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A negative permittivity metamaterial composed of planar resonators with randomly detuned resonant frequencies and randomly distributed in space

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Abstract

This paper investigates metamaterials composed of resonant particles with negative electric polarizability located in a three-dimensional net. The main problem in fabricating these materials is the spread of the resonant frequencies of particular planar resonators. This spread is caused by the tolerances of the fabrication process for planar resonators. The simulation shows that there is a limit to the dispersion of resonant frequencies that allow the metamaterial to behave as a metamaterial with negative effective permittivity. Two metamaterials with a negative real part of the effective permittivity were designed on the basis of simulations. The first metamaterial has a regular periodic structure. The second is a metamaterial in which the resonant particles are randomly distributed both in space and in orientation, and it offers an isotropic response. This metamaterial was fabricated by inserting planar resonators into plastic shells that can be poured into any volume and ensure a random distribution of the resonant particles in space. The results of the simulations have been verified by measurements.

Introduction

Regular three-dimensional (3D) periodic systems composed of resonant elements showing negative electric and/or magnetic polarizabilities are an analogy of crystalline materials. In the view of this, an amorphous material is represented by a system of resonant particles located with the random spread of positions and also of orientations. This system can be used as a metamaterial with an isotropic response, as has been shown experimentally in [1] for metamaterials with negative permeability, and in [2] for metamaterials with negative permittivity. An analysis and measurements of a system composed of broad side coupled split-ring resonators (BC SRR) showing negative permeability are presented in [3].

A number of techniques for achieving a metamaterial with an isotropic response have been presented both in the microwave band and in the terahertz/infrared band. A 3D isotropic left-handed metamaterial based on the rotated transmission-line matrix scheme, represented by a simplified planarized implementation, was presented in [4]. An approach with some similarities was applied in [5], where an almost isotropic 3D multilayered composite right-handed/left-handed metamaterial structure was proposed and designed. It was composed of conductive mesh plates and dielectric layers, including dielectric resonators with a high dielectric constant. A novel 3D structure that supports a negative refractive index was proposed in [6]. This 3D structure was composed of an array of metallic spheres arranged in a face-centered cubic lattice configuration, as in natural molecules. All the spheres are wired at the midpoint of the lattice. A metamaterial with an isotropic response composed of transmission lines was designed in [7]. The authors of [8] showed that the isotropic response of a 3D system of planar resonant particles can be obtained by locating resonators with proper symmetry on the faces of a cube. A novel fabrication technique for an isotropic IR metamaterial consisting of fourfold-symmetric 3D SRRs was proposed in [9]. The authors of [10] used a sample of randomly shaped and randomly oriented SiC microparticles, and showed that these dielectric particles might be a much simpler alternative to SRRs for metamaterial fabrication in the THz frequency band.

Several planar resonant elements showing a negative real part of electric polarizability (NEP) have been presented. Resonators based on SRRs with a purely electric response were presented in [11]. The authors of [12] proposed electric dipoles terminated by inducting loops in uniplanar form with an inductor having one turn, and a miniaturized version of it with two turns in two-sided form. Various resonators in double H shaped form were proposed in [2]. The authors of [13] presented very small resonators with an electrical response in the shape of a meander.

The analysis of the metamaterial with negative permeability presented in [3] has been rewritten to homogenize the metamaterial showing negative permittivity. This paper studies

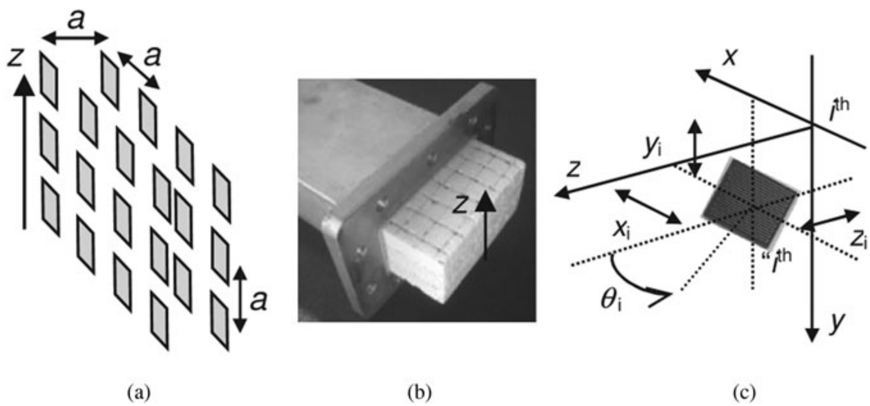

Fig. 1. A 3D periodical system of elements aligned in the same direction (a), elements inserted into three polystyrene slices. The direction of z-axis is shown (b). Sketch of the i^{th} NEP resonant element located in the coordinate system in the position defined by x_i, y_i, z_i, (the i^{th} node – nominal position) and at orientation angle θ_i (c).

the behavior of the system of NEP resonant elements presented in [13]. The simulation is verified by measuring the scattering parameters of systems of resonant elements inserted in a waveguide. The aim of this study is to define the parameters of the resonant elements in order to design a metamaterial showing a negative real part of the effective permittivity, both in a regular 3D structure and in particular, as an amorphous metamaterial showing negative permittivity with an isotropic response.

The paper is organized as follows. The section "An analysis of a general system of resonators" rewrites the homogenization analysis presented in [3]. The section "Selection of NEP resonant elements" presents particular resonant elements that show NEP. The resonant element presented in [13] was selected based on the comparison of values of the polarizabilities as the most robust that provides the metamaterial with negative real part of effective permittivity. The section "Analysis and experiment – A metamaterial with negative permittivity" presents an analysis of the regular 3D structure and of the amorphous metamaterial. The conditions under which the negative permittivity is obtained are specified. The analysis is verified by experiment. Finally, the conclusions are drawn.

An analysis of a general system of resonators

An analysis of systems of NEP resonant elements was performed by the method presented in [3], rewritten to a description of the electric field instead of the magnetic field, i.e. by a dual approach. Two systems are studied. In the first material, resonant particles are located in a regular periodic 3D net with the period a, all aligned in the direction parallel to the z-axis, see Fig. 1(a). In the second system, the resonators are located in random positions dispersed around the lattice nodes in perturbations of Δx, Δy, Δz, and are oriented randomly around the z-axis in perturbation of $\Delta\theta$. The variation of the second spherical angle φ is not relevant as the electric moment of the NEP element does not depend on this angle. The resonant element can be in the plane $x - y$ oriented at any angle φ as the electric field is always parallel to its main axis, see Fig. 1. This corresponds to the TE_{10} mode in rectangular waveguide where the electric field is parallel to z-axis, see Fig. 1 (b). A sketch of the i^{th} resonator located in a nominal position in the coordinate system is shown in Fig. 1(c). The resonators are modeled by small electric dipoles. Their resonant frequencies differ from each other due to imperfect fabrication. The distribution of the resonant frequencies is assumed to be normal.

The total electric field averaged over the cell volume $V = a^3$ is aligned in the z direction $\hat{\boldsymbol{E}} = \hat{E}\,\boldsymbol{z}_o$. The local electric field at the origin of the coordinate system that excites the reference dipole numbered as "0" is composed as the superposition of all fields $\boldsymbol{E}_i$ excited by dipoles located in positions i

$$\boldsymbol{E}_{0loc} = \hat{\boldsymbol{E}} + \sum_{i \neq 0} \left(\boldsymbol{E}_i - \hat{\boldsymbol{E}}_i\right) - \hat{\boldsymbol{E}}_0. \tag{1}$$

In fact, equation (1) compares the differences between the local fields and the averaged fields [14]. Fields $\mathbf{E}_i$ are expressed by [15]

$$\boldsymbol{E}_i = \frac{1}{4\pi\varepsilon_o}\begin{bmatrix} k^2(\boldsymbol{n}_i \times \boldsymbol{p}_i) \times \boldsymbol{n}_i \dfrac{e^{-jkr_i}}{r_i} \\ +[3\boldsymbol{n}_i(\boldsymbol{n}_i \cdot \boldsymbol{p}_i) - \boldsymbol{p}_i]\left(\dfrac{1}{r_i^3} + \dfrac{jk}{r_i^2}\right)e^{-jkr_i} \end{bmatrix}, \tag{2}$$

where $\boldsymbol{n}_i$ and r_i are the direction vector and the distance between nodes 0 and i. In (1), the fields are summed over all dipoles with the exception of $i = 0$. The reference dipole at position $i = 0$ with electric moment $\boldsymbol{p}_0$ excites a field with an averaged value [15]

$$\hat{\boldsymbol{E}}_0 = -\frac{1}{3\varepsilon_o a^3}\boldsymbol{p}_0. \tag{3}$$

The electric moments of dipoles oriented in directions defined by unit vectors $\boldsymbol{p}_{oi}$ are determined by the local electric fields $\boldsymbol{E}_{iloc}$

$$\boldsymbol{p}_i = \alpha_i(\omega, \omega_{oi})\,(\boldsymbol{E}_{iloc} \cdot \boldsymbol{p}_{oi})\,\boldsymbol{p}_{oi}, \tag{4}$$

where $\alpha_i(\omega, \omega_{oi})$ is the electric polarizability of the resonator at frequency ω, and ω_{oi} is the resonant frequency.

The unknown components of vectors $\boldsymbol{E}_{iloc}$ are determined by solving a system of algebraic equations (1). The z component of the effective permittivity tensor is defined by the electric moment of the reference dipole, and assuming $\hat{\boldsymbol{E}} = 1$ as a source quantity

$$\varepsilon_{zz}(\omega, \omega_o) = 1 + \frac{\alpha_0(\omega, \omega_{o0})\,(\boldsymbol{E}_{0loc} \cdot \boldsymbol{p}_{o0})\,p_{o0z}}{\varepsilon_o a^3}, \tag{5}$$

where p_{o0z} is the z component of unit vector $\boldsymbol{p}_{o0}$. Finally, the effective permittivity is determined by integrating (5) multiplied by the distribution function over the resonant frequencies. The electric polarizability of the dipole is taken in the Lorentz form

$$\alpha_i(\omega, \omega_{oi}) = \frac{A\omega^2}{\omega_{oi}^2 - \omega^2 + j\omega\delta}, \tag{6}$$

Fig. 2. View of particular NEP elements. An element is taken from [11] (a), an electric dipole terminated by an inductor [12] (b), a double H-shaped resonator (DHR) [2] (c), a meander shaped resonator [13] (d).

Table 1. Amplitudes *A* of the polarizabilities (6) of the resonant elements shown in Fig. 2.

Figure 2	*a*	*b*	*c*	*d*
$A(m^2F)$	1.8×10^{-19}	2.1×10^{-19}	1.63×10^{-19}	1.28×10^{-18}

where δ represents losses and A is amplitude. These values were determined as proposed in [16] by simulating a single resonator in the R32 waveguide by the CST Microwave Studio and by measurements giving similar values.

In (1), the resonant frequencies are randomly chosen with normal probability of distribution. The perturbations of the resonator positions and of the orientations are randomly chosen independently for each dipole within the defined limits with a uniform distribution of probability. Finally, (5) is averaged over a number of realizations chosen high enough to obtain convergence. This averaging replaces N-dimensional integration. Generally, between 100 and 1000 realizations are taken. The number depends on the convergence of the simulation process. Convergence is ensured by getting smooth dependencies of permittivity on frequency. For most applications, it is sufficient to take into account one central cell surrounded by one layer of neighboring resonant particles from all sides (26 cells) to obtain smooth dependence of permittivity on frequency.

Selection of NEP resonant elements

There is a number of NEP resonant elements that have been presented in the literature. Some of them were taken here into consideration, those from ref. [11, 13] and those already investigated by the author [2, 12]. These elements are shown in Fig. 2. Measurements, as described in [16], of all these resonators showed NEP. The amplitudes A for all resonators under consideration are shown in Table 1. This table shows that the NEP from Fig. 2(d) has an amplitude A almost one order higher than the other resonators. Therefore this resonant element is the most suitable for the application in a metamaterial showing negative permittivity between elements presented in Fig. 2.

The meander NEP resonators [13] were designed and fabricated using a Rogers RT Duroid 5870 substrate that was 0.254 mm in thickness with permittivity 2.33. The dimensions of the substrate were $6.7 \times 6.7\ \text{mm}^2$, and the strips and slots were 0.15 mm in width. The resonant frequency measurements gave an average value of 2.965 GHz, with a standard deviation of $\sigma = 0.015$ GHz. The polarizability was measured as stated in [16] for a single resonant particle located at the center of an R32 waveguide. Fitting the resulting frequency dependence (averaged over six measurements of six resonators) determined values from (6) $A = 1.28 \times 10^{-18}\ \text{m}^2\text{F}$, $\delta = 0.355$ GHz. The same measurements of the double H-shaped resonator (DHR) element [2] resulted in an average value of the resonant frequency 3.844 GHz and standard deviation of the resonant frequencies $\sigma = 0.005$ GHz. Measured values from (6) are $A = 1.63 \times 10^{-19}\ \text{m}^2\text{F}$, $\delta = 0.11$ GHz. Similar values were determined for the resonators from Figs 2(a) and 2(b), see Table 1.

The suitability of the application of particular resonant elements to the design the metamaterial with negative permittivity has been proved by the presented homogenization method. The positions of the resonators were perturbed by spreading them randomly around the net nodes in all three directions by ±a/2, see Fig. 1(c). The two particular systems were analyzed. In the first system resonant elements were aligned in the same direction $-\Delta\theta = 0°$. In the second system, the resonators were randomly oriented in the perturbation of the full span that corresponds to $\Delta\theta = 180°$ taken with respect to the z-axis.

The dependence of the complex permittivity on frequency was calculated and plotted in Fig. 3. The metamaterials composed of both DHR and meander elements aligned in the same direction showed negative effective permittivity, see Fig. 3. The polarizability value of the DHR resonators is not high enough to ensure that a material composed of these resonant elements randomly located both in space and in orientation exhibits negative permittivity, Fig. 3(a). On the other hand, Fig. 3(b) proves that the real part of the complex effective permittivity of NEP in the shape of meander resonators calculated for randomly located resonators aligned in the full span corresponding to $\Delta\theta = 180°$ reaches negative values.

An analysis of the resonators shown in Figs 2(a) and 2(b) proved in addition that they do not provide the negative real part of the effective permittivity if they are randomly distributed in space and in orientation.

Finally, this discussion showed that only meander-shaped resonators [13] out of the selected resonators can be used to design a metamaterial exhibiting negative effective permittivity.

Analysis and experiment – A metamaterial with negative permittivity

The scattering parameters of the designed metamaterial prisms were measured as inserted into a waveguide R32 with metal walls, and were compared with the results of the simulation

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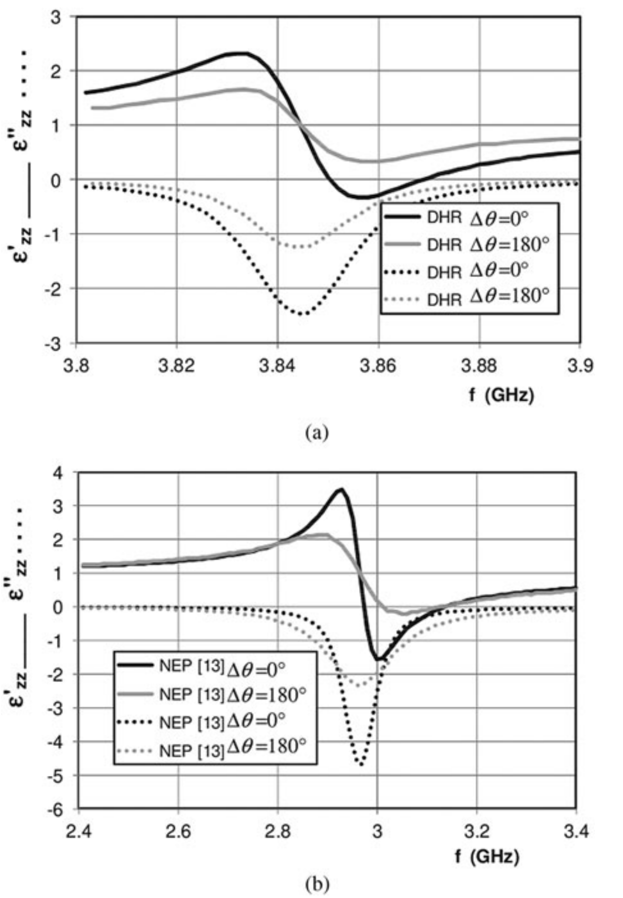

Fig. 3. Simulated complex effective permittivity of a system of DHRs [2] aligned in the same directions ($\Delta\theta$ =0°) and in a random distribution of directions ($\Delta\theta$ =180°) (a). The same plots for NEP [13] (b). Resonators are randomly spread in space in all three directions by ±a/2, the spread of resonant frequencies is defined for NEP by σ = 0.015 GHz, and for DHR by σ = 0.005 GHz.

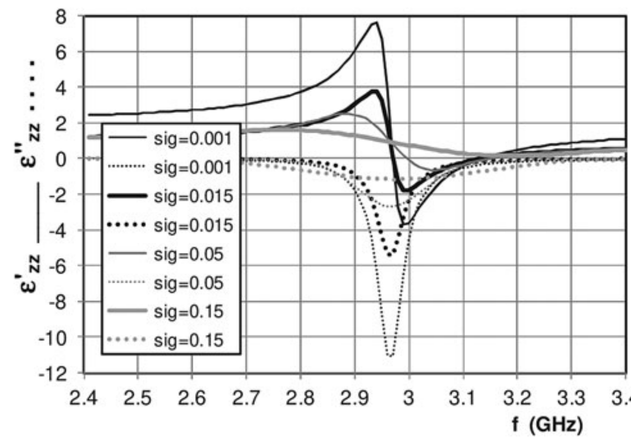

Fig. 4. Calculated effective permittivity of the regular 3D periodic system of electric dipoles from Fig. 2d aligned in the same direction. Standard deviation σ is marked as "sig".

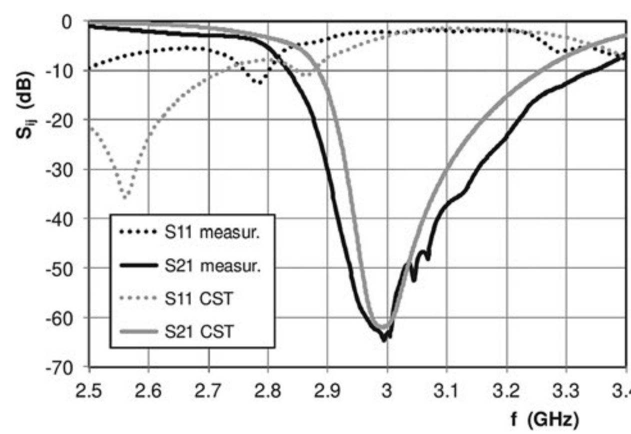

Fig. 5. Measured, simulated by CST scattering parameters of the metamaterial block composed by a 3D periodic system of electric dipoles aligned in the same direction. The metamaterial prism is located in waveguide R32.

done by the CST Microwave Studio. CST analyzes the prism of the metamaterial located in the waveguide as a homogeneous fictive material defined by the calculated complex permittivity, and permeability equal to 1.

A regular 3D metamaterial

The presented homogenization procedure was used to analyze the response of the regular 3D periodic system composed of NEP elements from Fig. 2(d) aligned in the same direction, Fig. 1(a). The experimental realization of this system is shown in Fig. 1(b). The resonant elements are inserted into three slices of polystyrene with a period of 11 mm. The calculated complex effective permittivity of this metamaterial is plotted in Fig. 4 with the standard deviation of the distribution of the resonant frequencies taken as a parameter. The system defined by $\sigma = 0.015$ GHz is a metamaterial that exhibits a negative real part of the effective permittivity in the interval from 2.97 to 3.13 GHz, with a maximum value of −1.4 at 3.0 GHz. With increasing σ, the frequency band of negative permittivity shrinks. Finally, for $\sigma > 0.15$ GHz this band disappears.

Figure 5 shows the experimental verification of the simulation. The behavior of this metamaterial is determined by measuring the scattering parameters in an R32 (WG10) waveguide with metallic walls. The measured scattering parameters plotted in Fig. 5 were averaged over six permutations of three polystyrene slices, see Fig. 1(b). The agreement between the measured and calculated scattering parameters S_{21} is relatively good. For S_{11}, there is a fit only in the frequency band of the resonance. The CST model used the block of a fictional homogeneous material defined by effective permittivity taken from Fig. 4 calculated for σ = 0.015 GHz.

An amorphous metamaterial

The amorphous metamaterial, see Fig. 6(a), was assembled by planar resonators from Fig. 1(d) put into spherical plastic shells 11 mm in outer diameter shown in Fig. 6(b). The diameter of the shells determines the system period *a*. The permittivity of the shell material is 3.3, and the losses are $\tan\delta = 0.02$. The system consisting of 264 NEP elements in shells fills the sample holder that is R32 (WG10) waveguide with transverse dimensions 72 × 72 mm^2 and length 72 mm (a cube of edge equal to 72 mm). Now the resonators are fully randomly distributed and oriented. The scattering parameters were measured at reference planes at the faces of the holder.

(a) (b)

Fig. 6. The amorphous metamaterial in the partly disassembled measurement setup [1] (a). The two halves of the plastic shell with the fabricated NEP elements (b).

The model of the amorphous material for the purposes of the analysis presented here is composed of a system of resonators located in a 3D periodical net with the period a. This defines the basic geometry of the system. Now the positions of the resonators are perturbed by spreading them randomly around the net nodes in all three directions by ±a/2, see Fig. 1(c), with a uniform distribution of probability. The orientations of the resonators are randomly chosen with a uniform distribution of probability within the position angle measured with respect to the z-axis in the perturbation of the full span corresponding to $\Delta\theta = 180°$. These conditions correspond to the real metamaterial implementation with no preference for any orientation in its realization, see Fig. 6(a).

The behavior of the resonators in this material is significantly influenced by the presence of the shells made of a plastic material with permittivity equal to 3.3. This material detunes the resonant frequency of the resonant particles downwards so that their average value is 2.727 GHz. The standard deviation of the distribution of the resonant frequencies was unchanged, and remained $\sigma =$ 0.015 GHz. The losses of this material widen the particle polarizability response. The polarizability measurements [16] of NEP located in a plastic shell gave values for (6) $A = 1.4 \times 10^{-18}$ m^{2}F, $\delta = 0.45$ GHz. Now the polarizability (6) includes both the NEP resonator and the plastic shell. The presented homogenization takes this combination as a single resonator.

Figure 7 shows the results of the analysis of the amorphous material presented above, i.e. a metamaterial with resonators randomly located in space. The lines in Fig. 7(a) represent the real and imaginary parts of the effective permittivity (equal in all directions for the isotropic - amorphous – material studied here) calculated for different values of the standard deviation of resonant frequencies σ. The dispersion of the resonator orientations is described by the perturbation in the full span $\Delta\theta = 180°$. It follows from the plot that the standard deviation of the resonant frequencies must be lower than or maximally equal to 0.015 GHz, in order to obtain a metamaterial with a negative real part of the permittivity. Therefore the fabricated resonant particles fulfill this requirement. This amorphous metamaterial exhibits the negative real part of the effective permittivity in the interval from 2.76 to 2.78 GHz, with a maximum value of −0.06 at 2.77 GHz. Better results would be obtained with the use of resonators with a smaller standard deviation value, see Fig. 7(a). Figure 7(b) shows the dependence of simulated effective permittivity on frequency, with the width of the perturbation of position angles Δθ as a parameter. The interval of the negative real part of the effective permittivity is naturally narrower as the perturbation of the angles increases. $\sigma =$ 0.015 GHz is assumed in Fig. 7(b).

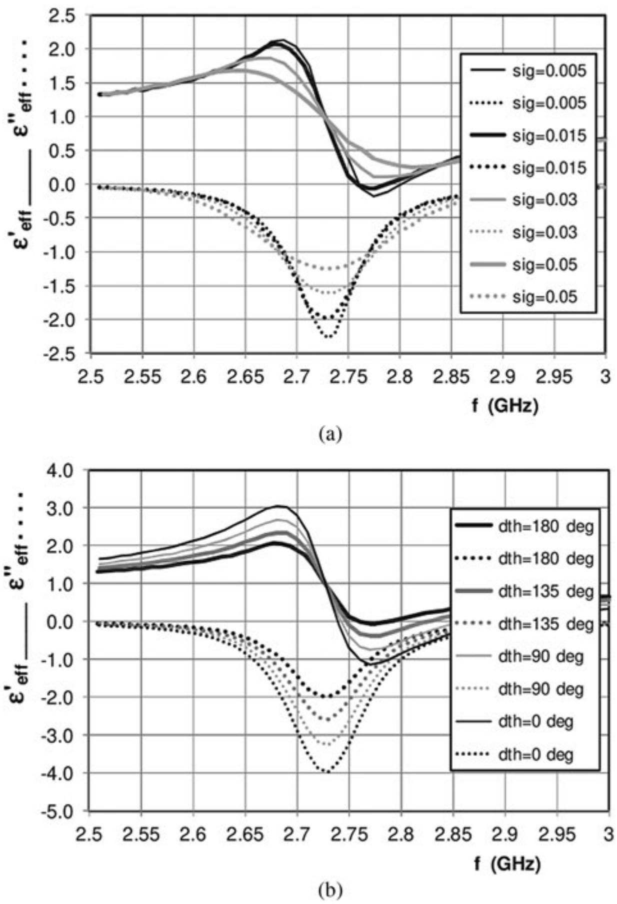

Fig. 7. Calculated effective permittivity of the amorphous metamaterial for shown values of the standard deviation of the resonant frequency distribution marked as "sig", $\Delta\theta = 180°$ is taken (a), for shown values of $\Delta\theta$ marked as "dth", $\sigma = 0.015$ GHz is assumed (b).

An analysis of the magnetic metamaterial with negative permeability [3] has shown that a lower value of system period a must be used in order to obtain a higher negative effective permeability value. The resonators are more tightly coupled, which increases the value of the effective permeability. The same is valid for an electric metamaterial with negative permittivity.

A block of homogeneous metamaterial with the same dimensions as the fabricated specimen was analyzed by the CST Microwave Studio. The complex permittivity for this fictional material was taken from the analysis plotted in Fig. 7(a) for $\sigma =$ 0.015 GHz. The simulated and measured scattering parameters

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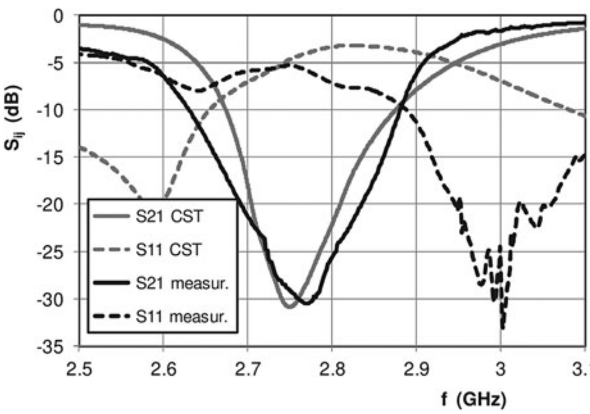

Fig. 8. Measured, simulated by CST scattering parameters of the block of amorphous metamaterial located in a raised R32 (WG10) waveguide.

are plotted in Fig. 8. The measured data in Fig. 8 were taken as the average value from 11 subsequent measurements for differently stirred resonators in the volume of the cube. They can be compared, but the comparison is not perfect. The reason is that fabricated resonators with dimensions of 6.7 mm are not negligibly small in comparison with the cell dimensions, which are equal to the system period of 11 mm. The cube inserted into the waveguide filled with a finite number of resonators located in shells has a limited volume. Additionally, the homogenization procedure used here models the resonators as point electric dipoles. This also reduces the preciseness of the results, because this condition is not exactly kept.

Conclusion

This paper has presented a modified method for homogenizing a metamaterial composed of resonant elements exhibiting NEP. The method takes into account the random distribution of the resonant frequencies of the resonant elements, together with the random variation of their positions and orientations in space. The method was applied to the analysis of a metamaterial with the negative real part of the effective permittivity.

The behavior of the metamaterial composed of a system of resonant elements is significantly influenced by the fabrication imperfections of used elements. Fluctuations in the geometry and in the material parameters of resonant elements have been described by the random distribution of the resonant frequencies of particular resonators.

The frequency band width of the effective permittivity resonance is determined by the dependence of the polarizability of the resonant particle on the frequency. This frequency dependence is superimposed by the spread of the resonant frequencies of particular resonators, which are caused by the spread of the geometry and the material parameters of the resonators. This effect has a major influence on the system behavior. The metamaterial exhibits a negative real part of the effective permittivity up to a certain value of the standard deviation of the resonant frequencies. This is the main limiting factor of the design of the investigated metamaterial.

The results for the analysis of the system of planar resonators located in the 3D periodic system and directed in the same orientation correspond quite well with the measured data. These data were obtained by measuring the scattering parameters of the assembled metamaterial inserted into a waveguide. This metamaterial shows the negative real part of the effective permittivity, and its response is of course not isotropic.

The random distribution of resonant elements in space was used to design an amorphous metamaterial with an isotropic response, and with a negative real part of the effective permittivity. This metamaterial composed of a system of electric planar resonators placed in plastic shells was analyzed and fabricated. It was assembled in a very simple way. The required volume was just filled with plastic spherical shells containing resonant particles.

The results have been verified experimentally. The scattering parameters measured on the fabricated metamaterial located in the waveguide fit quite well the scattering parameters of the homogeneous isotropic fictitious metamaterial with complex permittivity calculated by the homogenization procedure presented here.

The analysis showed the limits in the standard deviation of the spread of the resonant frequencies of the planar resonators. These limits are 0.15 GHz for the 3D regular periodic system, and 0.015 GHz for the amorphous metamaterial. The metamaterial composed of resonant elements with the higher standard deviation does not exhibit negative permittivity.

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References

1. **Zehentner J and Machac J** (2008) Widening the negative effective parameter frequency band of resonant SNG metamaterials, PIERS 2008, *Progress in Electromagnetics Research Symposium, Hangzhou, China, March* 2008, pp. 381–386.
2. **Machac J, Rytir M, Protiva P and Zehentner J** (2008) A Double H-Shaped Resonator for an Isotropic ENG Metamaterial, *38th European Microwave Conference, Amsterdam, October* 2008, pp. 547–550.
3. **Machac J** (2017) Amorphous metamaterial with negative permeability. *IEEE Antennas and Wireless Propagation Letters* **16**, 2138–2141.
4. **Zedler M, Caloz C and Russer P** (2007) A 3-D isotropic left-handed metamaterial based on the Rotated Transmission-Line Matrix (TLM) scheme. *IEEE Transactions on Microwave Theory and Techniques* **55**, 2930–2941.
5. **Sato Y, Ueda T, Kado Y and Itoh T** (2011) Design of isotropic 3-D multilayered CRLH metamaterial structures using conductive mesh plates and dielectric resonators, *Asia-Pacific Microwave Conference 2011, Melbourne, Australia, December 2011*, pp. 526–529.
6. **Sanada A** (2008) *A 3D isotropic left-handed metamaterial composed of wired metallic spheres, 2008 IEEE MTT-S International Microwave Symposium Digest, Atlanta, GA, USA, June* 2008, pp. 339–342.
7. **Selvanayagam M and Eleftheriades GV** (2013) Dual-Polarized volumetric transmission-line metamaterials. *IEEE Transactions on Antennas and Propagation* **61**, 2550–2560.
8. **Baena JD, Jelinek L and Marques R** (2007) Towards a systematic design of isotropic bulk magnetic metamaterials using the cubic point groups of symmetry. *Physical Review B* **76**, 245115.
9. **Tanaka T and Ishikawa A** (2014) Fabrication of isotropic infrared metamaterials, *8th International Congress on Advanced Electromagnetic Materials in Microwaves and Optics – Metamaterials 2014, Copenhagen, Denmark, August 2014*, pp. 418–420.
10. **Wheeler MS, Aitchison JS, Chen JIL, Ozin GA and Mojahedi M** (2009) Infrared magnetic response in a random silicon carbide micropowder. *Physical Review B* **79**, 073103.
11. **Tao H, Padilla WJ, Zhang; X and Averitt RD** (2011) Recent progress in electromagnetic metamaterial devices for terahertz applications. *IEEE Journal of Selected Topics in Quantum Electronics* **17**, 92–101.

12. **Machac J, Protiva P and Zehentner J** (2007) *Isotropic epsilon-negative particles, 2007 IEEE MTT-S International Microwave Symposium, Honolulu, Hi., USA, June*, TH4D-03, CD ROM.
13. **Imhof PD, Ziolkowski RW and Mosig JR** (2006) *Highly subwavelength unit cells to achieve epsilon negative (ENG) metamaterial properties, 2006 IEEE Antennas and Propagation Society International Symposium*, pp. 1927–1930, DOI: 10.1109/APS.2006.1710951.
14. **Tretyakov S** (2003) *Analytical Modeling in Applied Electromagnetics*. Boston, London: Artech House, Inc.
15. **Jackson JD** (1998) *Classical Electrodynamics*, 3rd Edn., John Wiley & Sons, Inc.
16. **Jelinek L and Machac J** (2014) A polarizability measurement for electrically small particles. *IEEE Antennas and Wireless Propagation Letters* **13**, 1051–1053.

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