section, we consider the parametric instability, and the group velocity terms in (28) and (83) will be neglected, assuming that $\partial/\partial t_1 \gg (\partial\omega/\partial k)\,\partial/\partial z_1$. At the onset of a parametric instability, this inequality is always valid. We also introduce a small frequency mismatch Δ in the resonance matching conditions: $\Delta = \omega_2+\omega_3-\omega_1$. Then (28) and (83) can be written as

$$\left(\frac{\partial}{\partial t_1}+\Gamma_H\right)\tilde{F}(\omega_2) = i\eta_2'\,\tilde{E}_x(\omega_1)\tilde{F}^*(\omega_3)e^{it_1\Delta}, \tag{28$'$}$$

$$\left(\frac{\partial}{\partial t_1}+\Gamma_L\right)\tilde{F}(\omega_3) = i\eta_3'\,\tilde{E}_x(\omega_1)\tilde{F}^*(\omega_2)e^{it_1\Delta}, \tag{83$'$}$$

where η_2' and η_3' are the coefficients in (28) and (83) respectively. One can easily eliminate $\tilde{F}(\omega_2)$ from the above two equations to get

$$\frac{\partial^2 A}{\partial t_1^2}+(i\Delta+\Gamma_H-\Gamma_L)\frac{\partial A}{\partial t_1}-\Omega A = 0, \tag{86}$$

where $A = \tilde{F}^*(\omega_3)\;e^{\Gamma_L t_1}$ and

$$\Omega = \frac{e^2\,k_3^2\omega_p^4\,|\tilde{E}_x(\omega_1)|^2}{2m^2\omega_1^2\omega_2^2\omega_3\lambda_3\,\partial D_3/\partial\omega_3}\left(\frac{1}{\omega_p^2-\omega_3^2}-\frac{1}{\omega_1\omega_2}\right). \tag{87}$$

We consider the threshold and growth rate of the instability. Putting $\partial/\partial t_1 = i(\omega_3-\omega)+\Gamma_L$, (86) gives the following dispersion relation:

$$(\omega-\omega_3)^2+(\omega-\omega_3)(i\Gamma_H+i\Gamma_L-\Delta)-\Gamma_L\,\Gamma_H-i\Gamma_L\Delta+\Omega = 0, \tag{88}$$

which is the standard form for the dispersion relation of the parametric instability (Nishikawa 1968). After separating this equation into real and imaginary parts, with $\omega - \omega_3 = x + iy$, and eliminating x, we get

$$(y+\Gamma_H)(y+\Gamma_L)\left\{1+\frac{\Delta^2}{(2y+\Gamma_H+\Gamma_L)^2}\right\} = \Omega. \tag{89}$$

If the value of Ω is not too large, (89) can be satisfied by a negative value of y. For instability ($y > 0$), it can be seen by inspection that Ω should be greater than at least $\Gamma_H\Gamma_L$ (when $\Delta = 0$). The threshold value of Ω for instability is obtained by setting $y = 0$ in (89):

$$\Omega_{th} = \Gamma_H\Gamma_L\left\{1+\frac{\Delta^2}{(\Gamma_H+\Gamma_L)^2}\right\}. \tag{90}$$

In the following, we shall put $\Delta = 0$ and take the threshold value as $\Gamma_H\Gamma_L$, and then the condition for instability is given as $\Omega > \Gamma_H\Gamma_L$, which can be written in the form

$$\frac{2e^2\,|\tilde{E}_x(\omega_1)|^2}{c^2m^2\nu_H\nu_L} > \frac{(1+k_3^2/\alpha_3^2)\omega_1^2\omega_3\lambda_3}{c^2\alpha_3 w_3^3 k_3^2\left(\dfrac{1}{\omega_p^2-\omega_3^2}-\dfrac{1}{\omega_1\omega_2}\right)}. \tag{91}$$

The maximum growth rate y_m above the threshold is obtained when $\Delta = 0$ in (89):

$$y_m = \tfrac{1}{2}(\{(\Gamma_H-\Gamma_L)^2+4\Omega\}^{1/2}-(\Gamma_H+\Gamma_L)) \tag{92}$$

We assume that the value of Ω is slightly above the threshold $\Gamma_H\Gamma_L$, i.e. $\Omega = \Gamma_H\Gamma_L + \delta\Omega$ with $\delta\Omega \ll \Gamma_H^2$, Γ_L^2. Expanding the square root in (92), we have

$$y_m \approx \frac{\delta\Omega}{\Gamma_H+\Gamma_L}; \tag{93}$$

i.e. the growth rate just above the threshold is inversely proportional to $\Gamma_H + \Gamma_L$. Using (26b) and (84), we have

$$\frac{\nu_L}{2(\Gamma_H+\Gamma_L)} = \frac{1}{\omega_p^2}\left\{\frac{\nu_H/\nu_L}{w_2^2}+\frac{(1+k_3^2/\alpha_3^2)}{\alpha_3\omega_3^3\,\partial D_3/\partial\omega_3}\right\}^{-1} \tag{94}$$

Equations (91) and (94) are the main results of this work. The right-hand sides of these equations represent respectively the dimensionless threshold and growth rate for the resonant triads that satisfy the matching conditions (2). We first solve the resonance conditions in (3) numerically, and obtain the resonant triads from which the thresholds and growth rates given by (91) and (94) respectively are determined. We find that there are numerous events satisfying the resonance conditions. From our numerical work, we discover that ck_3/ω_p should be smaller than 1.3 and ck_1/ω_p should be greater than 1.3 to form a resonant triad satisfying the conditions (3). Using (5) and (8) gives the following relationship with the angle of incidence θ_0:

$$\frac{ck_1}{\omega_p} = \tan\theta_0.$$

Thus, for a parametric decay instability considered in this work to take place, the angle of incidence should be greater than $\tan^{-1} 1.3 = 52°$. Otherwise the

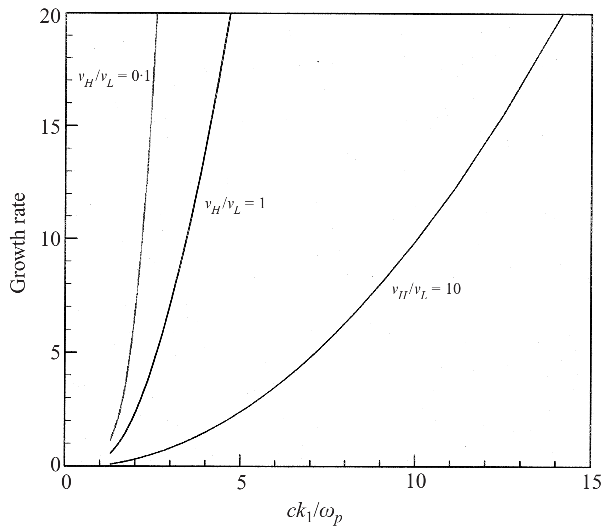

Figure 2. Growth rate (right-hand side of (94)) versus wavenumber for the light wave for various values of ν_H/ν_L.

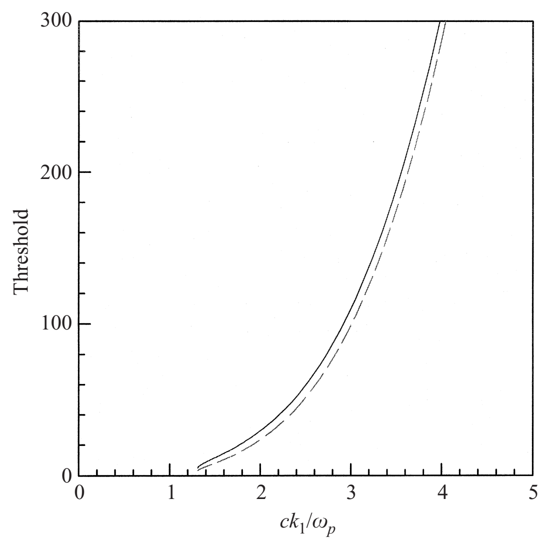

Figure 3. Threshold (right-hand side of (91)) versus wavenumber for the light wave. The solid (broken) line represents the case where the boundary is free (fixed).

resonance conditions as depicted in Fig. 1 are not met. For this particular angle of incidence $\theta_0 = 52°$, the value of the pump frequency is $\omega_1 = 1.6\omega_p$.

For our calculated resonant triads, we have plotted the right-hand sides of (44) (growth rate) and (91) (threshold) respectively in Figs 2 and 3. Figure 2 roughly represents the qualitative features of the instability just above the threshold. It exhibits the tendency that the instability growth rate increases as the incident angle becomes larger. In the approximation used to derive (94), the detailed coupling mechanisms are glossed over.

In Fig. 3, we have plotted the thresholds both when the boundary is rippling (solid curve) and when it is fixed (broken curve). The range of angles of incidence investigated in Fig. 3 is from $\theta_0 = 53°$ to $75°$, over which the threshold is steadily increasing. The value $\theta_0 = 75°$ corresponds to the pump frequency $\omega_1 = 3.8\omega_p$. The rippling effect of the boundary enhances the threshold value as compared with the fixed boundary by a typical amount

$$\Delta E_x \approx 3\frac{cm(\nu_H \nu_L)^{1/2}}{e} \approx 3 \times (\nu_H \nu_L)^{1/2} \times 10^{-3}\,\mathrm{V\,M}^{-1},$$

which might be barely observable in an experiment for a rather dense plasma of low temperature.

Finally, we point out that the coupling coefficient Ω, (87), has a contribution only from the $\mathbf{v} \times \mathbf{B}$ nonlinear force (the $(\omega_p^2 - \omega_3^2)^{-1}$ term) and from $\xi^{(1)}\,\partial\xi^{(1)}/\partial z_0$ nonlinearity, (76). The latter is due to the rippling effect. It should be pointed out that the convective ponderomotive force (the $\mathbf{v} \cdot \nabla \mathbf{v}$ nonlinearity) plays no role in the resonant interaction that we consider in this work. Except the explicit rippling nonlinearity, $\xi^{(1)}\,\partial\xi^{(1)}/\partial z_0$, the free-boundary effects implied in the solutions in (71) are suppressed since only the quadratic terms play a role in this parametric interaction. The rippling nonlinearity $\xi^{(1)}\,\partial\xi^{(1)}/\partial z_0$ is the beat of two high-frequency components of the surface elevation, (79a,b), which would be rather small and could not possibly introduce a drastic change in our parametric interactions.

7. Summary

We have investigated a decay instability in which a high-frequency light wave decays into another light wave of high frequency and a low-frequency TM-mode surface wave. We have shown that a p-polarized light wave transmitted from vacuum into a plasma at a particular angle of incidence can propagate as a bulk electromagnetic plasma wave without attenuation. In deriving the coupled-mode equations, we have presented a method of dealing with the kinematics of the rippling free boundary. The nonlinear boundary conditions are formulated in terms of the surface charge and volume currents. The surface-wave equation, whose solutions involve a power series in the perpendicular coordinate, can be conveniently solved in operational form, and a two-scale analysis can be carried out straightforwardly. We have calculated the thresholds of the parametric instability, and have found that the thresholds are slightly higher for a moving boundary than for a fixed boundary. This parametric instability might be interesting in the case of oblique incidence into an underdense plasma. The result obtained in this work that the parametric instability occurs for an angle of incidence greater than 52° might be worth checking experimentally.

It can be seen that the dominant nonlinearity is the Lorentz force $\mathbf{v} \times \mathbf{B}$ in this parametric instability, and instability is possible even though the nonlinear current $(n\mathbf{v})$ is absent.

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References

Aliev, Y. M. and Brodin, G. 1990 *Phys. Rev.* **A42**, 2374.

Atanassov, V., Mateev, E. and Zhelyazkov, I. 1982 *J. Plasma Phys.* **26**, 217.

Boardman, A. D. 1982 *Electromagnetic Surface Modes*. Wiley, New York.

Brodin, G. and Lundberg, J. 1991 *J. Plasma Phys.* **46**, 299.

Buti, B. 1985 *Advances in Space Plasma Physics*, p. 167. World Scientific, Singapore.

Chaker, M., Moisan, M. and Zakrzevski, Z. 1986 *Plasma Chem. Plasma Process.* **6**, 79.

Davidson, R. C. 1972 *Methods in Nonlinear Plasma Theory*. Academic Press, New York.

Dodd, R. K., Eilbeck, J. C., Gibbon, J. D. and Morris, H. C. 1982 *Solitons and Nonlinear Wave Equations*. Academic Press, New York.

Halevi, P. 1992 *Spatial Dispersion in Solids and Plasmas*. Elsevier, Amsterdam.

Infeld, E. 1989 *Phys. Rev.* **A39**, 723.

Jackson, J. D. 1975 *Classical Electrodynamics*, 2nd edn, Chap. 7. Wiley, New York.

Kruskal, M. and Schwarzschild, M. 1954 *Proc. R. Soc. Lond.* **A223**, 348, London.

Lee, H. J. 1995 *J. Korean Phys. Soc.* **28**, 51.

Lee, H. J. and Hong, H. S. 1998 *Phys. Plasmas* **5**, 4094.

Lindgren, T., Larsson, J. and Stenflo, L. 1982 *Plasma Phys.* **24**, 1177.

Nishikawa, K. 1968 *J. Phys. Soc. Japan* **24**, 918.

Sedov, L. I. 1972 *A Course in Continuum Mechanics*, Vol. 2, Chap. 7. Wolters-Noordhoff, Groningen.

Shivarova, A., Stoychev, T. and Russeva, S. 1975 *J. Phys. D: Appl. Phys.* **8**, 383.

Simon, A. and Thompson, W. B. (eds) (1975) *Advances in Plasma Physics*, Vol. 6. Interscience, New York.

Vladimirov, S. V., Yu, M. Y. and Tsytovich, V. N. 1994 *Phys. Rep.* **241**, 1.

Weiland, J. and Wilhelmsson, H. 1977 *Coherent Nonlinear Interaction of Waves in Plasmas*. Pergamon, Oxford.

Zhelyazkov, I. 1987 Nonlinear wave phenomena in bounded plasmas. In: *Proceedings of International Conference on Plasma Physics*, p. 694. World Scientific, Singapore.