MONEY AND NOMINAL BONDS

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This paper studies an economy with ex post heterogeneity and nominal bonds in a model à la Lagos and Wright (2005). It is shown that a strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare-improving. This result comes from protection against the inflation tax.

Keywords: Money, Nominal Bonds, Taxation

1. INTRODUCTION

Berentsen, Camera, and Waller (2007) (hereafter, BCW) show that in new generation models of monetary economics with preference shocks, the existence of a banking sector can help to reduce the inefficiency generated from the fact that some agents are cash-constrained, whereas others hold idle money. This source of inefficiency has been investigated by Bewley (1980), Green and Zhou (2005), and Levine (1991). Other attempts to address this inefficiency include models with illiquid assets [Kocherlakota (2003)], collateralized credit [Shi (1996)], or inside money [Cavalcanti and Wallace (1999), Cavalcanti, Erosa, and Temzelides (1999), and He, Huang and Wright (2005)].

BCW demonstrate that financial intermediation improves the allocation and welfare. This is because sellers can deposit idle cash (and earn interest) and not from relaxing borrowers' liquidity constraints.

An alternative approach to reduce this inefficiency consists of replacing banks with nominal risk-free bonds. Within the basic framework of BCW and Lagos and Wright (2005) (hereafter, LW), we allow agents to acquire nominal government-issued bonds once they realize that they hold idle money. A crucial assumption here is that individuals cannot sell bonds, that is, they cannot borrow, which will make clear that the welfare-improving role of bonds is a result of protection from the inflation tax and not from relaxing agents' cash constraints. As in Kocherlakota (2003), it is assumed that bonds are *illiquid* in the sense that they are not accepted in exchange for goods.

The LW framework is useful because it allows one to introduce heterogenous preferences for consumption and production while keeping the distribution of money holdings analytically tractable. In Shi (1997), money holdings are

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degenerate because the fundamental decision-making unit is a household with a continuum of agents rather than an individual. For a detailed discussion of the two approaches, see Lagos and Wright (2004).

The main result of this paper is that a strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare-improving.

The paper is organized as follows. Section 2 describes the basic framework and the agents' decision problem. Stationary equilibria are characterized in Section 3. Section 4 states the results. Section 5 examines a modification of the tax system. The conclusions end the paper.

2. THE MODEL

The basic setup is LW. Time is indexed by $t = 1, 2, ..., \infty$ and in each period t there are two perfectly competitive markets that open sequentially.¹ There is a [0, 1] continuum of infinitely lived agents and one perishable good that can be produced and consumed by all agents. At the opening of the first market, agents get a preference shock such that they can either consume or produce. With probability $n \in \mathbf{R}(0, 1)$, an agent can produce but cannot consume, whereas with probability 1-n, the agent can consume but cannot produce. We refer to consumers as buyers and producers as sellers. Attempts at endogenizing the fraction of agents in the market include Berentsen, Rocheteau, and Shi (2007), Li (1995, 1997), and Shi (1997).

Agents get utility u(q) from q consumption in the first market, where u'(q) > 0, u''(q) < 0, $u'(0) = \infty$, and $u'(\infty) = 0$. Furthermore, we assume that the elasticity of utility e(q) = qu'(q)/u(q) is bounded. Producers incur utility cost c(q) from producing q units of output with c'(q) > 0 and $c''(q) \ge 0$. Let q^* denote the solution to $u'(q^*) = c'(q^*)$. Agents in the first market are *anonymous*. Consequently, trade credit is ruled out so that transactions are subject to a *quid pro quo* restriction and there is a role for money (Kocherlakota, 1998 and Wallace, 2001).

In the second market, all agents consume and produce, getting utility U(x) from x consumption, with U'(x) > 0, $U'(0) = \infty$, $U'(\infty) = 0$ and $U''(x) \le 0$. Let x^* be the solution to $U'(x^*) = 1$. Consumption goods can be produced from labor using a linear technology. Hence, all agents will choose to carry the same amount of money out of market 2, independent of their trading history. Agents discount between market 2 and the next-period market 1, but not between market 1 and market 2. This is not restrictive because, as in Rocheteau and Wright (2005), all that matters is the total discounting between one period and the next.

At the beginning of market 1, after the idiosyncratic shocks are realized, sellers hold idle cash, while buyers may want more money than what they are carrying. Before trade of goods takes place in the first market, sellers can (and they will) invest their money in a risk-free asset *b* bearing the gross nominal rate of return 1 + i with $i \ge 0.^2$

As in Zhu and Wallace (2007), this asset is a one-period, risk-free bond that matures (automatically turns into money) in the second market. Suppose that there are vending machines maintained by the government that offer such



FIGURE 1. Timing of events.

bonds in exchange for money. It is assumed that these vending machines have a record-keeping technology of their activity and that they can observe the owner's name and address, which is printed on the certificate. Certificates can be counterfeit at no cost, and counterfeits automatically perish after they change hands. It is also assumed that the technology for detecting counterfeits is not available in the good market, so agents do not accept bonds in transactions. In this sense, bonds are illiquid and money is the only medium of exchange.³

It is assumed that $b \in \mathbf{R}_+$, so that individuals can invest but not borrow. Interest payments are financed by *lump-sum* taxes levied by the government in market 2. The change in the nature of taxes does not affect the main results of the analysis, and this will be discussed later in this paper.

It is assumed a central bank exists that controls the money supply at time *t*, $M_t > 0$. We also assume that $M_t = \gamma M_{t-1}$, where $\gamma > 0$ is constant and new money is injected, or withdrawn if $\gamma < 1$, as lump-sum transfers $\pi M_{t-1} = (\gamma - 1)M_{t-1}$ to all buyers; things are basically the same if transfers also go to sellers, as long as they are lump-sum. We restrict attention to policies where $\gamma \ge \beta$, with $\beta \in \mathbf{R}(0, 1)$ denoting the discount factor. Let $\pi_b M_{t-1} = \pi M_{t-1}/(1-n)$ be the per-buyer money transfer. The time subscript *t* is omitted and shorten t + 1 to +1, etc. in what follows.

The timing of the events is shown in Figure 1. At the beginning of market 1, agents observe their preference shock and buyers receive the lump-sum money transfers π_b . Then, sellers have the opportunity to invest their cash in nominal bonds before trade of goods begins. In the second market, agents produce, pay taxes, receive the principal plus interest on bonds, and consume. The structure of this economy is shown in Figure 2.

In period t, let $\phi = 1/P$ be the real price of money and P the price of goods in market 2. We study state equilibria, where aggregate real money balances are constant. We refer to this as stationary equilibrium:

$$\phi M = \phi_{-1} M_{-1}, \tag{1}$$

which implies that $\phi_{-1}/\phi = M/M_{-1} = \gamma$. The Fisher equation holds, hence it is equivalent to either set the nominal interest or the inflation rate.



In nominal terms, the government budget constraint is

$$PG + Bi = T, (2)$$

where *B* is the government debt outstanding at the beginning of market 2, *T* is a lump-sum nominal tax, and *PG* is spending for government consumption. Equation (2) states that government expenditure (PG + Bi) is financed by tax revenues (*T*). To simplify the analysis, we assume G = 0.

3. STATIONARY EQUILIBRIA

Consider a stationary equilibrium. Let $V(m_1)$ denote the expected value from trading in market 1 with m_1 money balances conditional on the idiosyncratic shock. Let $W(m_2, b)$ denote the expected value from entering the second market with m_2 units of money and b units of nominal bonds. In what follows, we look at a representative period t and work backward from the second to the first market.

In the second market, agents produce h units of good using h hours of labor, pay taxes, receive repayment of the investment plus interest, consume x, and adjust their money balances. The real wage per hour is normalized to 1. Hence, the representative agent's problem is

$$W(m_2, b) = \max_{x, h, m_{1,+1}} [U(x) - h + \beta V_{+1}(m_{1,+1})],$$
(3)

such that

$$x = h + \phi(m_2 - m_{1,+1}) + \phi(1+i)b - \phi T,$$
(4)

where $m_{1,+1}$ is the money taken into period t + 1. Eliminate h from (3) using (4) and get

$$W(m_2, b) = \phi[m_2 + (1+i)b - T] + \max_{x, m_{1,+1}} [U(x) - x - \phi m_{1,+1} + \beta V_{+1}(m_{1,+1})].$$
(5)

The first-order conditions (FOCs) with respect to x and $m_{1,+1}$ are

$$U'(x) = 1, \quad \beta V'_{+1}(m_{1,+1}) = \phi,$$
 (6)

where the term $\beta V'_{+1}(m_{1,+1})$ is the marginal benefit of taking money out of market 2 and ϕ is its marginal cost. In competitive markets (i.e., under price-taking), uniqueness of $m_{1,+1}$ is a direct consequence of u''(q) < 0, so all agents in the second market choose the same $m_{1,+1}$.⁴

There are two main results from (6). First, the quantity of goods *x* consumed by every agent is equal to the efficient level x^* where x^* is such that $U'(x^*) = 1$. Second, $m_{1,+1}$ is independent of *b* and m_2 . The quasi-linearity assumption in (3) eliminates wealth effects on money demand in market 2. Hence, agents who bring too much cash into the second market spend some buying goods, whereas those with too little cash sell goods. As a result, the distribution of money holdings is degenerate at the beginning of the following period.

The envelope conditions are

$$W_m(m_2, b) = \phi, \quad W_b(m_2, b) = \phi(1+i).$$
 (7)

Let q_b and q_s denote the quantities consumed by a buyer and produced by a seller trading in market 1, respectively. Let p be the nominal price of goods in market 1. It is straightforward to show that buyers will never acquire nominal bonds. We drop the argument b in $W(m_2, b)$ where relevant for notational simplicity.

An agent who has m_1 money at the opening of market 1 has expected lifetime utility

$$V(m_1) = (1 - n) [u(q_b) + W(m_1 + \pi_b M_{-1} - pq_b, 0)] + n[-c(q_s) + W(m_1 - b + pq_s, b)],$$

where pq_b is the amount of money spent as a buyer, and pq_s the money received as a seller. From linearity of W(m, b), expression (5) can be rewritten as

$$W(m_2, b) \equiv W(0, 0) + \phi[m_2 + (1+i)b],$$

which can be used to rewrite the indirect utility function as follows

$$V(m_1) = W(m_1, 0) + (1 - n) [u(q_b) + \phi(\pi_b M_{-1} - pq_b)]$$

+ n[-c(q_s) + \phi(pq_s + ib)]. (8)

Once the production and consumption shocks occur, agents become either a buyer or a seller.

If an agent is a seller in the first market, his problem is

$$\max_{q_s,b}[-c(q_s) + W(m_1 - b + pq_s, b)]$$
(9)

such that

$$b \le m_1. \tag{10}$$

The FOCs are

$$-c'(q_s) + pW_m = 0,$$

$$-W_m + W_b - \lambda_b = 0$$
(11)

where λ_b is the Lagrangian multiplier on the bonds constraint. By virtue of (7), if i > 0 then $\lambda_b > 0$, hence (10) binds. So sellers invest all their money in government bonds. Again, using (7) the FOC for q_s reduces to

$$c'(q_s) = p\phi. \tag{12}$$

Sellers produce a quantity such that the ratio of marginal costs across markets $(c'(q_s)/1)$ is equal to the relative price of goods $(p\phi)$. As a result of the linearity of the envelope conditions, q_s is independent of m_1 and b. Consequently, each seller in market 1 produces the same amount of goods no matter how much money he holds or what financial decisions he makes.

If an agent is a buyer in the first market, his problem is

$$\max_{q_b} [u(q_b) + W(m_1 + \pi_b M_{-1} - pq_b)],$$
(13)

such that

$$pq_b \le m_1 + \pi_b M_{-1},$$
 (14)

where (14) means that buyers cannot spend more money than what they bring into the first market, m_1 , plus the transfer $\pi_b M_{-1}$. Using (7), the buyer's FOC is

$$u'(q_b) - \phi p - \lambda_c p = 0, \tag{15}$$

then eliminate p using (12) and get

$$u'(q_b) = \left[1 + \frac{\lambda_c}{\phi}\right] c'(q_s), \tag{16}$$

where λ_c is the multiplier on the cash constraint.

If the constraint (14) is not binding, (i.e., $\lambda_c = 0$), condition (16) reduces to $u'(q_b) = c'(q_s)$, so trade is efficient. Conversely, if $\lambda_c > 0$ then the constraint binds and $u'(q_b) > c'(q_s)$. Hence, no trade is efficient and the buyer consumes $q_b = (m_1 + \pi_b M_{-1})/p$.

Differentiating (8) with respect to m_1 yields

$$V'(m_1) = W_m(m_1) + (1-n) \left[u'(q_b) \frac{\partial q_b}{\partial m_1} - \phi p \frac{\partial q_b}{\partial m_1} \right]$$

+ $n \left[-c'(q_s) \frac{\partial q_s}{\partial m_1} + \phi \left(p \frac{\partial q_s}{\partial m_1} + i \frac{\partial b}{\partial m_1} \right) \right],$ (17)

where $V'(m_1)$ is the marginal value of money. Because the quantity of goods produced by sellers is independent of their money holdings, it holds that $\partial q_s / \partial m_1 = 0$. Note that sellers can derive no benefits from holding cash in the first market, so they always spend all their balances in nominal bonds if i > 0; this means $\partial b / \partial m_1 = 1$. (If i > 0 then $W_b > W_m$; hence (10) binds.)

4. WELFARE ANALYSIS

Using (7), (12), and rearranging equation (17) can be rewritten as

$$V'(m_1) = \phi \left[(1-n) \frac{u'(q_b)}{c'(q_s)} + n(1+i) \right].$$
 (18)

The first term within brackets, $(1 - n) u'(q_b)/c'(q_s)$, refers to buyers and is the same as in the basic LW model. Now, the second term, n(1 + i), refers to sellers and indicates that they can invest a unit of money and receive 1 + i. Hence, the effect of nominal bonds on the marginal value of money is positive because sellers can earn an interest on idle balances.

In order to characterize monetary equilibria, we have to derive hours of work in the second market. Because all buyers have the same amount of money at the opening of market 1 and face the same problem, q_b coincides for all of them. In a symmetric equilibrium, the same applies to sellers. Hence, the market 1 clearing condition implies

$$q_s = \frac{1-n}{n} q_b, \tag{19}$$

and efficiency is achieved at

$$u'(q^*) = c'\left(\frac{1-n}{n}q^*\right),$$
 (20)

where q^* is the quantity such that (20) is satisfied. The buyer's hours of work in the second market are

$$h_b = x^* + \phi m_{1,+1} + \phi T, \qquad (21)$$

where x^* is the quantity of goods such that the first equation in (6) is satisfied. A buyer enters the second market with no cash; hence, he has to work $x^* + \phi m_{1,+1} + \phi T$ hours in order to consume x^* quantity of goods, pay taxes T, and take $m_{1,+1}$ units of money out of the second market. Similarly, hours of work for a seller are

$$h_s = x^* + \phi m_{1,+1} + \phi T - \phi [pq_s + (1+i)b].$$
⁽²²⁾

A seller enters the second market with pq_s units of money, receives interest plus notional (1 + i) b, consumes x^* , pays taxes T, and takes $m_{1,+1}$ units of money into the next period. Directly from (21) and (22), it holds that sellers work less than buyers in market 2, that is, $h_s < h_b$.

Aggregate hours of work in the second market are

$$h = nh_s + (1 - n)h_b, (23)$$

which, using (19), (21), and (22) and rearranging, can be rewritten as

$$h = x^* - \phi i B + \phi T, \tag{24}$$

by virtue of $M = [1 + (1 - n)\pi_b]M_{-1}$, symmetric conditions $m_{1,+1} = M$, $b = m_1 = M_{-1}$, $nb = nM_{-1} = B$, and using the fact that buyers in market 1 spend all their money, that is, $pq_b = (1 + \pi_b)M_{-1}$.

Now, use the budget constraint (2) to eliminate *B* from (24), and impose symmetric conditions h = H and x = X to get aggregate hours of work in market 2

$$H = X^*$$

where X^* is such that $U'(X^*) = 1$.

In steady-state monetary equilibria, inflation equals the money growth rate (i.e., $\gamma = 1 + \pi$), and the real interest rate is $i_R = 1/\beta - 1$. Substitute these terms directly into the Fisher equation, $1 + i = (1 + i_R)(1 + \pi)$, and get

$$i = \frac{\gamma - \beta}{\beta}.$$
 (25)

Now, use the second expression in (6) lagged one period and (19) to rewrite (18) as follows:

$$\frac{\phi_{-1}}{\beta} = \phi \left\{ (1-n) \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)} + n(1+i) \right\},$$

then take the steady state, eliminate i using (25), and rearrange to get the equilibrium condition

$$\frac{\gamma - \beta}{\beta} = \frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)} - 1.$$
(26)

DEFINITION 1. A symmetric steady state monetary equilibrium is an interest rate i satisfying (25) and a quantity q_b satisfying (26).

At this point of the analysis, the main result of this paper can be stated:

PROPOSITION 1. A strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare improving.

Proof. Assume a strictly positive interest rate, that is, i > 0. Now, let \tilde{q}_b denote the quantity of goods consumed in an economy without nominal bonds (see LW). This implies

$$\frac{\gamma - \beta}{\beta} = (1 - n) \left[\frac{u'(\tilde{q}_b)}{c'\left(\frac{1 - n}{n} \tilde{q}_b\right)} - 1 \right].$$
(27)

Because $n \in \mathbf{R}(0, 1)$, the expression within brackets must be lower, for given $\gamma > \beta$, in an economy with nominal bonds than without. Comparison of equations (27) and (26) implies $\tilde{q}_b < q_b$ for any i > 0.

BCW get exactly the same result with financial intermediation. In their framework buyers can (they will) borrow, while here they are not allowed to do so. So it is clear that the welfare improving role of bonds is due to protection from the inflation tax and not from relaxing agents' liquidity constraints.

5. TAX SYSTEM

In this section, we explore a modification of the tax system. Instead of lump-sum taxes, it is assumed that interest payments are financed by *distortionary* labor income taxes. Although this affects hours worked and entails an inefficient level of consumption in market 2, Proposition 1 still holds.

As before, we assume G = 0. Thus, the government budget constraint (2) becomes

$$Bi = Pt_h H, (28)$$

where $t_h \in \mathbf{R}(0, 1)$ is the proportional income tax on aggregate hours of work in market 2. By working backward from the second to the first market, it is straightforward to show that the marginal value of money is

$$V'(m_1) = \frac{\phi}{1 - t_h} \left[(1 - n) \frac{u'(q_b)}{c'(q_s)} + n(1 + i) \right],$$
(29)

which differs from (18) as we have distortionary taxes here.

The agent's hours of work in market 2 are

$$h_b = \frac{x + \phi m_{1,+1}}{1 - t_h},$$

if he is a buyer, and

$$h_s = \frac{x + \phi m_{1,+1} - \phi [pq_s + (1+i)b]}{1 - t_h},$$

if he is a seller. Consequently, using (23) and rearranging, one gets

$$h = \frac{x - \phi i B}{1 - t_h},\tag{30}$$

then eliminate *B* using the budget constraint (28), impose symmetric conditions h = H and x = X, and obtain aggregate hours of work in the second market

$$H = X, \tag{31}$$

where *X* in (31) is such that $U'(X) = 1/(1 - t_h)$, with *X* < *X*^{*}.

The equilibrium conditions with distortionary taxes are

$$i = \frac{\gamma - \beta}{\beta},\tag{32}$$

and

$$\frac{\gamma - \beta}{\beta} = \frac{u'(q_b)}{c'\left(\frac{1 - n}{n}q_b\right)} - 1.$$
(33)

As in the case of lump-sum taxes, a strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare-improving; to see this, note that equations (33) and (26) are identical. It then follows that the main result of this paper (Proposition 1) is robust to alternative specifications of the tax system.

6. CONCLUSIONS

This paper studied an economy with ex post heterogeneity and nominal bonds in a model à la Lagos and Wright (2005). It was shown that a strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare improving, as it allows agents to protect themselves against the inflation tax.

NOTES

1. Competitive pricing in LW is a feature of Rocheteau and Wright (2005) and BCW.

2. A similar framework in which agents can either lend or borrow is in BCW and Berentsen and Waller (2005).

3. An exhaustive discussion of illiquid bonds is in Kocherlakota (2003). Restrictions on bond circulation also have been introduced in Andolfatto (2006), Berentsen and Waller (2007), and Boel and Camera (2006).

4. See LW under bargaining and Rocheteau and Wright (2005) under price posting.

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