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HONORARY LECTURE

BAYESIAN MODELING OF ECONOMIES AND DATA REQUIREMENTS

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Marshallian demand, supply, and entry models are employed for major sectors of an economy that can be combined with factor market models for money, labor, capital, and bonds to provide a Marshallian macroeconomic model (MMM). Sectoral models are used to produce sectoral output forecasts, which are summed to provide forecasts of annual growth rates of U.S. real GDP. These disaggregative forecasts are compared to forecasts derived from models implemented with aggregate data. The empirical evidence indicates that it pays to disaggregate, particularly when employing Bayesian shrinkage forecasting procedures. Further, some considerations bearing on alternative model-building strategies are presented using the MMM as an example and describing its general properties. Last, data requirements for implementing MMMs are discussed.

Keywords: Marshallian Macroeconomic Model; Sectoral Disaggregation; Sectoral Forecasting; Bayesian Modeling

1. INTRODUCTION

For many years, theoretical and empirical workers have tried to model national economies in order to (a) understand how they operate, (b) forecast future outcomes, and (c) evaluate alternative economic policies. Although much progress has been made in the decades since Tinbergen's pioneering work, no generally accepted model has yet appeared. On the theoretical side, there are monetary, neo-monetary, Keynesian, neo-Keynesian, real-business-cycle, generalized real-business-cycle, and other theoretical models; see Belongia and Garfinkel (1992) for an excellent review of many of these models and Min (1992) for a description of a generalized real-business-cycle model. Some empirical testing of alternative models has appeared in the literature. However, Fair (1992) and Zellner (1992), in invited contributions to a St. Louis Federal Reserve Bank conference on

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alternative macroeconomic models, concluded that there is a great need for additional empirical testing of alternative macroeconomic models and production of improved models.

Over the years many structural econometric and empirical statistical models have been constructed and used. These include large structural econometric models, for example, the Tinbergen, Klein, Brookings–SSRC, Federal Reserve–MIT–PENN, OECD, and Project Link. Although progress has been made, there does not yet appear to be a structural model that performs satisfactorily in point and turning-point forecasting. Indeed, the forecasting performance of some of these models is not as good as that of simple benchmark models for example, random walk, autoregressive, Box–Jenkins univariate ARIMA, and autoregressive leading-indicator (ARLI) models [see, e.g., Cooper (1972), Garcia-Ferrer et al. (1987), Hong (1989), and Nelson and Plosser (1982)]. Further, some have implemented vector autoregressive (VAR) and Bayesian VAR models in efforts to obtain improved forecasts [see, e.g., Litterman (1986) and McNees (1986)]. However, these VAR's generally have not been successful in point and turning-point forecasting performance, as noted by Zarnowitz (1986) and McNees (1986). See also the simulation experiments performed by Adelman and Adelman (1959) and Zellner and Peck (1973), which revealed some rather unusual properties of two large-scale econometric models.

Given the need for improved models, in Garcia-Ferrer et al. (1987) an empirical implementation of the structural econometric time-series analysis (SEMTSA) approach of Zellner and Palm (1974, 1975), Palm (1976, 1977, 1983) and Zellner (1979, 1994) was reported. In line with the SEMTSA general approach, relatively simple forecasting equations, autoregressive leading-indicator (ARLI) models were formulated and tested in forecasting output growth rates for nine industrialized countries with some success. In later work, the sample of countries was expanded to 18 and the forecast period extended to include more out-of-sample growth rates of real GDP to be forecast; see Min and Zellner (1993). Building on work of Wecker (1979) and Kling (1987), a Bayesian decision theoretic procedure for forecasting turning points that yielded correct forecasts in about 70% of 211 turning point episodes was formulated and applied; see Zellner and Min (1999), Zellner et al. (1999), and the references cited in these papers. Further, the ARLI models were shown to be compatible with certain aggregate supply and demand, Hicksian "IS-LM," and generalized real-business-cycle models by Hong (1989), Min (1992), and Zellner (1999).

In a continuing effort to improve our models, in the present paper, we use a relatively simple Marshallian model in Section 2 that features demand, supply, and entry equations for each sector of an economy [see Veloce and Zellner (1985) for a derivation of this model and an application of it in the analysis of data for a Canadian industry]. The model is solved to produce a sectoral relation that can be employed to forecast sectoral output. These sectoral output forecasts are summed to produce forecasts of total output that are compared to forecasts derived from models implemented with aggregate data. Some possible advantages of disaggregation have been discussed by Orcutt et al. (1961), Espasa and Matea (1990), Espasa

(1994), and de Alba and Zellner (1991), among others. Actual comparisons of such forecasts for U.S. annual real GDP growth rates, 1980–1997, are reported in Section 4 after statistical estimation and forecasting techniques, employed to implement the MMM, are presented in Section 3. In Section 4, the data used in our empirical forecasting work are described, and forecasting results using the MMM and other models with and without disaggregation are reported. Also, MMM models' forecast performance is compared to that of various benchmark and ARLI models. In Section 5, some comments on data requirements, a summary of conclusions, and remarks on future research are presented.

2. MARSHALLIAN MACROECONOMIC MODEL

In the MMM, we have three basic, rather well-known equations, described and applied by Veloce and Zellner (1985): (i) demand for output, (ii) supply of output, and (iii) entry equations encountered in Marshall's famous economic analyses of the behavior of industries. Although many macro models have included demand and supply equations, they have not included an entry equation. For example, in some models there is just a representative firm, and one wonders what happens when the representative firm shuts down. In our MMM model, supply depends on the number of firms in operation and thus an equation governing the number of firms in operations—an entry equation—is introduced.

We use two variants of the MMM model: an aggregate, reduced-form variant and a disaggregated structural equation variant. In the aggregate variant, we adopt a "one-sector" view of an economy whereas in the disaggregated variant, we adopt a multisectoral view of an economy. With the multisectoral view, many assumed structures are possible, from the multisectoral view of traditional Leontieff input–output analysis to the simple view that we employ, namely, an economy in which each sector sells in a final-product market. Herein, we do not take up the interesting problem of classifying economies by the nature of their sectoral interrelations, but we do show that, by adopting our sectoral view, we are able to improve forecasts of aggregate output growth rates because disaggregation provides more observations to estimate relationships and permits use of sectoral-specific variables to help improve forecasts. Of course, if the disaggregated relations are misspecified and/or the disaggregated data are faulty, then there may be no advantages and perhaps some disadvantages in using disagregated data, as is evident. Also, there are some circumstances when data are good and relations are well formulated but disaggregation does not lead to improved forecasts. However, the issue cannot be completely settled theoretically and hence our current empirical work.

As explained by Veloce and Zellner (1985), the equations for a sector that we use are a demand equation for output, an industry supply equation for output, and a firm entry equation. Although we could elaborate the system in many ways, we proceed to determine how well this simplest system peforms empirically—our "Model T" that can be improved in many different ways in the future. When these three equations are solved for the implied equation for the sectoral output growth rate [see Veloce and Zellner (1985) for details], the result is the following differential equation for total industrial sales, denoted by $S = S(t)$:

$$(1/S)\, dS/dt = a(1 - S/F) + g, \tag{1}$$

where a and F are positive parameters and g is a linear function of the growth rates of the wage rate, the price of capital, and of demand shifters such as real income and real money balances. If $g = 0$ or $g = c$, a positive constant, then (1) is the differential equation with a logistic curve solution that is employed in many sciences, including economics. Also note that (1) incorporates both the rate of change of S and the level of S, a "cointegration" effect. Also see Veloce and Zellner (1985, p. 463), for analysis of (1) when $g = g(t)$, a special form of Bernouilli's differential equation and its solution.

In our empirical work, we use the discrete approximations to equation (1) shown in Table 1 and denoted by MMM(DA)I-IV. In these equations, the rate of growth of S, real output, is related to lagged levels of S, lagged rates of change of real stock prices, SR, and real money, m, and current rates of change of real wage rates, W, and real GDP, Y. The variables m and Y are "demand shifters," W is the price of labor, and SR is related to the price of capital. As noted in the literature and in our past work, the rates of change of m and SR are effective leading-indicator variables in a forecasting context and their use has led to improved forecasts in our past work; see references cited above for empirical evidence.

Under Sectoral forecast equations in Table 1 are three benchmark models that are used to produce sectoral one-year-ahead forecasts of the rates of change of output for each of our 11 sectors. The first is an AR(3) that has been used in many earlier studies as a benchmark model. The second is an AR(3) that incorporates lagged leading-indicator variables and current values of W and Y but no lagged level variables. The third "Distributed Lag" model is like the second except for the inclusion of lagged rates of change of W and of Y.

At the top of Table 1, under Reduced-form equations, are equations for the rate of change of Y, annual real GDP. The first is a benchmark AR(3) model. The second is an AR(3) with lagged leading-indicator variables that is denoted by AR(3)LI. The third model, denoted MMM(A), is the same as the AR(3)LI model except for the inclusion of two lagged Y variables, where Y = real GDP and t = a time trend.

For our aggregate analyses, we use the reduced-form equations in Table 1 to produce one-year-ahead forecasts of the rate of change of real GDP, Y, which we refer to as "aggregate forecasts." These are means of diffuse prior Bayesian predictive densities for each model that are simple one-year-ahead least-squares forecasts. As explained later, the MMM(A) reduced-form equations for the rates of change of Y and of W are employed in the estimation of the sectoral forecasting equations and in computing one-year-ahead forecasts of sectoral output growth rates. These sectoral growth-rate forecasts are transformed into forecasts of levels, added across the sectors and converted into a forecast of the rate of change of real GDP, Y. Root-mean-squared errors (RMSE's) and mean absolute errors (MAE's) are computed for each forecasting procedure and are shown in Table 2 and 3 below.

Table 1. Forecasting equations

Model	Reduced-form equations
	Real U.S. GDP
AR(3)(A):	$(1-L)\log Y_t = \alpha_0 + \alpha_1(1-L)\log Y_{t-1} + \alpha_2(1-L)\log Y_{t-2} + \alpha_3(1-L)\log Y_{t-3} + u_t$
AR(3)LI(A):	$(1-L)\log Y_t = \alpha_0 + \alpha_1(1-L)\log Y_{t-1} + \alpha_2(1-L)\log Y_{t-2} + \alpha_3(1-L)\log Y_{t-3} + \beta_1(1-L)\log SR_{t-1}$ $+ \beta_2(1-L)\log m_{t-1} + u_t$
MMM(A):	$(1-L)\log Y_t = \alpha_0 + \alpha_1(1-L)\log Y_{t-1} + \alpha_2(1-L)\log Y_{t-2} + \alpha_3(1-L)\log Y_{t-3} + \alpha_4 Y_{t-1} + \alpha_5 Y_{t-2} + \alpha_6 t$ $+ \beta_1(1-L)\log SR_{t-1} + \beta_2(1-L)\log m_{t-1} + u_t$
	Real Wage
AR(3) (A):	$(1-L)\log W_t = \alpha_0 + \alpha_1(1-L)\log W_{t-1} + \alpha_2(1-L)\log W_{t-2} + \alpha_3(1-L)\log W_{t-3} + u_t$
AR(3)LI(A):	$(1-L)\log W_t = \alpha_0 + \alpha_1(1-L)\log W_{t-1} + \alpha_2(1-L)\log W_{t-2} + \alpha_3(1-L)\log W_{t-3} + \beta_1(1-L)\log SR_{t-1}$ $+ \beta_2(1-L)\log m_{t-1} + u_t$
MMM(A):	$(1-L)\log W_t = \alpha_0 + \alpha_1(1-L)\log W_{t-1} + \alpha_2(1-L)\log W_{t-2} + \alpha_3(1-L)\log W_{t-3} + \gamma_1 W_{t-1} + \gamma_2 W_{t-2} + \gamma_3 t$ $+ \beta_1(1-L)\log SR_{t-1} + \beta_2(1-L)\log m_{t-1} + u_t$
Model	**Sectoral forecast equations**
AR(3)(DA):	$(1-L)\log S_t = \alpha_0 + \alpha_1(1-L)\log S_{t-1} + \alpha_2(1-L)\log S_{t-2} + \alpha_3(1-L)\log S_{t-3} + u_t$
AR(3)LI(DA):	$(1-L)\log S_t = \alpha_0 + \alpha_1(1-L)\log S_{t-1} + \alpha_2(1-L)\log S_{t-2} + \alpha_3(1-L)\log S_{t-3} + \beta_1(1-L)\log SR_{t-1}$ $+ \beta_2(1-L)\log m_{t-1} + \beta_3(1-L)\log W_t + \beta_4(1-L)\log Y_t + u_t$
Distrib.Lag(DA):	$(1-L)\log S_t = \alpha_0 + \alpha_1(1-L)\log S_{t-1} + \beta_1(1-L)\log SR_{t-1} + \beta_2(1-L)\log m_{t-1} + \beta_3(1-L)\log W_t$ $+ \beta_4(1-L)\log Y_t + \beta_5(1-L)\log W_{t-1} + \beta_6(1-L)\log Y_{t-1} + u_t$
MMM(DA)I:	$(1-L)\log S_t = \alpha_0 + \alpha_1 S_{t-1} + \beta_1(1-L)\log SR_{t-1} + \beta_2(1-L)\log m_{t-1} + \beta_3(1-L)\log W_t$ $+ \beta_4(1-L)\log Y_t + u_t$
MMM(DA)II:	$(1-L)\log S_t = \alpha_0 + \alpha_1 S_{t-1} + \alpha_2 S_{t-2} + \beta_1(1-L)\log SR_{t-1} + \beta_2(1-L)\log m_{t-1} + \beta_3(1-L)\log W_t$ $+ \beta_4(1-L)\log Y_t + u_t$
MMM(DA)III:	$(1-L)\log S_t = \alpha_0 + \alpha_1 S_{t-1} + \alpha_2 S_{t-2} + \alpha_3 S_{t-3} + \beta_1(1-L)\log SR_{t-1} + \beta_2(1-L)\log m_{t-1} + \beta_3(1-L)\log W_t$ $+ \beta_4(1-L)\log Y_t + u_t$
MMM(DA)IV:	$(1-L)\log S_t = \alpha_0 + \alpha_1 S_{t-1} + \alpha_2 S_{t-1}^2 + \beta_1(1-L)\log SR_{t-1} + \beta_2(1-L)\log m_{t-1} + \beta_3(1-L)\log W_t$ $+ \beta_4(1-L)\log Y_t + u_t$

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3. ESTIMATION AND FORECASTING METHODS

3.1. Notation and Equations

In what follows, we use the following notation: For each sector, we have

1. Endogenous or Random Current Exogenous Variables:

$$y_{1t} = (1 - L)\log S_t, \quad y_{2t} = (1 - L)\log W_t, \quad y_{3t} = (1 - L)\log Y_t,$$

   where S_t = sectoral real output, W_t = national real wage rate, and Y_t = real GDP.
2. Predetermined Variables:

$$\boldsymbol{x}'_{1t} = [1, S_{t-1}, S_{t-2}, S_{t-3}, (1 - L)\log SR_{t-1}, (1 - L)\log m_{t-1}],$$

   where SR_t = real stock price and m_t = real money.

We use these variables to form the following structural equation for each sector:

$$y_{1t} = y_{2t}\gamma_{21} + y_{3t}\gamma_{31} + x'_{1t}\beta_1 + u_{1t} \qquad t = 1, 2,.., T$$

or

$$\boldsymbol{y}_1 = Y_1\boldsymbol{\gamma}_1 + X_1\boldsymbol{\beta}_1 + \boldsymbol{u}_1, \tag{2}$$

where the vectors $\boldsymbol{y}_1$ and $\boldsymbol{u}_1$ are $Tx1$, Y_1 is $Tx2$ and X_1 is $Tx5$ and $(\boldsymbol{\delta}_1)' = (\gamma'_1, \beta'_1)$ is a vector of structural parameters.

The MMM unrestricted reduced-form equations, shown in Table 1, are denoted by

$$y_1 = X\pi_1 + v_1 \tag{3a}$$

and

$$Y_1 = X\Pi_1 + V_1, \tag{3b}$$

where $X = (X_1, X_0)$ with X_0 containing predetermined variables in the system that are not included in equation (2).

By substituting from (3b) in (2), we obtain the following well-known restricted reduced-form equation for y_1:

$$y_1 = X\Pi_1\gamma_1 + X_1\beta_1 + v_1 \tag{4a}$$

$$= \bar{Z}\delta_1 + v_1, \tag{4b}$$

where $\bar{Z} = (X\Pi_1, X_1)$, which is assumed to be of full column rank. Further, if we consider the regression of v_1 on V_1,

$$v_1 = V_1\eta_1 + e_1 = (Y_1 - X\Pi_1)\eta_1 + e_1, \tag{5}$$

we can substitute for v_1 in (4a) to obtain

$$\boldsymbol{y}_1 = X\Pi_1\boldsymbol{\gamma}_1 + X_1\boldsymbol{\beta}_1 + (Y_1 - X\Pi_1)\boldsymbol{\eta}_1 + \boldsymbol{e}_1 \tag{6}$$

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In (6), for given Π_1, we have a regression of y_1 on $X\Pi_1$, X_1, and $Y_1 - X\Pi_1$. Given that e_1 is uncorrelated with the the elements of V_1, the system (3b) and (6) is a nonlinear SUR system with an error covariance matrix restriction. Earlier, Pagan (1979) recognized a connection between the model in (2) and (3b) and the SUR model, given the "triangularity" of the system, and reported an iterative computational procedure for obtaining maximum likelihood estimates of the structural coefficients. In our case, we use (3b) and (6) as a basis for producing a convenient algorithm for computing posterior and predictive densities.

Note further that if $\gamma_1 = \eta_1$, equation (6) becomes

$$\boldsymbol{y}_1 = Y_1\boldsymbol{\gamma}_1 + X_1\boldsymbol{\beta}_1 + \boldsymbol{e}_1, \tag{7}$$

the same as (2) except for the error term. It is possible to view (7) as a regression with Y_1 containing observations on stochastic independent variables, given that the elements of e_1 and V_1 are uncorrelated. The above restriction, however, may not hold in general. Another interpretation that permits (7) to be viewed as a regression with stochastic input variables is that the variables y_{2t} and y_{3t} are stochastic exogenous variables vis à vis the sectoral model. In such a situation, equation (2) can be treated as a regression equation with stochastic independent variables. However, we are not sure that this exogeneity assumption is valid and thus will use not only least-squares techniques to estimate (2) but also special simultaneous-equation techniques.

3.2. Estimation Techniques

The sampling-theory estimation techniques that we employ in estimating the parameters of (2) are the well-known ordinary least-squares (OLS) and two-stage least-squares (2SLS) methods. As shown by Zellner (1998), in very small samples, but not in large samples, the OLS method produces an optimal Bayesian estimate relative to a generalized quadratic "precision of estimation" loss function when diffuse priors are employed. Also, the 2SLS estimate has been interpreted as a conditional Bayesian posterior mean using (4) conditional on $\Pi_1 = \hat{\Pi}_1 = (X'X)^{-1}X'Y_1$, a normal likelihood function, and diffuse priors for the other parameters of (4). A similar conditional result is obtained without the normality assumption using the assumptions of the Bayesian method of moments (BMOM) approach; see, for example, Zellner (1997a,b, 1998). Since the "plug in" assumption $\Pi_1 = \hat{\Pi}_1$ does not allow appropriately for the uncertainty regarding Π_1's value, the 2SLS estimate will not be optimal in small samples; see, for example, Monte Carlo experiments reported by Park (1982), Tsurumi (1990), and Gao and Lahiri (1999). However, since OLS and 2SLS are widely employed methods, we use them in our analyses of the models for individual sectors.

In the Bayesian approach, we decided to use the "extended minimum expected loss" (EMELO) optimal estimate put forward by Zellner (1986, 1998), which has performed well in Monte Carlo experiments by Tsurumi (1990) and Gao and Lahiri

(1999). It is the estimate that minimizes the posterior expectation of the following extended or balanced loss function:

$$\begin{aligned} L(\boldsymbol{\delta}_1, \hat{\boldsymbol{\delta}}_1) &= w(\boldsymbol{y}_1 - \bar{Z}\hat{\boldsymbol{\delta}}_1)'(\boldsymbol{y}_1 - \bar{Z}\hat{\boldsymbol{\delta}}_1) + (1-w)(\boldsymbol{\delta}_1 - \hat{\boldsymbol{\delta}}_1)'\bar{Z}'\bar{Z}(\boldsymbol{\delta}_1 - \hat{\boldsymbol{\delta}}_1) \\ &= w(\boldsymbol{y}_1 - \bar{Z}\hat{\boldsymbol{\delta}}_1)'(\boldsymbol{y}_1 - \bar{Z}\hat{\boldsymbol{\delta}}_1) + (1-w)(X\boldsymbol{\pi}_1 - \bar{Z}\hat{\boldsymbol{\delta}}_1)'(X\boldsymbol{\pi}_1 - \bar{Z}\hat{\boldsymbol{\delta}}_1), \end{aligned} \quad \textbf{(8)}$$

where w has a given value in the closed interval 0 to 1, $\hat{\boldsymbol{\delta}}_1$ is some estimate of $\boldsymbol{\delta}_1$, and in going from the first line of (8) to the second, the identifying restrictions, multiplied on the left by X, namely $X\boldsymbol{\pi}_1 = \bar{Z}\delta_1$, have been employed.

Relative to equation (4), the first term on the right side of (8) reflects goodness of fit and the second reflects precision of estimation or, from the second line of (8), the extent to which the identifying restrictions are satisfied when an estimate of $\boldsymbol{\delta}_1$ is employed. When the posterior expectation of the loss function in (8) is minimized with respect to $\hat{\boldsymbol{\delta}}_1$, the minimizing value is

$$\hat{\boldsymbol{\delta}}_1^* = (E\bar{Z}'\bar{Z})^{-1}[wE\bar{Z}'\boldsymbol{y}_1 + (1-w)E\bar{Z}'X\boldsymbol{\pi}_1]. \quad \textbf{(9)}$$

On evaluation of the moments on the right-hand side of (9), we have an explicit value for the optimal estimate. For example, with the assumption that, for the unrestricted reduced-form system in (3), the rows of (v_1, V_1) are i.i.d. $N(0, \Omega)$, where Ω is a pds (positive definite symmetric) covariance matrix, combining a standard diffuse prior for the reduced-form parameters with the normal likelihood function yields a marginal matrix t density for the reduced-form coefficients. Thus, the moments needed to evaluate (9) are readily available [see Zellner (1986) for details] and, surprisingly, the result is in the form of the double-K-class estimate,

$$\hat{\boldsymbol{\delta}}_1^* = \begin{bmatrix} \hat{\gamma}_1 \\ \hat{\boldsymbol{\beta}}_1 \end{bmatrix} = \begin{bmatrix} Y_1'Y_1 - K_1\hat{V}_1'\hat{V}_1 & Y_1'X_1 \\ X_1'Y_1 & X_1'X_1 \end{bmatrix}^{-1} \begin{bmatrix} (Y_1 - K_2\hat{V}_1)'\boldsymbol{y}_1 \\ X_1'\boldsymbol{y}_1 \end{bmatrix}, \quad \textbf{(10)}$$

with

$$\hat{V}_1 = Y_1 - X\hat{\Pi}_1, \quad \hat{\Pi}_1 = (X'X)^{-1}X'Y_1$$

and

$$K_1 = 1 - k/(T-k-m-2) \quad \text{and} \quad K_2 = K_1 + wk/(T-k-m-2). \quad \textbf{(11)}$$

K-class and double-K-class estimates are discussed in most econometrics texts [see, e.g., Judge et al. (1987)] and the choice of optimal values for the K's has been the subject of much sampling-theory research. The Bayesian approach provides optimal values of these parameters quite directly on use of goodness of fit, precision of estimation, or balanced loss functions.

When the form of the likelihood function is unknown and thus a traditional Bayesian analysis is impossible, we used the BMOM approach [Zellner (1998)] to obtain a postdata maxent density for the elements of $\Pi = (\Pi_1\, \pi_1)$ that was

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used to evaluate the expectation of the balanced loss function in (8), and we derived an optimal value of $\hat{\delta}_1$ that is also in the form of a double-K-class estimate, shown in (10), but with slightly different values of the K parameters, namely, $K_1 = 1 - k/(T-k)$ and $K_2 = K_1 + wk/(T-k)$. In our calculations based on the extended MELO estimate, we used the BMOM K values and $w = 0.75$, the value used by Tsurumi (1990) in his Monte Carlo experiments.

SUR estimates for the system were computed by assuming that the y_2 and y_3 variables in (2) are stochastic exogenous variables for each sector and treating the 11 sectoral equations as a set of seemingly unrelated regression equations. We estimated the parameters by "feasible" generalized least squares. The parameter estimates so obtained are means of conditional posterior densities in traditional Bayesian and BMOM approaches.

Complete shrinkage estimation utilized the assumption that all sectors' parameter vectors are the same. Under this assumption and the assumption that y_2 and y_3 are stochastic exogenous variables, estimates of the restricted parameter vector were obtained by least squares that are also posterior means in Bayesian and BMOM approaches.

Exact posterior densities for the structural parameters in (6) can be calculated readily in the Bayesian approach by using diffuse priors for the parameters of (6), given Π_1, that is, a uniform prior on elements of $\boldsymbol{\delta}_1$, $\boldsymbol{\beta}_1$, $\boldsymbol{\eta}_1$, and $\log \sigma_e$, where σ_e is the standard deviation of each element of $\boldsymbol{e}_1$. Further, the usual diffuse priors are employed for Π_1 and Ω_1, a marginal uniform prior on the elements of the reduced-form matrix Π_1 in (3) and a diffuse prior on Ω_1, the covariance matrix for the independent, zero-mean, normal rows of V_1. With use of these priors, the usual normal likelihood function for the system, and Bayes' theorem, we obtain the following joint posterior density for the parameters, where D denotes the given data [see Zellner et al. (1994) and Currie (1996)]:

$$\underbrace{f(\gamma_1, \boldsymbol{\beta}_1, \boldsymbol{\eta}_1 \mid \sigma_e, \Pi_1, D)}_{\text{MVN}}\,\underbrace{g(\sigma_e \mid \Pi_1, D)}_{\text{IG}}\,\underbrace{h(\Pi_1 \mid \Omega_1, D)}_{\text{MVN}}\,\underbrace{j(\Omega_1 \mid D)}_{\text{IW}}, \qquad \textbf{(12)}$$

where MVN denotes a multivariate normal density, IG is an inverted gamma density, and IW is an inverted Wishart density. A similar factorization of the joint BMOM postdata density is available; see Zellner (1997a,b).

Given equation (12), we can draw from the IW density and insert the drawn values in h and make a draw from it. Then, the Π_1 value so drawn is inserted in g and a draw made from it. Then drawn values of σ_e and Π_1 are inserted in f, and a draw of the structural coefficients in f is made. This direct Monte Carlo procedure can be repeated many times to yield moments, fractiles, and marginal densities for all parameters appearing in (12). Also, a similar approach, described in Section 3.3 can be employed to compute predictive densities. Some of these calculations have been performed using sectoral models and data that will be reported in a future paper.

3.3. Forecasting Techniques

For one-year-ahead forecasts of the rates of growth of real GDP using the aggregate models in Table 1, we employed least-squares forecasts that are means of Bayesian predictive densities when diffuse priors and the usual normal likelihood functions are employed. Predictive means are optimal in terms of providing minimal expected loss vis à vis squared-error predictive loss functions. Further, since these predictive densities are symmetric, the predictive mean is equal to the predictive median that is optimal relative to an absolute error predictive loss function.

One-year-ahead forecasts for the sectoral models in Table 1 were made using one-year-ahead MMM(A) reduced-form forecasts of the y_{2T+1} and y_{3T+1} variables on the right-hand side of equation (2) and using the parameter estimates provided by the methods described earlier. That is, the one-year-ahead forecast is given by

$$\hat{y}_{1T+1} = \hat{y}_{2T+1}\hat{\gamma}_{21} + \hat{y}_{3T+1}\hat{\gamma}_{31} + \mathbf{x}'_{1T+1}\hat{\boldsymbol{\beta}}_1. \tag{13}$$

The η shrinkage technique, derived and utilized by Zellner and Hong (1989), involves shrinking a sector's forecast toward the mean of all 11 sectors' forecasts by averaging a sector's forecast with the mean of all sectors' forecasts as follows:

$$\hat{y}_{1T+1} = \eta\hat{y}_{1T+1} + (1 - \eta)\bar{\mathbf{y}}_{T+1},$$

where $\hat{y}_{1T+1}$ is the sector forecast, $\bar{y}_{T+1}$ is the mean of all the sectors' forecasts, and η is assigned a value in the closed interval 0 to 1.

Gamma shrinkage, discussed and applied by Zellner and Hong (1989), involves assuming that the individual sector's coefficient vectors are distributed about a mean, say $\boldsymbol{\theta}$, and then using an average of an estimate of the sector's coefficient vector with an estimate of the mean $\boldsymbol{\theta}$ of the parameter vectors. That is,

$$\hat{\boldsymbol{\delta}}_\eta = (\hat{\boldsymbol{\delta}}_1 + \gamma\hat{\boldsymbol{\theta}})/(1 + \gamma\hat{\boldsymbol{\theta}}) \tag{14}$$

with $0 < \gamma < \infty$. This coefficient estimate can be employed to produce one-year-ahead forecasts using the structural equations for each sector and MMM(A) reduced-form forecasts of the endogenous variables $(1 - L)$ log W_{T+1} and $(1 - L)$ log Y_{T+1}. Various values of η and γ are employed in forecasting sectoral growth rates that are used to construct an aggregate forecast of the growth rate of real GDP.

We also can compute a predictive density for a sector's one-year-ahead growth rate as follows: From (6), we can form the conditional density

$$q(y_{1T+1} \mid \Pi_1, \gamma_1, \boldsymbol{\beta}_1, \boldsymbol{\eta}_1, \sigma_e, y_{2T+1}, y_{3T+1}, D),$$

which will be in a normal form, given error-term normality. Thus, each draw from (12) and a draw from the predictive density for (y_{2T+1}, y_{3T+1}) can be inserted in q and and a value of y_{1T+1} drawn from q. Repeating the process will produce a sample of draws from q from which the complete predictive density, its moments, and so on, can be computed. Two such predictive densities, one for the durables

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sector and the other for the services sector, are presented below. The densities are slightly skewed to the left and rather spread out. However, the means that are optimal relative to squared-error loss are not too far from the actual values being forecasted. Also, these densities are valuable in making probability statements about future outcomes, including turning-point forecasts.

We now turn to consider plots of the data and reports of forecasting results.

4. DISCUSSION OF DATA AND FORECASTING RESULTS

In Figure 1A are shown plots of the real rates of growth of GDP, M1, currency, stock prices, and wage rates, 1949–1997. Peaks and troughs in the plots occur roughly at about 4-to 6-year intervals. Note the sharp declines in real GDP growth rates in 1974 and 1982 and a less severe drop in 1991. The money and stock-price growth-rate variables tend to lead the real GDP growth-rate variable, as observed in the earlier work of many. The two money growth-rate variables show similar patterns before the 1990's, but during the 1990's their behavior is somewhat different, perhaps because of a change in policy that permitted interest to be paid on demand deposits. In our forecasting results, we find that use of the currency variable yields somewhat better results than use of the M1 variable.

Figure 1B presents a plot of the output growth rates for 11 sectors of the U.S. economy. It is seen that, except for the agriculture and mining sectors, the sectoral output growth rates tend to move together over the business cycle, whereas the agricultural and mining sectors show extreme variation. In contrast, the other sectors have much smaller interquartile ranges and fewer outlying growth rates. See also the boxplots for the sectoral growth rates in Figure 1C.

In Figure 2A are shown the one-year-ahead, aggregate forecasts plotted as solid lines and the actually observed rates of growth plotted as circles. In the first panel of Figure 2A, labeled AR(3), an aggregate AR(3) model for the real GDP growth rates (see Table 1) was employed with data 1949–79 to generate one-year-ahead forecasts year by year, 1980–1997, with estimates being updated each year. The plot shows dramatically the failure of the AR(3) model to forecast turning points successfully. Very large errors occurred in 1982 and 1991. Use of the AR(3)LI model (see Table 1) that incorporates two lagged leading-indicator variables, the real rates of growth of currency and of stock prices, produced the forecasts shown by the solid lines in the second panel of Figure 2A. There are improvements in forecasts for 1982 and 1984 vis à vis use of the AR(3) model. However, there is still a large error in the 1991 forecast. Use of the MMM(A) model (see Table 1) that incorporates two lagged level GDP variables and a linear time trend in the AR(3)LI model produced the forecasts shown in the third panel of Figure 2A. Here, there are improvements, as compared to the use of the AR(3) model in most years, especially 1982, 1990, and 1991. Also, use of the MMM(A) model led to improved forecasts as compared to those provided by the AR(3)LI, especially in the 1990's.

In Figure 2B are shown the disaggregated, one-year-ahead forecasts, plotted as solid lines, and the observed real GDP growth-rate data, plotted as circles. Here, for

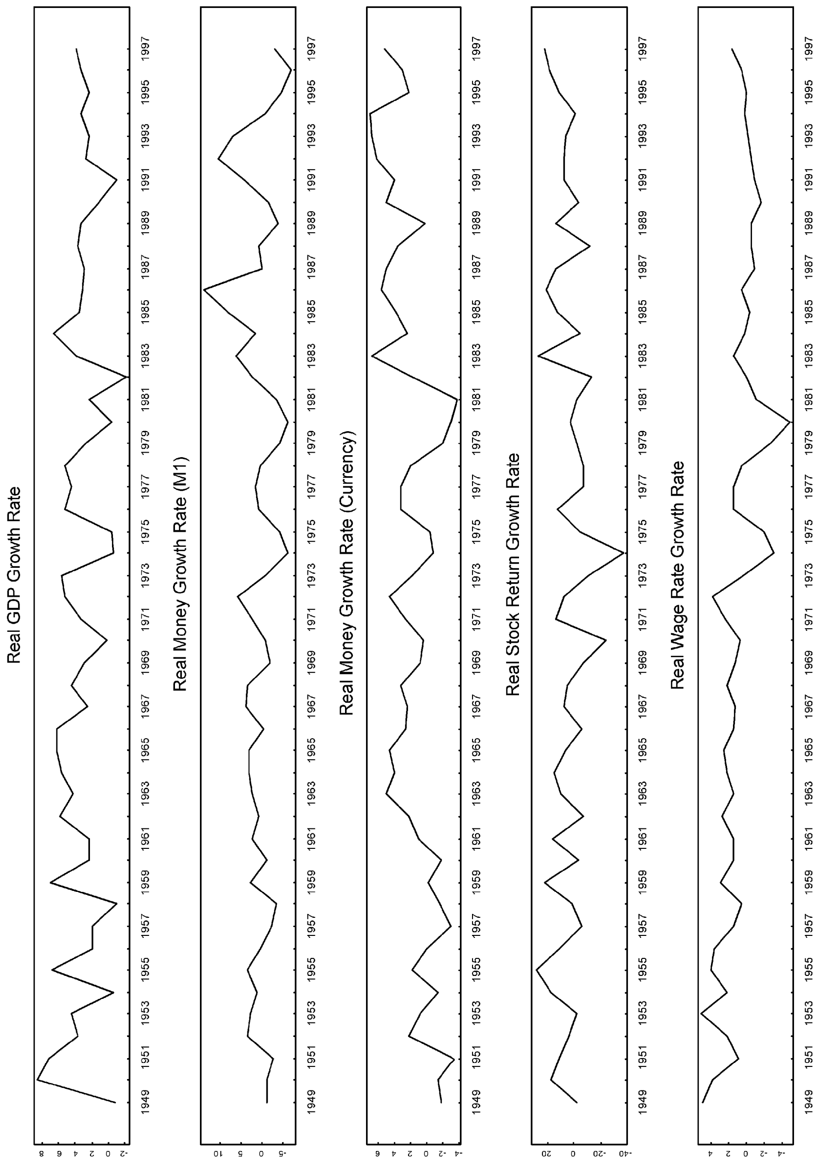

FIGURE 1A. Real growth rates, 1949–1997.

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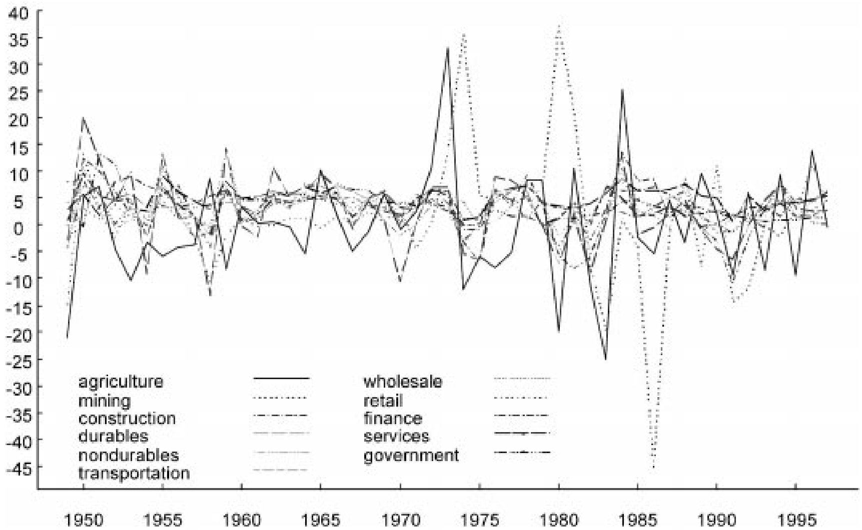

FIGURE 1B. U.S. sectoral real output growth rates.

each year, the 11 sectoral forecasts are employed to generate a forecast of the growth rate of aggregate real GDP using annually updated estimates of relations. Even though the sectoral AR(3) forecasts were employed, there is little improvement as compared with the aggregate AR(3) forecasts, shown in Figure 2A. The AR(3)LI and distributed-lag (DL) models were used to generate forecasts for each of the 11 sectors and these were employed to calculate a forecast of the annual growth rates of real GDP, with results shown in the second panel of the first column of Figure 2B. The forecast performance of the DL model is seen to be better than that of the AR(3) model and about the same as that of the AR(3)LI model. With use of disaggregation and of the MMM(DA) models I–IV (see Table 1 for their definitions) that include lagged level variables, the forecasting results shown in Figure 2B were obtained. The MMM(DA) models outperformed the AR(3) model by a wide margin and the disaggregated DL and AR(3)LI models by smaller margins. Also, from a comparison of Figures 2A and 2B, the MMM(DA) models performed better than all of the aggregate models.

With respect to the four MMM(DA) models, it appears that MMM(DA)III has a slight edge over the other three MMM(DA) models. It caught the 1982 downturn and subsequent upturn rather well and its performance in later years, particularly the 1990's, is slightly better than that of the alternative models considered in Figures 2A and 2B. However, it missed the 1991 trough growth rate.

When the lagged rate of growth of real $M1$ is used as a leading-indicator variable, rather than the lagged rate of growth of real currency, the results in Figures 3A and 3B were obtained. The results in Figure 3A are similar to those in Figure 2A

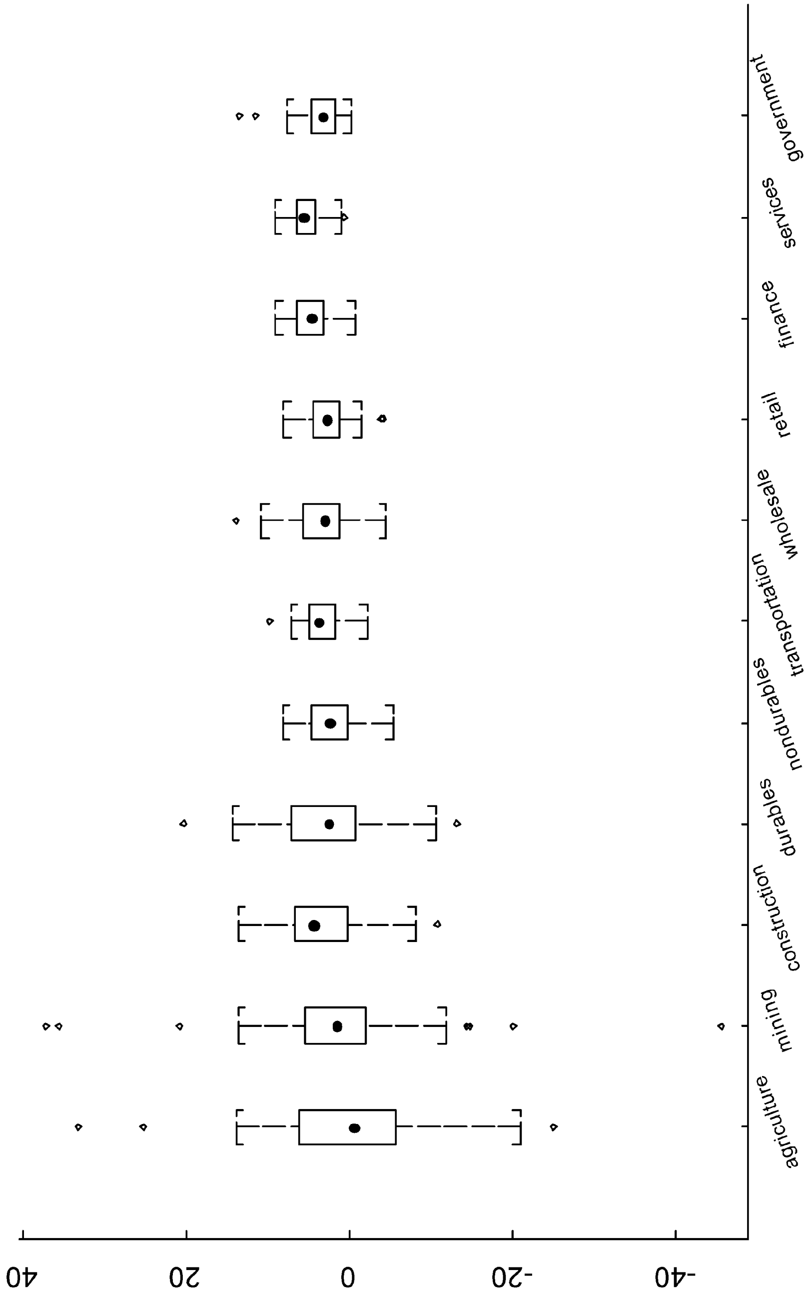

Figure 1C. Real output growth rates by sector.

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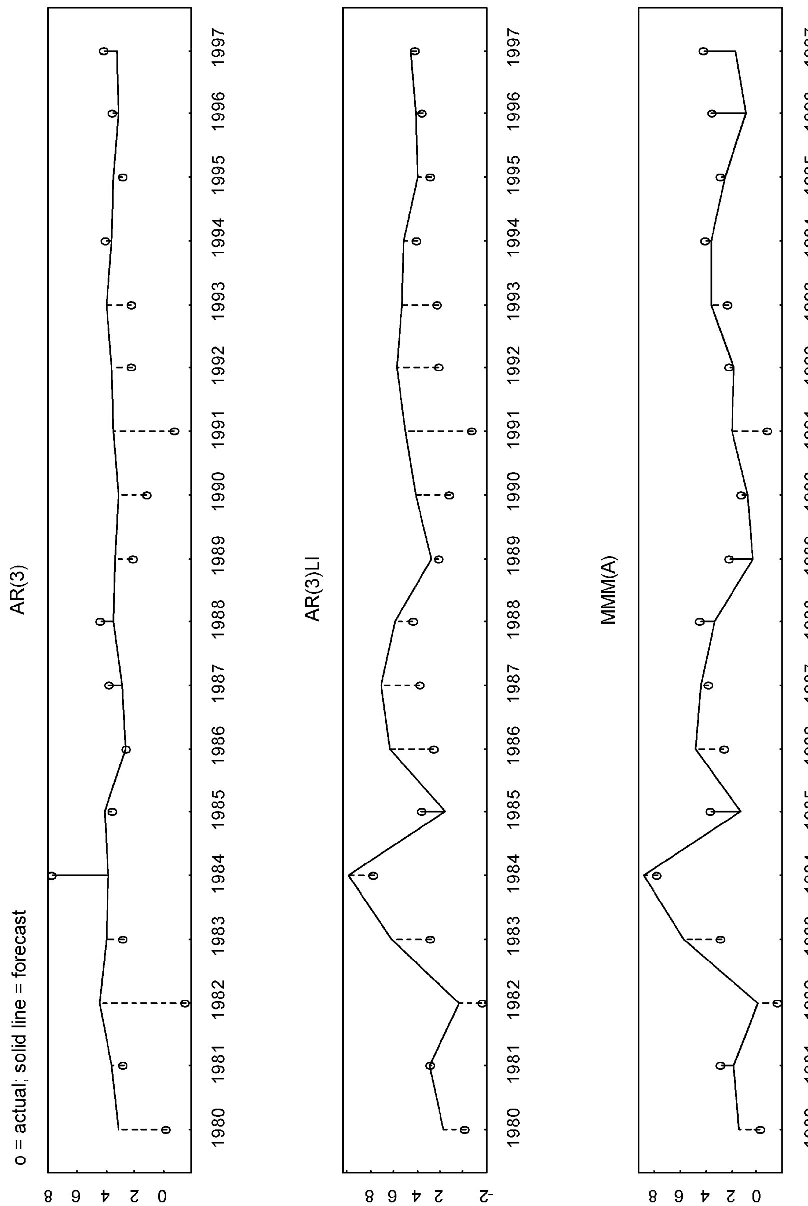

FIGURE 2A. Aggregate GDP growth-rate forecasts using the currency variable.

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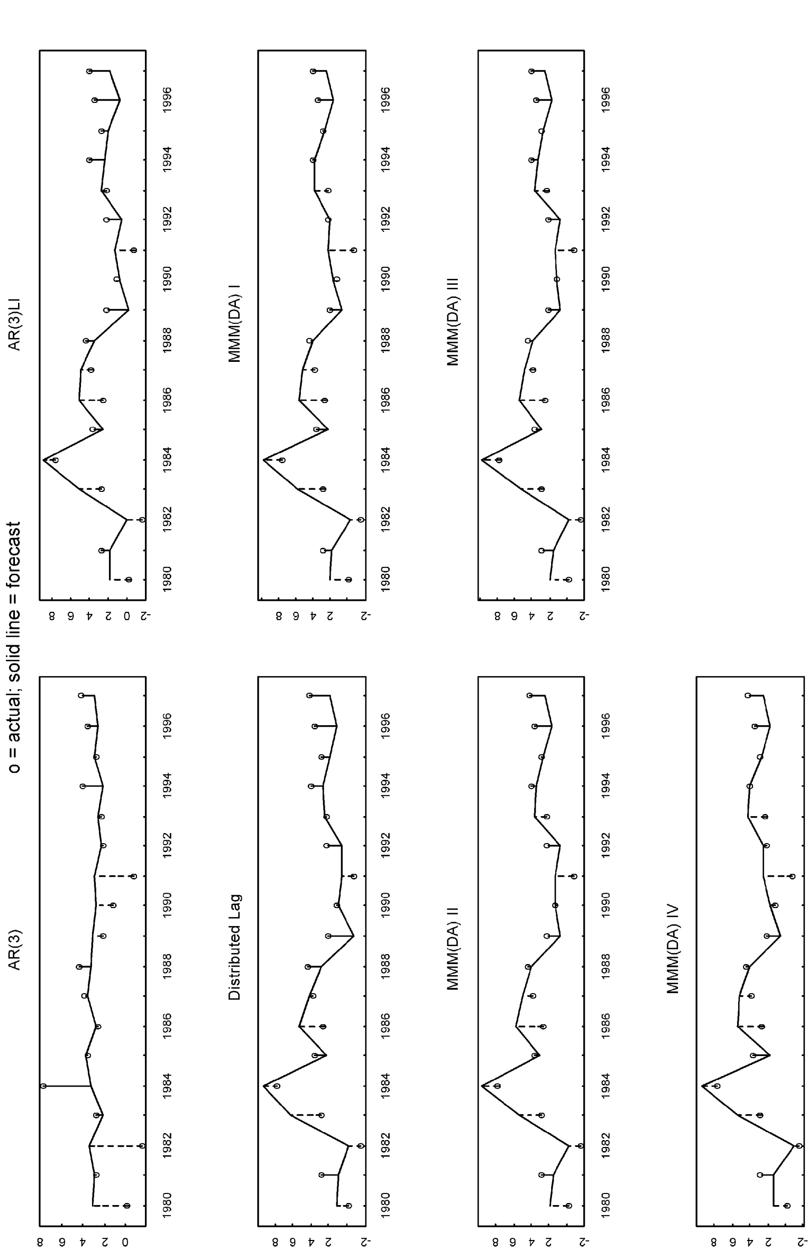

FIGURE 2B. Disaggregated GDP growth-rate forecasts using the currency variable (complete shrinkage).

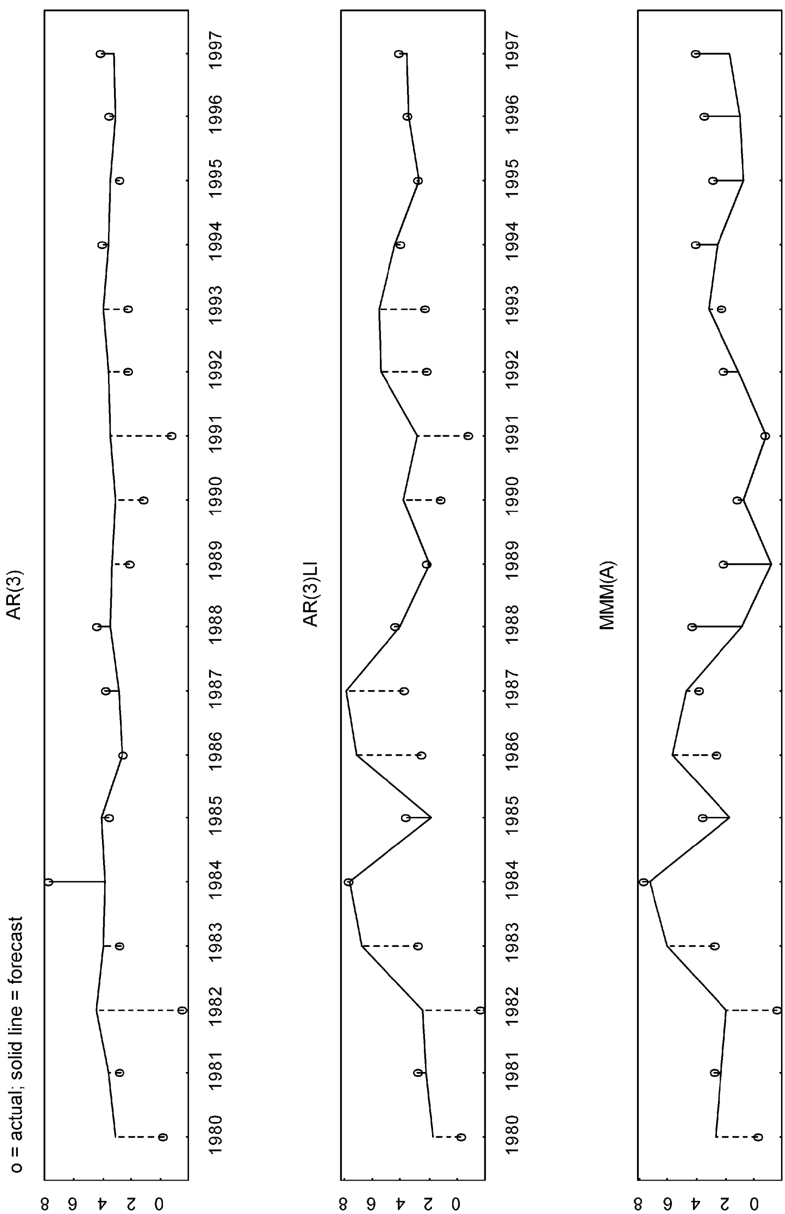

FIGURE 3A. Aggregate GDP growth-rate forecasts using the $M1$ variable.

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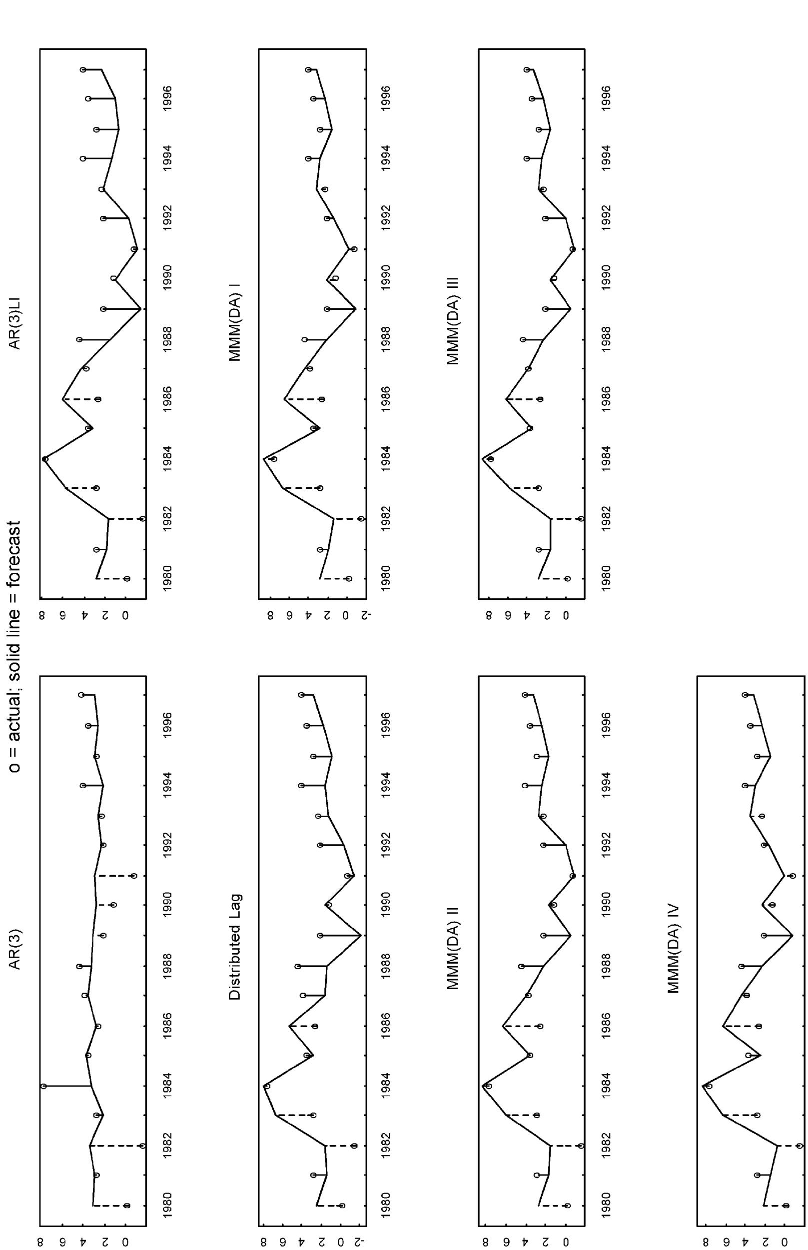

FIGURE 3B. Disaggregated GDP growth-rate forecasts using the $M1$ variable (complete shrinkage).

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in that both the AR(3)LI and MMM(A) models' forecasting performance was much better than that of the AR(3) model. Use of the $M1$ variable rather than the currency variable led to a slight deterioration of the forecasting performance of the MMM(A). To a lesser degree, the same conclusion holds for the AR(3)LI model's performance. In Figure 3B, the use of the $M1$ variable produced results similar to those reported in Figure 2B. Note, however, that use of $M1$ and the models other than the AR(3) led to better forecasts of the low 1991 real GDP growth rate and slightly worse forecasts of the low 1982 growth rate.

Shown in Tables 2 A and 2B are the RMSE's and MAE's associated with various models' one-year-ahead forecasts of annual real GDP growth rates, 1980–1997, using data from 1952–1979 to estimate models, which were then reestimated year by year in the forecast period. Currency was used as the money variable. From Table 2A, it is seen that the MMM(A) model has an RMSE of 1.72 and an MAE of 1.48, lower than those associated with the AR(3) and AR(3)LI models. For the rates of change of the real wage rate, the AR(3) model's RMSE of 1.43 and MAE of 0.98 are somewhat smaller than those of the MMM(A) and AR(3)LI models. These results indicate that the MMM(A) model for the growth rate of the real wage needs improvement, perhaps by inclusion of demographic and other variables.

As regards the disaggregated forecasts for the rate of growth of real GDP, shown in Table 2B, it is seen that all of the disaggregated forecasts have smaller RMSE's and MAE's than those for the aggregate and disaggregated AR(3) model. For example, the disaggregated AR(3) model has RMSE = 2.26 and MAE = 1.65, whereas the disaggregated AR(3)LI, Distributed Lag, and MMM(DA) models have RMSE's ranging from 1.40 to 1.98 and MAE's ranging from 1.17 to 1.62. As regards just the MMM(DA) models shown in Table 2B, their associated RMSE's and MAE's ranged from 1.40 to 1.92 and 1.17 to 1.62, respectively. The lowest RMSE and MAE are encountered for the MMM(DA) III model fitted using the SUR approach, namely RMSE = 1.40 and MAE = 1.17. However, quite a few other MMM(DA) models had RMSE's in the 1.4–1.5 range and MAE's in the 1.2–1.4 range.

Table 2A. Forecast RMSE's and MAEs for aggregate models using currency as money variable (percentage points), 1952–1997 ⇒ 1980–1997

Error	AR(3)(A)	AR(3)LI(A)	MMM(A)
	Real income Y_t (Real GDP)		
RMSE	2.32	2.61	1.72
MAE	1.71	2.19	1.48
	Real wage rate W_t		
RMSE	1.43	1.71	1.49
MAE	0.98	1.10	1.11

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Table 2B. Forecast RMSE's and MAE's for disaggregated models using currency as money variable (percentage points),* 1952–1979 ⇒ 1980–1997

Error	AR(3)(DA)	AR(3)LI(DA)	Distrib. Lag(DA)	MMM(DA) I	MMM(DA) II	MMM(DA) III	MMM(DA) IV
			OLS				
RMSE	2.26	1.62	1.61	1.61	1.52	1.47	1.80
MAE	1.65	1.32	1.35	1.31	1.28	1.25	1.47
			Extended MELO				
RMSE	2.26	1.58	1.62	1.55	1.55	1.50	1.80
MAE	1.65	1.23	1.34	1.26	1.31	1.26	1.46
			2SLS				
RMSE	2.26	1.60	1.63	1.59	1.49	1.48	1.78
MAE	1.65	1.31	1.38	1.29	1.25	1.24	1.45
			SUR				
RMSE	2.21	1.70	1.66	1.68	1.61	1.40	1.92
MAE	1.52	1.41	1.36	1.39	1.38	1.17	1.60
			Complete Shrinkage				
RMSE	2.11	1.73	1.82	1.76	1.57	1.59	1.70
MAE	1.45	1.57	1.60	1.46	1.37	1.38	1.43
			γ-Shrinkage, $\gamma = 0$ (same as OLS), $\gamma = 0.25$				
RMSE	2.21	1.62	1.61	1.61	1.49	1.46	1.74
MAE	1.59	1.36	1.38	1.34	1.26	1.25	1.41
			$\gamma = 0.5$				
RMSE	2.18	1.62	1.63	1.62	1.49	1.46	1.71
MAE	1.56	1.39	1.42	1.36	1.27	1.27	1.38
			$\gamma = 1$				
RMSE	2.15	1.64	1.66	1.64	1.49	1.48	1.69
MAE	1.52	1.44	1.46	1.38	1.29	1.29	1.39
			$\gamma = 2$				
RMSE	2.13	1.66	1.70	1.67	1.51	1.50	1.68
MAE	1.49	1.48	1.51	1.41	1.32	1.32	1.40
			$\gamma = 5$				
RMSE	2.11	1.69	1.75	1.71	1.53	1.54	1.68
MAE	1.47	1.52	1.56	1.44	1.34	1.35	1.41

TABLE 2B. (Continued.)

Error	AR(3)(DA)	AR(3)LI(DA)	Distrib. Lag(DA)	MMM(DA) I	II	III	IV
$\gamma = 10^6$ (same as complete shrinkage)							
η-Shrinkage, $\eta = 0$ (same as OLS), $\eta = 0.25$							
RMSE	2.26	1.66	1.62	1.59	1.49	1.44	1.77
MAE	1.63	1.36	1.40	1.34	1.25	1.24	1.48
$\eta = 0.5$							
RMSE	2.28	1.73	1.70	1.60	1.48	1.42	1.76
MAE	1.63	1.43	1.50	1.37	1.23	1.23	1.52
$\eta = 0.75$							
RMSE	2.33	1.82	1.82	1.62	1.48	1.42	1.79
MAE	1.68	1.55	1.64	1.41	1.21	1.23	1.57
$\eta = 1$							
RMSE	2.40	1.93	1.98	1.65	1.49	1.44	1.83
MAE	1.76	1.67	1.79	1.44	1.22	1.25	1.62

*Using MMM(A) reduced-form equations to forecast real income and real wage growth rate.

TABLE 3A. Forecast RMSE's and MAE's for aggregate models using $M1$ as the money variable (percentage points), 1952–1979 ⇒ 1980–1997

	AR(3)(A)	AR(3)LI(A)	MMM(A)
Real income Y_t (Real GDP)			
RMSE	2.32	2.57	2.23
MAE	1.71	1.98	1.90
Real wage rate W_t			
RMSE	1.43	1.73	1.66
MAE	0.98	1.07	1.29

In Tables 3A and 3B, results similar to those presented in Tables 2A and 2B are shown for models incorporating a lagged rate of change of real $M1$ rather than the real currency variable. In general the use of the $M1$ variable resulted in a generally small deterioration in forecasting precision for all the models. However, again, disaggregation led to improved forecasting precision for the AR(3)LI and MMM models in all cases. Use of the MMM(DA)III model generally led to slightly lower RMSE's and MAE's than other MMM(DA) models. The lowest RMSE

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Table 3B. Forecast RMSE's and MAE's for disaggregated models using $M1$ as the money variable* (percentage points), 1952–1979 ⇒ 1980–1997

Error	AR(3)(DA)	AR(3)LI(DA)	Distrib. Lag(DA)	MMM(DA) I	MMM(DA) II	MMM(DA) III	MMM(DA) IV
			OLS				
RMSE	2.26	2.03	2.01	2.04	1.96	1.89	2.17
MAE	1.65	1.77	1.74	1.78	1.76	1.67	1.88
			Extended MELO				
RMSE	2.26	1.97	2.07	1.95	2.00	1.93	2.14
MAE	1.65	1.74	1.81	1.73	1.83	1.76	1.89
			2SLS				
RMSE	2.26	2.06	2.06	1.99	1.91	1.89	2.12
MAE	1.65	1.78	1.76	1.75	1.73	1.69	1.85
			SUR				
RMSE	2.21	2.21	2.07	2.14	2.01	2.00	2.30
MAE	1.52	1.87	1.74	1.87	1.79	1.75	1.96
			Complete Shrinkage				
RMSE	2.11	2.21	2.34	2.05	1.94	1.93	1.89
MAE	1.45	1.81	2.02	1.70	1.55	1.56	1.63
			γ-Shrinkage, $\gamma = 0$ (same as OLS), $\gamma = 0.25$				
RMSE	2.21	2.04	2.02	2.01	1.91	1.85	2.06
MAE	1.59	1.75	1.67	1.74	1.68	1.60	1.78
			$\gamma = 0.5$				
RMSE	2.18	2.06	2.04	1.99	1.89	1.84	2.01
MAE	1.56	1.73	1.65	1.71	1.62	1.55	1.71
			$\gamma = 1$				
RMSE	2.15	2.08	2.09	1.99	1.88	1.84	1.95
MAE	1.52	1.74	1.71	1.69	1.56	1.52	1.66
			$\gamma = 2$				
RMSE	2.13	2.12	2.16	2.00	1.88	1.85	1.91
MAE	1.49	1.75	1.80	1.69	1.53	1.53	1.65
			$\gamma = 5$				
RMSE	2.11	2.16	2.24	2.02	1.90	1.89	1.89
MAE	1.47	1.76	1.91	1.69	1.53	1.54	1.64

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Table 3B. (Continued.)

Error	AR(3)(DA)	AR(3)LI(DA)	Distrib. Lag(DA)	MMM(DA) I	MMM(DA) II	MMM(DA) III	MMM(DA) IV
$\gamma = 10^6$ (same as complete shrinkage)							
η-Shrinkage, $\eta = 0$ (same as OLS), $\eta = 0.25$							
RMSE	2.26	2.04	2.03	2.00	1.93	1.85	2.13
MAE	1.63	1.73	1.70	1.72	1.70	1.60	1.82
$\eta = 0.5$							
RMSE	2.28	2.07	2.15	1.99	1.92	1.84	2.13
MAE	1.63	1.71	1.82	1.66	1.63	1.52	1.79
$\eta = 0.75$							
RMSE	2.33	2.14	2.33	2.00	1.92	1.86	2.16
MAE	1.68	1.74	2.00	1.64	1.61	1.53	1.78
$\eta = 1$							
RMSE	2.40	2.23	2.57	2.03	1.95	1.89	2.22
MAE	1.76	1.82	2.19	1.70	1.65	1.61	1.86

*Using MMM(A) reduced-form equations to forecast real income and real wage rate growth.

and MAE, namely, 1.84 and 1.52, respectively, are associated with the use of MMM(DA)III and η shrinkage with a value of $\eta = 0.50$, which can be compared to the aggregate MMM(A) model's RMSE of 2.23 and MAE of 1.90 and the aggregate AR(3)LI model's RMSE of 2.32 and MAE of 1.98. The RMSE and MAE for the aggregate AR(3) benchmark model are 2.32 and 1.71, respectively. Clearly, use of disaggregation has led to improved forecasting performance again—about a 20% reduction in both RMSE and MAE.

In Tables 2 and 3, use of alternative methods of estimation, OLS, Extended MELO, and 2SLS did not have much influence on the precision of forecasts. It may be that for the present model, the rates of change of real income and of the real wage rate are stochastic exogenous or independent variables in the sector models and thus endogeneity is not a problem. However, these two variables must be forecast in order to forecast sectoral-output growth rates and thus there is a need for the reduced-form equations shown in Table 1, whether these variables are stochastic exogenous or endogenous variables.

For the MMM(DA)III model, diffuse prior, predictive densities for the sectoral-output growth rates for the Services and Durables sectors were calculated for 1980 and are shown in Figure 4. From the BMOM predictive density, 1,000 draws are plotted using the methods described in Section 3. The Services predictive density has a mean equal to 3.14 percentage points and a standard deviation equal to 2.05 percentage points. The actual growth rate for the Services sector output

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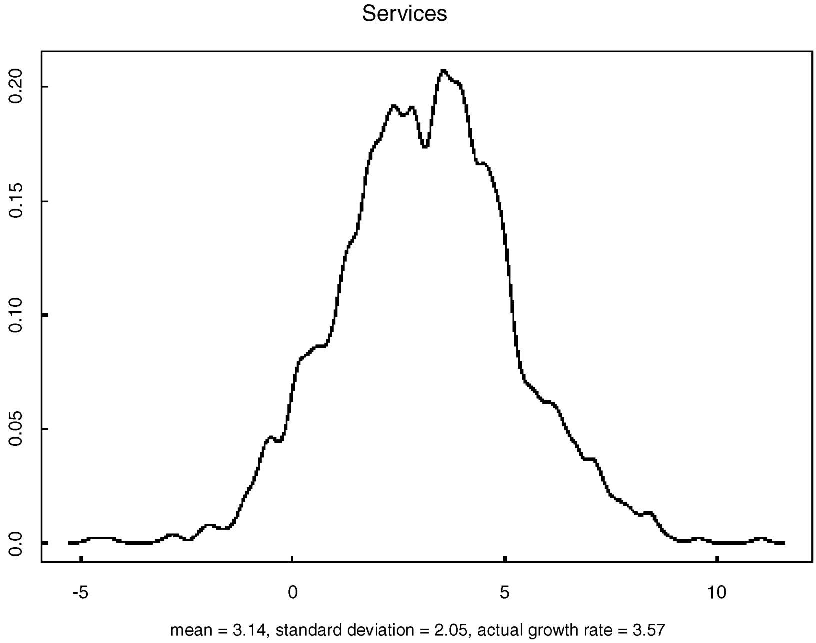

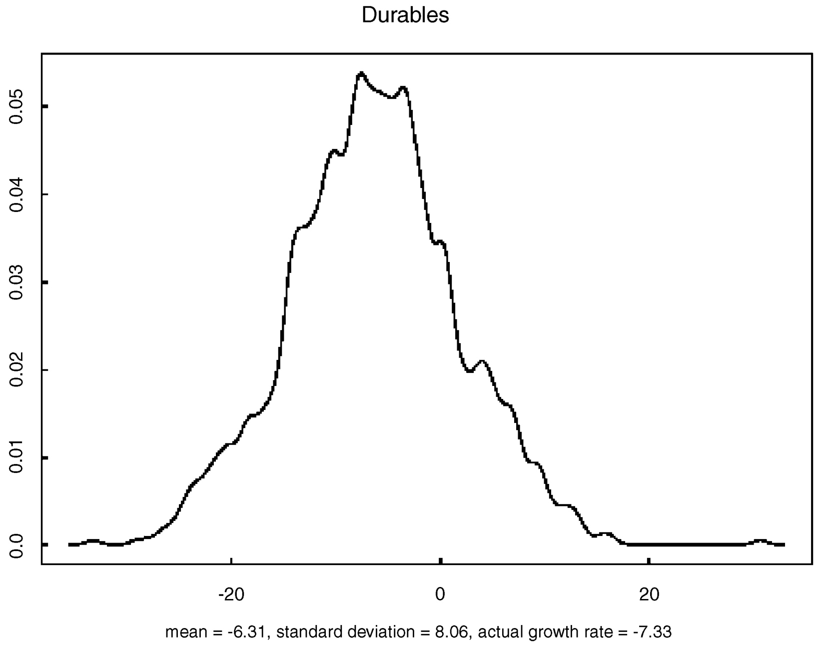

FIGURE 4. Posterior predictive densities for services and durables, 1980.

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in 1980 is 3.57 percentage points. For the Durables sector, the 1980 predictive mean is 6.31 percentage points, with a standard deviation of 8.06. The actual 1980 growth rate for this sector is 7.33. Both predictive densities appear to be slightly skewed to the left and rather spread out. Use of informative priors will result in reduced dispersion. As is well known, such densities can be employed in making probability statements about possible outcomes, for example, a downturn in the growth rate and in implementing a decision-theoretic approach for making optimal turning-point forecasts. Also, these predictive densities and the predictive densities for other models can be used to form Bayes' factors for model comparison and/or model combination. That these predictive densities can be computed relatively easily using the "direct" Monte Carlo approach described in Section 3 is fortunate.

Last, in Table 4, we present some MAE's of forecast for various types of forecasts of one-year-ahead growth rates of real GDP for the United States compiled by Zarnowitz (1986). For several different periods and forecasting units, the averages of their MAE's associated with annual forecasts of the growth rate of real GNP in 1972 dollars are given in Table 4.

Many of the MAE's in Table 4 are of magnitude comparable to those associated with the MMM(DA) annual one-year-ahead, reproducible forecasts for the period 1980–1997 shown in Table 2B. Some forecasters use informal judgment along with models and data to produce forecasts. Adding "outside" information through the use of informative prior distributions may improve the precision of MMM(DA) forecasts. See Zellner et al. (1999) for some examples of the use of judgmental information in forecasting turning points in output growth rates. Also, averaging forecasts from different sources may improve forecast precision, as many have pointed out. Note that the MAE's labeled (b) and (d) in Table 4 are such averages. On the other hand, on-line forecasters have problems associated with the use of preliminary estimates of economic variables that we do not have in our forecasting experiments using revised data throughout. The results of some on-line forecasting experiments would be of great value in assessing the importance of the "preliminary data" problem.

TABLE 4. MAE's for annual forecasts of growth rates of real GNP made by various forecasters (percentage points)

Period	MAE	Average
1953–1967	1.3(e), 1.0(d)	1.2
1962–1976	1.1(a), 1.4(d)	1.2
1969–1976	1.2(a), 1.0(b), 1.6(d), 0.9(c)	1.2
1977–1984	1.2(a), 1.0(b), 1.0(d), 1.0(c)	1.0

Source: Zarnowitz (1986, Table 1, p. 23). The forecasts are those of (a) the Council of Economic Advisers, (b) ASA&NBER Surveys, (c) *Wharton Newsletter*, University of Pennsylvania, (d) University of Michigan, and (e) an average of forecasts from the following sources: *Fortune Magazine*, Harris Bank, IBM, NICB, National Securities and Research Corp., University of Missouri, Prudential Insurance Co., and University of California at Los Angeles.

5. SUMMARY AND CONCLUSIONS

In the present research, we found that several disaggregated MMM forecasting equations performed the best in our forecasting experiments. Given the theoretical appeal of the Marshallian sector models, it is indeed satisfying that they yielded forecasts of the growth rates of aggregate real GDP, 1980–1997, that were quite a bit better than those yielded by several aggregate benchmark models and competitive with other forecasting models and techniques. Shrinkage techniques and use of currency as the money variable in our models led to improved forecasts. However, the performance of various of our sector models, particularly those for the agricultural and mining sectors, has to be improved. In addition, factor market models for labor, capital, money, and bonds, as well as other equations, are in the process of being formulated to complete our MMM.

Bayesian and certain non-Bayesian point forecasts performed about equally well for our disaggregated MMM models. However, the Bayesian approach provides exact finite sample posterior densities for parameters and predictive densities. The latter are very useful for forecasting turning points and making probabilistic statements about various future outcomes. The "direct" Monte Carlo numerical procedure for computing finite sample Bayesian posterior densities for parameters and predictive densities for simultaneous-equation models appears convenient and useful.

We recognize that writing a single structural equation in restricted reduced form and allowing for error-term effects in the equation, along with other unrestricted reduced-form equations, yields a nonlinear SUR system with an important restriction on the error-term covariance matrix. This form for the system is very useful in terms of understanding it, analyzing it, and computing posterior and predictive densities.

Although much can be said on the topic of data improvement, here we remark only that better data on the numbers of firms and plants in operation within sectors, sectoral stock price and wage rate indices, weather variables, and quality corrected output price data would be very useful and may lead to improved forecasts. Futher, having monthly or quarterly data for individual sectors would be useful in dealing with temporal aggregation problems. However, seasonality must be treated carefully. Mechanical seasonal adjustment procedures may not be the best alternative. Improvement of preliminary estimates of variables is another important issue in on-line forecasting. Preliminary estimates of variables that are contaminated with large errors obviously can lead to poor forecasts.

Last, with better data for sectors of a number of economies and reasonably formulated MMMs, past work on use of Bayesian shrinkage forecasting and combining techniques can be extended to produce improved point and turning-point forecasts for many countries.

REFERENCES

Adelman, I. & F. Adelman (1959) The dynamic properties of the Klein–Goldberger model. *Econometrica* 27, 569–625.

Belongia, M. & M. Garfinkel (eds.) (1992) *The Business Cycle: Theories and Evidence*. *Proceedings of the 16th Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis*. Boston: Kluwer Academic.

Cooper, R. (1972) The predictive performance of quarterly econometric models of the United States. In B. Hickman (ed.), *Econometric Models of Cyclical Behavior*, vol. 2, pp. 813–936. New York: Columbia University Press.

Currie, J. (1996) *The Geographic Extent of the Market: Theory and Application to the U.S. Petroleum Markets*. Ph.D. Dissertation. University of Chicago.

de Alba, E. & A. Zellner (1991) Aggregation, Disaggregation, Predictive Precision and Modeling. Manuscript, H.G.B. Alexander Research Foundation, University of Chicago.

Espasa, A. (1994) Comment on Zellner (1994). *Journal of Forecasting* 13, 234–235.

Espasa, A. and Matea, M.L. (1990) Underlying Inflation in the Spanish Economy: Estimation and Methodology. Bank of Spain working paper.

Fair, R.C. (1992) How might the debate be resolved. In M. Belongia & M. Garfinkel (eds.), *The Business Cycle: Theories and Evidence, Proceedings of the 16th Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis*, pp. 133–147. Boston: Kluwer Academic.

Gao, C. & K. Lahiri (1999) A Comparison of Some Recent Bayesian and Non-Bayesian Procedures for Limited Information Simultaneous Equations Models. Paper presented at the American Statistical Association's meeting, Baltimore, Maryland.

Garcia-Ferrer, A., R. Highfield, F. Palm, & A. Zellner (1987) Macroeconomic forecasting using pooled international data. *Journal of Business and Economic Statistics* 5, 55–67.

Hong, C. (1989) *Forecasting Real Output Growth Rates and Cyclical Properties of Models: A Bayesian Approach*. Ph.D. Dissertation, University of Chicago.

Judge, G., W. Griffiths, R. Hill, H. Lütkepohl, & T. Lee (1987) *The Theory and Practice of Econometrics*. New York: Wiley.

Kling, J. (1987) Predicting the turning points of business and economic time series. *Journal of Business* 60, 201–238.

Litterman, R. (1986) Forecasting with Bayesian vector autoregressions: Five years of experience. *Journal of Business and Economic Statistics* 4, 25–38.

McNees, S. (1986) Forecasting accuracy of alternative techniques: A comparison of U.S. macroeconomic forecasts. *Journal of Business and Economic Statistics* 4, 5–23.

Min, C. (1992) *Economic Analysis and Forecasting of International Growth Rates Using Bayesian Techniques*. Ph.D. Dissertation, University of Chicago.

Min, C. & A. Zellner (1993) Bayesian and non-Bayesian methods for combining models and forecasts with applications to forecasting international growth rates. *Journal of Econometrics* 56, 89–118.

Nelson, C. & C. Plosser (1982) Trends and random walks in macroeconomic time series. *Journal of Monetary Economics* 10, 139–162.

Orcutt, G., M. Greenberger, J. Korbel, & A. Rivlin (1961) *Microanalysis of Socioeconomic Systems*. New York: Harper.

Pagan, A. (1979) Some Consequences of Viewing LIML as an Iterated Aitken Estimator. Working paper 18, Australian National University.

Palm, F. (1976) Testing the dynamic specification of an econometric model with an application to Belgian data. *European Economic Review* 8, 269–289.

Palm, F. (1977) On univariate time series methods and simultaneous equation econometric models. *Journal of Econometrics* 5, 379–388.

Palm, F. (1983) Structural econometric modeling and time series analysis: An integrated approach. In A. Zellner (ed.), *Applied Time Series Analysis of Economic Data*, pp. 199–233. Washington, DC: Bureau of the Census, U.S. Department of Commerce.

Park, S. (1982) Some sampling properties of minimum expected loss (MELO) estimators of structural coefficients. *Journal of Econometrics* 18, 295–311.

Tsurumi, H. (1990) Comparing Bayesian and non-Bayesian limited information estimators. In S. Geisser, S. Hodges, J. Press, & A. Zellner (eds.), *Bayesian and Likelihood Methods in Statistics and Econometrics: Essays in Honor of George A. Barnard*, pp. 179–202. Amsterdam: North-Holland.

Veloce, W. & A. Zellner (1985) Entry and empirical demand and supply analysis for competitive industries. *Journal of Econometrics* 30, 459–471.

Wecker, W.E. (1979) Predicting the turning points of business and economic time series. *Journal of Business* 52, 35–50.

Zarnowitz, V. (1986) The record and improvability of economic forecasting. *Economic Forecasts* 3, 22–31.

Zellner, A. (1979) Statistical analysis of econometric models. *Journal of the American Statistical Association* 74, 628–651.

Zellner, A. (1986) Further results on Bayesian minimum expected loss (MELO) estimates and posterior distributions for structural coefficients. In D. Slottje (ed.), *Advances in Econometrics*, vol. 5, pp. 171–182. Greenwich, CT: JAI Press.

Zellner, A. (1992) Comment on Ray C. Fair's thoughts on "How Might the Debate Be Resolved?" In M. Belongia & M. Garfinkel (eds.), *The Business Cycle: Theories and Evidence, Proceedings of the 16th Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis*, pp. 148–157. Boston: Kluwer Academic.

Zellner, A. (1994) Time series analysis, forecasting and econometric modeling: The structural econometric modeling, time series analysis approach. *Journal of Forecasting* 13, 215–233.

Zellner, A. (1997a) *Bayesian Analysis in Econometrics and Statistics: The Zellner View and Papers*. Cheltenham: Edward Elgar.

Zellner, A. (1997b) The Bayesian Method of Moments (BMOM): Theory and applications. In T. Fomby & R. Hill (eds.), *Advances in Econometrics*, vol. 12, pp. 85–105. Greenwich, CT: JAI Press.

Zellner, A. (1998) The finite sample properties of simultaneous equations' estimates and estimators: Bayesian and non-Bayesian approaches. L. Klein (ed.), *Annals Issue of Journal of Econometrics* 83, 185–212.

Zellner, A. (1999) Bayesian and Non-Bayesian Approaches to Scientific Modeling and Inference in Economics and Econometrics. Paper presented at the Ajou University Research Conference in honor of Tong Hun Lee, South Korea. Published in *Korea Journal of Money and Finance* 5, 11–56.

Zellner, A. & C. Hong (1989) Forecasting international growth rates using Bayesian shrinkage and other procedures. *Journal of Econometrics* 40, 183–202.

Zellner, A. & C. Min (1999) Forecasting turning points in countries' growth rates: A response to Milton Friedman. *Journal of Econometrics* 40, 183–202.

Zellner, A. & F. Palm (1974) Time series analysis and simultaneous equation econometric models. *Journal of Econometrics* 2, 17–54.

Zellner, A. & F. Palm (1975) Time series analysis of structural monetary models of the U.S. economy. *Sankya*, Series C, 37 12–56.

Zellner, A. & S. Peck (1973) Simulation experiments with a quarterly macroeconometric model of the U.S. economy. In Power, A. & R. Williams (eds.), *Econometric Studies of Macro and Monetary Relations*, pp. 149–168. Amsterdam: North-Holland; reprinted in A. Zellner (1987) *Basic Issues in Econometrics*. Chicago: University of Chicago Press.

Zellner, A., J. Tobias, & H. Ryu (1999) Bayesian method of moments analysis of time series models with an application to forecasting turning points in output growth rates. *Estadistica*, pp. 3–63, 49–51, 152–157, with discussion.

Zellner, A., C. Min, D. Dallaire & J. Currie (1994) Bayesian Analysis of Simultaneous Equation, Asset Pricing and Related Models Using Markov Chain Monte Carlo Techniques. Manuscript, H.G.B. Alexander Research Foundation, University of Chicago.