Scientific Structuralism: Presentation and Representation

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This paper explores varieties of scientific structuralism. Central to our investigation is the notion of 'shared structure'. We begin with a description of mathematical structuralism and use this to point out analogies and disanalogies with scientific structuralism. Our particular focus is the semantic structuralist's attempt to use the notion of shared structure to account for the theory-world connection, this use being crucially important to both the contemporary structural empiricist and realist. We show why minimal scientific structuralism is, at the very least, a powerful methodological standpoint. Our investigation also makes explicit what more must be added to this minimal structuralist position in order to address the theory-world connection, namely, an account of representation.

- 1. Introduction. The focus of this paper is the recent revival of interest in structuralist approaches to science and, in particular, the structural empiricist and structural realist positions in philosophy of science. Our aim is to appeal to the notion of shared structure to classify the varieties of scientific structuralism and to offer a 'minimal' construal that is best viewed from a methodological stance.
- 2. Mathematical and Scientific Structuralism: Analogies and Disanalogies. Since much of what is taken as distinctive of scientific structuralism is built up out of an analogy with mathematical structuralism, we begin first

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1. Discussions of structuralism in the philosophy of science literature have, quite naturally, centered on those sciences, like physics, that are formulated in mathematical terms; ours will do the same.

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with a brief description² of what we mean by this. We take mathematical structuralism to be the following philosophical position: the subject matter of mathematics is structured systems and their morphology, so that mathematical 'objects' are nothing but 'positions in structured systems', and mathematical theories aim to describe such objects and systems by their shared structure, that is, by their being instances of the same kind of structure.

For example, the theory of natural numbers, as framed by the Peano axioms, describes the various systems that have a Natural-Number structure. These structured systems are, for example, the von Neumann ordinals, the Zermelo numerals, and so forth; they are *models* (in the Tarskian sense of the term) of the Natural-Number structure. The 'objects' that the theory of natural numbers talks about are then the positions in the various models. For example, the von Neumann ordinal '2' is a position in the model 'von Neumann ordinals'; the Zermelo numeral '2' is a position in the model 'Zermelo numerals'; and the theory of natural numbers describes the number '2' in terms of the shared structure of these, and other, models that have the same kind of structure. If all models that exemplify this structure are isomorphic, then the Natural-Number structure and its morphology are said to present its *kinds of* objects, that is, are said to determine its 'objects' only 'up to isomorphism'.

As explained by Benacerraf (1965), mathematical structuralism implies that there are no natural numbers as *particular objects*, that is, as existing things whose 'essence' or 'nature' can be individuated independently of the role they play in a structured system of a given kind. This is because the relevant criterion of individuation, namely, Leibniz's Principle of the Identity of Indiscernibles, does not hold. For example, in one system of the natural numbers the property $2 \in 4$ holds for the natural number '2' while in another it does not. Yet clearly, since the systems are isomorphic, we want to say that we are talking about the same natural number '2'. In our terminology, we express this by saying that we are talking about '2' as a *kind of* object. More generally, we say that there are only mathematical 'objects' as kinds of objects, that is, that there are 'objects' that can be individuated only up to isomorphism as positions in a structured system of a given kind.

Thus, taking 'structured system' to mean 'model', we say that a mathematical theory, while framed by its axioms, can be *characterized* by its models, and the *kinds of objects* that the theory talks about can be *presented* by their being positions in models that have the same kind of structure.

2. For a more detailed analysis of the interpretations and varieties of mathematical structuralism, see Landry and Marquis (2005).

Analogously, according to the semantic view of scientific theories, theories (regardless of how, or whether, they are formally framed) are to be characterized as a collection of models that share the same kind of structure, and the kinds of objects that the theory talks about can be presented as positions in such models. In developing our characterization of minimal scientific structuralism we too accept this analogy with mathematical structuralism. However, we pause here to note two important disanalogies. First, in physical theorizing it is important to keep clear the semantic distinction between kinds of objects and particular objects. For the mathematical structuralist, this distinction is not possible; mathematical objects are kinds of objects rather than particular objects. This is what it means to say that mathematical 'objects' are characterized only by what can be said of their shared structure. For example, when we speak about the natural number '2' we do not intend to refer to (or mean) any particular instance of the natural number 2: we are speaking about the 'object' 2 as a kind of object.

The second disanalogy is that in physical theorizing we also need the *ontological* distinction between theoretical objects and their physical realization. We need to maintain a level of description in which a physical theory can *talk about* electrons, as theoretical objects, without its having to *be about* electrons, as objects that are physically realized in the world. To talk about electrons (or unicorns) is not thereby to bring them into existence as physical objects. Again, in mathematics there is no such distinction; for a sentence to be about an object is nothing more than for a sentence to talk about an object. Thus, for a mathematical object, 'to be', as Quine (1948) explains, is to be a value in the range of a bound variable.

We rely on the terminology of 'presentation' versus 'representation' to express these important analogies and disanalogies. At the semantic level, we say that in mathematics the kinds of objects that the theory talks about are *presented* via the shared structure holding between the mathematical models. Likewise for physical theories; theoretical objects, as kinds of physical objects, may be *presented* via the shared structure holding between the theoretical models. However, at the ontological level, a physical theory, insofar as it is successful,³ must also *represent* particular physical objects and/or phenomena and not merely present kinds of physical objects.

- **3. Applicability in Terms of Shared Structure.** Building on the analogy with mathematical structuralism, the semantic view of scientific theories
- 3. To remain agnostic about whether and/or how such representations need 'save the phenomena' or 'get a hold on reality', i.e., to remain agnostic about whether and/or how theories, to be successful, need be 'empirically adequate' or 'true', we leave the notion of success as unanalyzed.

is put to use to account for the applicability of a scientific theory to the phenomena. As Suppes has pointed out (1960, 1962), scientific theorizing consists of "a hierarchy of theories and their models" (Suppes 1962, 255) that bridges the gap between the high level theory and the lower level phenomena that the theory is intended to be about. There is a theory, characterized by the collection of its models, associated with each layer so that the relationship of shared structure between each layer (e.g., between the theory and the data) can be formally analyzed and experimentally evaluated. So arranged, the *formal analysis* (by model-theoretic methods) of the *applicability* of the theory to the phenomena is made by appealing to isomorphisms to formally express the claim that their models have the same structure. The question for us is just how far this analysis can take us.

In this light, it is important to note that data models, for Suppes, are models in the Tarskian sense—they are models of *a theory of* data. As such, data models are far removed from 'mere descriptions of what is observed', that is, from what we might call 'the phenomena'. As Suppes notes, "the precise definition of models of data for any given experiment requires that there be a theory of data in the sense of the experimental procedure, as well as in the ordinary sense of the empirical theory of the phenomena being studied" (Suppes 1962, 253). Thus, two things are required to connect the high level theory to the phenomena: an experimental theory of the data and an empirical theory of the phenomena.

Suppes (1960, 1962, and 1967) details the evaluative criteria of those theories (theories of experimental design and of ceteris paribus conditions) that go into the construction of the experimental theory of the data. But he is clear that, since there are no models (in the Tarskian sense) of these theories, one can formally characterize the experimental theory of the data only by the collection of its data models; and so one's formal analysis

- 4. See Suppes (1962) for a description of the "criteria of evaluation" and, particularly, see page 259 for a list of the "typical problems" associated with each.
- 5. See Suppes (1967, 59) for the claim that "[t]he definition of isomorphism of models in the given context makes the intuitive idea of *same structure* precise." In an earlier work (1960) Suppes also states that once the [empirical] theory is axiomatized within a standard set-theoretical framework, the mathematical methods of using "representation theorems" and "embedding theorems" to capture facts about isomorphisms can be extended from mathematics to the empirical sciences.
- 6. Van Fraassen makes a similar distinction, namely, "the point long emphasized by Patrick Suppes that the theory is not confronted with the raw data but with models of the data, and the construction of these data models is a sophisticated and creative process" (van Fraassen 1989, 229). However, he then collapses this distinction by claiming that models of data are "the dress in which the debutante phenomena make their debut" (ibid.).

must *begin with* models of data. To then connect the data to the phenomena one must establish that their models have the same structure. But without an (empirical) theory of the phenomena, one cannot speak of 'the structure of the phenomena', for example, one cannot characterize the structure of the phenomena in terms of the shared structure of its models. Suppes thus remains silent on the issue of why we should suppose that models of data have the same structure as the phenomena.

It is here, then, that we are presented with three options in accounting for applicability in terms of shared structure: (i) from a *methodological* stance, we may forgo talk of the structure of the phenomena and simply begin with structured data, that is, with data models; (ii) from an *empirical* stance we may say that what structures the phenomena into data models is the high level theory; and finally, (iii) from a *realist* stance we may say that what structures the phenomena is the world.⁷ Regardless of one's stance, it should be clear that without a theory of the phenomena one cannot formalize (again, by model theoretic methods) the treatment of the structure of the phenomena in terms of data models alone, and so one cannot use the semantic view's account of shared structure between models to fully account for the applicability of a theory to the phenomena and, thereby, to establish a theory-world connection.

Data models, then, represent a significant cut-off point; below the level of data models we require more than comparisons of shared structure between models to relate the levels of the hierarchy to one another. In recognition of this we separate the scientific structuralist's challenge of establishing a theory-world connection into two components: (a) to give an account of applicability in terms of the shared structure between models of the theory and data models wherein models of the theory present the kinds of objects that the data models are intended to talk about so that their 'objects' have the same kind of structure, and (b) to give an account of representation in terms of the shared structure between data models and the phenomena so that the phenomena that the theory is about are appropriately structured (by the theory or by the world).

- **4.** Beyond the Mathematical Analogy: From Presentation to Representation. How do theories connect to the world? As noted, viewing this challenge in light of a semantic structuralist characterization of a theory, the connection can be broken down into two main components: connecting theoretical models to data models and connecting data models to the phenomena. We argue that while the first connection can be accounted
- 7. See French (2000, 116-117) for his distinction between the empiricist and realist stance.

for solely in terms of *presentation* of shared structure, ⁸ the second demands the addition of something more. We need an account of *representation*: we need an account of how a physical theory, that *talks about* kinds of objects, comes to *be about* particular objects. We hold that appeals to shared structure are not up to this task. Thus, to establish a theory-world connection, it is necessary to go further than characterizing a theory as a collection of Tarskian models that presents the kinds of objects that the theory talks about.⁹

To move from presentation to representation, and so to move from Quine's semantic 'is' to an ontological 'is', 10 one needs something more than semantic scientific structuralism. The question of the reality of particular physical objects and/or the truth of physical propositions cannot be settled semantically, that is, cannot be settled merely by appeal to a Tarskian notion of a model and/or a Tarskian notion of truth: it depends crucially on some extrasemantic process whereby the connection between what we say and what there is is both established and justified. This is what we mean when we say that an account of representation 11 is required.

- 8. Of course, a great deal of both theoretical and experimental work needs to be done in order to arrive at the data models, but this is not what we are concerned with here. Nor is our concern with the particular strategies of how we 'structure' the data (e.g., bottom-up, top-down, or even boot-strapping strategies). While no doubt (as Giere 1985; Cartwright, Shomar, and Suarez 1995; Suarez 2003; etc., have pointed out) such strategic investigations are necessary for the practical problem of constructing the hierarchy, we can, once this hierarchy is so constructed, place the theoretical hierarchy above the data models and so consider the connection between a theory and the data models
- 9. To appreciate the same point, though expressed differently, see Giere's (1995) discussion of why Tarskian semantics is not appropriate for the representational role of models of physical theories. Note, too, that even though Ladyman (1998) accepts that the semantic view is the most appropriate frame for the structural realist position, he agrees with Giere that Tarskian semantics cannot do the job of closing the gap between the theory and the world. He further agrees with Giere that "once the semantic approach is adopted the crucial issue is whether or not theoretical models tell us about *modalities*" (Ladyman 1998, 416).
- 10. By the ontological 'is' we do not mean the metaphysical 'is' that ranges over the noumena; we are happy for this 'is' to range over just the phenomena, and too we allow for the phenomena to be observable or unobservable. That is, we take no stand on the Kantian realism/idealism debate or the realism/constructive empiricism/instrumentalism debates.
- 11. Taking this challenge to close the gap between theories and the world as being met by an account of truth (as opposed to being met by an account of representation) that would serve to fill out the Tarskian notion of truth and so lend itself to the realist-empiricist debate, Da Costa and French (1990, 251) note various approaches that appeal to additional aspects of truth that can then be used to close this gap, e.g., Putnam's warranted assertability criterion, Fine's NOA, and their own pragmatic po-

The term 'model' in science is, of course, replete with connotations of representation, and the temptation in the past has perhaps been for the semantic view of theories, with its use of Tarskian models (which, to repeat, are truth makers and *not* representations), to piggyback on this required representational role.

In our view this is not acceptable: if the semantic view of theories is to do better than the syntactic view in tackling the problem of the theoryworld connection, then it owes us an account of how its models (Tarskian or otherwise) gain their representational significance.¹² Indeed, as we will now see, it is the differences in how representation is treated that lead to the different varieties of scientific structuralism.

What we call *minimal* structuralism is committed only to the claim that the kinds of objects that a theory talks about are presented through the shared structure of its theoretical models and that the theory applies to the phenomena just in case the theoretical models and the data models share the same kind of structure. No ontological commitment—nothing about the nature, individuality, or modality of particular objects—is entailed. Viewed methodologically, to establish the connection between the theoretical and data models, minimal structuralism considers only the appropriateness of the kind of structure and owes us no story connecting data models to the phenomena. In adopting a methodological stance, we forgo talk of 'the structure of the phenomena' and simply begin with data models. We notice that our theoretical models are appropriately structured (present objects of the appropriate kind) and shared structure is what does the work connecting our data models up through the hierarchy to the theoretical models, and so we suggest the methodological strategy of seeking out, exploring, and exploiting the notion of the appropriate kind of shared structure, both up and down the hierarchy, and sideways¹³ across both different and successive theories.

There are various ways of going beyond this methodologically viewed

sition. In contrast to such truth-seeking approaches, examples of representational approaches include Giere (1988) and van Fraassen (1989), which assert some sort of correspondence (e.g., similarity or isomorphism) between the physical system under investigation and some part of at least one of the theoretical models. Other 'representational' approaches also include accounts, such as Cartwright et al. (1995), which seek to begin with phenomenological models and build up representational relations by, for example, forgoing notions of similarity or isomorphism (see Suarez 2003) and, instead, considering the inference patterns among these and theoretical models.

^{12.} Indeed, one could argue that at least the logical positivists saw the need for such even if their notion of cognitive significance was not up to the task.

^{13.} See Bokulich (2003) for an excellent account of how analyses of horizontal models, i.e., models that are developed by way of analogy with models of a neighboring theory (623), are just as significant for picking out the appropriate kind of structure as are models of the theory and/or models of the data.

minimal structuralism, depending, in part, on how one wishes to make the theory-world connection. That is, depending on how one chooses to close the gap between the data models and the phenomena, a theory that presents us with the appropriate kinds of objects can also be claimed to represent (the structure of) physical objects in the world. Recall that we offered two alternatives to our methodological stance: from an empirical stance, one may hold that what structures the phenomena is the high-level theory, whereas from a realist stance one may hold that what structures the phenomena is the world. Such additional stances are all very well and good, but if we are to be motivated to move beyond the more modest methodological stance we need reasons. In particular, if we are to adopt either the empiricist or the realist alternative, we need a justification for the claim that data models share the same structure as the phenomena and, as a result, that the former can be taken as representations of the latter.

Adopting an empiricist stance, van Fraassen, as a "structural empiricist," suggests that we simply identify the phenomena with the data models: "the data model . . . is, as it were, a secondary phenomenon created in the laboratory that becomes the primary phenomenon to be saved by the theory" (van Fraassen 2002, 252). In this way, the step from presentation to representation is made almost trivially: the data models act as the 'phenomena to be saved' and so all we need to connect the theory to data models qua 'the phenomena' is a guarantee of their shared structure. Van Fraassen makes this connection by using embedability as a guarantee of the shared structure between theoretical models and 'the phenomena', maintaining that "certain parts of the [theoretical] models [are] to be identified as empirical substructures, and these [are] the candidates for representation of the observable phenomena which science can confront within our experience" (van Fraassen 1989, 227). This empiricist version of scientific structuralism avoids the question of why it should be assumed that the phenomenon is represented by data models by simply collapsing any distinction between the two and so offers no justification for why such an identification should be presumed possible. We think it is necessary, for any attempt which aims to move beyond a methodological stance, to provide an account of what allows us, in the first place, to make the identification between the phenomena and data models.

Structural realists, such as French and Ladyman,¹⁴ who adopt a realist stance and so presume that the world structures the phenomena, invoke the 'no miracles' argument to explain the necessity of identifying the structure of data models and the structure of the phenomena; it is used

14. See French (2000) and Ladyman (1998).

to argue that if there was no shared structure between the (data models of the) theory and the world (the phenomena) the success of science would be a miracle. Thus, while no detailed account of how the data models come to share structure with the phenomena is given, the possibility (or, indeed, necessity) of making the identification is itself justified by appeal to at least an argument. Structural realism, insofar as it identifies the structure of data models and the structure of the phenomena, is, thus, in all its forms, committed to the claim that the kinds of objects presented by our theory accurately represent the structure of particular objects of which 'the world' is claimed to consist. The forms of structural realism differ in just how far this representation is claimed to take us.

The epistemological structural realist says that, with respect to the particular objects, all that can be known is that they are instances of the structural kinds given by our theories; all that can be known is their structure. They remain open to the possibility, however, that the particular objects in the world have other properties that are not represented by the theory. Ontological structural realism can be understood as rejecting this last claim and asserting that the particular objects in the world have no properties beyond those that make them instances of certain structural kinds; all there is is structure. In both cases, the claim that structural properties play a representational role at all is justified entirely by appeal to the 'no miracles argument'. As minimal scientific structuralists, we eschew this representational role; we accept that if (models of) scientific theories *present* us with kinds of objects, then all that can be known of 'objects', as instances of those kinds, is their structure. But, in adopting a methodological stance, we remain open to the possibility (epistemic, ontic, or modal¹⁵) that particular objects may have properties that are not structured by how we present them.

- **5.** Conclusion. We have made use of an analogy with mathematical structuralism in order to set apart what we call minimal scientific structuralism. On this account, a theory is characterized by the collection of its models, and the kinds of objects that the theory talks about are presented through
- 15. Ladyman is developing an alternative modal form of ontological structural realism which aims to account for the notion of necessity in 'structural' terms. He claims that "the abstract mathematical structures it [the theoretical parts of a theory] employs... must have some grip on reality. It is clear that the 'grip on reality' in question must go beyond a correct description of the actual phenomena to the representation of modal relations between them" (Ladyman 1998, 418). For a further motivation of this approach, see Saunders (1993, 320), who suggestively remarks that "it has long been apparent that no workable account of nomological necessity can be made out at the level of unstructured particulars (except in the context of an unfathomable and antiquated notion of the 'rule of law')".

the shared structure of those models. And, in so characterizing the 'structure' of a scientific theory, the applicability of the theory to the data can be expressed in terms of the shared structure of their respective models.

No further analyses (either formal¹⁶ or philosophical) are needed for meeting the challenges facing the minimal scientific structuralist. To account, however, for the connection between the theory and the world one must move past minimal scientific structuralism; here the issue of representation becomes crucial, and so more than a methodological stance must be adopted. Just how such representation is to be accomplished and what justification we might give for believing that it is, is what divides scientific structuralism into its different varieties. The empirical stance, taken by van Fraassen, simply asserts the identity of the data models and the phenomena. The realist stance, adopted by the structural realist, offers only the 'no miracles' argument as evidence for the claim that the structure of the data models is shared by the structure of the phenomena. In any case, neither the framework of the semantic view of theories nor the appeal to shared structure alone offers the scientific structuralist a quick route to representation. One can choose to take whatever additional stance one likes, but a stance itself is not a justification.

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- 16. Da Costa and Chauqui (1988), Da Costa and French (1990) and Da Costa, Bueno, and French (1997) all seek to provide such *formal* accounts; they seek to provide an abstract analysis of what a 'structure' is qua a partial structure, and insofar as a theory is a family of partial structures, to thereby provide an abstract analysis of what shared structure is, by analyzing both in set-theoretic terms. This, as explicitly claimed in Da Costa and French (1990), so as to be in line with both Bourbaki's set-theoretic account of mathematical structuralism wherein a kind of structure is a type of set-structured system and Suppes' (1957, 1960, 1962) slogan that "To axiomatize a theory is to define a set-theoretical predicate" (249). Van Fraassen, too, seeks to provide such a formal analysis; in his 1970 paper, he makes use of isomorphisms between models as state-spaces (interpreted by Beth semantics); and in his 1980 book, he makes use of the embedability of empirical substructures into theoretical 'structures', again as state-spaces. We note, however, that unlike Da Costa et al., he remains open to the possibility that there may be frameworks other than set-theoretic (see van Fraassen 1980, 65–66).

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